Uncertainty Quantification for an Eckstein-Keane-Wolpin model with correlated input parameters

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Abbreviations

[see UQ book , center table, two horizontal lines to the top and at the bottom...]

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1 Introduction

Structure: Need for UQ (incl. SA) - GSA - Importance measures - Dependent inputs - Goal of the thesis - Structure en passant

[*Uncertainty quantification* (UQ) studies the components of a mathematical model that might contribute to its discrepancy from the real world. Modeling choices that cause such a discrepancy may be made because aspects of the real world are unknown to the modeler or because he intentionally chooses to simplify the model. Reasons for the latter can, for instance, be an emphasis on specific aspects of reality, algebraic ease, computation time, or numeric. UQ is a toolbox of methods that can help a researcher to reflect upon potential deficiencies of his model and to avoid the risk of overconfidence in his forecasts. Therefore, UQ is not only useful for improving the model but also crucial for providing differentiated, realistic forecasts, and well-founded thinking about useful policies.

In Economics, but of course also in other quantitative disciplines, one major challenge for using models to understand and quantify real-world phenomena, mechanisms, and effects is that model input parameters are often not well-known. In this case, usually, only some parameters that can describe a (joint) probability distribution of the model input parameters are available to the researcher. Consequently, the model outputs or *quantities of interest* (QoIs) inherit this lack of knowledge from the input parameters. Put differently, the uncertainty in the input parameters is propagated through the model towards the QoIs. This gives rise to important questions like "Given the uncertainty of the input parameters, what is the probability distribution of the model outputs?". For instance, a model might predict an interesting outcome given the means of the parameter estimates. However, there might be a considerable probability that the model predicts an entirely different outcome. Of course, such findings should be reported if possible. Other important questions are "To what extent does the uncertainty of one or more specific input parameters contribute to the uncertainty of some QoI?" and "In which direction does each parameter affect the QoI globally?". These questions aim to inspect the influence of each parameter's or parameter group's uncertainty on the uncertainty of the respective QoI. They are also posed to investigate what effect higher-probability parameter values besides the usual measures of location can have on the QoI. Answers to these questions can then be used to reduce the model uncertainty in two ways: On the one hand, by devoting additional research to important input parameters that have a large influence on the QoI and its uncertainty and, on the other hand, they allow making informed decisions on whether or not to consider the uncertainty of less important input parameters or to even simplify their representation in the model. These questions are a subset of what is covered by the discipline of UQ. The field provides the suitable methods to answer the above questions profoundly. Their importance implies that these methods should be an essential part of every quantitative, model-based research and practice.

The goal of this thesis is to answer the above questions for an important and well-known economic model, thereby providing a showcase for the application of uncertainty quantification to Economics.

In section 2, I outline the general discipline of uncertainty quantification and present the subfields on Parameter Uncertainty that are featured in this thesis. These subfields are uncertainty propagation, *global sensitivity analysis* (GSA) and surrogate models. Uncertainty Propagation is the construction of probability distribution functions for QoIs by propagating the model input uncertainty. GSA contains measures that indicate how much of the QoIs uncertainty can be attributed to specific parameters and parameter groups. The thesis employs

' indices and univariate effects. uncertainty propagation and GSA can be computed in different ways. Besides Monte Carlo and quasi-Monte Carlo methods, one can also use surrogate models. A surrogate model substitutes the computation of a QoI via a model by approximating the QoI with one explicit, algebraic function of the model input parameters. They are used to save computation time if the evaluation of the original model is computationally expensive. This thesis uses both (quasi-)Monte Carlo methods and surrogate models to compute each result.

Section 3 gives an overview of the economic literature that uses UQ methods. It shows that, typically, economic research does not address the questions mentioned above, as the respective literature is small. Exceptions are Harenberg et al. (2019) [Name other authors].

Section 4 presents the model for which the UQ is implemented. It is the well-known Dynamic Discrete Occupational Choice model developed by Keane and Wolpin (1994) in the field of labour economics. The QoI is the effect of a $2000 subsidy on occupation choices. This model output is chosen because the result is of high relevance for policy-makers and thus well-suited to illustrate the benefits of performing a structured UQ in quantitative studies of economic models that may have important real-world implications.

In Section 5, the results for the uncertainty propagation and global sensitivity analysis using Monte Carlo and quasi Monte Carlo methods are presented. [Add one sentence for the results.]

Section 6 shows the results for the same measures but computed by using a surrogate model. [Add one sentence for the results.]

Section 7 compares the two approaches and discusses the results. [Add two to three sentences for the results.]

Section 8 offers conclusions and indicates directions for future research.]

2 Uncertainty Quantification Framework

[intrusive vs. non-intrusive.]

mention European guidelines.

2.1 Uncertainty Propagation

2.2 Global Sensitivity Analysis

2.2.1 Sobol' Indices

$$S_i = \frac{\mathrm{Var}_i[Y|X_i]}{\mathrm{Var}[Y]} \tag{1}$$

$$S_i = \frac{\mathrm{Var}_i[\mathbb{E}_{\sim i}[Y|X_i]]}{\mathrm{Var}[Y]} \tag{2}$$

$$S_{ij} = \frac{\mathrm{Var}_{ij}[\mathbb{E}_{\sim\{i,j\}}[Y|X_i, X_j]]}{\mathrm{Var}[Y]} - S_i - S_j \tag{3}$$

$$S_{\mathrm{u}} = \frac{\mathrm{Var}_{\mathrm{u}}[\mathbb{E}_{\sim \mathrm{u}}[Y|X_{\mathrm{u}}]]}{\mathrm{Var}[Y]} - \sum_{\mathrm{w} \subset \mathrm{u}} S_{\mathrm{w}} \tag{4}$$

$$S_i^{\mathrm{T}} = \sum_{i \in \mathrm{u}} S_{\mathrm{u}} \tag{5}$$

$$\mathrm{Var}[Y] = \mathrm{Var}_i[\underset{\sim i}{\mathbb{E}}[Y|X_i]] + \underset{i}{\mathbb{E}}[\mathrm{Var}_{\sim i}[Y|X_i]] \tag{6}$$

$$1 = \frac{\mathrm{Var}_i[\mathbb{E}_{\sim i}[Y|X_i]]}{\mathrm{Var}[Y]} + \frac{\mathbb{E}_i[Var_{\sim i}[Y|X_i]]}{\mathrm{Var}[Y]} \tag{7}$$

$$1 = S_i + S_{\sim i}^T \tag{8}$$

$$S_{\sim i}^T = \frac{\mathbb{E}_i[\mathrm{Var}_{\sim i}[Y|X_i]]}{\mathrm{Var}[Y]} \tag{9}$$

$$S_i^T = \frac{\mathbb{E}_{\sim i}[\mathrm{Var}_i[Y|X_{\sim i}]]}{\mathrm{Var}[Y]} \tag{10}$$

$$S_{\mathrm{u}}^{clo} = \frac{\mathrm{Var}_{\mathrm{u}}[\mathbb{E}_{\sim \mathrm{u}}[Y|X_{\mathrm{u}}]]}{\mathrm{Var}[Y]} \tag{11}$$

$$Y = \mathcal{M}(\mathrm{x}) = \mathcal{M}_0 + \sum_{i=1}^{M} \mathcal{M}_i(x_i) + \sum_{1 \leq i \leq j \leq M} \mathcal{M}_{ij}(x_i, x_j) + ... + \mathcal{M}_{12..M}(\mathrm{x}) \tag{12}$$

$$S_i = \frac{\mathrm{Cov}[\mathcal{M}_i(x_i), Y]}{\mathrm{Var}[Y]} \tag{13}$$

$$S_i = \frac{\mathrm{Var}[\mathcal{M}_i(x_i)]}{\mathrm{Var}[Y]} + \frac{\mathrm{Cov}[\mathcal{M}_i(x_i)]}{\mathrm{Var}[Y]} \tag{14}$$

