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Assignment 3

Problem 1

KB

1. Mythical $\Rightarrow$ Immortal
2. $\neg$ Mythical $\Rightarrow$ $\neg$ Immortal $\wedge$ Mammal
3. Immortal $\vee$ Mammal $\Rightarrow$ Horned
4. Horned $\Rightarrow$ Magic

CNF

a) $\neg$ Mythical $\vee$ Immortal

b) Mythical $\vee$($\neg$ Immortal $\wedge$ Mammal)

c) $\neg$ (Immortal $\vee$ Mammal) $\vee$ Horned

d) Horned $\Rightarrow$ Magic

Using the statements above.

- From a and b: Immortal $\vee$ Mammal
- From 3/c: we can assume is horned
- From 4/d: we can assume is magic

We can prove that unicorns are horned and magic. However we cant prove if he is mythical.

Problem 2

Vocabulary:

- Student(x): x is a student
- Takes(x,y): student x takes course y
- Failed(x,y): student x failed course y
- Person(x): x is a person
- Vegetarian(x): x is a vegetarian
- Likes(x,y): person x likes person y
- Smart(x): person x is smart
- Homework(x,y): student x does homework for student y

a) $\exists x$ (Student(x) $\wedge$ $\neg$[Takes(x,History) $\wedge$ Takes(x,Biology)])

b) $\exists x$(Student(x) $\wedge$ Failed(x,History) $\wedge$ $\forall y$(Failed(y,History) $\Rightarrow$ x=y))

c) $\forall x$(Person(x) $\wedge$ $\forall y$[Vegetarian(y) $\wedge$ $\neg$ Likes(x,y)])$\Rightarrow$ Smart(x)

d) $\forall x \forall y$(Person(x) $\wedge$ Smart(y) $\wedge$ Vegetarian(y)) $\Rightarrow$ $\neg$ Likes(x,y)

e) $\exists x$ Student(x) $\wedge$ ($\forall y$ Student(y)$\wedge$ $\neg$Homework(y,y)) $\Rightarrow$ Homework(x,y)

Problem 3

Given: $\exists x, y$[Gray(x) $\wedge$ Pink(y) $\wedge$ Likes(x,y)]
Contradiction (Assume): $\forall x, y$[$\neg$ Gray(x) $\vee$ $\neg$ Pink(y)$\vee$ $\neg$ Likes(x,y)]

Knowledge Base:

1. Pink(Sam)
2. Gray(Clyde)
3. Likes(Clyde, Oscar)
4. Pink(Oscar) $\vee$ Gray(Oscar)
5. Pink(Oscar) $\Rightarrow$ $\neg$ Gray(Oscar) $\rightarrow$ $\neg$ Pink(Oscar) $\vee$ $\neg$ Gray(Oscar)
6. Likes(Oscar, Sam)
7. $\neg$ Gray(x) $\vee$ $\neg$ Pink(y) $\vee$ $\neg$ Likes(x,y)

Refutation:

8. From 6,7: $\neg$ Gray(Oscar) $\vee$ $\neg$ Pink(Sam)
9. From 1,8: $\neg$ Gray(Oscar)
10. From 4,9: Pink(Oscar)
11. From 2,7,10: $\neg$ Likes(Clyde, Oscar) !Contradicts 3!

Problem 4

Start true $\subset$ HIRE. Choose provisional rule:

- $r_{GPA} = \frac{4}{7} = 0,57$
- $r_{CU} = \frac{1}{4} = 0,25$
- $r_{UST} = \frac{1}{3} = 0,33$
- $r_{REC} = \frac{4}{8} = 0,5$
- $r_{HKU} = \frac{2}{4} = 0,67$
- $r_{EXP} = \frac{3}{4} = 0,75$

We choose EXP, and generate the rule: EXP $\subset$ HIRE.
This rule still covers the negative instance 8.

- $r_{GPA} = \frac{4}{4} = 1$
- $r_{UST} = \frac{5}{8} = 0,625$
- $r_{HKU} = \frac{5}{7} = 0,71$
- $r_{CU} = \frac{4}{7} = 0,57$
- $r_{REC} = \frac{3}{3} = 1$

Between GPA and REC and choose GPA because is based on a larger sample, then this yields the rule: EXP $\wedge$ GPA $\subset$ HIRE.
Then we have to create more more rules.

Example	Attributes						Hire
	GPA	UST	HKU	CU	REC	EXP	
2	1	1	0	0	0	0	0
4	1	1	0	0	1	0	1
6	1	0	0	1	1	0	0
7	1	0	1	0	1	0	0
8	0	1	0	0	0	1	0
9	0	0	1	0	0	0	0
10	0	0	0	1	1	0	0
11	0	0	0	1	1	0	0

Table 1: Reduced data. OK set.

Start with true $\subset$ OK. Generate a new rule:

- $r_{GPA} = \frac{1}{4} = 0,25$
- $r_{UST} = \frac{1}{3} = 0,33$
- $r_{HKU} = \frac{0}{2} = 0$
- $r_{CU} = \frac{0}{3} = 0$
- $r_{REC} = \frac{1}{5} = 0,5$
- $r_{EXP} = \frac{0}{1} = 0$

We choose UST, and generate the rule: UST $\subset$ OK.
This rule still covers the negative instance 1 and 8.Then we have to create more rules.

