LUNAR SOLUTION ELP 2000-82B

1. Introduction

Lunar solution ELP 2000-82B, including 36 data files and a FORTRAN program file, allows to compute a high precision geocentric lunar ephemeris. The solution involves the series of the semi-analytic theory ELP 2000-82, the constants of the lunar ephemeris ELP 2000 fitted to the numerical integration DE 200/LE 200 of the Jet Propulsion Laboratory and the arguments of the semi-analytic theory ELP 2000-85.

2. Description of the data files

2.1 Names and contents

The data files contain the series of the semi-analytic theory ELP 2000-82. For each data file, name and contents are:

ELP1	Main problem. Longitude periodic terms (sine)
ELP2	Main problem. Latitude (sine)
ELP3	Main problem. Distance (cosine)
ELP4	Earth figure perturbations. Longitude
ELP5	Earth figure perturbations. Latitude
ELP6	Earth figure perturbations. Distance
ELP7	Earth figure perturbations. Longitude/t
ELP8	Earth figure perturbations. Latitude/t
ELP9	Earth figure perturbations. Distance/t
ELP10	Planetary perturbations. Table 1. Longitude
ELP11	Planetary perturbations. Table 1. Latitude
ELP12	Planetary perturbations. Table 1. Distance
ELP13	Planetary perturbations. Table 1. Longitude/t
ELP14	Planetary perturbations. Table 1. Latitude/t
ELP15	Planetary perturbations. Table 1. Distance/t
ELP16	Planetary perturbations. Table 2. Longitude
ELP17	Planetary perturbations. Table 2. Latitude
ELP18	Planetary perturbations. Table 2. Distance
ELP19	Planetary perturbations. Table 2. Longitude/t

ELP20	Planetary perturbations. Table 2. Latitude/t
ELP21	Planetary perturbations. Table 2. Distance/t
ELP22	Tidal effects. Longitude
ELP23	Tidal effects. Latitude
ELP24	Tidal effects. Distance
ELP25	Tidal effects. Longitude/t
ELP26	Tidal effects. Latitude/t
ELP27	Tidal effects. Distance/t
ELP28	Moon figure perturbations. Longitude
ELP29	Moon figure perturbations. Latitude
ELP30	Moon figure perturbations. Distance
ELP31	Relativistic perturbations. Longitude
ELP32	Relativistic perturbations. Latitude
ELP33	Relativistic perturbations. Distance
ELP34	Planetary perturbations (solar eccentricity). Longitude/t^2
ELP35	Planetary perturbations (solar eccentricity). Latitude/t^2
ELP36	Planetary perturbations (solar eccentricity). Distance/t^2

2.2 Records description

The first record of each file contains a title. Each following record contains one term of the series, according to the following formulations and FORTRAN formats.

a) Files ELP1 to ELP3

The adopted formulation for the series is:

$$\sum A \begin{matrix}\sin \\ \cos\end{matrix} \left(i_1 D + i_2 l' + i_3 l + i_4 F \right)$$

sine for files ELP1 and ELP2, cosine for file ELP3.

Each record gives:

$$i_1,\ i_2,\ i_3,\ i_4,\ A,\ B_i\ (i = 1 \ldots 6)$$

where the six quantities B_i are the derivatives of A: $\dfrac{\partial A}{\partial \sigma_i}$ for longitude and latitude and $a_0 \dfrac{\partial}{\partial \sigma_i} \left(\dfrac{A}{a_0} \right)$ for distance, with respect to six constants $\sigma_i = (m, \Gamma, E, e', \alpha, \mu)$ (see sect. 5 for definitions).

The format is:

4I3, 2X, F13.5, 6(2X, F10.2)

b) Files ELP4 to ELP9

The adopted formulation for the series is:

$$\sum A \sin(i_1 \zeta + i_2 D + i_3 l' + i_4 l + i_5 F + \phi)$$

Each record gives:

$$i_1,\ i_2,\ i_3,\ i_4,\ i_5,\ \phi,\ A,\ P$$

where ϕ is a phase and P is an approximate value of the period of the term as far as this period is less than 99 999.999 years. For longer periods, the limit value is given.

The format is:

5I3, 1X, F9.5, 1X, F9.5, 1X, F9.3

c) Files ELP10 to ELP15

The adopted formulation for the series is:

$$\sum A \sin(i_1 M_e + i_2 V + i_3 T + i_4 M_a + i_5 J + i_6 S + i_7 U + i_8 N + i_9 D + i_{10} l + i_{11} F + \phi)$$

Each record gives:

$$i_1,\ i_2,\ i_3,\ i_4,\ i_5,\ i_6,\ i_7,\ i_8,\ i_9,\ i_{10},\ i_{11},\ \phi,\ A,\ P$$

where ϕ and P have the same meaning as in *b)*.

The format is:

11 I3, 1X, F9.5, 1X, F9.5, 1X, F9.3

d) Files ELP16 to ELP21

The adopted formulation for the series is:

$$\sum A \sin(i_1 M_e + i_2 V + i_3 T + i_4 M_a + i_5 J + i_6 S + i_7 U + i_8 D + i_9 l' + i_{10} l + i_{11} F + \phi)$$

Each record gives:

$$i_1,\ i_2,\ i_3,\ i_4,\ i_5,\ i_6,\ i_7,\ i_8,\ i_9,\ i_{10},\ i_{11},\ \phi,\ A,\ P$$

where ϕ and P have the same meaning as in *b)*.

