• • •

*

*

N_{side}

map

θ

π

ϕ

$\phi \in [0, 2\pi[$

$$|s| \geq 0$$

$$s = 0$$

> 0

$a_{\ell m}$

$$C(\ell)$$

$B(\ell)$

$P_{\ell m}$

$$0 < |s| \leq 100$$

$N_{\mathrm{side}} > 8192$

$$\ell < 46340$$

$\partial X/\partial\theta$

$\partial^2 X/(\partial\theta\partial\phi\sin\theta)$

$$\partial^2 X/\partial\theta^2$$

$$X = Q, U$$

N_{side}

$$N_{\text{side}} = 128$$

y

$$C_{12}^{XY}(\ell) = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a_{1,\ell m}^{X} a_{2,\ell m}^{Y*},$$

X

Y

T, E, B

$$X \neq Y$$

$$C_{\{12\}}^{\{XY\}}(\ell) \equiv \frac{C_{12}^{XY}(\ell) + C_{12}^{YX}(\ell)}{2} = \frac{C_{12}^{XY}(\ell) + C_{21}^{XY}(\ell)}{2}$$

ℓ

m

p

$$d = 1$$

C_ℓ^{TT}

$$d = 4$$

$$p = 3$$

$$C_\ell^{EE}$$

$$C_\ell^{BB}$$

$$C_\ell^{TE}$$

$d \geq 6$

$$C_\ell^{TB}$$

$$C_\ell^{EB}$$

$d \geq 9$

$$C_{\ell}^{ET}$$

$$C_\ell^{BT}$$

$$C_{\ell}^{BE}$$

$d \geq 4$

$$(C_\ell^{TE} + C_\ell^{ET})/2$$

$$(C_\ell^{TB} + C_\ell^{BT})/2$$

$$(C_\ell^{EB} + C_\ell^{BE})/2$$

$$\ell = 128$$

$$a_{\ell m}^{T}$$

$$a_{\ell m}^{E}$$

$$a_{\ell m}^{B}$$

|

$$P_{\ell m}(\theta)$$

$$z = \sin(\text{latitude}) = \cos(\theta).$$

<

$< z <$

$\leq z \leq$

$0 < z < 0.5$

$Y_{\ell m}$

$${}_{s}a_{\ell m}$$

$$T(p)$$

$$Q(p)$$

$$U(p)$$

$(\partial T/\partial\theta, \partial T/\partial\phi/\sin\theta)$

$(\partial T/\partial\theta, \partial Q/\partial\theta, \partial U/\partial\theta;$

$\partial T/\partial\phi/\sin\theta,\ldots)$

$(\partial^2 T/\partial\theta^2, \partial^2 T/\partial\theta\partial\phi/\sin\theta,$

$\partial^2 T/\partial\phi^2/\sin^2\theta)$

$$(\partial^2 T/\partial\theta^2, \partial^2 Q/\partial\theta^2, \partial^2 Q/\partial\theta^2, \ldots)$$

