SOLVING FOR CONSTRAINTS WITH RIGID BODIES - A NOTE

AMIR VAXMAN, GAME PHYSICS (INFOMGP), PERIOD 3, 2018/2019

Last Update: 13/Mar/2019

Suppose that we have a bilateral constraint $C(\mathbf{x}_1, \mathbf{x}_2)$, for points $\mathbf{x}_1$ and $\mathbf{x}_2$ on two meshes M_1 and M_2 with masses m_1 and m_2 and inertia tensors I_1 and I_2 (each around the COM of each body, like we usually treat collision). The points $\mathbf{x}$ do not have to be vertices on the meshes; just points on them.

In the case of an equality constraint, we have to maintain:

(1) $C = 0$ (valid position)
(2) $\frac{dC}{dt} = 0$ (valid velocity).

For the velocity constraint, we have to maintain (by the chain rule):

$$\frac{dC}{dt} = \frac{dC}{d\mathbf{x}_1}\frac{d\mathbf{x}_1}{dt} + \frac{dC}{d\mathbf{x}_2}\frac{d\mathbf{x}_2}{dt} = \frac{dC}{d\mathbf{x}_1}\overline{v}_1 + \frac{dC}{d\mathbf{x}_2}\overline{v}_2 = 0,$$

where $\overline{v}_1 = v_1^{||} + \omega_1 \times r_1$ (linear COM velocity, angular velocity, and arm to COM). You should massage this expression to separate the non-velocity terms and the velocity terms into the form:

$$\frac{dC}{dt} = J \cdot \begin{pmatrix} v_1^{||} \\ \omega_1 \\ v_2^{||} \\ \omega_2 \end{pmatrix} = J \cdot v$$

You get (for this single constraint) that J is a 1×12 matrix (row vector) and the velocities vector is 12×1 (3 components for 4 velocities).

The mass matrix M is of the following form:

$$M = \begin{pmatrix} m_1 & & & & & & & \\ & m_1 & & & & & & \\ & & m_1 & & & & & \\ & & & I_1 & & & & \\ & & & & m_2 & & & \\ & & & & & m_2 & & \\ & & & & & & m_2 & \\ & & & & & & & I_2 \end{pmatrix}$$

Like this, we get that $M \cdot v$ is the impulse and torque vector.

Finally, to solve the system you have to compute the scalar λ:

$$\lambda = -(1+CR)\frac{J \cdot v}{J \cdot M^{-1} J^T}$$

.

The resulting impulses and torques are given by $M\Delta v = J^T \cdot \lambda$. CR is the coefficient of restitution that gives an extra velocity bias, in case you want things to "bounce".

For position correction, in practical 2 we hack and *ignore* the effect of angular velocity (at least in the basic form of the practical, without the extension). Like this, we get a different J (we'll call it J') since now we approximate:

$$\frac{dC}{dt} = \frac{dC}{d\mathbf{x}_1}\frac{d\mathbf{x}_1}{dt} + \frac{dC}{d\mathbf{x}_2}\frac{d\mathbf{x}_2}{dt} \approx \frac{dC}{d\mathbf{x}_1}v_1^{\|} + \frac{dC}{d\mathbf{x}_2}v_2^{\|} = J'v^{\|}$$

where now we simply get $J' = \left(\frac{dC}{d\mathbf{x}_1}, \frac{dC}{d\mathbf{x}_2}\right)$, which is a 1×6 vector, fitting the fact that $v^{\|}$ is a 6×1 vector. To solve for position correction, we solve:

$$\lambda' = -\frac{C(\mathbf{x})}{J' \cdot M^{-1} J'^T}$$

And the $\Delta p = M^{-1} J'^T \lambda'$.

In case of equality constraints, you do not do anything until $C(\mathbf{x}) < 0$ (the constraint is violated). Once that happens, you proceed exactly like the equality constraint on both velocity and position corrections.