The format is the same as in *c)*.

e) Files ELP22 to ELP36

The description is the same as in *b)*, the value of i_1 (ζ coefficient) being always 0.

2.3 Units

For longitude and latitude, coefficients A are given in arcseconds; for distance they are given in kilometers. Coefficients B_i are given in the same unit as A, the constants m, Γ, E, e', α, μ being dimensionless. Phases are given in degrees. Periods are in years.

t is barycentric time TDB expressed in Julian centuries and reckonned from J2000 (Julian TDB date 2 451 545.0).

3. A few comments about ELP 2000-82 series

Series of files ELP7 to ELP9, ELP13 to ELP15, ELP19 to ELP21, ELP25 to ELP27 must be multiplied by t, series of files ELP34 to ELP36 must be multiplied by t^2 in order to restore mixed terms (Poisson terms).

The mean mean longitude:

$$W_1 = W_1^{(0)} + \nu t + W_1^{(2)} t^2 + W_1^{(3)} t^3 + W_1^{(4)} t^4$$

must be added to the sum of the longitude series (Fourier and Poisson terms) so as to obtain the longitude of the Moon referred to the mean dynamical ecliptic of date and departure point (see sect. 4 and sect. 8 for definitions).

All the perturbations (files ELP4 to ELP33) have been obtained at the first approximation as it is defined in (Chapront-Touzé and Chapront, 1980) except for the main perturbations due to the secular terms in solar eccentricity. Nevertheless, lunar mean motions, including approximate values of planetary and Earth figure perturbations, have been introduced in the integration.

The adjustment of integration constants, as defined in (Chapront-Touzé and Chapront, 1980) has been performed separately for the different kinds of perturbations. It gives rise to supplementary Fourier terms which have been added respectively to files ELP4 to ELP6 (for Earth figure perturbations), ELP16 to ELP18 (for planetary perturbations), ELP22 to ELP24 (for tidal effects), ELP28 to ELP30 (for Moon figure perturbations), ELP31 to ELP33 (for relativistic perturbations).

The *Earth figure perturbations* (files ELP4 to ELP9) include perturbations due to J_2 and J_3. The motion of the true equator has been taken into account through: the linear term of the precession included in argument ζ (see sect. 4); the four main terms of the nutation in longitude and the three main terms of the nutation in obliquity from Woolard (coefficients computed for J2000); the linear term of obliquity ϵ_A from (Lieske et al, 1977)(†).

The *planetary perturbations. Table 1* (files ELP10 to ELP15) include indirect and direct planetary perturbations without any adjustment of the integration constants. The indirect planetary perturbations have been computed with Bretagnon's solution VSOP80(*) for the Earth-Moon barycenter up to the third order in masses. As far as Table 1 is concerned, short periodic perturbations of the Earth-Moon barycenter due to the lunar action and relativistic perturbations have not been included in VSOP80. Furthermore, the secular terms of variables $K = e' \cos \varpi'$, $H = e' \sin \varpi'$, $Q = \gamma' \cos \Omega'$, $P = \gamma' \sin \Omega'$ for the Earth-Moon barycenter have been dropped out from VSOP80, these terms being taken into account elsewhere. The direct planetary perturbations have been computed with Bretagnon's solution VSOP80 for the planets up to the first order for all planets, up to the third order (Fourier terms only) for the major planets. Especially, the linear terms of H, K, P, Q have been taken into account. The effect of the relativistic terms of the planets has been found negligible.

The *planetary perturbations. Table 2* (files ELP16 to ELP21) include several perturbations:

a) The main effect of the linear terms of the solar eccentricity. If we note the solar eccentricity as:

$$e' = e'^{(0)} + e'^{(1)}t + e'^{(2)}t^2 \ldots$$

the main effect of the linear term on a coordinate X is $\dfrac{\partial X}{\partial e'} e'^{(1)}t$. The main effects of the secular terms of the solar perigee have been taken into account in the arguments themselves (see sect. 4).

b) The secondary effects of the linear terms of the solar eccentricity and perigee as described in (Chapront-Touzé, 1982).

c) The secondary effects of the linear and quadratic terms of variables P and Q of VSOP80 for the Earth-Moon barycenter. The so-called secondary effects come from Coriolis forces, the Moon motion being referred to the mean ecliptic of date and not to a fixed plane.

d) The indirect planetary perturbations due to the short periodic terms of the Earth-Moon barycenter produced by the Moon action.

e) The complete effect of the adjustment of the integration constants concerning the perturbations of *Planetary perturbations. Table 1* and *Planetary perturbations. Table 2* has been added to *Planetary perturbations. Table 2.*

(†) Lieske, J.H., Lederle, T., Fricke, W., Morando, B.: 1977, *Astron. Astrophys.*, **58**, 1

(*) Bretagnon, P.: 1980, Théorie planétaire VSOP80, magnetic tape

The *planetary perturbations (solar eccentricity)* (files ELP34 to ELP36) include only the main effect of the quadratic term of the solar eccentricity that is $\dfrac{\partial X}{\partial e'}\, e'^{(2)} t^2$.

