CMBの温度・偏光揺らぎにおける弱い重力レンズ効果

再構築法の開発

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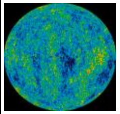

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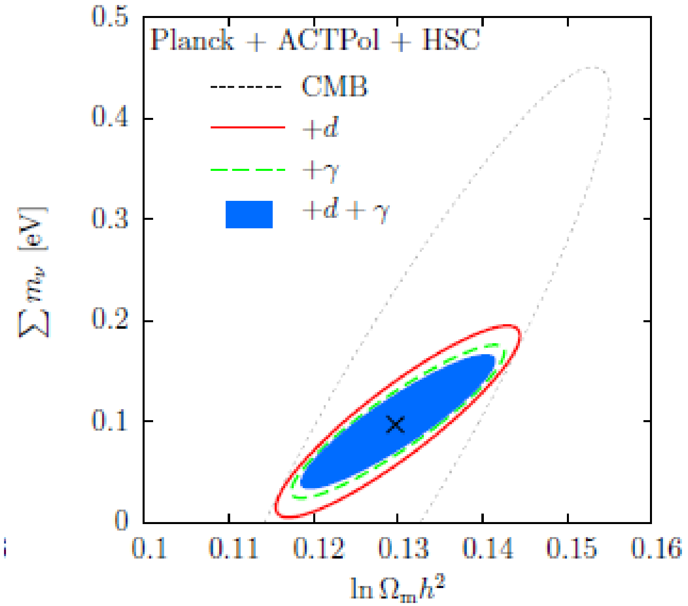
CMB lensing の宇宙論への応用

暗黒エネルギー、ニュートリノ質量など

- 比較的高赤方偏移の揺らぎに感度をもつ
- 光源の性質がよく分かっている
- 他の観測と相補的



TN+'10

原始重力波

- 重力レンズ由来のB-modeの除去

カールモード

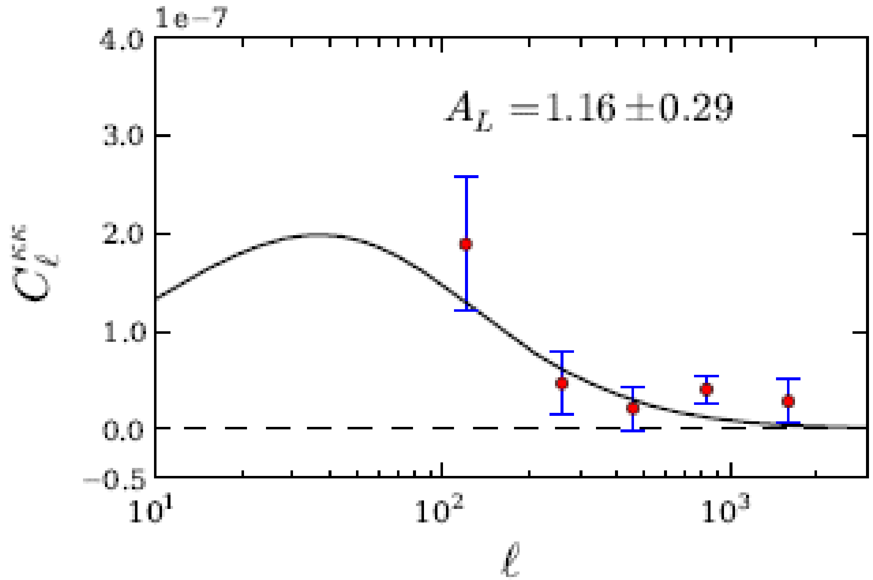
- 宇宙紐、重力波など

CMB lensing の観測の現状について

現在

$C_\ell^{g\kappa}$
- Smith+ ’07 (3.4σ)
- Hirata+ ’08 (2.5σ)

$C_\ell^{\kappa\kappa}$
- Smidt+ ’10 (~2σ)
- Das+ ’11 (~4σ)



Das+ ’11

今後

✓ Ground
- PolarBear (2011-)
- ACTPol (2012-)

✓ Space
- Planck (2010-)
- CMBPol (?)