$$S_i = \frac{\mathrm{Var}_i[\mathcal{M}_i(x_i)]}{\mathrm{Var}[Y]} \tag{15}$$

$$S_{ij} = \frac{\mathrm{Var}_{ij}[\mathcal{M}_{ij}(x_i, x_j)]}{\mathrm{Var}[Y]} \tag{16}$$

$$\mathrm{Var}[Y] = \sum_{i=1}^{M} \mathrm{Var}[\mathcal{M}_i(x_i)] + \sum_{1 \leq i \leq j \leq M} \mathrm{Var}[\mathcal{M}_{ij}(x_i, x_j)] + ... + \mathrm{Var}[\mathcal{M}_{12..M}(\mathrm{x})] \tag{17}$$

$$\begin{split} S_i^T = S_i + \sum_{j \neq i} S_{ij} + \sum_{1 \leq i \leq j \leq M, \{j,k\} \neq i} S_{ijk} + ... = \sum_{i \in \mathrm{w}} S_{\mathrm{w}} = \\ \frac{1}{\mathrm{Var}[Y]} \sum_{i \in \mathrm{w}} \mathrm{Var}_i[\mathcal{M}_{\mathrm{w}}(x_{\mathrm{w}})] \end{split} \tag{18}$$

$$S_{\mathrm{u}}^{clo} = \frac{\mathrm{Var}_{\mathrm{u}}[\mathcal{M}_{\mathrm{u}}(\mathrm{x}_{\mathrm{u}})]}{\mathrm{Var}[Y]} + \sum_{\mathrm{w} \subseteq \mathrm{u}} \frac{\mathrm{Var}_{\mathrm{w}}[\mathcal{M}_{\mathrm{w}}(\mathrm{x}_{\mathrm{w}})]}{\mathrm{Var}[Y]} \tag{19}$$

2.2.2 Univariate Effects

2.3 Surrogate Models and Spectral Expansions

[Scheidegger: Also called Interpolator in the literature]

[Univariate Effects as a measure for comparative statics]

[Philipp: Please add a plot to your thesis (not our notebook) that implements the idea of the uncertainty cone in Figure 1. 2 in our textbook. For example, Figure 1 from KW97 could use such a cone for hte out of support predictions in the occupational shares.]

3 Uncertainty Quantification in the Economic Literature

The need for UQ as an essential part of quantitative economic studies has long been recognized in the economics profession.[1] Also GSA in particular has had strong advocates.[2] However, the demanded evolution of research practice has only been met by a few publications until today. This literature review summarizes these publications with regards to the UQ subfields that are emphasized in the prior section. These are uncertainty propagation and GSA. Table 1 gives an overview of the major measures, methods and topics in the literature. I find 14 contributions that meet the described criteria. Arguably, because UQ is more accomplished in climatology, a large share of research comes from climate economics. The first publications tend to use the conceptually simple Monte Carlo uncertainty propagation. The majority of papers use surrogate models to save computation time. The later contributions focus on GSA. Harenberg et al. (2019) gives a well-argued explanation about why GSAs are better than LSAs. The recent works use more sophisticated methods like polynomial chaos expansions (as first applied in Harenberg et al. (2019)) or intrusive approaches (see, for instance, Scheidegger and Bilionis (2019)). This section concludes by explaining the choice of measures and methods made in this thesis and by comparing them to those used in the literature.

Table 1. Overview of UQ literature by keywords

Content	Number of articles
Climate economics	8
Uncertainty propagation	4
Sobol' indices	6
Univariate effects	4
Monte Carlo sampling	5
Surrogate models	8
Polynomial chaos expansions	2
Intrusive methods	2
	14

Harrison and Vinod (1992) suggest to use uncertainty propagation via Monte Carlo sampling for applied general equilibrium modeling to inspect the uncertainty in model inputs. As a showcase, they propagate the distributions of 48 elasticities through a taxation model by drawing 15,000 input parameter vectors. They analyse their results graphically, using a histogram for their QoI as well as confidence intervals for its mean. For further use, N denotes the size of a Monte Carlo sample.

[1]See Hansen and Heckman (1996), Kydland (1992) and Canova (1994), amongst others.
[2]See Canova (1995) and Gregory and Smith (1995).

Canova (1994) proposes to perform a Monte Carlo uncertainty propagation to reflect upon the calibration of dynamic general equilibrium models. The author also addresses challenges and methods for parameter calibration. Canova illustrates his approach by plotting distributions and computing moments and prediction intervals for QoIs in an asset-pricing (N=10,000) and a real business cycle model (N=1,000). Moreover, he analyzes the QoIs' sensitivity towards the uncertainty of individual input parameters by propagating different specifications of input distributions.

More recent examples for Monte Carlo uncertainty propagation concern investigate climate models, such as Webster et al. (2012). Examples using Latin hypercube sampling are Mattoo et al. (2009) and Hope (2006).

Recently, Harenberg et al. (2019) compare measures from LSA to measures from GSA for multiple QoIs of the canonical, macroeconomic real business cycle model. Thereby, they provide a context for GSA within UQ. The computed sensitivity measures are Sobol' indices and univariate effects. They are obtained by polynomial chaos expansions. For this purpose, Harenberg et al. introduce the leave-one-out error estimator (see page XX) as a measure to select an orthogonal polynomial as the surrogate model. The authors come to the following conclusion: On the one hand, a LSA can easily be misleading because its perspective is not broad enough. In particular, they criticise the one-at-a-time approach on which LSAs rely. One-at-a-time methods base on changing one uncertain parameter while keeping the others constant. The choice of parameter combinations tends to be arbitrary. These methods are typically used in economics. The authors conclude that LSA is neither adequate for identifying the inputs that drive the uncertainty, nor does it allow to analyse interactions. On the other hand, a GSA can provide profound insights, and polynomial chaos expansions are a fast way to compute approximations for the respective global sensitivity measures.

Ratto (2008) presents global sensitivity measures for multiple variants of DSGE models computed by Monte Carlo methods and surrogate models. The first measure bases on the Smirnov test (see, e.g., Hornberger and Spear (1981)): The QoI range is partitioned into a desired set S, and an undesired set $\overline{S}$. Then a Monte Carlo sample of parameter vectors from the input distribution is propagated through the model. From the QoI realizations for each set, two cumulative distribution functions for each input parameter, one conditioned on QoI realizations in set S, and the other conditioned on realizations in set $\overline{S}$, are generated. For each input independently, it is tested whether the distributions differ. If they do, the parameters and their specific regions that lead to the undesired QoI realizations can directly be identified. The second measure is first-order Sobol' indices. Ratto computes them by employing two different surrogate models. The first surrogate is obtained by state-dependent regression. The idea is to regress the QoI on (combinations of) input parameters. The second surrogate is a polynomial representation of the first one. The author finds that the surrogates provide a good fit for the Monte Carlo sample except for the distribution tails. The fit varies conditional on different input parameters. Ratto compares his results for the first-order Sobol' indices computed by both surrogates. The results show some differences in size but not in the ranking.

Saltelli and D'Hombres (2010) criticise the arbitrary input value choices in the sensitivity analysis design of the influential Stern (2007) report about the consequences of climate change. Particularly, Stern argue that their cost-benefit analysis' results about the economic impact of climate change are robust towards the uncertainty in their input parameters. Yet, Saltelli and D'Hombres (2010) contradict Stern's assertion by presenting a more thorough sensitivity analysis with parameter choices that better represent the original input distribution.

A series of papers (Anderson et al. (2014), Butler et al. (2014), Miftakhova (2018)) conducts sensitivity analyses for the Dynamic Integrated Climate-Economy (DICE) model in Nordhaus (2008). Each work concludes that a GSA is superior to a LSA for the same reasons as Harenberg et al. (2019). Furthermore, all contributions find that leaving some hypothetically low-impact parameters out of the sensitivity analyses lead Nordhaus to neglect the uncertainty in important parameters.