The *tidal effects* (files ELP22 to ELP27) have been computed with an acceleration model from (Williams et al, 1978)(*) and constants given in sect. 6.

The *Moon figure perturbations* (files ELP28 to ELP30) include the effects of the harmonic development of the lunar potential up to the third order, with coefficients given in sect. 6 and solution due to M. Moons for the libration(†).

The *relativistic perturbations* include the main relativistic perturbations and the indirect relativistic perturbations produced by the relativistic terms of VSOP80 for the Earth-Moon barycenter in the indirect planetary perturbations of the Moon.

4. Arguments

In files ELP1 to ELP3, Delaunay arguments D, l', l, F are polynomial functions of time under the general formulation:

$$\lambda = \lambda^{(0)} + \lambda^{(1)} t + \lambda^{(2)} t^2 + \lambda^{(3)} t^3 + \lambda^{(4)} t^4 \tag{1}$$

where t is barycentric time TDB in Julian centuries reckonned from J2000 (Julian TDB date 2 451 545.0) They are derived from W_1 (mean mean longitude of the Moon), W_2 (mean longitude of the lunar perigee), W_3 (mean longitude of the lunar ascending node), T (mean heliocentric mean longitude of the Earth-Moon barycenter) and ϖ' (mean longitude of the perihelion of the Earth-Moon barycenter) by:

$$\begin{aligned} D &= W_1 - T + 180^\circ \\ l' &= T - \varpi' \\ l &= W_1 - W_2 \\ F &= W_1 - W_3 \end{aligned}$$

W_1, W_2 and W_3 are angles of the inertial mean ecliptic of date referred to the departure point γ'_{2000} (see definition in sect. 8). T and ϖ' are angles of the inertial mean ecliptic of J2000 referred to the inertial mean equinox γ^I_{2000} of J2000.

In ELP 2000-82 and ELP 2000-85, the constant parts $W_1^{(0)}$, $W_2^{(0)}$, $W_3^{(0)}$, $T^{(0)}$ and $\varpi'^{(0)}$ of W_1, W_2, W_3, T and ϖ' are literal constants which must fitted to observations. The coefficients of t in W_1 and T, denoted respectively as ν and n', are constants of the theory, i.e. constants which have received an assigned value in the theory. The coefficients of t in W_2 and W_3, respectively $W_2^{(1)}$ and $W_3^{(1)}$, and the coefficients of t^2, t^3, t^4 in W_1, W_2 and W_3, respectively $W_1^{(2)}$, $W_2^{(2)}$, $W_3^{(2)}$, $W_1^{(3)}$, $W_2^{(3)}$, $W_3^{(3)}$, $W_1^{(4)}$, $W_2^{(4)}$, $W_3^{(4)}$, are computed values yielded by the theory.

The various contributions to $W_2^{(1)}$ and $W_3^{(1)}$ and the total values, computed for the values of the constants adopted in the theory (see sect. 6), are given in table A. The derivatives of the main problem contributions to $\dfrac{W_2^{(1)}}{\nu}$ and $\dfrac{W_3^{(1)}}{\nu}$ with respect to the same set of constants $\sigma_i = (m, \Gamma, E, e', \alpha, \mu)$ as the coefficients of the main problem series are given in table B.

(*) Williams, J.G., Sinclair, W.S., Yoder, C.F.: 1978, *Geophys. Res. Let.*, **5**, 11, 943
(†) Moons, M.: 1982, *Celes. Mech.*, **26**, 131

Table A. Computed values of the mean motions of perigee and node (″/cy) for the constants of sect. 6

Contribution of		$W_2^{(1)}$	$W_3^{(1)}$
Main problem		14 642 537.936 8	−6 967 167.264 3
Earth figure	Without C.A.	615.883 3	−588.200 7
	C.A.	17.520 1	−4.335 0
Planetary perturbations	Indirect	−21.612 7	6.252 6
(Table 1)	Direct	267.973 6	−142.823 6
Planetary perturbations	Solar eccentricity	0.033 9	0.000 0
(Table 2)	Moon on barycenter	3.349 2	−8.232 9
	C.A.	−2.638 8	0.730 6
Tidal effects	Without C.A.	0.066 3	0.000 2
	C.A.	0.000 7	−0.000 2
Lunar figure	Without C.A.	−2.268 9	−16.810 8
	C.A.	0.521 7	−0.133 5
Relativity	Main effect	4.452 8	1.253 4
	Indirect effect	0.689 7	−0.198 9
	C.A.	−3.345 4	0.847 4
Total		14 643 418.562 3	−6 967 918.915 7

C.A. means Constant Adjustment

Table B. Derivatives of the main problem contributions to $\dfrac{W_2^{(1)}}{\nu}$ and $\dfrac{W_3^{(1)}}{\nu}$

σ_i	$\dfrac{\partial}{\partial\sigma_i}\dfrac{W_2^{(1)}}{\nu}$	$\dfrac{\partial}{\partial\sigma_i}\dfrac{W_3^{(1)}}{\nu}$
m	0.311 079 095	−0.103 837 907
Γ	−0.004 482 398	0.000 668 287
E	−0.001 102 485	−0.001 298 072
e'	0.001 056 062	−0.000 178 028
α	0.000 050 928	−0.000 037 342
μ	−0.000 000 418	0.000 000 220

σ_i are dimensionless

The contributions to $W_1^{(2)}$, $W_2^{(2)}$ and $W_3^{(2)}$ and the total values from ELP 2000-85, computed for the values of the constants adopted in the theory (see sect. 6), are given in table C.

