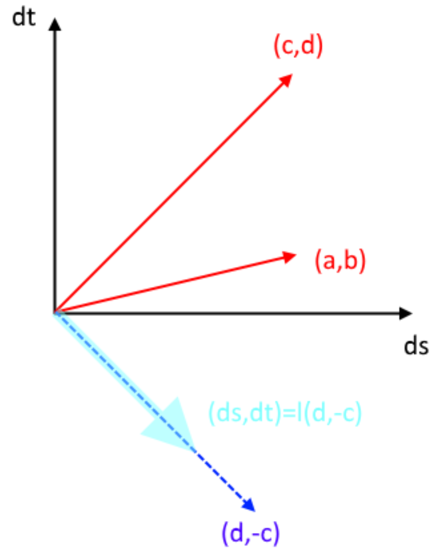

$$x = x(s,t)$$
$$(u,v) = \boldsymbol{f}(s,t)$$

$$du = \left(\frac{\partial u}{\partial s}, \frac{\partial u}{\partial t}\right) \bullet (ds, dt) = (a,b) \bullet (ds, dt)$$
$$dv = \left(\frac{\partial v}{\partial s}, \frac{\partial v}{\partial t}\right) \bullet (ds, dt) = (c,d) \bullet (ds, dt)$$

As above, in the coordinate system $dsdt$, (a,b) **does not** represent the axis of du, but geometrically, the gradient of the plane $du = (a,b) \bullet (ds, dt)$

To achieve $(du, dv) = (1,0)$:

1. $(c,d) \bullet (ds,dt) = 0$, (ds,dt) can only move in direction perpendicular to (c,d), i.e. $(ds,dt) = l(d,-c)$
2. $(a,b) \bullet l(d,-c) = l(ad-bc) = 1$, then $l = 1/(ad-bc)$, and $(ds,dt) = (d,-c)/(ad-bc)$

From a second thought

If $(\boldsymbol{u},\boldsymbol{v})$ represents the axis of du and dv in the coordinate system $dsdt$, there's $(\boldsymbol{u},\boldsymbol{v})\begin{pmatrix} du \\ dv \end{pmatrix} = \begin{pmatrix} ds \\ dt \end{pmatrix}$

Since $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} du \\ dv \end{pmatrix} = (1/(ad-bc))\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} du \\ dv \end{pmatrix} = \begin{pmatrix} ds \\ dt \end{pmatrix}$, then $\boldsymbol{u} = (d,-c)/(ad-bc)$

Then, move 1 along axis of du means moving $(d,-c)/(ad-bc)$ in the coordinate system of $dsdt$, so the same conclusion is made: $(ds,dt) = (d,-c)/(ad-bc)$

Finally, $dx = \left(\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}\right) \bullet (1,0) = \frac{\partial x}{\partial u} = \left(\frac{\partial x}{\partial s}, \frac{\partial x}{\partial t}\right) \bullet (d,-c)/(ad-bc)$