Anderson et al. (2014) use Sobol' Indices, the δ-sensitivity measure, and correlation measures for paired QoIs in their GSA. The δ-sensitivity measure (see, e.g., Borgonovo (2006)) is given by half the expectation value of the absolute difference between the unconditional distribution of a QoI and the QoI distribution conditioned on one specific, fixed input (group). Estimates for these measures are computed with the algorithm used in Plischke et al. (2013) applied to a Monte-Carlo sample (N=10,000). In Anderson et al. (2014), the δ-sensitivity measure is the main measure of sensitivity and used to rank the parameters in terms of their contributions to the model uncertainty. The authors also use a surrogate model obtained through Cut-HDMR (Cut-High Dimensional Model Representation; see, e.g., Ziehn and Tomlin (2009)) for graphical analyses of the interaction between input parameters.

Butler et al. (2014) also generate importance rankings for the uncertainty in input parameters. However, they use first, second and total order Sobol' indices instead of the δ-sensitivity measure. They compute the Sobol' indices based on Sobol' sequences (Sobol' (1967)) for the results and based on Latin Hypercube sampling (McKay et al. (1979)) as a check. The results in Butler et al. (2014) and Anderson et al. (2014) can not be compared as they analyse different QoIs.

Gillingham et al. (2015) conduct an UQ for six major climate models. They select three input parameters that are present in each model. The authors generate a surrogate model from regressing several model outputs separately on a linear-quadratic-interaction specification of the three input parameters on 250 grid points. Then they draw 1,000,000 parameter vectors from the probability density function of the input parameters and evaluate the sample with the surrogate model. They find that the parametric uncertainty contributes to more than 90% whereas the differences in the six models contribute to less than 10% of the QoI variances for the year 2100. They also present QoI values for multiple percentiles of each input parameter.

Miftakhova (2018) applies the GSA procedure outlined by Harenberg et al. (2019). The importance ranking that she obtains from the polynomial-chaos-expansions-based Sobol' indices is different from the ranking that Anderson et al. (2014) obtain from the δ-sensitivity measure. Yet, this is not mentioned by Miftakhova.[3] However, the author emphasizes that the standard procedure for obtaining Sobol' indices from a variance decomposition as used by Anderson et al. (2014) and Butler et al. (2014) is not feasible for the DICE model because a set of input parameters is calibrated jointly in order to let the model match some observables. Therefore, although these input parameters are not correlated in the classical sense, they are dependent. Hence, the variance-based Sobol decomposition is not applicable because the summands are not orthogonal to each other or, in other words, the input-specific variance terms contain a covariance component. Thus, they do not add to the total model variance. Miftakhova (2018) shows how the set of dependent input parameters can be changed to a set of independent parameters by changing the model structure: She includes uncertain observables as independent parameters and reformulates dependent input parameters as endogenous variables. These endogenous variables are functions of the remaining, formerly dependent parameters and the new input parameters.[4]

Most recently, Scheidegger and Bilionis (2019) made a noteworthy contribution that naturally connects the solution process of economic models to UQ with surrogate models. The difference to the prior contributions is that their method is intrusive instead of non-intrusive (see page XX). In particular, they conduct an uncertainty propagation and compute univariate effects. Scheidegger and Bilionis' approach is to solve very-high-dimensional dynamic programming problems by approximating and interpolating the value function with a combination of the active subspace method (see, e.g., Constantine (2015)) and Gaussian process regression (see, for example, Rasmussen and Williams (2005)) within each iteration of the value function iteration algorithm. The authors can apply their method up to a 500-dimensional stochastic growth model. Therefore, they can solve models that contain substantial parameter heterogeneity. The link to UQ is that one can also "directly solve for all possible steady state policies as a function of the economic states and parameters in a single computation" (Scheidegger and Bilionis, 2019, p. 4) from the Value function interpolant. In other words, this step yields the QoI expressed by a surrogate model. Thus, to add an UQ, one has to, first, specify the uncertain parameters as continuous state variables, and second, assign a probability distribution to each of these parameters. Then (assuming the uncertain input parameters are independent), one provides a sample from each parameter's distribution as input to the Gaussian process regression to obtain a surrogate model. Following these steps, QoIs can be expressed as functions of the uncertain input parameters without much additional effort. Finally, by

[3] I do not have access to the numerical codes. Thus the reasons for the discrepancies remain unclear.

[4] For a discussion of more general methods to compute Sobol' indices in the presence of dependent input parameters see, e.g., Chastaing et al. (2015) and Wiederkehr (2018), with references therein.

using a processed value function interpolant as a surrogate model, Scheidegger and Bilionis propagate the model uncertainty and depict univariate effects.

Building on the contributions by Harenberg et al. (2019) and Scheidegger and Bilionis (2019), Usui (2019) conducts a GSA based on Sobol' indices and univariate effects to study rare natural disasters in a dynamic stochastic economy. Because the repeated model evaluations required to construct an adequate surrogate model are too computationally expensive, they choose to apply a method similar to Scheidegger and Bilionis' intrusive framework. However different to Scheidegger and Bilionis (2019), they generate numerical approximates for their policy functions by time iteration collocation (see, e.g., Judd (1998)) with adaptive sparse grid (see Scheidegger et al. (2018)) instead of Gaussian machine-learning.

This thesis uses uncertainty propagation to obtain the probability distribution of a QoI given the total parameter uncertainty. This allows to compute simple descriptive statistics and to use visualizations for analysing the distribution's skewness and kurtosis.

[Consider Canidou.2012 presentation for delta vs Sobol measures]

[Change for dependent input parameters: Additionally, the thesis follows a global instead of a local level of sensitivity analysis for the reasons explained in Harenberg et al. (2019). It does not compute the Smirnov-test-based measure in Ratto (2008) because I want to make more general statements about the uncertainty of a QoI rather than to focus on two specific partitions of the QoI's range. I also prefer Sobol' indices over the δ-sensitivity measure used in Anderson et al. (2014) because of two reasons: First, it is straightforward to compute Sobol' indices because the input parameters in the analysed model are assumed to be independent. This assumption is discussed critically at a later stage. Second, Sobol' indices are easy to interpret because they are scaled by the variance of the model given its total parametric uncertainty. As a second measure, the thesis computes univariate effects to show the relationship between a QoI and one input parameter over the whole parameter range. The two analysis parts are conducted based on both Monte Carlo sampling and polynomial chaos expansions. Therefore, these methods can be compared for a moderately large computational model with 26 uncertain input parameters. On the one hand, the slow convergence and high computation time of the Monte Carlo method is compared with the potential imprecision of the model approximation by orthogonal polynomials and the numerical methods involved. On the other hand, the reversed opposite properties are what makes these methods appealing to use. Especially attractive is the elegant derivation of Sobol' indices and univariate effects from orthogonal polynomials.]

So far, UQ has exclusively been applied to climate or macroeconomic models. This thesis is the first UQ for a labour economic model. Additionally, it is the first contribution that computes sensitivity indices for dependent input parameters. Therefore, the contributions of this thesis are, first, setting an example for best practices using UQ in microeconomic research and, second, providing an example for computing sensitivity indices in a more realistic model setup.