Table C. Computed values of $W_1^{(2)}$, $W_2^{(2)}$ and $W_3^{(2)}$ ($''/\mathrm{cy}^2$) for the constants of sect. 6

Contribution of		$W_1^{(2)}$	$W_2^{(2)}$	$W_3^{(2)}$
Earth figure		0.192 5	0.100 3	−0.095 8
Planetary perturbations	Indirect	0.002 0	−0.005 7	0.001 6
(Table 1)	Direct	0.000 5	−0.000 8	0.000 2
Planetary perturbations (Table 2)	Solar eccentricity	5.864 0	−38.547 5	6.502 6
Tidal effects		−11.947 3	0.176 1	−0.046 4
Total		−5.888 3	−38.277 6	6.362 2

The contributions to $W_1^{(3)}$, $W_2^{(3)}$ and $W_3^{(3)}$ and the total values from ELP 2000-85 are given in table D.

Table D. Computed values of $W_1^{(3)}$, $W_2^{(3)}$ and $W_3^{(3)}$ ($''/\mathrm{cy}^3$) for the constants of sect. 6

Contribution of		$W_1^{(3)}$	$W_2^{(3)}$	$W_3^{(3)}$
Earth figure		−0.000 027	−0.000 014	0.000 013
Planetary perturbations (Table 2)	Solar eccentricity	0.007 015	−0.045 039	0.007 613
Tidal effects		−0.000 384	0.000 006	−0.000 001
Total		0.006 604	−0.045 047	0.007 625

The contributions to $W_1^{(4)}$, $W_2^{(4)}$ and $W_3^{(4)}$ from ELP 2000-85 are given in table E.

Table E. Computed values of $W_1^{(4)}$, $W_2^{(4)}$ and $W_3^{(4)}$ ($''/\mathrm{cy}^4$) for the constants of sect. 6

Contribution of		$W_1^{(4)}$	$W_2^{(4)}$	$W_3^{(4)}$
Planetary perturbations (Table 2)	Solar eccentricity	−0.000 031 69	0.000 213 01	−0.000 035 86

The coefficient of t in ϖ' and the coefficients of t^2, t^3, t^4 in T and ϖ' are computed values yielded by a planetary theory. ELP 2000-85 uses values from (Laskar, 1986)(*)

In files ELP4 to ELP36, Delaunay arguments D, l', l, F are reduced to their linear parts under the general formulation:

$$\lambda = \lambda^{(0)} + \lambda^{(1)} t \tag{2}$$

In files ELP4 to ELP9, ζ is deduced from W_1, reduced to its linear part under formulation (2), by:

$$\zeta = W_1 + p\,t$$

where p is the precession constant in J2000.

(*) Laskar, J.: 1986, *Astron. Astrophys.*, **157**, 59

In files ELP10 to ELP21, T is reduced to its linear part under formulation (2); M_e, V, M_a, J, S, U and N are the linear parts of the mean mean longitudes of the planets Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune under formulation (2). Table F gives the corresponding values of $\lambda^{(0)}$ and $\lambda^{(1)}$ from the planetary theory VSOP82 (Bretagnon, 1982)(†).

Table F. Planetary longitudes in J2000 and mean motions (″/cy) from VSOP82

Planet	$\lambda^{(0)}$	$\lambda^{(1)}$
M_e	252°15′03″.259 86	538 101 628.688 98
V	181°58′47″.283 05	210 664 136.433 55
M_a	355°25′59″.788 66	68 905 077.592 84
J	34°21′05″.342 12	10 925 660.428 61
S	50°04′38″.896 94	4 399 609.659 32
U	314°03′18″.018 41	1 542 481.193 93
N	304°20′55″.195 75	786 550.320 74

5. Derivatives

a) First set of constants S_1

Derivatives given in files ELP1 to ELP3 and in table B are derivatives with respect to the set of constants S_1 (m, Γ, E, e', α, μ) with:

$m = \dfrac{n'}{\nu}$ (n' = mean motion of T, ν = mean motion of W_1 in J2000 as defined in sect. 4)

Γ = the half coefficient of $\sin F$ in the latitude

E = the half coefficient of $\sin l$ in the longitude

e' = eccentricity of the heliocentric orbit of the Earth-Moon barycenter

$\alpha = \dfrac{a_0}{a'}$ where a_0 is the keplerian semi-major axis of the Moon related to ν by:

$$\nu^2 a_0^3 = G(m_T + m_L) \tag{3}$$

a' is the semi-major axis of the heliocentric orbit of the Earth-Moon barycenter related to n' by:

$$n'^2 a'^3 = G(m_S + m_T + m_L)$$

where m_S, m_T, m_L are respectively Sun, Earth and Moon masses.

