Să se determine polinomul P astfel încât:

$$P(a) = f(a) \quad , P'(a) = f'(a)$$
$$P(b) = f(b)$$

REZOLVARE

- Polinomul P verifică 3 condiții $\Rightarrow P \in \widetilde{\Pi}_2$ (se pot determina 3 coeficienți)
- Considerăm $P(x) = \ell(x) \cdot f(a) + h(x) \cdot f'(a) + g(x) \cdot f(b)$, unde $\ell, h, g \in \widetilde{\Pi}_2$
- Polinoamele ℓ, h, g verifică:

$$\begin{cases} \ell(a) = 1 \\ \ell'(a) = 0 \\ \ell(b) = 0 \end{cases} \qquad \begin{cases} h(a) = 0 \\ h'(a) = 1 \\ h(b) = 0 \end{cases} \qquad \begin{cases} g(a) = 0 \\ g'(a) = 0 \\ g(b) = 1 \end{cases}$$

- Deducem că

$$\ell(x) = (x-b)(\alpha_1 x + \beta_1) \in \widetilde{\Pi}_2$$
$$h(x) = \alpha_2 (x-a)(x-b) \in \widetilde{\Pi}_2$$
$$g(x) = \alpha_3 (x-a)^2 \in \widetilde{\Pi}_2$$

- Vom determina constantele $\alpha_1, \alpha_2, \alpha_3$ și β_1

$$\begin{cases} \ell(a) = 1 \\ \ell'(a) = 0 \end{cases} \underset{\ell'(x) = 2\alpha_1 x + \beta_1 - \alpha_1 b}{\Longleftrightarrow} \begin{cases} (a-b)(\alpha_1 a + \beta_1) = 1 \\ \alpha_1 (2a-b) + \beta_1 = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha_1 a + \beta_1 = \frac{1}{a-b} \\ \alpha_1 (2a-b) + \beta_1 = 0 \end{cases}$$

$$\Rightarrow \alpha_1 (a-b) = \frac{-1}{a-b} \Rightarrow \alpha_1 = \frac{-1}{(a-b)^2} , \beta_1 = \frac{2a-b}{(a-b)^2}$$

$$\Rightarrow \boxed{\ell(x) = (x-b) \frac{1}{(a-b)^2} (-x + 2a - b)}$$

$$h'(a) = 1 \underset{h'(x) = \alpha_2 (2x-a-b)}{\Longleftrightarrow} \alpha_2 (2a-a-b) = 1 \Rightarrow \alpha_2 = \frac{1}{a-b}$$

$$\Rightarrow \boxed{h(x) = \frac{1}{a-b} (x-a)(x-b)}$$

$$g(b) = 1 \iff \alpha_3 (b-a)^2 = 1 \implies \alpha_3 = \frac{1}{(b-a)^2}$$

$$\implies \boxed{g(x) = \frac{1}{(b-a)^2} (x-a)^2}$$

$$\implies P(x) = \frac{-1}{(a-b)^2} (x-b)(x-2a+b) f(a) + \frac{1}{a-b} (x-a)(x-b) f'(a) +$$

$$+ \frac{1}{(b-a)^2} (x-a)^2 f(b)$$