4 Model and Estimation

This section introduces the economic model whose uncertainty is quantified. It is the partial equilibrium, dynamic model of occupational choice developed in Keane and Wolpin (1994) (henceforth KW94). In their survey of dynamic discrete choice strucutural models, Aguirregabiria and Mira (2010) assign this model to the more general class of Eckstein-Keane-Wolpin models. I largely follow their notation to ease comparisons with other models and, most importantly, to ease the explanation of the estimation method. Besides applications to labour economics, Eckstein-Keane-Wolpin models are used to explain educational and occupational choices at the individual level. The model class is structural. This means that, from the perspective of an econometrician, the model structure allows for the estimation of relationships between observable and unobservable state variables. These relationships are governed by exogenous parameters. These parameters may, for example, be utility parameters or distributional parameters which describe the processes of unobserved shocks. Therefore, the exogenous parameters can be estimated given a dataset of observable endogenous variables. Besides the observable states, the observable endogenous variables may also comprise of other parameters like, for instance, payoffs. Estimates for the exogenous parameters allow to use simulations (of states) in order to analyse counterfactual policy scenarios. These policies are represented by changes in some exogenous parameters. For example, Keane and Wolpin (1997) obtain the following two results based on data from the NLSY79: First, unobserved heterogeneity in the endowment at age sixteen accounts for almost 90% of the variance in lifetime utility whereas 10% is explained by shocks to productivity. And second, a college tuition subsidy of 2,000 USD increases high school and college graduation by 3.5% and 8.4%, respectively. As the research code for Keane and Wolpin (1997) is currently in alpha-version, this thesis studies the predecessor model in KW94. The main differences are that the model in KW94 does not contain unobserved permanent agent heterogeneity and that its choice-specific utility functions feature less covariates. This difference in complexity implies a decrease of the computational burden for the UQ but also a worse fit to the data. In fact, this thesis does not use estimates from real data but estimates from data simulated on arbitrary parameters choices that are taken from KW94.

The section proceeds as follows: First, I introduce the KW94 model specification embedded in the more general Eckstein-Keane-Wolpin framework. In the next step, the estimation method simulated maximum likelihood is presented. This approach is used for the structural estimation of the exogenous model parameters. After remarks on the numerical implementation, I show the estimation results. These include the estimates, the standard errors and the correlations for all parameters. These results constitute the mean vector and the covariance matrix that are used to characterize the joint input distribution for the UQ in the next section. The section ends by describing the QoI choice.

4.1 Keane and Wolpin (1994)

Aguirregabiria and Mira (2010) define Eckstein-Keane-Wolpin models by four characteristics. The first characteristic is that these models allow for permanent unobserved heterogeneity between agents. The simpler model by KW94 considered here does not use this option in contrast to Keane and Wolpin (1997). The other three characteristics are as follows:

1. Unobservable shocks ε_t do not have to be additively seperable from the remainder of the utility functions.

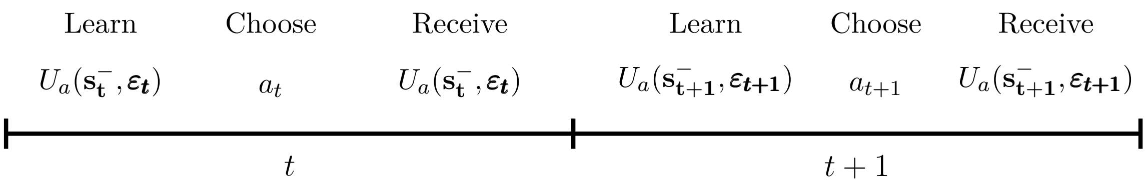
2. Shocks ε_t can be correlated across choices a_t.

3. Observable payoffs, or wages, $W^-_{a,t}$ are not conditionally independent on the unobservable shocks ε_t given the observable choices a_t and the observable part of the state vector $\mathbf{s_t^-}$. The reason is that wage shocks enter the wage function directly. This can be observed by the agent prior to his decision. This decision can then lead to the non-observation of alternative-specific wages.

This paragraph describes the Eckstein-Keane-Wolpin model framework without permanent agent heterogeneity as in KW94 in the context of occupational choices. In this setting, agents only differ in their draws of unobserved shocks ε_t.

A representative agent decides for action, or occupation, a_t from the set of alternatives A in each time period t. These alternatives are mutually exclusive. From each decision, agents obtain the alternative-specific utility U_a. Notation U_a indicates that the utility depends on occupation choice a_t. In each time period, Utilities U_a are subject to random shocks $\varepsilon_{a,t}$ which are also alternative-specific. For some occupation alternatives, utility and prior decisions may be intertemporally connected: Agents receive a higher utility if they accumulated skills in past occupations that are useful for these alternatives. Other occupations may not reward experience. $\boldsymbol{S_t}$ denotes the state space. The state space is the set of information in each period t relevant for the present and future utilities for each occupation choice a_t. The observable part of the state space comprises the time period, the work experience and the choice in the previous period. It is denoted by vector $\boldsymbol{s_t^-}$. The unobservable part of the state space is denoted by vector $\boldsymbol{\varepsilon_t}$ and consists of the alternative-specific shocks $\boldsymbol{\varepsilon_{a,t}}$. The sequence of events is depicted in Figure 1.

Figure 1. Sequence of events



At the beginning of each period t, the agent recognizes the reward shocks $\boldsymbol{\varepsilon_{a,t}}$ (as opposed to the observer), and the shocks become part of the unobserved state space $\boldsymbol{\varepsilon_t}$. Thus, the utilities $U_a(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t})$ are known to the agent in period t. However, he can only form expectations about rewards in the future as the alternative-specific shocks $\boldsymbol{\varepsilon_{a,t}}$ are stochastic. The specification in KW94 assumes the rewards shocks $\boldsymbol{\varepsilon_{a,t}}$ to be serially uncorrelated. Therefore, prior shocks do not enter the state space. Next, the agent chooses his occupation a_t based on the state space information. Then he receives the occupation-specific reward U_a. This flow repeats for each $t < T$.

Agents are rational and forward-looking. Future utilities are subject to time discount factor $\delta \in [0, 1]$. Hence, they choose their optimal sequence of occupations by maximizing the remaining expected, discounted life-time utility. This maximal value is given by value function $V(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t})$.

$$V(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) = \max_{\{a\}_{t=0}^{T}} \left\{ \sum_{t=0}^{T} \delta^t \int_{\boldsymbol{\varepsilon_t}} U(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}, a_t) f(\boldsymbol{\varepsilon_t}) d^{|A|} \boldsymbol{\varepsilon_t} \right\} \tag{20}$$

Value V depends directly on time t because T is finite. Together with the discount factor δ, this typically induces life-cycle behaviour. For example, agents invest more in the earlier time periods and work (and consume) more in the following periods. As $\boldsymbol{\varepsilon_{a,t}}$ are the only random parameters and serially independent, the expectation of $U(\boldsymbol{s_t^-}, \boldsymbol{\varepsilon_t}, a_t)$ is given by the $|A|$-dimensional integral of U multiplied by the joint probability density function $f(\boldsymbol{\varepsilon_t})$ with respect to $\boldsymbol{\varepsilon_t}$. $|A|$ denotes the number of occupation choices.

Coursely sketched, the approach to solve the above maximization problem is given by the dynamic programming problem characterized by the Bellman equation (Bellman (1957)).[5]

$$V(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) = \max_{a_t} \left\{ U(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}, a_t) + \delta \int_{\boldsymbol{\varepsilon_t}} \max_{a_{t+1}} V_{a_{t+1}}(\mathbf{s_{t+1}^-}, \boldsymbol{\varepsilon_{t+1}}) f(\boldsymbol{\varepsilon_{t+1}}) d^{|A|} \boldsymbol{\varepsilon_{t+1}} \right\} \tag{21}$$

The Bellman equation states, that solving for the whole sequence of policy functions $\{a^*\}_{t=0}^{T}$ is equivalent to solving iteratively for each optimal, period-specific policy function $a_t^*(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t})$. For this purpose, choose a_t for each period such that the current period utility and the discounted expected future lifetime utility (given the optimal choice of a_{t+1}) are maximized. The finite time horizon eases the problem as the value function for the last period T simplifies to $V(\mathbf{s_T^-}, \boldsymbol{\varepsilon_T}) = \max_{a_T} U(\mathbf{s_T^-}, \boldsymbol{\varepsilon_T}, a_T)$. With this condition the problem can be solved for all states by iterating backwards. Given initial states and random draws for the unobservable shocks $\boldsymbol{\varepsilon_t}$ for each period, these policy equations are used to simulate the occupational paths for a number of agents.