We shall call respectively B_1, B_2, B_3, B_4, B_5, B_6 the derivatives of a coefficient A with respect to m, Γ, E, e', α, μ as they are given in files ELP1 to ELP3 (N.B. In file ELP3, B_i are "$a_0\times$ derivatives of $\dfrac{A}{a_0}$"). We shall also call B_i the derivatives of $\dfrac{W_2^{(1)}}{\nu}$ and $\dfrac{W_3^{(1)}}{\nu}$ as they are given in table B.

b) Second set of constants S_2

Table G gives the expressions of the derivatives of a coefficient A for the main problem contribution in the longitude, latitude and distance and those of the derivatives of the mean motions $W_2^{(1)}$ and $W_3^{(1)}$ with respect to the set of constants S_2: (ν, Γ, E, n', e', μ, μ', G') in function of B_i. To obtain the derivatives of the longitude, latitude or distance themselves, it is necessary to use both the derivatives of the coefficients and those of the mean motions.

(†) Bretagnon, P.: 1982, *Astron. Astrophys.*, **114**, 278

Table G. Derivatives of the coefficients of the longitude, latitude and distance and of the mean motions of perigee and node with respect to the constants S_2

	ν	Γ	E	n'	e'	μ	μ'	G'
Longitude or latitude	$-\frac{m}{\nu}\left(B_1+\frac{2}{3}\frac{\alpha}{m}B_5\right)$	B_2	B_3	$\frac{1}{\nu}\left(B_1+\frac{2}{3}\frac{\alpha}{m}B_5\right)$	B_4	B_6	$\frac{m^2}{3\alpha^2}B_5$	0
Distance	$-\frac{m}{\nu}\left(B_1+\frac{2}{3}\frac{\alpha}{m}B_5+\frac{2}{3}\frac{A}{m}\right)$	B_2	B_3	$\frac{1}{\nu}\left(B_1+\frac{2}{3}\frac{\alpha}{m}B_5\right)$	B_4	$B_6+\frac{1}{3}\frac{A}{1-\mu}$	$\frac{m^2}{3\alpha^2}B_5$	$\frac{1}{3}\frac{A}{G'}$
$W_i^{(1)}$ $(i=2,3)$	$\frac{1}{\nu}\left(W_i^{(1)}-\nu m\left(B_1+\frac{2}{3}\frac{\alpha}{m}B_5\right)\right)$	νB_2	νB_3	$B_1+\frac{2}{3}\frac{\alpha}{m}B_5$	νB_4	νB_6	$\frac{\nu m^2}{3\alpha^2}B_5$	0

6. Constants of the theory

The values of the constants involved in the series of the semi-analytical theory ELP 2000-82 are the following:

a) Main problem

ν	sidereal mean motion of the Moon	$1\,732\,559\,343''.18$/cy
$2E$	coefficient of $\sin l$ in longitude	$22\,639''.55$
2Γ	coefficient of $\sin F$ in latitude	$18\,461''.40$
e'	solar eccentricity (Newcomb)	0.016 709 24
n'	sidereal mean motion of the Sun (Newcomb)	$129\,597\,742''.34$/cy
μ	$=m_L/(m_T+m_L)$	0.012 150 568
μ'	$=(m_T+m_L)/(m_S+m_T+m_L)$	$3.040\,423\,956\,10^{-6}$
G'	$=Gm_T$	$3.986\,005\,10^{14}$ $\mathrm{m^3/s^2}$

From these values we derive, by means of equation (3), the value $a_0 = 384\,747\,980.674$ m of the keplerian semi-major axis of the Moon.

b) Earth figure

J_2		0. 001 082 63
J_3		$-0.254\,10^{-5}$
ϵ	obliquity of the ecliptic in J2000	$23°26'21''.448$
a_T	equatorial radius of the Earth	6 378 140 m
p	precession constant in J2000	$5\,029''.096\,6$/cy

c) Planetary perturbations

Constants of the theory VSOP80 (Bretagnon, 1980) have been used for the orbits of the planets.

d) Lunar figure perturbations

We have used parameters from (Ferrari et al, 1980)(*), β and γ being corrected from lunar mean tidal distorsions by means of the model from (Yoder, 1979)(†).

