Curs 7

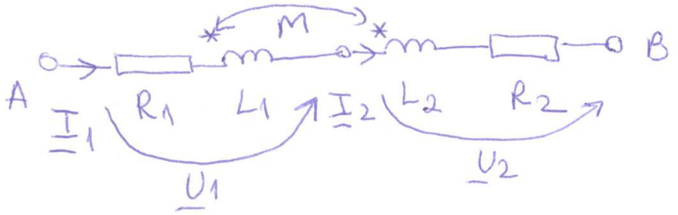
2.6 Equivalent ~~circuits~~ impedances for circuits with magnetic coupling

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} \leq 1$$

k – coupling coefficient (coef. de cuplaj)

L_1, L_2 – self inductance (inductivități proprii)

a) Series connection



$\underline{I}_1 = \underline{I}_2 = \underline{I}$

(conexiune serie)

$\underline{U}_1 = R_1 \cdot \underline{I}_1 + j\omega L_1 \cdot \underline{I}_1 + j\omega M \cdot \underline{I}_2$

– series aiding connection, if the crt. enters/leaves both * of the connection (cuplaj adițional)

– series bucking connection (opposition) (cuplaj diferențial)

$\underline{U}_2 = R_2 \cdot \underline{I}_2 + j\omega L_2 \cdot \underline{I}_2 + j\omega M \cdot \underline{I}_1$

$\underline{U} = \underline{U}_1 + \underline{U}_2 = [\underbrace{(R_1 + R_2)}_{R_e \text{ equivalent } R} + j\omega \underbrace{(L_1 + L_2 + 2M)}_{L_e \text{ equivalent inductance}}] \cdot \underline{I}$ (KVL / TKII)

$\underline{Z}_e = R_e + j\omega L_e \quad \left(\underline{Z}_e = \frac{\underline{U}}{\underline{I}}\right)$

$$\left.\begin{aligned} L_e &= L_1 + L_2 + 2M \\ L_e' &= L_1 + L_2 - 2M \end{aligned}\right\} \Rightarrow M = \frac{L_e - L_e'}{4}$$

L_e' → bucking connection

$L_e' = \underbrace{(L_1 - M)}_{L_{e_1}'} + \underbrace{(L_2 - M)}_{L_{e_2}'}$

$\begin{cases} L_{e_1}' > 0, \ L_{e_2}' < 0 \text{ or} \\ L_{e_1}' < 0, \ L_{e_2}' > 0 \end{cases}$

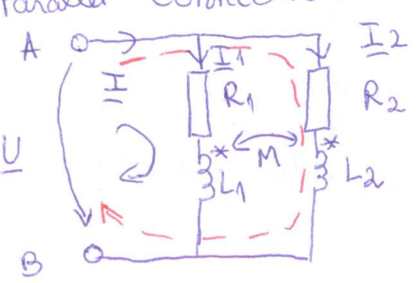
Important to know: always $L_e' > 0$!

Proof: $(\sqrt{L_1} - \sqrt{L_2})^2 \geq 0 \Rightarrow L_1 + L_2 - 2\sqrt{L_1 L_2} \geq 0$

$\Rightarrow L_1 + L_2 \geq 2\sqrt{L_1 L_2} \geq 2M$ (see def. of k)

$\Rightarrow L_1 + L_2 - 2M \geq 0$

b) Parallel connection



$\underline{Z}_e = \frac{\underline{U}}{\underline{I}}$; $\underline{I} = \underline{I}_1 + \underline{I}_2$ (KCL / TKI)

$$\begin{cases} \underline{U} = (R_1 + j\omega L_1)\underline{I}_1 + j\omega M \cdot \underline{I}_2 & \text{(KVL ②)} \\ \underline{U} = (R_2 + j\omega L_2)\underline{I}_2 + j\omega M \cdot \underline{I}_1 & \text{(KVL ②)} \end{cases}$$

Notații: $\underline{Z}_1 = R_1 + j\omega L_1$
$\underline{Z}_2 = R_2 + j\omega L_2$
$\underline{Z}_M = j\omega M$

$$\Rightarrow \begin{cases} \underline{U} = \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_M \cdot \underline{I}_2 \quad (*) \\ \underline{U} = \underline{Z}_2 \cdot \underline{I}_2 + \underline{Z}_M \cdot \underline{I}_1 \quad (**) \\ \underline{I} = \underline{I}_1 + \underline{I}_2 \quad (***) \end{cases}$$

$$(*) = (**) \Rightarrow \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_M \cdot \underline{I}_2 = \underline{Z}_2 \cdot \underline{I}_2 + \underline{Z}_M \cdot \underline{I}_1 \Rightarrow \frac{\underline{Z}_1 - \underline{Z}_M}{\underline{Z}_2 - \underline{Z}_M} \cdot \underline{I}_1 = \underline{I}_2$$