$${}_{s}Y_{\ell m}$$

S

$-s$

S

$$ {}_sS(p) = \sum_{\ell m} {}_sa_{\ell m} \quad {}_sY_{\ell m}(p) $$

$$\ell \geq |m|, \ell \geq |s|$$

$${}_sS^* = {}_{-s}S$$

$$
{}_sY^*_{\ell m} = (-1)^{s+m}\,{}_{-s}Y_{\ell -m}
$$

$$ {}_{s}a^{*}_{\ell m} = (-1)^{s+m} {}_{-s}a_{\ell -m} $$

$${}_{|s|}a^{+}_{\ell m}$$

$$= -({}_{|s|}a_{\ell m} + (-1)^s {}_{-|s|}a_{\ell m})/2,$$

$${}_{|s|}a_{\ell m}^{-}$$

$$= -({}_{|s|}a_{\ell m} - (-1)^s {}_{-|s|}a_{\ell m})/(2i),$$

$m \geq 0$

$m < 0$

$$|s|\,a^{+}_{\ell-m}$$

$$= (-1)^m {}_{|s|}a^{+*}_{\ell m},$$

$$|s|\, a^{-}_{\ell-m}$$

$$= (-1)^m {}_{|s|}a_{\ell m}^{-*}.$$

$$_{|s|}S^{+}$$

$$= ({}_{|s|}S + {}_{-|s|}S)/2,$$

$$_{|s|}S^{-}$$

$$= ({}_{|s|}S - {}_{-|s|}S)/(2i).$$

$${}_2a^+$$

$_2a^-$

$_2S^+$

$_2S^-$

a^E, a^B, Q

U

$${}_0a^{+}_{\ell m} = -a^{T}_{\ell m}$$

$$ {}_0a^-_{\ell m} = 0 $$

$${}_0S^+ = T$$

$${}_0S^- = 0.$$

$$= 0$$

$${}_{|s|}a^{+}_{\ell m}$$

$${}_{|s|}a^{-}_{\ell m}$$

$$|s| S^{+}$$

$$|s|S^{-}$$

$$|S|$$

$$index = \ell^2 + \ell + m + 1$$

$$a_{\ell m}^{T}$$

$$a_{\ell m}^{E}$$

$$a_{\ell m}^{B}$$

$$a_{\ell m} \longrightarrow a_{\ell m} b(\ell)$$

B_ℓ

$\ell \in \{$

}

$d < p$

$$N_{\text{side}} = 64$$

$$\ell_{\max} = m_{\max} = 128$$

(θ, ϕ)

$$(x, y, z)$$

$$x = \sin\theta\cos\phi$$

$$y = \sin\theta \sin\phi$$

$$z = \cos\theta$$

$$[0,\ \pi]$$

$$[0,\ 2\pi]$$

$$(0,0,1)$$

$\cos^{-1}$

$p > 1$

$$q < p$$

$$z = \sin(\text{latitude}) = \cos(\theta)$$

$$|b| \leq 15^o$$

Will print out:

```
------------------
Number of OpenMP threads in use:  2
Number of CPUs available:  2
------------------
```

on a bi-pro (or dual core) computer

$$\{x_i\}$$

n

$$m = \sum_i x_i / n$$

$$a = \sum_i |x_i - m|/n$$

$$\sigma^2 = \sum(x_i - m)^2/(n-1)$$

σ

$$s = \sum(x_i - m)^3/(n\sigma^3)$$

$$k = \sum(x_i - m)^4/(n\sigma^4) - 3$$

Will return:

```
a
bbbbbbbb
C 10 3
```

convert_nest2ring

$convert\_ring2nest$

> 1

$$N_{\text{side}} = 256$$

ψ, θ, φ

ψ

θ

φ

$4N_{\text{side}}$

$$\ell \leq 4N_{\rm side}$$

C_ℓ

$$C_\ell^{TT}$$

$$C_\ell^{EE}$$

$$C_\ell^{BB}$$

$$C_\ell^{TE}$$

$$C_\ell^{TB}$$

C_ℓ^{EB}

$$C_{\ell}^{ET}$$

$$C_{\ell}^{BT}$$

$$C_{\ell}^{BE}$$

$\ell =$

$$\ell \leq 1024$$

\

$$= 1$$

$$= 2$$

$$[16, 2048]$$

$\leq$

$$10^{-30}$$

$$10^{30}$$

$$10^{-200}$$

10^{200}

32767

$$2^{31} - 1 \simeq 2.1\ 10^{9}$$

$$2^{63} - 1 \simeq 9.2\ 10^{18}$$

$$\simeq 3.40\ 10^{38}$$

$$\simeq 1.80\ 10^{308}$$

$\pi/4$

$\pi/2$

$$\pi \simeq 3.14159\ldots$$

$$2\pi$$

4π

$$\sqrt{2}$$

$$\gamma \simeq 0.577\ldots$$

$$1/\sqrt{4\pi}$$

$2/3$

$\pi/180$

$180/\pi$

$$-1.6375\ 10^{30}$$

ϕ

$2 \times dphi$

$phi0$

$\leq 4N_{\rm side}$

$$\phi = 0$$

土

2^{31}

$$2^{31} - 1$$

A

S

$\mathbf{a}, \mathbf{m}$

W

$$\mathbf{a}^{(n)} = \mathbf{a}^{(n-1)} + \mathbf{A}.\left(\mathbf{w}.\mathbf{m} - \mathbf{S}.\mathbf{a}^{(n-1)}\right),$$