(*) Ferrari, A.J., Sinclair, W.S., Sjogren, W.L., Williams, J.G., Yoder, C.F.: 1980, *J. Geophys. Res.*, **85**, 3939

(†) Yoder, C.F.: 1979 in *Natural and Artificial Satellite Motion*, University of Texas Press, p.210

β	$0.632\,108\,10^{-3}$	S_{31L}	$0.561\,07\,10^{-5}$
γ	$0.228\,443\,10^{-3}$	C_{32L}	$0.488\,84\,10^{-5}$
J_{2L}	$0.202\,15\,10^{-3}$	S_{32L}	$0.168\,7\,10^{-5}$
J_{3L}	$0.121\,26\,10^{-4}$	C_{33L}	$0.143\,6\,10^{-5}$
C_{22L}	$0.223\,04\,10^{-4}$	S_{33L}	$-0.334\,35\,10^{-6}$
C_{31L}	$0.307\,1\,10^{-4}$	a_L	1 738 000 m

e) Tidal effects

k_2	Love number	0.30
δ	phase	0.040 7

7. Constants fitted to DE200/LE200

Corrections to the numerical values of constants S_2 given in sect. 6. (constants S_2 are defined in sect. 5.) and values in J2000 of the arguments W_1, W_2, W_3, T and ϖ' have been derived from a fit, over a time span of one century, of ELP 2000-82 to the numerical integration DE200/LE200 of the Jet Propulsion Laboratory (Standish, 1981)(*). Note that, for this fit, the values $G' = 3.986\,004\,48\,10^{14}$ m^2/s^3 and $\mu = 0.012\,150\,581\,6$ have been used instead of the values quoted in sect. 6.

The additive corrections to the constants S_2 of sect. 6, with their standard deviations, are the following:

$$\delta\nu = (0''.556\,04 \pm 0''.000\,49)\,/\mathrm{cy}$$
$$\delta E = (0''.017\,89 \pm 0''.000\,03)$$
$$\delta\Gamma = (-0''.080\,66 \pm 0''.000\,03)$$
$$\delta n' = (-0''.064\,2 \pm 0''.003\,7)\,/\mathrm{cy}$$
$$\delta e' = (-0''.128\,79 \pm 0''.000\,30)$$

These corrections produce the following additive corrections to the mean motions of perigee and node given in table A:

$$\delta W_2^{(1)} = 1''.700\,9\,/\mathrm{cy}$$
$$\delta W_3^{(1)} = -0''.446\,5\,/\mathrm{cy}$$

The values of tables C, D, E are not modified.

The additive corrections to the coefficients A of the series of the main problem are given by:

a) For longitude and latitude (files ELP1 and ELP2)

$$\delta A = -m\Big(B_1 + \frac{2}{3}\frac{\alpha}{m}B_5\Big)\frac{\delta\nu}{\nu} + \Big(B_1 + \frac{2}{3}\frac{\alpha}{m}B_5\Big)\frac{\delta\,n'}{\nu} + \Big(B_2\delta\Gamma + B_3\delta E + B_4\delta e'\Big)/206\,264.81$$

b) For distance (file ELP3)

$$\delta A = -m\Big(B_1 + \frac{2}{3}\frac{\alpha}{m}B_5 + \frac{2}{3}\frac{A}{m}\Big)\frac{\delta\nu}{\nu} + \Big(B_1 + \frac{2}{3}\frac{\alpha}{m}B_5\Big)\frac{\delta\,n'}{\nu} + \Big(B_2\delta\Gamma + B_3\delta E + B_4\delta e'\Big)/206\,264.81$$

The coefficients of the other series are not modified.

(*) Standish, E.M.: 1981, Numerical integration DE200/LE200, magnetic tape

The fitted values in J2000 of the arguments W_1, W_2, W_3, T and ϖ' ,with their standard deviations, are the following:

$$W_1^{(0)} = 218°18'59''.955\,71 \pm 0''.000\,18$$
$$W_2^{(0)} = 83°21'11''.674\,75 \pm 0''.000\,94$$
$$W_3^{(0)} = 125°02'40''.398\,16 \pm 0''.001\,29$$
$$T^{(0)} = 100°27'59''.220\,59 \pm 0''.002\,05$$
$$\varpi'^{(0)} = 102°56'14''.427\,53 \pm 0''.017\,80$$

The complete expressions of the arguments W_1, W_2, W_3, T, ϖ' and of Delaunay arguments, involving all the corrections induced by the fit to DE200/LE200, are:

$$W_1 = 218°18'59''.955\,71 + 1\,732\,559\,343''.736\,04\,t - 5''.888\,3\,t^2 + 0''.006\,604\,t^3 - 0''.000\,031\,69\,t^4$$
$$W_2 = 83°21'11''.674\,75 + 14\,643\,420''.263\,2\,t - 38''.277\,6\,t^2 - 0''.045\,047\,t^3 + 0''.000\,213\,01\,t^4$$
$$W_3 = 125°02'40''.398\,16 - 6\,967\,919''.362\,2\,t + 6''.362\,2\,t^2 + 0''.007\,625\,t^3 - 0''.000\,035\,86\,t^4$$
$$T = 100°27'59''.220\,59 + 129\,597\,742''.275\,8\,t - 0''.020\,2\,t^2 + 0''.000\,009\,t^3 + 0''.000\,000\,15\,t^4$$
$$\varpi' = 102°56'14''.427\,53 + 1\,161''.228\,3\,t + 0''.532\,7\,t^2 - 0''.000\,138\,t^3$$
$$D = 297°51'00''.735\,12 + 1\,602\,961\,601''.460\,3\,t - 5''.868\,1\,t^2 + 0''.006\,595\,t^3 - 0''.000\,031\,84\,t^4$$
$$l' = 357°31'44''.793\,06 + 129\,596\,581''.047\,4\,t - 0''.552\,9\,t^2 + 0''.000\,147\,t^3$$
$$l = 134°57'48''.280\,96 + 1\,717\,915\,923''.472\,8\,t + 32''.389\,3\,t^2 + 0''.051\,651\,t^3 - 0''.000\,244\,70\,t^4$$
$$F = 93°16'19''.557\,55 + 1\,739\,527\,263''.098\,3\,t - 12''.250\,5\,t^2 - 0''.001\,021\,t^3 + 0''.000\,004\,17\,t^4$$