- $r_{GPA} = \frac{4}{4} = 1$
- $r_{HKU} = \frac{5}{6} = 0,83$
- $r_{CU} = \frac{4}{5} = 0,8$
- $r_{REC} = \frac{3}{3} = 1$
- $r_{EXP} = \frac{6}{7} = 0,857$

Between GPA and REC and choose GPA because is based on a larger sample, then this yields the rule: UST $\wedge$ GPA $\subset$ OK. We dont have any postive instances left, so we are done.

When combining we have the rule: EXP$\wedge$ UST $\wedge$ GPA $\subset$ HIRE.

Problem 5

$$P(j|m) = \frac{P(j \wedge m)}{P(m)}$$

$$P(m) = P(m|a) + P(m|\overline{a}) = 0.70 + 0.01 = 0.71$$

$$P(j \wedge m) = \sum_a \sum_b \sum_e P(j, m, a, b, e) =$$

$$\sum_a \sum_b \sum_e P(j|a)P(m|a)P(a|b,e)P(b)P(e) =$$

$$\sum_a P(j|a)P(m|a) \sum_b \sum_e P(a|b,e)P(b)P(e) =$$

$$\sum_a P(j|a)P(m|a)\ [P(a|b,e)P(b)P(e) + P(a|b,\overline{e})P(b)P(\overline{e}) + P(a|\overline{b},e)P(\overline{b})P(e) + P(a|\overline{b},\overline{e})P(\overline{b})P(\overline{e}) +$$

$$P(\overline{a}|b,e)P(b)P(e) + P(\overline{a}|b,\overline{e})P(b)P(\overline{e}) + P(\overline{a}|\overline{b},e)P(\overline{b})P(e) + P(\overline{a}|\overline{b},\overline{e})P(\overline{b})P(\overline{e})] =$$

$$(0.9 * 0.7 + 0.05 * 0.01) * [0.95 * 0.001 * 0.002 + 0.94 * 0.001 * (1 - 0.002) + 0.29 * (1 - 0.001) * 0.002 + 0.001 * (1 - 0.001) * (1 - 0.002) + (1 - 0.95) * 0.001 * 0.002 + (1 - 0.94) * 0.001 * (1 - 0.002) + (1 - 0.29) * (1 - 0.001) * 0.002 + (1 - 0.001) * (1 - 0.001) * (1 - 0.002)] = 0.6305$$

$$P(j|m) = \frac{0.6305}{0.71} \approx 0.8880282 \approx 89\%$$

Problem 6

1. Yes. Possible paths:
   Test1 $\rightarrow$ Disease1 $\rightarrow$ Symptom2 $\leftarrow$ Disease2 $\leftarrow$ Test2
   Test1 $\rightarrow$ Disease2 $\leftarrow$ Test2
   We can D-separate the nodes.

2. No. Possible paths
   Disease1 $\leftarrow$ Test1 $\rightarrow$ Disease2
   Disease1 $\rightarrow$ Symptom2 $\leftarrow$ Disease2
   We cant d-separate. Test1 and Symptom2 have relation with both nodes.

3. Yes. Possible paths
   Disease1$\rightarrow$ Symptom2 $\leftarrow$ Disease2 $\rightarrow$ Symptom3 $\leftarrow$ Disease3
   Disease1 $\leftarrow$ Test1 $\rightarrow$ Disease2 $\leftarrow$ Test3 $\rightarrow$ Disease3
   We cant d-separate. We can D-separate the nodes by Disease2.

4. E = Test1. Knowing this we turn Disease1 and Disease2 independents by removing the common cause.

5. E = Disease2, Test1,Test3. Knowing this we make sure Disease1 and Disease3 independents by removing the Indirect Evidential Effect.

Problem 7

	Fed: contract	Fed: do nothing	Fed: expand
Pol: contract	1,7	4,9	6,6
Pol: idle	2,8	5,5	9,4
Pol: expand	3,3	7,2	8,1

Figure 1: Red best action to Fed. Green Best action to Pol.

The Nash equilibria of this game is when Politicians expand, and the Fed contract (3,3).

Problem 8

We can formulate the problem in the following table. We can find Nash Equilibria in positions (0,0) and (6,6).

	0	1	2	3	4	5	6
0	**3,3**	0,5	0,4	0,3	0,2	0,1	0,0
1	5,0	2.5,2.5	0,4	0,3	0,2	0,1	0,0
2	4,0	4,0	2,2	0,3	0,2	0,1	0,0
3	3,0	3,0	3,0	1.5,1.5	0,2	0,1	0,0
4	2,0	2,0	2,0	2,0	1,1	0,1	0,0
5	1,0	1,0	1,0	1,0	1,0	0.5,0.5	0,0
6	0,0	0,0	0,0	0,0	0,0	0,0	**0,0**

Table 2: In bold we can see Nash Equilibria