これらの観測は非常に精度よく重力レンズを測れる予定なので、
宇宙論への応用が可能になってくる

研究の動機

曲がり角

揺らぎの線形理論では、曲がり角はスカラー量（lensing potential）の勾配として表せる

$$d_i(\vec{n}) = \partial_i \phi(\vec{n})$$

これまでの曲がり角の測定では、この関係式が念頭に置かれてきた

(e.g., Hu & Okamoto '02)

カール成分

一般には、勾配成分とカール成分の二成分に分解できる

2D Levi-Chivita tensor

$$d_i(\vec{n}) = \underbrace{\partial_i \phi(\vec{n})}_{\text{Gradient part}} + \epsilon_{ij} \underbrace{\partial_j \omega(\vec{n})}_{\text{Curl part}}$$

ベクトル・テンソル揺らぎ（宇宙ひも、重力波）はカール成分を作る

カール成分を宇宙論へ応用するために、その推定法を導出したい

目的 1

Find an algorithm for reconstructing deflection angle including both gradient and curl mode

- Previous works which consider curl-type deflection angle
  - Hirata & Seljak ’03 ・Based on the likelihood estimator
  - Cooray+ ’05 ・Based on the optimal quadratic estimator proposed by Hu & Okamoto ’02
- Our work
  - ✓ Our estimator is based on Okamoto & Hu ’03 (OH03), but including curl-type deflection angle (extension of Cooray+’05 in full sky)
  - ✓ Then, we show that the gradient- and curl-type deflection angle can be reconstructed with unbiased condition

目的 2

- Sources of curl-type deflection angle

An example: cosmic string

- ✓ Cosmic string can be produced by the phase transition in the early universe

- ✓ The primordial CMB temperature anisotropies produced by cosmic strings are less than ~10% (corresponds to a constraint on dimensionless string tension: $G\mu < O(10^{-7})$)

(e.g., Wyman+'05, Seljak+'06, Bevis+ '07)

- ✓ Cosmic string induces vector/tensor perturbations and would produce curl-type deflection angle : cosmic string would be constrained from curl mode

Estimate expected detectability of cosmic string by reconstructing the curl-type deflection angle

Brief Review of OH'03

➢ Definition of estimator

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}$ is $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}$, or $\tilde{B}_{\ell m}$

To determine the functional form of f theoretically, the following conditions are imposed :

1. Unbiased

Ensemble average over the estimator $\hat{\phi}^{XY}_{\ell m}$ *with fixing the lensing potential* should be equals to the lensing potential

$$\langle \hat{\phi}^{XY}_{\ell m} \rangle_{CMB} = \phi_{\ell m}$$

2. Optimal

Choosing f so that N_ℓ is minimized

$$\left\langle \hat{\phi}^{(XY)}_{\ell m} \left(\hat{\phi}^{(XY)}_{\ell m} \right)^* \right\rangle = \underline{N^{\phi,(XY)}_\ell} + C^{\phi\phi}_\ell$$

Brief Review of OH'03

➢ Functional form of f

- ✓ described by the observed (lensed) power spectra, $\hat{C}^{XY}_{\ell}$, and unlensed Cl's

$$f^{XY}_{\ell L_1 L_2} = (2\ell + 1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_{\ell}}$$

Summation : $\sum_{L_1}\sum_{L_2} \Phi^{XY}_{\ell L_1 L_2} F^{XY}_{\ell L_1 L_2}$

$$F^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell + L_1 + L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

* The quantity Φ depends on unlensed Cl's

➢ Reconstruction

Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2}\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$

- ✓ In principle, we can reconstruct the lensing potential from observed CMB maps.