8. Coordinate systems

The depature point γ'_{2000} is the point of the inertial mean ecliptic of date defined by:

$$\mathrm{N}\gamma'_{2000} = \mathrm{N}\gamma^I_{2000}$$

where γ^I_{2000} is the inertial mean equinox of J2000 and N the node of the inertial mean ecliptics of date and of J2000 (see fig. 1).

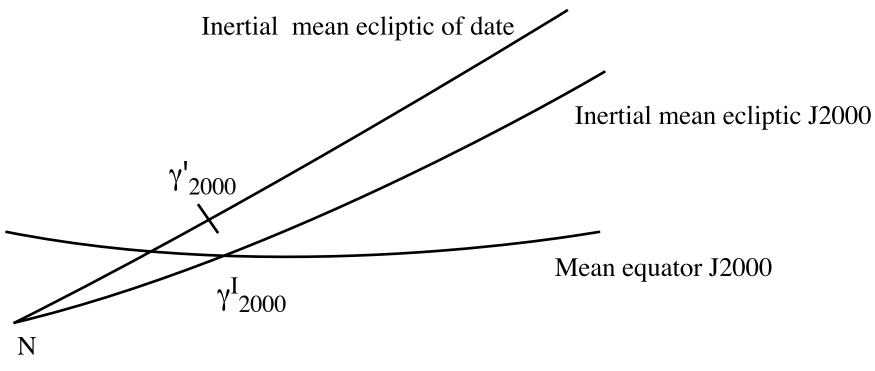

Fig. 1 Position of the departure point

The natural coordinate system of the lunar theories ELP 2000-82 and ELP 2000-85 consists of the inertial mean ecliptic of date and departure point γ'_{2000}. In this coordinate system, longitude V is obtained by adding W_1 to the series of files ELP1, ELP4, ELP10, ELP16, ELP22, ELP28 and ELP31, to the series of files ELP7, ELP13, ELP19 and ELP25 multiplied by t, and at last to the series of file ELP 34 multiplied by t^2. Latitude U is obtained by adding the series of files ELP2, ELP5, ELP11, ELP17, ELP23, ELP29 and ELP32 to the series of files ELP8, ELP14, ELP20 and ELP26 multiplied by t and at last to the series of file ELP35 multiplied by t^2. Distance r, which does not depend on the coordinate system, is obtained by adding the series of files ELP3, ELP6, ELP12, ELP18, ELP24, ELP30 and ELP33 to the series of files ELP9, ELP15, ELP21 and ELP27 multiplied by t and at last to the series of file ELP36 multiplied by t^2. Rectangular coordinates x, y, z are given by:

$$\begin{aligned} x &= r \cos V \cos U \\ y &= r \sin V \cos U \\ z &= r \sin U \end{aligned}$$

Longitude and latitude referred to the inertial mean ecliptic and equinox of date are respectively:

$$V_d = V + p_A$$

and U. p_A is the accumulated precession between J2000 and the date. By truncating Laskar's series (Laskar, 1986), we have:

$$p_A = 5\,029''.096\,6\,t + 1''.112\,0\,t^2 + 0''.000\,077\,t^3 - 0''.000\,023\,53\,t^4$$

Rectangular coordinates x^E_{2000}, y^E_{2000} and z^E_{2000} referred to the inertial mean ecliptic and equinox of J2000 are given by:

$$\begin{pmatrix} x^E_{2000} \\ y^E_{2000} \\ z^E_{2000} \end{pmatrix} = \begin{pmatrix} 1-2P^2 & 2PQ & 2P\sqrt{1-P^2-Q^2} \\ 2PQ & 1-2Q^2 & -2Q\sqrt{1-P^2-Q^2} \\ -2P\sqrt{1-P^2-Q^2} & 2Q\sqrt{1-P^2-Q^2} & 1-2P^2-2Q^2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where P and Q are Laskar's series reproduced here up to degree five:

$$\begin{aligned} P = {} & 0.101\,803\,91\,10^{-4}\,t + 0.470\,204\,39\,10^{-6}\,t^2 - 0.541\,736\,7\,10^{-9}\,t^3 \\ & - 0.250\,794\,8\,10^{-11}\,t^4 + 0.463\,486\,10^{-14}\,t^5 \\ Q = {} & -0.113\,469\,002\,10^{-3}\,t + 0.123\,726\,74\,10^{-6}\,t^2 + 0.126\,541\,70\,10^{-8}\,t^3 \\ & - 0.137\,180\,8\,10^{-11}\,t^4 - 0.320\,334\,10^{-14}\,t^5 \end{aligned}$$