This paragraph addresses the alternative-specific utility functions $U_a(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t})$ that finally pin down the model structure in KW94. There are four different occupations, b, w, e and

[5] For more details, see Raabe (2019), p. 9-19.

h, of which occupations b and w are defined by the same type of utility function. In the following, I will roughly explain how the first two utility functions model characteristics for working in the blue and the white collar sector and how the latter two equations sketch receiving institutional education and staying at home. The parametrization that distinguishes the blue from the white collar sector and additional intuition is given later in subsection Estimation Results and Table 2. It is assumed that there is a direct mapping from USD to utility. Based on this, the utility functions for occupation b and w, U_b and U_w, equal the occupation-specific wage, $W_{b,t}$ and $W_{w,t}$, in USD. The wage equations are given by the Mincer equation for earnings (Mincer (1958)):

$$\begin{aligned} U_b(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) &= W_{b,t}^- = \exp\Big\{\beta^b + \beta_e^b x_{e,t} + \beta_b^b x_{b,t} + \beta_{bb}^b x_{b,t}^2 + \beta_w^b x_{w,t} + \beta_{ww}^b x_{w,t}^2 + \varepsilon_{b,t}\Big\} \\ U_w(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) &= W_{w,t}^- = \exp\Big\{\beta^w + \beta_e^w x_{e,t} + \beta_w^w x_{w,t} + \beta_{ww}^w x_{w,t}^2 + \beta_b^w x_{b,t} + \beta_{bb}^w x_{b,t}^2 + \varepsilon_{w,t}\Big\} \end{aligned} \tag{22}$$

Both equations comprise of a constant term, years of schooling $x_{e,t}$, linear and quadratic terms of occupation experience, and cross-occupational experience and the respective shocks in $\boldsymbol{\varepsilon_{a,t}}$. $\boldsymbol{\beta}$ is the vector of coefficients that multiply the previously defined terms.[6] These coefficieints are called covariates by many structural economists.

The utilities for education, or schooling, and staying at home are given by the following functions in (23). These functions are also called non-pecuniary rewards.

$$\begin{aligned} U_e(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) &= \beta^e + \beta_{he}^e \mathbf{1}(x_{e,t} \geq 12) + \beta_{re}^e (1 - \mathbf{1}(a_{t-1} = e)) + \varepsilon_{e,t} \\ U_h(\mathbf{s_t^-}, \boldsymbol{\varepsilon_t}) &= \beta^h + \varepsilon_{h,t} \end{aligned} \tag{23}$$

β^e is the consumption reward of schooling. Function $\mathbf{1}(x_{e,t} \geq 12)$ indicates whether an agent has completed high school. β_{he}^e is the tuition fee on higher or post-secondary education and β_{re}^e is an adjustment cost for returning to school when the agent chose another occupation the previous period ($a_{t-1} \neq e$). β^h is the mean reward for staying at home.

It is assumed that $\boldsymbol{\varepsilon_{a,t}}$ follows a joint normal distribution, such that $\boldsymbol{\varepsilon_{a,t}} \sim \mathcal{N}(0, \boldsymbol{\Sigma_\varepsilon})$. $\boldsymbol{\Sigma_\varepsilon}$ denotes the covariance matrix for shocks $\boldsymbol{\varepsilon_{a,t}}$. σ_a^2 and $\sigma_{a(j),a(k\neq j)}^2$ denote the alternative-specific variances and covariances in $\boldsymbol{\Sigma_\varepsilon}$. Shocks are serially uncorrelated. Indices j and k are used to denote subsets of a.

Finally, there is a bijective mapping from time periods t to age 16 to 65. The next subsection describes the estimation method.

[6] The notation for $\boldsymbol{\beta}$ includes two references. The superscript indicates the occupation-specific utility that contains the coefficients. The subscript indicates the occupation-specific experience or abbreviates the condition that regulates the coefficients. Thus, coefficients for constant terms do not have a subscript. Coefficients for quadratic terms are marked by twice the respective subscript.

4.2 Simulated Maximum Likelihood Estimation

To estimate the exogenous model parameters, the approach that this thesis and also KW94 uses is the simulated maximum likelihood method (Albright et al. (1977))[7].

This method can be applied to a set of longitudinal data on occupational choices a_t and, if available, wages $W^-_{a,t}$ of a sample of $i \in I$ individuals starting from age 16. To distinguish from its functional form, let $\mathcal{W}^-_{a(k),t}$ henceforth denote the measured wages. For each period t, the recorded choices a_0, ..., a_{t-1} imply the occupation-specific experiences $x_{a,t}$. Together with t, they constitute the observable state vector $\mathbf{s^-_t}$. Consequently, the measured, observable endogenous variables are $\mathbf{m} \stackrel{\text{def}}{=} (\mathbf{s^-_t}, \mathcal{W}^-_{a,t})$. Given this setup, the goal is to estimate the exogenous model parameters $\boldsymbol{\theta} = (\delta, \boldsymbol{\beta}, \boldsymbol{\Sigma_\varepsilon})$.[8] Thus, in the following, every probability is a function of the exogenous model parameters $\boldsymbol{\theta}$. The approach to compute the likelihood function $L_{\mathbf{m}}(\boldsymbol{\theta})$ of the observables in the data begins with the individual latent variable representation in period t.

$$a_t = \underset{a}{\operatorname{argmax}} \, V_a(\mathbf{s^-_t}, \boldsymbol{\varepsilon_t}) \tag{24}$$

As a_t and $\mathbf{s^-_t}$ are known, the next step is to derive the unobservable shocks $\boldsymbol{\varepsilon_t}$ in terms of a_t and $\mathbf{s^-_t}$. Therefore, write the set of shocks for which the alternative-specific value function $V_{a(j)}$ is higher than the other value functions $V_{a(k \neq j)}$ in time t as

$$\boldsymbol{\varepsilon_t}(a_t(j), \mathbf{s^-_t}) \stackrel{\text{def}}{=} \{\boldsymbol{\varepsilon_t} | V_{a_t(j)}(\mathbf{s^-_t}, \boldsymbol{\varepsilon_t}) = \max_a V_a(\mathbf{s^-_t}, \boldsymbol{\varepsilon_t})\}). \tag{25}$$

Note that the set condition is a function of the unobservable model parameters $\boldsymbol{\theta}$.

Consider first the case of non-working alternatives $a_t(j) \in [e, h]$. The probability of choosing $a_t(j)$ is the probability of set $\boldsymbol{\varepsilon_t}(a_t(j), \mathbf{s^-_t})$. This probability equals the integral of the probability distribution function $f(\boldsymbol{\varepsilon_t})$ over all elements of set $\boldsymbol{\varepsilon_t}(a_t(j), \mathbf{s^-_t})$ with respect to $\boldsymbol{\varepsilon_t}$. Formally,

$$\mathrm{p}\Big(a_t(j) | \mathbf{s^-_t}\Big) = \int_{\boldsymbol{\varepsilon_t}(a_t(j), \mathbf{s^-_t})} f(\boldsymbol{\varepsilon_t}) d^{|A|} \boldsymbol{\varepsilon_t}. \tag{26}$$

The second case is $a_t(k) \in [b, w]$. Assuming the dataset contains wages for the working alternatives $a_t(k)$, the probabilities of choosing $a_t(k)$ take a few steps more to compute. In the first step, note from the wage equations that the the alternative-specific shocks $\boldsymbol{\varepsilon_{a,t}}$ are log normally distributed. Second, in contrary to the non-working alternatives, using (22), the shocks can directly be expressed as a function of the alternative-specific model parameters $\boldsymbol{\beta_{a(k)}}$ by inserting the inferred alternative-specific experiences $\boldsymbol{x_{a(k),t}}$ into $W_{a(k),t}$ and subtracting the expression from the observed wage $\mathcal{W}^-_{a(k),t}$ for each individual. Both

[7] see Aguirregabiria and Mira (2010), p. 42-44 and Raabe (2019), p. 21-26 for more details.