Lensing field as a quadratic statistics

Average with fixed lensing fields

Similar analogy to OH'03

$$\langle \widetilde{\Theta}_{L_1M_1} \widetilde{\Theta}_{L_2M_2} \rangle_{CMB} = C^{\Theta\Theta}_{L_1} \delta_{L_1L_2} \delta_{M_1M_2} (-1)^{M_1} + \sum_{\ell m} (-1)^m [\Phi^{\Theta\Theta}_{\ell L_1 L_2} \phi_{\ell m} + \Omega^{\Theta\Theta}_{\ell L_1 L_2} \omega_{\ell m}] \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}$$

[Key property]

$\Phi_{\ell L_1 L_2} = 0$, for $\ell + L_1 + L_2 =$ odd

$\Omega_{\ell L_1 L_2} = 0$, for $\ell + L_1 + L_2 =$ even

$\Rightarrow$ $\phi_{\ell m}$, $\omega_{\ell m}$ are expressed independently

For ω,

$$\omega_{\ell m} = (2\ell + 1)(-1)^m \times \sum_{L_1 M_1} \sum_{L_2 M_2} \frac{G^{\Theta\Theta}_{\ell L_1 L_2}}{[\Omega G]^{\Theta\Theta}_{\ell}} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \langle \widetilde{\Theta}_{L_1M_1} \widetilde{\Theta}_{L_2M_2} \rangle_{CMB}$$

Arbitrary function

Estimator including curl mode

Definition of estimators

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

$$\hat{\omega}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}$ is $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}$, or $\tilde{B}_{\ell m}$

To determine the functional form of f and g theoretically, the following conditions are imposed :

1. Unbiased

Note: In Cooray +'05, they claim their estimator is not satisfied this condition, but I checked their flat sky estimator also satisfies the condition.

Ensemble average over the estimators $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ *with fixing the lensing fields* should be equals to the lensing fields, respectively

$$\langle \hat{\phi}^{XY}_{\ell m} \rangle_{CMB} = \phi_{\ell m} \qquad \langle \hat{\omega}^{XY}_{\ell m} \rangle_{CMB} = \omega_{\ell m}$$

2. Optimal

Choosing f and g so that N_ℓ is minimized

$$\left\langle \hat{\phi}^{(XY)}_{\ell m} \left(\hat{\phi}^{(XY)}_{\ell m} \right)^* \right\rangle = N^{\phi,\,(XY)}_\ell + C^{\phi\phi}_\ell \qquad \left\langle \hat{\omega}^{(XY)}_{\ell m} \left(\hat{\omega}^{(XY)}_{\ell m} \right)^* \right\rangle = N^{\omega,\,(XY)}_\ell + C^{\omega\omega}_\ell$$

Estimator including curl mode

Functional form of f and g

- ✓ Both f and g are described by the observed (lensed) and unlensed Cl's
- ✓ Thanks to the property of parity, the estimators, $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ are reconstructed in a similar way that of OH'03, and f is the same as that of OH'03
- ✓ The functional form of g is similar to that of f

$$f^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_{\ell}} \qquad F^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}} \qquad G^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

