Supported Probability Distributions

Discrete Probability Distributions

Name	PMF/CDF	Support	Parameters	Mean	Standard deviation
Binomial	$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $F_X(x) = \sum_{i=0}^{\lfloor x \rfloor} \binom{n}{i} p^i (1-p)^{n-i}$	$x \in \{0, ..., n\}$	$n \in \mathbb{N}_0$ $p \in [0, 1]$	np	$\sqrt{np(1-p)}$
Geometric	$p_X(x) = (1-p)^{x-1} p$ $F_X(x) = 1 - (1-p)^x$	$x \in \{1, 2, 3, ...\}$	$p \in (0, 1]$	$\frac{1}{p}$	$\sqrt{\frac{1-p}{p^2}}$
Negative binomial	$p_X(x) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$ $F_X(x) = \sum_{i=r}^{x} \binom{i-1}{r-1} (1-p)^{i-r} p^r$	$x \in \{r, r+1, ...\}$	$r \in \mathbb{N}$ $p \in (0, 1)$	$\frac{r}{p}$	$\sqrt{\frac{r(1-p)}{p^2}}$
Poisson	$p_X(x) = \frac{\lambda^x \exp(-\lambda)}{x!} = \frac{(vt)^x \exp(-vt)}{x!}$ $F_X(x) = \exp(-\lambda) \sum_{i=0}^{\lfloor x \rfloor} \frac{\lambda^i}{i!}$	$x \in \{0, 1, 2, ...\}$	$\lambda > 0$ **or** $v > 0, \quad t > 0$	$\lambda = vt$	$\sqrt{\lambda} = \sqrt{vt}$

Continuous Probability Distributions

Name	PDF/CDF	Support	Parameters	Mean	Standard deviation
Exponential	$f_X(x) = \lambda \exp(-\lambda x)$ $F_X(x) = 1 - \exp(-\lambda x)$	$x \in [0, \infty)$	$\lambda > 0$	λ^{-1}	λ^{-1}
Gamma	$f_X(x) = \dfrac{\lambda^k x^{k-1} \exp(-\lambda x)}{\Gamma(k)}$ $F_X(x) = \dfrac{\gamma(k,\, \lambda x)}{\Gamma(k)}$	$x \in [0, \infty)$	$k > 0$ $\lambda > 0$	$\dfrac{k}{\lambda}$	$\sqrt{\dfrac{k}{\lambda^2}}$
Beta	$f_X(x) = \dfrac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}$ $F_X(x) = I_x(\alpha,\beta)$	$x \in (a, b)$	$\alpha > 0$ $\beta > 0$ $(a < b) \in \mathbb{R}$	$\dfrac{a\beta + b\alpha}{\alpha+\beta}$	$\sqrt{\dfrac{\alpha\beta(b-a)}{(\alpha+\beta)^2(\alpha+\beta+1)}}$
Gumbel Min	$f_X(x) = \dfrac{1}{a_n} \exp(z - \exp(z))$ $F_X(x) = \exp(-\exp(z))$ with $z = \dfrac{x - b_n}{a_n}$	$x \in (-\infty, \infty)$	$b_n \in \mathbb{R}$ $a_n > 0$	$-b_n + a_n\gamma$ $\gamma \approx 0.577216$	$\sqrt{\dfrac{\pi^2 a_n^2}{6}}$
Gumbel (Max)	$f_X(x) = \dfrac{1}{a_n} \exp(-z - \exp(-z))$ $F_X(x) = \exp(-\exp(-z))$ with $z = \dfrac{x - b_n}{a_n}$	$x \in (-\infty, \infty)$	$b_n \in \mathbb{R}$ $a_n > 0$	$b_n + a_n\gamma$ $\gamma \approx 0.577216$	$\sqrt{\dfrac{\pi^2 a_n^2}{6}}$