[8] Improvements in this thesis' estimation over KW94 are that, first, it is not assumed that the standard errors of the parameters estimates are uncorrelated, and second, that β is not left out of the estimation.

wages are logarithmized. Thus,

$$\varepsilon_{a(k),t} = \ln(\mathcal{W}^{-}_{a(k),t}) - \ln(W^{-}_{a(k),t}). \tag{27}$$

Third, the alternative-specific shocks $\boldsymbol{\varepsilon_{a,t}}$ are not distributed independently. Since $\varepsilon_{a(k),t}$ can be inferred from the observed wage $\mathcal{W}^{-}_{a(k),t}$, the information can be used to form the expectation about the whole error distribution. Therefore, using the conditional probability density function $f(\boldsymbol{\varepsilon_t}|\varepsilon_{a(k),t})$, the probability of choosing occupation $a_t(k)$ conditional on observed states and wages writes

$$\mathrm{p}\Big(a_t(k)|\mathbf{s}^{-}_{\mathbf{t}}, W^{-}_{a(k),t}\Big) = \int_{\boldsymbol{\varepsilon_t}(a_t(k),\mathbf{s}^{-}_{\mathbf{t}})} f(\boldsymbol{\varepsilon_t}|\varepsilon_{a(k),t}) d^{|A|}\boldsymbol{\varepsilon_t}. \tag{28}$$

Applying integration by substitution yields the following expression for the probability of the observed wage:[9]

$$\mathrm{p}\Big(\mathcal{W}^{-}_{a(k),t}|\mathbf{s}^{-}_{\mathbf{t}}\Big) = \omega_t^{-1}\frac{1}{\sigma_{a(k)}}\phi\Bigg(\frac{\varepsilon_{a(k),t}}{\sigma_{a(k)}}\Bigg) \tag{29}$$

Here, ω_t^{-1} is the Jacobian of the transformation from observed wage $\mathcal{W}^{-}_{a(k),t}$ to error $\varepsilon_{a(k),t}$ in (27) and ϕ is the standard normal probability density function. Finally, the joint probability of observing choice $a_t(k)$ and wage $\mathcal{W}^{-}_{a(k),t}$ conditional on the observed states is given by the product of the two probabilities in (28) and (29):

$$\mathrm{p}\Big(a_t(k), \mathcal{W}^{-}_{a(k),t}|\mathbf{s}^{-}_{\mathbf{t}}\Big) = \mathrm{p}\Big(a_t(k)|\mathbf{s}^{-}_{\mathbf{t}}, \mathcal{W}^{-}_{a(k),t}\Big)\mathrm{p}\Big(\mathcal{W}^{-}_{a(k),t}|\mathbf{s}^{-}_{\mathbf{t}}\Big) \tag{30}$$

Based on these results, the likelihood contribution of one individual i can be written as the product of the probability to observe the measured endogenous variables for one individual and for one period over all time periods:

$$L^i_{\mathbf{m}}(\boldsymbol{\theta}) = P\Big(\{a^i_{t,}, \mathcal{W}^{-,i}_{a,t,}\}^T_{t=0}\Big) = \prod_{t=0}^{T}\mathrm{p}\Big(a^i_{t}, \mathcal{W}^{-,i}_{a,t}|\mathbf{s}^{-,\mathbf{i}}_{\mathbf{t}}\Big) \tag{31}$$

Therefore, the sample likelihood is given by the product of the individual likelihoods over the whole sample of individuals:

$$L_{\mathbf{m}}(\boldsymbol{\theta}) = P\Big(\Big\{\{a^i_{t,}, \mathcal{W}^{-,i}_{a,t,}\}^T_{t=0}\Big\}_{i\in I}\Big) = \prod_{i\in I}\prod_{t=0}^{T}\mathrm{p}(a^i_{t}, \mathcal{W}^{-,i}_{a,t}|\mathbf{s}^{-,\mathbf{i}}_{\mathbf{t}}) \tag{32}$$

Since the probabilities are functions of the exogenous parameters $\boldsymbol{\theta}$, the simulated maximum likelihood estimator $\hat{\boldsymbol{\theta}}$ is the vector of exogenous parameters that maximizes (32). As maximum likelihoods estimates are asymptotically normal[10], these results are taken as the

[9] See Raabe (2019), p. 29 and p.39-40 for the complete derivation.

[10] This property is an advantage of this thesis' estimation approach. It facilitates the uncertainty quantification via Monte Carlo sampling because there is a simple closed form for the (marginal) probability density available.

mean vector for the input parameters in the uncertainty quantification.

The procedure to estimate the parameter vector $\boldsymbol{\theta}$ using the expressions for the likelihood is as follows: First, The optimization algorithm of choice proposes a parameter vector. Second, the model is solved via backward induction. Third, using the policy functions, the likelihood is computed. These steps are repeated until the optimizer has found the maximal likelihood.

Finally, the calculation of the estimator's covariance is described.[11] The result is used as the covariance matrix for the input parameters in the UQ.

The asymptotic covariance of a maximum likelihood estimator equals the inverse of the Fisher information matrix: $\mathrm{Var}(\theta) = \mathrm{I}(\theta)^{-1}$. In this thesis, the information matrix $\mathrm{I}(\boldsymbol{\theta})$ is given by the variance of the scores of the parameters.[12] The scores $\mathrm{s}(\boldsymbol{\theta})$ are the first derivatives of the likelihood function. This can be written in terms of sample and individual likelihoods. Formally, the relationships are given by

$$\mathrm{s}(\boldsymbol{\theta}) \overset{\text{def}}{=} \frac{\partial L_{\mathbf{m}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \sum_{i \in I} \frac{L^i_{\mathbf{m}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \overset{\text{def}}{=} \sum_{i \in I} \mathrm{s}_i(\boldsymbol{\theta}). \tag{33}$$

Having multiple individual likelihood contributions, the scores are in the form of the Jacobian matrix. Using the property that the expected values of scores, $\mathbb{E}[\mathrm{s}(\theta)]$, are zero at the maximum likelihood estimator, the variance of the scores is given by (34). It is equal to the inverse of the Fisher information matrix.

$$\mathrm{I}^{-1}(\boldsymbol{\theta}) = \mathrm{Var}(\mathrm{s}(\boldsymbol{\theta})) = \mathbb{E}[\mathrm{s}(\boldsymbol{\theta})\mathrm{s}(\boldsymbol{\theta})']. \tag{34}$$

Hence, the estimator for the asymptotic covariance of the maximum likelihood estimator is given by

$$\hat{\mathrm{Cov}}_J(\hat{\boldsymbol{\theta}}) = \left(\frac{1}{N} \sum_{i \in I} \mathrm{s}_i(\hat{\boldsymbol{\theta}}) \mathrm{s}_i(\hat{\boldsymbol{\theta}})' \right)^{-1} \tag{35}$$

The intuition behind the above expression is the following: Estimator $\hat{\boldsymbol{\theta}}$ maximizes the sample likelihood. This is equivalent to $\hat{\boldsymbol{\theta}}$ setting the sample scores to zero. However, the individual likelihood may not be zero at the optimal parameter vector for the sample likelihood. This variation is captured by the variance of the individual scores evaluated at $\hat{\boldsymbol{\theta}}$. The relations in (33) and (34) then imply that the inverse of the variance of the individual scores is equivalent to the variance of the maximum likelihood estimator.

[11] see Verbeek (2012), p. 184-186.

[12] The computation of $\mathrm{Cov}(\boldsymbol{\theta})$ by using the Jacobian of the individual likelihood contributions is chosen over other approaches because, first, it yields no error in the inversion step of $I(\boldsymbol{\theta})$ and, second, the results are reasonably close to the similar specification in KW94.

4.3 Numerical Implementation

Besides the standard python libraries, the thesis uses the packages *respy* and *estimagic* to compute the QoI and to estimate the distribution of the input parameters. All other programs can be found in the *Master's Thesis Replication Repository*.