* The quantity Ω depends on unlensed Cl's but the dependence is different from Φ

Summary of Our Estimator

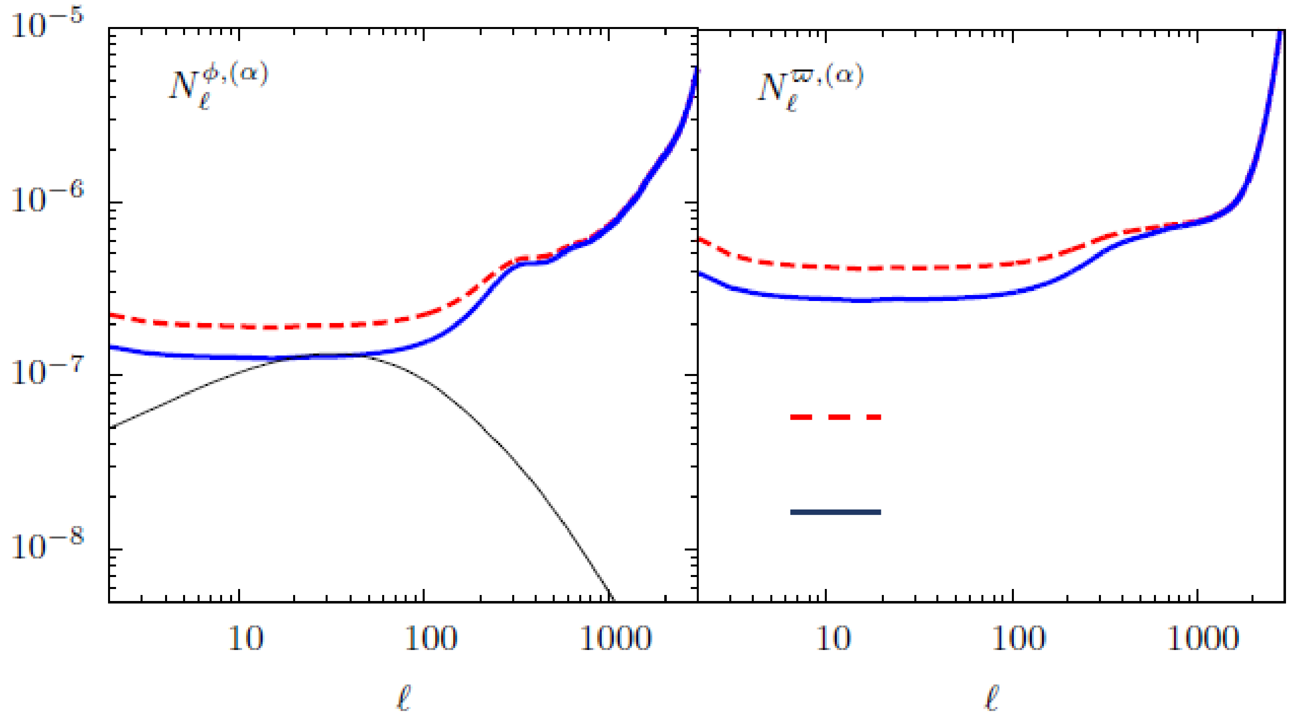
Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$G^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}}$$

$$\omega^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2}\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$

