1) Să se calculeze

$$\left[x_0, x_1, \dots, x_n ; \frac{1}{ax+b}\right]$$

unde $a \neq 0$ și $a x_i + b \neq 0$

Rezolvare

Vom demonstra prin inducție matematică

$n=1$

$$\left[x_0, x_1 ; \frac{1}{ax+b}\right] = \frac{\frac{1}{ax_1+b} - \frac{1}{ax_0+b}}{x_1 - x_0} = \frac{a(x_0 - x_1)}{(x_1-x_0)(ax_0+b)(ax_1+b)} =$$

$$= (-1)^1 \frac{a^1}{(ax_0+b)(ax_1+b)}$$

$n=2$

$$\left[x_0, x_1, x_2 ; \frac{1}{ax+b}\right] = \frac{\left[x_1, x_2 ; \frac{1}{ax+b}\right] - \left[x_0, x_1 ; \frac{1}{ax+b}\right]}{x_2 - x_0} =$$

$$= \frac{(-1)^1 \frac{a}{(ax_1+b)(ax_2+b)} - (-1)^1 \frac{a}{(ax_0+b)(ax_1+b)}}{x_2 - x_0} =$$

$$= (-1)^1 a \frac{(ax_0+b) - (ax_2+b)}{(x_2-x_0)(ax_0+b)(ax_1+b)(ax_2+b)} = (-1)^2 \frac{a^2}{(ax_0+b)(ax_1+b)(ax_2+b)}$$

Deducem că $\left[x_0, x_1, \dots, x_n ; \frac{1}{ax+b}\right] = (-1)^n \frac{a^n}{\prod\limits_{i=0}^{n} (ax_i+b)}$

Vom "schița" o demonstrație prin inducție matematică

Presupunem că $\left[x_0, x_1, \dots, x_{n-1} ; \frac{1}{ax+b}\right] = \frac{(-1)^{n-1} a^{n-1}}{\prod\limits_{i=0}^{n-1} (ax_i+b)}$

Are loc

$$\left[x_0, x_1, \dots, x_n ; \frac{1}{ax+b}\right] = \frac{\left[x_1, \dots, x_n ; \frac{1}{ax+b}\right] - \left[x_0, \dots, x_{n-1} ; \frac{1}{ax+b}\right]}{x_n - x_0}$$

$$= \frac{\frac{(-a)^{n-1}}{\prod_{i=1}^{n}(ax_i+b)} - \frac{(-a)^{n-1}}{\prod_{i=0}^{n-1}(ax_i+b)}}{x_n - x_0} = (-a)^{n-1} \frac{(ax_0+b) - (ax_n+b)}{(x_n - x_0) \prod\limits_{i=0}^{n} (ax_i+b)} =$$

$$= (-a)^n \frac{1}{\prod\limits_{i=0}^{n} (ax_i+b)}$$