As standard deviations σ_a are restricted to positive numbers, drawing them from the unrestricted estimated joint normal distribution can lead to false results. Therefore, covariance matrix $\mathbf{\Sigma_{\boldsymbol{\epsilon}}}$ is written in terms of the lower triangular matrix $\mathbf{\Sigma^c_{\boldsymbol{\epsilon}}}$ obtained from the Cholesky decomposition of $\mathbf{\Sigma_{\boldsymbol{\epsilon}}}$. The contained Cholesky factors are unrestricted and denoted by c_i and $c_{i,j}$. i and j are positional indices. Hence, estimates for the Cholesky parameters and their variation replace the respective estimates for $\mathbf{\Sigma_{\boldsymbol{\epsilon}}}$ in the specification presented in the previous subsection.

4.4 Estimation Results

This subsection presents estimates $\hat{\boldsymbol{\theta}}$ for the exogenous parameters and the standard errors SE($\hat{\boldsymbol{\theta}}$). It also shows the correlations between important estimates.

The second column in Table 2 contains the estimates for the exogenous model parameters $\boldsymbol{\theta}$. They are obtained from a simulated dataset of 1000 individuals based on the arbitrary parametrization that is used in Data Set One in KW94.[13] This parametrization has the following economic implications: Occupation in the white collar sector is more skill-intensive or, more technically, has higher returns to education and occupational experience than occupation in the blue collar sector. Moreover, experience in the blue collar sector is rewarded in the white collar sector but not vice versa. Under this specific parametrization, the diagonal elements of the lower triangular matrix c_i coincide with the standard deviations of the utility shocks $\boldsymbol{\varepsilon_{a,t}}$ and the non-diagonal elements $c_{i,j}$ equal the correlations between different alternative-specific shocks $\boldsymbol{\varepsilon_{a,t}}$. The parameter estimates $\hat{\boldsymbol{\theta}}$ are precise. This means, they equal the parameters with which the model is simulated.

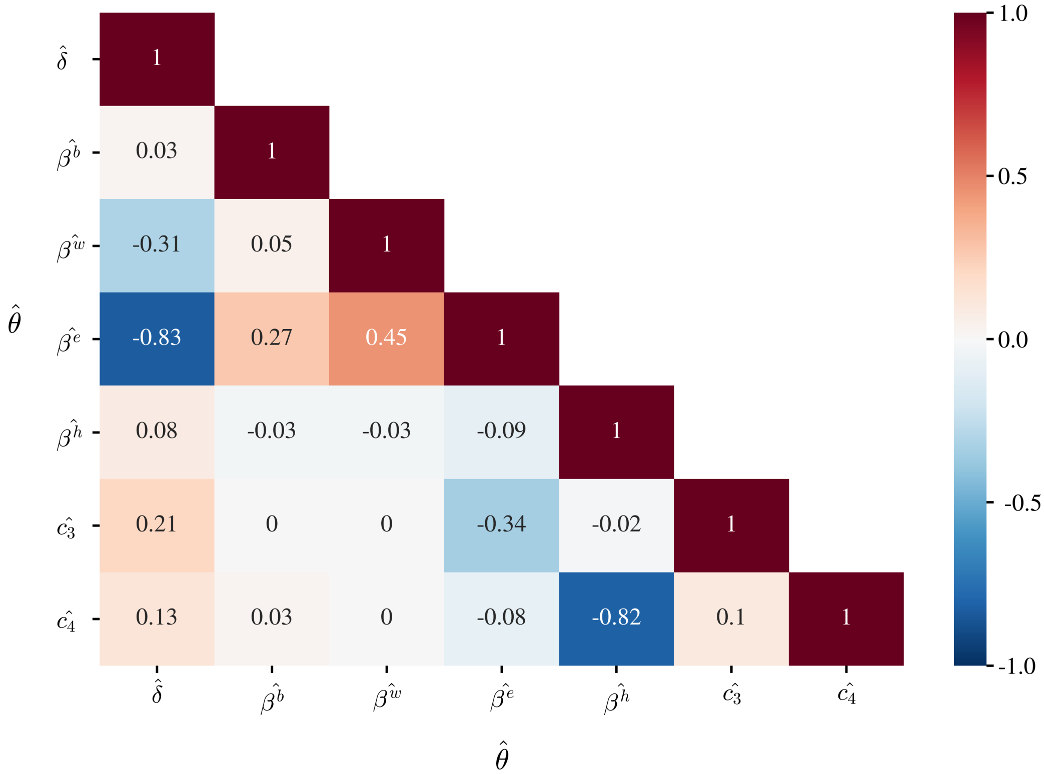
The third column shows this thesis' estimation of the standard errors SE($\hat{\boldsymbol{\theta}}$). The fourth column shows the standard errors computed in KW94. Given the differences between both estimation specifications, namely the inclusion of β and of correlations between standard errors in this thesis, the estimates are reasonably similar. However, the one exception that stands out are the results for the non-diagonal Choleksy factors $c_{i,j}$. I argue that this thesis' estimates are more precise than the estimates in KW94, and therefore, it is correct to use them in the subsequent Uncertainty Quantification. This claim is based on two reasons. These indicate that KW94 in fact do not estimate the Cholesky factors but the standard errors and correlations of shocks $\boldsymbol{\varepsilon_{a,t}}$. Both expressions are equal for the parametrization in Table 2. Nonetheless, they are conceptually different. Therefore, measures for their variation which, by construction, also consider parameter values other

[13] see table 1, p. 658; In contrary to the computation of Corr($\hat{\boldsymbol{\theta}}$), it is sufficient to find the average likelihood instead of the sample likelihood in (32).

than the mean estimates, have to differ. The arguments are: First, own estimates of SE($\hat{\boldsymbol{\theta}}$) for the model in terms of standard deviations and correlations of shocks $\boldsymbol{\varepsilon_{a,t}}$ are close to those in KW94. Second, the estimates in KW94 for the non-diagonal elements $c_{i,j}$,except of the estimate for $c_{1,2}$, would be unlikely corner solutions. For instance, fix the variance for the shocks in U_e, $\overline{\sigma_e^2}$, and write this variance in terms of the Cholesky factors such that $\overline{\sigma_e^2} = c_{3,1}^2 + c_{3,2}^2 + c_3^2$[14]. There are no restrictions that can force the maximum likelihood estimator to attribute σ_e^2 mainly to c_3. In fact and in line with the first argument, the size of the estimates for the standard errors of $c_{i,j}$ in KW94 correspond to the size that one would expect for standard deviations of correlation coefficients that range from -1 to 1. Cholesky coefficients, however, are unresticted and therefore their standard errors have not to be in this specific size. These arguments undermine the credibility of the estimation results in KW94 and as a consequence, the thesis proceeds with the own estimates.

Figure 2 depicts the correlations between the estimates of important parameters in $\boldsymbol{\theta}$. In general, the share of high correlations is considerable. The coefficients that stand out are corr($\hat{\delta}, \hat{\beta^e}$), corr($\hat{\delta}, \hat{\beta^h}$), corr($\hat{\beta^e}, \hat{\beta^w}$), corr($\hat{c_3}, \hat{\beta^e}$) and corr($\hat{c_4}, \hat{\beta^h}$) with -0.83, -0.31, 0.45, -0.34 and -0.82, respectively.

Figure 2. Correlations between estimates for important input parameters



The intuition behind these results can be obtained from the following insight: Negative correlations imply similar effects and positive correlations imply opposing effects on the likelihood of observed endogenous variables $\mathbf{m}$. For instance, consider an individual that

[14] In general, $\sigma_{i,j}^2 = \sum_{n=1}^{j} c_{j,n} c_{i,n}$.

decides for a long occupation in the education sector in the first years and then continues to work in the white collar sector for the rest of his life. The likelihood to observe this individual increases when δ rises because all individuals get more patient and therefore, ceteris paribus, they invest more in education. However, the same likelihood also increases if the educational utility constant β^e rises. Hence, because they can compensate each other, the likelihood around the optimal parameter $\hat{\boldsymbol{\theta}}$ decreases less for changes of both parameters in opposing directions than for changes in the same direction. Therefore, parameters δ and β^e are negatively correlated in terms of the score function in (33) around $\hat{\boldsymbol{\theta}}$. It follows from (35) that their standard errors are negatively correlated.