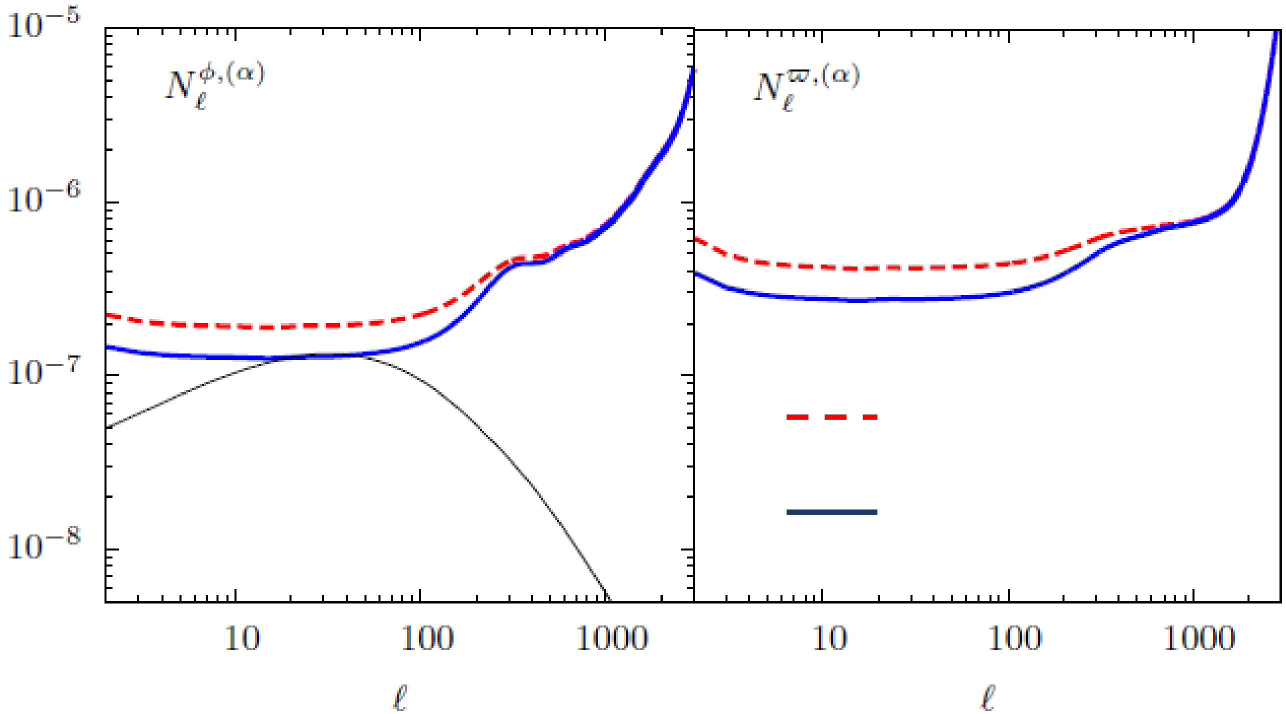
Noise Spectra : Planck



Note: For ACTPol, the noise improvement by including polarization is significant compared to that of Planck

The noise of curl mode is comparable to that of gradient mode

Noise Spectra : ACTPol



Note 1: For ACTPol, the noise improvement by including polarization is significant compared to that of Planck

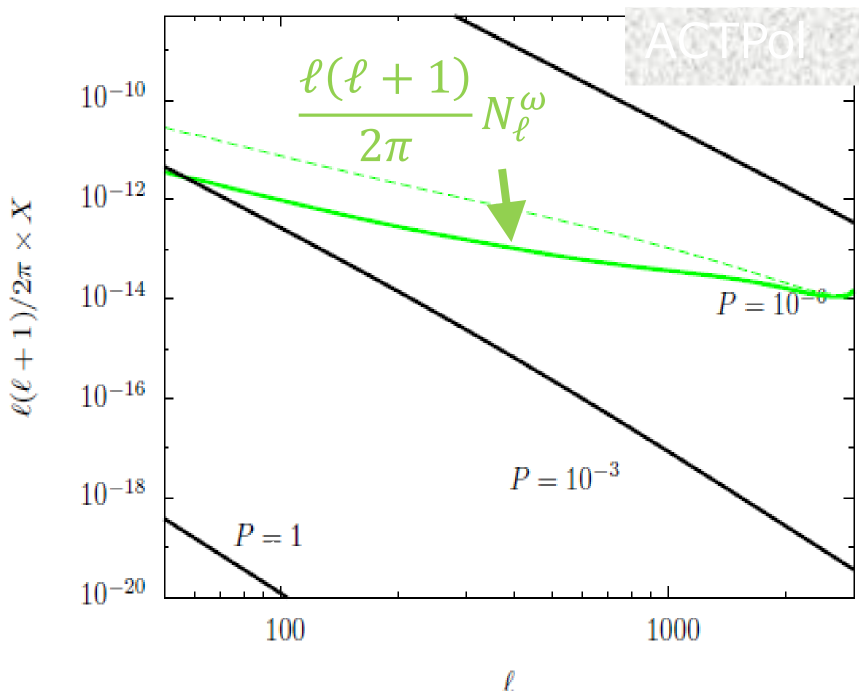
Note 2: The noise of curl mode is comparable to that of gradient mode

Implications for cosmic strings

Assumptions

- ✓ Nambu-string
- ✓ VOS model (Martins+'02)
- ✓ Energy loss rate (Martins+'02,'04) $\sim 0.23 P v_{rms} \rho_{str}/\xi$
- ✓ Number of string in the region $[z, z+\delta z]$ is $\delta z(\frac{dV}{dz})/\xi^3$
- ✓ Straight string

Results



$$\ell^2 C_\ell^{\omega\omega} \propto (G\mu)^2 P^{-\frac{5}{2}} \ell^{-5}$$

If $P < 10^{-3}$ and $G\mu > 10^{-7}$, the curl-type deflection angle induced by cosmic string would be detected

Summary

- ✓ We show an algorithm for reconstructing deflection angle including both gradient and curl mode

  Then, thanks to property of parity, the gradient and curl mode can be reconstructed similar to that of OH'03.

- ✓ Assuming ACTPol, we roughly estimate the expected constraint on cosmic string using the curl mode.

  Using ACTPol data, if $G\mu > O(10^{-7})$ and $P < O(10^{-3})$, the curl-type deflection angle from cosmic string would be detected

Curl mode has no contribution from liner-matter density fluctuations, so in this respect, considered as pure signal of string, which is an advantage of this method compared to other probes of string