Converting rectangular coordinates x^E_{2000}, y^E_{2000} and z^E_{2000} to rectangular coordinates x^Q_{2000}, y^Q_{2000} and z^Q_{2000} referred to the FK5 equator and equinox γ_{FK5} (i.e. mean equator and rotational mean equinox of J2000) involves the obliquity ϵ^I of the inertial mean ecliptic of J2000 on the mean equator of J2000 and the arc $\gamma^I_{2000}\gamma_{FK5}$ (see fig. 2). The values of these quantities derived from the fit of ELP 2000-82 to DE200/LE200 are:

$$\begin{aligned} \epsilon^I &= 23°26'21''.408\,83 \pm 0''.000\,06 \\ \gamma^I_{2000}\gamma_{FK5} &= 0''.098\,45 \pm 0''.000\,16 \end{aligned}$$

hence:

$$\begin{pmatrix} x^Q_{2000} \\ y^Q_{2000} \\ z^Q_{2000} \end{pmatrix} = \begin{pmatrix} 1.000\,000\,000\,000 & 0.000\,000\,437\,913 & -0.000\,000\,189\,859 \\ -0.000\,000\,477\,299 & 0.917\,482\,137\,607 & -0.397\,776\,981\,701 \\ 0.000\,000\,000\,000 & 0.397\,776\,981\,701 & 0.917\,482\,137\,607 \end{pmatrix} \begin{pmatrix} x^E_{2000} \\ y^E_{2000} \\ z^E_{2000} \end{pmatrix}$$

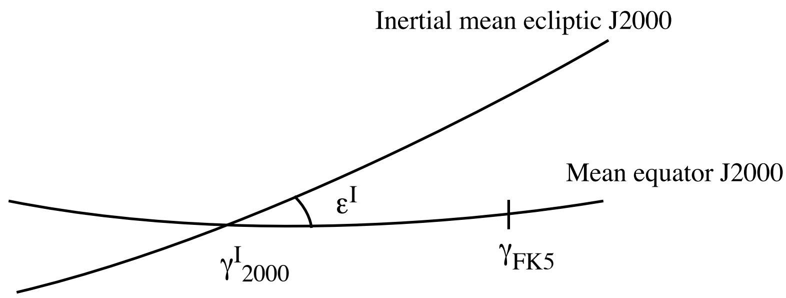

Fig. 2 Conversion of J2000 ecliptic coordinates to FK5 equatorial coordinates

9. Subroutine ELP82B

Subroutine ELP82B, in file ELP82B.FOR, is an elementary FORTRAN subroutine which allows to compute rectangular geocentric lunar coordinates referred to the inertial mean ecliptic and equinox of J2000 (coordinates x^E_{2000}, y^E_{2000}, z^E_{2000} of sect. 8). It uses files ELP1 to ELP36 and the expressions of lunar arguments of sect. 7 and takes into account the corrections to the constants of the theory given in sect. 7.

Table H. Check values: rectangular coordinates referred to the inertial mean ecliptic and equinox J2000

Julian TDB date (Gregorian date)	x^E_{2000} km	y^E_{2000} km	z^E_{2000} km	PREC $''$
2 469 000.5	−361 602.985 36	44 996.995 10	−30 696.653 16	0
(17 oct. 2047,0h)	−361 602.984 81	44 996.996 25	−30 696.651 52	$5.\,10^{-5}$
2 449 000.5	−363 132.342 48	35 863.653 78	−33 196.004 09	0
(13 jan. 1993,0h)	−363 132.343 05	35 863.651 87	−33 196.003 75	$5.\,10^{-5}$
2 429 000.5	−371 577.581 61	75 271.143 15	−32 227.946 18	0
(12 apr. 1938,0h)	−371 577.580 19	75 271.146 65	−32 227.946 80	$5.\,10^{-5}$
2 409 000.5	−373 896.158 93	127 406.791 29	−30 037.792 25	0
(9 jul. 1883,0h)	−373 896.155 45	127 406.791 53	−30 037.792 89	$5.\,10^{-5}$
2 389 000.5	−346 331.773 61	206 365.403 64	−28 502.117 32	0
(5 oct. 1828,0h)	−346 331.778 62	206 365.403 82	−28 502.117 73	$5.\,10^{-5}$

The inputs are:

• The Julian TDB date: TJJ

• The truncation level of the series in radian: PREC (If PREC> 0, the coefficients whose magnitude is smaller than PREC radians for longitude and latitude, and $a_0\times$PREC km for distance are disregarded(*); if PREC= 0, all the terms are kept)

• A number of logical unit for reading the files: NULOG (for example NULOG=3).

The outputs are:

(*) If PREC> $0''.01$ is sufficient, the user will find files and subroutines for computing different kinds of lunar coordinates in (Chapront-Touzé and Chapront, 1991) (see sect. 10)

• The table of rectangular coordinates in km referred to the inertial mean ecliptic and equinox of J2000: R(3)

• An error index: IERR (IERR=3 if one of the files ELP1 to ELP36 is not corrrectly read, else IERR=0).

Check values are given in table H.

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