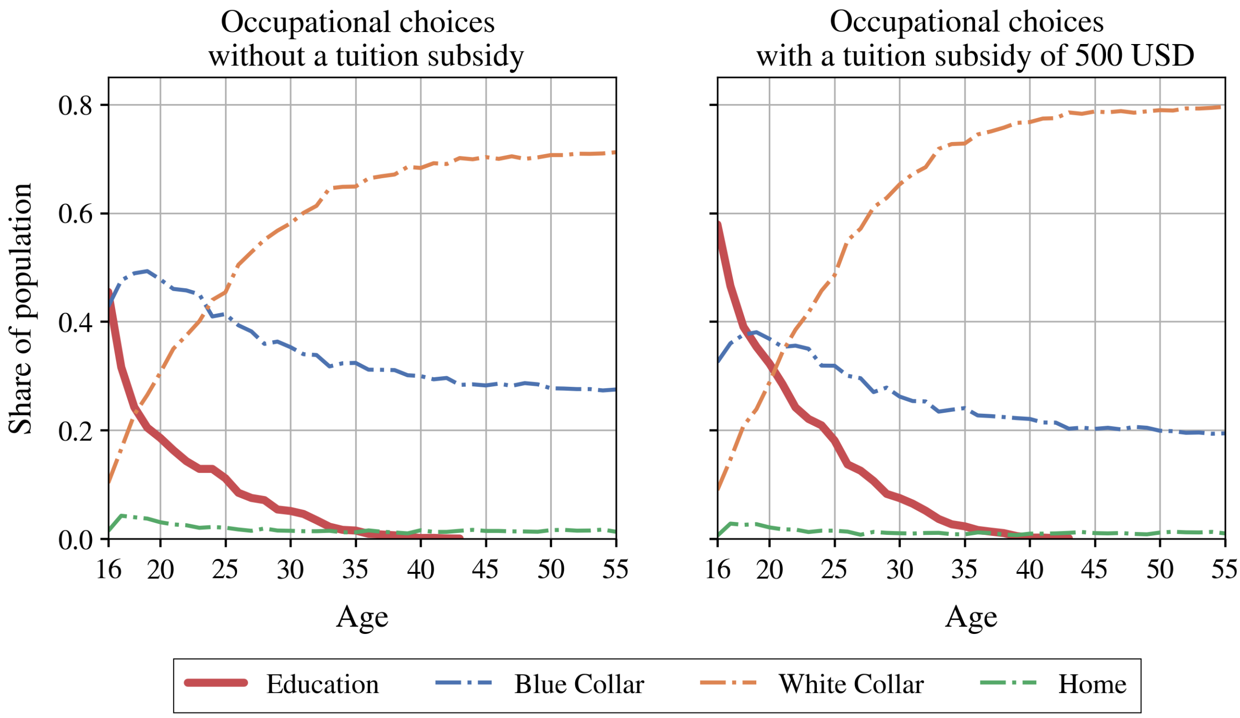
The above example provides intuition for $\text{corr}(\hat{\delta}, \hat{\beta^e}) = -0.83$ An analogous reasoning for the same example can explain $\text{corr}(\hat{\delta}, \hat{\beta^w}) = -0.31$. Yet, this correlation is smaller because $U_{w,t}$ has less covariates than $U_{e,t}$. $\text{corr}(\hat{c_3}, \hat{\beta^e}) = -0.34$ and $\text{corr}(\hat{c_4}, \hat{\beta^h}) = -0.82$ can be explained by a similar argument: Individuals decide for occupation in education or home sector if the respective utilities are high. This can be achieved by high constant terms or by high positive shocks. The latter can only happen to some individuals if the Cholesky factors are because the factors are components of the respective shock variance. Shocks $\boldsymbol{\varepsilon_{a,t}}$ are known to the agents prior to their decision a_t. Negative shocks have a smaller impact on choosing occupation e or h because individuals tend to decide against these alternatives anyway. Thus, c_3 and β^e, and c_4 and β^h can impact the likelihood in the same direction. And therefore, their standard errors are negatively correlated. Moreover, the latter relationship is stronger because $U_{h,t}$ has a lower level and less covariates.

4.5 Quantity of Interest

The QoI is the effect of a 500 USD subsidy on annual tuition costs for higher education on the average years of education. Formally, $\beta^{e,pol}_{he} = \beta^e_{he} - 500$. In KW94, the effect is an increase of 1.44 years (see table 4, p. 668). The same figure computed with *respy* is 1.5.

Figure 3 depicts a comparison between the shares of occupations in the different sectors for a sample of 1000 individuals over their relevant life-time between two scenarios. The left graph shows the occupation paths under baseline parametrization $\hat{\theta}$ and the right graph the paths for the same model with tuition subsidy.

Figure 3. Comparison of occupation paths between scenarios



The red, blue, white and green lines mark the shares of individuals occupied in the education, blue collar, white collar and home sector, respectively. Both graphs show the typical life-cycle behaviour. Many agents tend to invest in their education early and continue in the white collar sector. Another large group works in the blue collar and some of them switch to the white collar sector, as well. This switch is caused by accumulated experience that is also rewarded in the white collar sector and by positive shocks. As the relative participation in the home sector is low, this sector is relatively irrelevant for all ages.

The QoI, the impact of a 500 USD tuition subsidy for higher education on average schooling years, is chosen because it is relevant to society in many areas, for example, education, inequality and economic growth. The discussion section expands on this point. The QoI's relevance allows to illustrate the importance of UQ in economics in the context of political decisions.

The next section shows the results of the first part of the UQ, the Uncertainty Propagation.

Table 2. Model Parametrization

Parameter	Mean	Standard error (SE)	SE in KW94
General			
δ	0.95	0.000 84	-
Blue Collar			
β^b	9.21	0.013	0.014
β^b_e	0.038	0.0011	0.0015
β^b_b	0.033	0.000 44	0.000 79
β^b_{bb}	−0.0005	0.000 013	0.000 019
β^b_w	0.0	0.000 67	0.0024
β^b_{ww}	0.0	0.000 029	0.000 096
White Collar			
β^w	8.48	0.0076	0.0123
β^w_e	0.07	0.000 47	0.000 96
β^w_w	0.067	0.000 55	0.000 90
β^w_{ww}	−0.001	0.000 017	0.000 070
β^w_b	0.022	0.000 33	0.0010
β^w_{bb}	−0.0005	0.000 021	0.000 030
Education			
β^e	0.0	330	459
β^e_{he}	0.0	155	410
β^e_{re}	−4000	202	660
Home			
β^h	17 750	390	1442
Lower Triangular Cholesky Matrix			
c_1	0.2	0.0015	0.0056
c_2	0.25	0.0013	0.0046
c_3	1500	108	350
c_4	1500	173	786
$c_{1,2}$	0.0	0.0064	0.023
$c_{1,3}$	0.0	143	0.412
$c_{2,3}$	0.0	116	0.379
$c_{1,4}$	0.0	232	0.911
$c_{2,4}$	0.0	130	0.624
$c_{3,4}$	0.0	177	0.870

5 Uncertainty Propagation

6 Global Sensitivity Analysis

[inlcude relative LOO error panel like in Miftakhova]

6.1 Sobol' Indices

6.2 Univariate Effects

[PCEs do not use Monte Carlo sampling, at least no converging one, i.e. a small number of evalations is enough]

7 Discussion

none

8 Conclusion

none

[Go over (especially capitalizazion of) References]

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Appendix

none

none

Affidavit

"I hereby confirm that the work presented has been performed and interpreted solely by myself except for where I explicitly identified the contrary. I assure that this work has not been presented in any other form for the fulfillment of any other degree or qualification. Ideas taken from other works in letter and in spirit are identified in every single case."