$$\overset{(***)}{\Rightarrow} \underline{I} = \underline{I}_1 + \underline{I}_2 = \underline{I}_1 \left(1 + \frac{\underline{Z}_1 - \underline{Z}_M}{\underline{Z}_2 - \underline{Z}_M}\right) = \underline{I}_1 \cdot \frac{\underline{Z}_1 + \underline{Z}_2 - 2\underline{Z}_M}{\underline{Z}_2 - \underline{Z}_M} \quad (1)$$

$$\underline{U} = \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_M \cdot \frac{\underline{Z}_1 - \underline{Z}_M}{\underline{Z}_2 - \underline{Z}_M} \cdot \underline{I}_1 \Rightarrow \frac{\underline{U}}{\underline{I}_1} = \frac{\underline{Z}_1 \cdot \underline{Z}_2 - \cancel{\underline{Z}_1 \underline{Z}_M} + \cancel{\underline{Z}_M \underline{Z}_1} - \underline{Z}_M^2}{\underline{Z}_2 - \underline{Z}_M}$$

$$\Rightarrow \frac{\underline{U}}{\underline{I}_1} = \frac{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}{\underline{Z}_2 - \underline{Z}_M} \quad (2)$$

$$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \Rightarrow \frac{\underline{U}}{\underline{I}} = \underline{Z}_e = \frac{\underline{U}}{\underline{I}_1 \cdot \frac{\underline{Z}_1 + \underline{Z}_2 - 2\underline{Z}_M}{\underline{Z}_2 - \underline{Z}_M}} = \frac{\frac{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}{\cancel{\underline{Z}_2 - \underline{Z}_M}}}{\frac{\underline{Z}_1 + \underline{Z}_2 - 2\underline{Z}_M}{\cancel{\underline{Z}_2 - \underline{Z}_M}}}$$

$$\Rightarrow \boxed{\underline{Z}_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}{\underline{Z}_1 + \underline{Z}_2 \mp 2\underline{Z}_M}}$$

→ aiding connection
→ bucking connection

2.7 Energy transfer between magnetically coupled circuits (Transfer de energie între circuite cuplate magnetic)

Desen: idem conexiunea paralel

$$\begin{cases} \underline{I}_1 = I_1 \cdot e^{j\theta_1} \\ \underline{I}_2 = I_2 \cdot e^{j\theta_2} \\ \theta = \theta_2 - \theta_1 \end{cases}$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = P + j \cdot Q$$

$$\underline{S}_1 = \underline{U} \cdot \underline{I}_1^* = [(R_1 + j\omega L_1) \cdot \underline{I}_1 + j\omega M \underline{I}_2] \cdot \underline{I}_1^* = (R_1 + j\omega L_1) \cdot I_1^2 + j\omega M I_2 e^{j\theta_2} \cdot I_1 e^{-j\theta_1} =$$

$$= (R_1 + j\omega L_1) \cdot I_1^2 + j\omega M \cdot I_1 \cdot I_2 \cdot e^{j\theta} = (R_1 + j\omega L_1) \cdot I_1^2 + j\omega M I_1 I_2 (\cos\theta + j\sin\theta)$$

$$= (R_1 + j\omega L_1) \cdot I_1^2 + j\omega M I_1 I_2 \cos\theta - \omega M I_1 I_2 \cdot \sin\theta$$

$$= \underbrace{(R_1 \cdot I_1^2 - \omega M I_1 I_2 \cdot \sin\theta)}_{P_1} + j\underbrace{(\omega L_1 \cdot I_1^2 + \omega M I_1 I_2 \cos\theta)}_{Q_1}$$

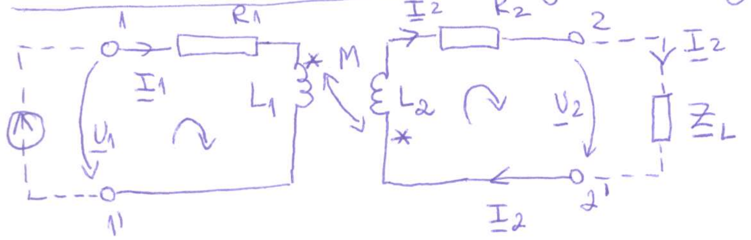
$$\underline{S}_2 = \underline{U} \cdot \underline{I}_2^* = \dots \Rightarrow P_2 = R_2 \cdot I_2^2 + \omega M \cdot I_1 I_2 \sin\theta$$

$$\boxed{P_{tr} = \omega M I_1 I_2 \sin\theta}$$

→ active power transfered between magnetically coupled circuits (putere activă transferată între circuite cuplate magnetic)