Name	PDF/CDF	Support	Parameters	Mean	Standard deviation
Fréchet	$f_X(x) = \frac{k}{a_n}\left(\frac{a_n}{x}\right)^{k+1}\exp\left(-\left(\frac{a_n}{x}\right)^k\right)$ $F_X(x) = \exp\left(-\left(\frac{a_n}{x}\right)^k\right)$	$x \in [0, \infty)$	$a_n \in (0, \infty)$ $k \in (0, \infty)$	$a_n\Gamma\left(1 - \frac{1}{k}\right)$ for $k > 1$	$a_n\left[\Gamma\left(1-\frac{2}{k}\right) - \Gamma^2\left(1-\frac{1}{k}\right)\right]^{1/2}$ for $k > 2$
Weibull	$f_X(x) = \frac{k}{a_n}\left(\frac{x}{a_n}\right)^{k-1}\exp\left(-\left(\frac{x}{a_n}\right)^k\right)$ $F_X(x) = 1 - \exp\left(-\left(\frac{x}{a_n}\right)^k\right)$	$x \in [0, \infty)$	$a_n \in (0, \infty)$ $k \in (0, \infty)$	$a_n\Gamma(1 + 1/k)$	$a_n\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2}$
GEV	$f_X(x) = \frac{1}{\sigma}\,(t(x))^{\xi+1}\exp(-t(x))$ $F_X(x) = \exp(-t(x))$ with $t(x) = \left(1 + (\frac{x-\mu}{\sigma})\xi\right)^{-1/\xi}$	$x \in [\mu - \sigma/\xi, \infty)$ $x \in (-\infty, \mu - \sigma/\xi]$	$\mu \in \mathbb{R}$ $\sigma > 0$ $\xi \in \mathbb{R}$	$\mu + \sigma\frac{\Gamma(1-\xi) - 1}{\xi}$ for $\xi \neq 0, \xi < 1$	$\sqrt{\sigma^2\,(g_2 - g_1^2)/\xi^2}$ for $\xi \neq 0, \xi < 1/2$ where $g_k = \Gamma(1 - k\xi)$
Pareto	$f_X(x) = \frac{\alpha\, x_{\mathrm{m}}^{\alpha}}{x^{\alpha+1}}$ $F_X(x) = 1 - \left(\frac{x_{\mathrm{m}}}{x}\right)^{\alpha}$	$x \in [x_m, \infty)$	$x_m > 0$ $\alpha > 0$	$\frac{\alpha\, x_{\mathrm{m}}}{\alpha - 1}$ for $\alpha > 1$	$\sqrt{\frac{x_{\mathrm{m}}^2\alpha}{(\alpha-1)^2(\alpha-2)}}$ for $\alpha > 2$
Rayleigh	$f_X(x) = \frac{x}{\sigma^2}\exp(-x^2/2\sigma^2)$ $F_X(x) = 1 - \exp(-x^2/2\sigma^2)$	$x \in [0, \infty)$	$\sigma > 0$	$\sigma\sqrt{\frac{\pi}{2}}$	$\sqrt{\frac{4-\pi}{2}\sigma^2}$

Name	PDF/CDF	Support	Parameters	Mean	Standard deviation
Chi-squared	$f_X(x) = \frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} x^{\left(\frac{k}{2}-1\right)} \exp\left(-\frac{x}{2}\right)$ $F_X(x) = \frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$	$x \in [0, \infty)$	$k \in \mathbb{N}_{>0}$	k	$\sqrt{2k}$
Uniform	$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$ $F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a,b) \\ 1 & x \geq b \end{cases}$	$x \in [a,b]$	$-\infty < a < \infty$ $-\infty < b < \infty$	$\frac{1}{2}(a+b)$	$\sqrt{\frac{1}{12}(b-a)^2}$
Standard normal	$\phi(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right)$ $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp\left(-t^2/2\right) \mathrm{d}t$	$u \in \mathbb{R}$	–	0	1
Normal	$f_X(x) = \frac{1}{\sqrt{2\sigma^2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ $F_X(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$	$x \in \mathbb{R}$	$\mu \in \mathbb{R}$ $\sigma > 0$	μ	σ
Log-normal	$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)$ $F_X(x) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[\frac{\ln x - \mu}{\sqrt{2}\sigma}\right]$	$x \in (0, \infty)$	$\mu \in \mathbb{R}$ $\sigma > 0$	$e^{\mu+(\sigma^2/2)}$	$\sqrt{(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}}$

In order to compute some of the previous expressions the following special functions are required:

- the error function,

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\mathrm{d}t$$

- The beta function,

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$$

- The regularized beta function,

$$I_x(a, b) = \frac{B(x; a, b)}{B(a, b)} = \frac{\int_0^x t^{a-1}\,(1-t)^{b-1}\mathrm{d}t}{B(a, b)}$$

- The gamma function,

$$\Gamma(t) = \int_0^\infty x^{t-1} \exp(-x)\ \mathrm{d}x$$

- The lower incomplete gamma function,

$$\gamma(s, x) = \int_0^x t^{s-1} \exp(-t)\ \mathrm{d}t$$