$$\mathbf{a}^{(0)} = \mathbf{A}.\mathbf{w}.\mathbf{m}.$$

$(\mathbf{w.m})$

$$\left(\mathbf{w}.\mathbf{m} - \mathbf{S}.\mathbf{a}^{(n-1)}\right)$$

x

$$\sigma \equiv \sqrt{\sum_{p=1}^{N} \frac{(x(p)-\bar{x})^2}{N-1}}$$

$$\bar{x} \equiv \sum_{p=1}^{N} \frac{x(p)}{N}$$

$$\mathbf{w}^2 \neq \mathbf{w}$$

$$\ell_{\max}$$

$$T_{\mathrm{CPU}} \propto 1 + 2\times$$

$$= 3$$

q

$$\{1, 2, 3\}$$

$$p = q = 3$$

$$q = 2$$

$q = 1$

$$= -({}_{|s|}a_{\ell m} + (-1)^s {}_{-|s|}a_{\ell m})/2$$

$$= -1.6375 \ 10^{30}$$

$(1/N_{\text{procs}})$

$$P^{\mathrm{T}}_{\ell m}$$

$$P_{\ell m}^{\mathrm{P}}$$

$$a_{lm}$$

$$= 8$$

$$u = p + 4N_{\text{side}}^2$$

$$12N_{\mathrm{side}}^2 - 1$$

$$p = 0$$

$N_{\mathrm{side}} =$

$$N_{\text{pix}}$$

$$\{1, \ldots, 2^{28}\}$$

$$N_{\mathrm{side}} = \sqrt{N_{\mathrm{pix}}/12}$$

$$\{1, \ldots, 2^{28} = 268435456\}$$

$$N_{\text{pix}} = 12 N_{\text{side}}^2$$

$$N_w = \frac{(N_{\rm side}+1)(3N_{\rm side}+1)}{4} \simeq \frac{N_{\rm pix}}{16}$$

N_w

$$N_{\text{template}} = \frac{1 + N_{\text{side}}(N_{\text{side}} + 6)}{4}$$

N_{template}

$$w_{\text{pix}}(\ell)$$

$$a_{\ell m}^{(\mathrm{pix})}$$

$$a_{\ell m}^{(\mathrm{pix})} = a_{\ell m} w_{\mathrm{pix}}(\ell)$$

$$\ell_{\max} \leq 4N_{\text{side}}$$

$$N_{\text{pix}} - 1$$

$$\lambda_{\ell m}$$

$$Y_{\ell m}(\theta, \phi) = \lambda_{\ell m}(\theta) e^{im\phi}$$

$p = 1$

$\lambda_{00}(\theta_1)$

$$\lambda_{10}(\theta_1)$$

$$\lambda_{20}(\theta_1)$$

$$\lambda_{11}(\theta_1)$$

$$\lambda_{21}(\theta_1)$$

$$\lambda_{00}(\theta_2)$$

Z

$\pi/5$

$3\pi/4$

$x, y, z > 0$

$$2^{128} - 1 \approx 3.410^{38}$$

$$N_{\mathrm{side}} = 32$$

$P_{\ell n}$

$$\ell_{\max} = 64$$

$$\propto 1/\sqrt{\text{n\_obs}}$$

$$\sum_{j=0}^{d^2-1} A_{ij}\ f_j = b_i$$

2

$$b_i \equiv \sum_{p \in \mathcal{P}} s_i(p) w(p) m(p),$$

$$A_{ij} \equiv \sum_{p \in \mathcal{P}} s_i(p) w(p) s_j(p),$$

$\mathcal{P}$

w

$$s_0(p) = 1$$

$$s_1(p) = x, \;\; s_2(p) = y, \;\; s_3(p) = z$$

$$m'(p) = m(p) - \sum_{i=0}^{d^2-1} f_i s_i(p).$$

$| < 10^{-10}$

$$|b| > 15^o$$

$Nmmax$

$0\ldots$

$$kphi0 = 0$$

$$z = 0.5$$

8

$$kphi0 = 1$$

$\ell \leq \ell_{\max}$

$$\ell^3_{\max}$$

$$\ell_{\max} = m_{\max} = 64$$

$$\psi = \pi/3, \theta = 0.5, \varphi = 0$$

$$z = \cos(\theta) \geq 2/3, \qquad 0 < \phi \leq \pi/2,$$

$$2/3 > z \geq 0, \qquad \phi = 0, \quad \text{or} \quad \phi = \frac{\pi}{4N_{\rm side}}.$$

/

$nside$

>

(

$)^2$

≥ 4

$\mathbf{v}_1$

$$\mathbf{v}_2$$

$$\mathbf{v}_3 = \mathbf{v}_1 \times \mathbf{v}_2$$

$$C_\ell^T$$

$$C_\ell^E$$

$$C_\ell^B$$

$$C_\ell^{T\times E}$$

$\propto 1024$

> 12000

1.

3.

P_{lm}

l