$$P = P_1 + P_2 = R_1 I_1^2 + R_2 \cdot I_2^2$$

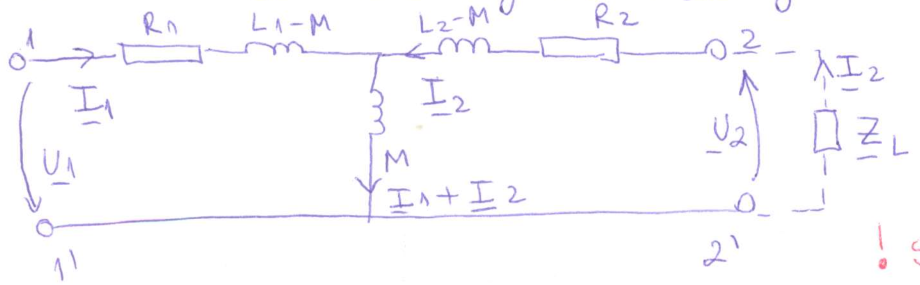
2.8 Air-core transformer (Transformator fără miez magnetic)



$$\begin{cases} \underline{U}_1 = R_1 \underline{I}_1 + j\omega L_1 \cdot \underline{I}_1 + j\omega M \cdot \underline{I}_2 \text{ (KVL)} \\ 0 = R_2 \underline{I}_2 + j\omega L_2 \cdot \underline{I}_2 + j\omega M \cdot \underline{I}_1 + \underline{Z}_L \cdot \underline{I}_2 \text{ (KVL)} \\ \underline{U}_2 = \underline{Z}_L \cdot \underline{I}_2 \end{cases}$$

$$\Rightarrow \begin{cases} \underline{U}_1 = R_1 \underline{I}_1 + j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2 & \mid \pm j\omega M \cdot \underline{I}_1 \\ -\underline{U}_2 = R_2 \cdot \underline{I}_2 + j\omega L_2 \cdot \underline{I}_2 + j\omega M \cdot \underline{I}_1 & \mid \pm j\omega M \cdot \underline{I}_2 \end{cases}$$

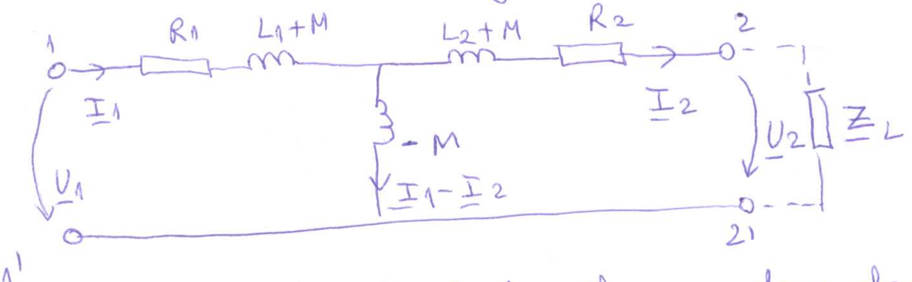
Semnul + la M:
$$\begin{cases} \underline{U}_1 = R_1 \cdot \underline{I}_1 + j\omega(L_1 - M) \cdot \underline{I}_1 + j\omega M(\underline{I}_1 + \underline{I}_2) \\ -\underline{U}_2 = R_2 \cdot \underline{I}_2 + j\omega(L_2 - M) \cdot \underline{I}_2 + j\omega M(\underline{I}_1 + \underline{I}_2) \end{cases} \quad (*)$$



The resulting equivalent circuit (circuitul electric echivalent pt sistemul *)

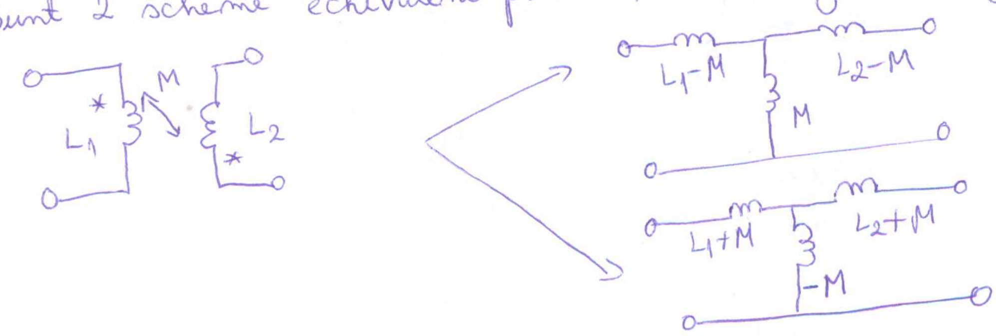
! Sensul lui $\underline{U}_2$ și $\underline{I}_2$ s-a schimbat

Semnul − la M:
$$\begin{cases} \underline{U}_1 = R_1 \cdot \underline{I}_1 + j\omega(L_1 + M) \cdot \underline{I}_1 - j\omega M(\underline{I}_1 - \underline{I}_2) \\ -\underline{U}_2 = R_2 \cdot \underline{I}_2 + j\omega(L_2 + M) \cdot \underline{I}_2 + j\omega M(\underline{I}_1 - \underline{I}_2) \end{cases} \quad (**)$$



The resulting circuit (circuitul el echivalent pt sistemul **)

Sunt 2 scheme echivalente pentru un transformator fără miez:



$$\underline{Z}_e = \underline{Z}_1 = \frac{\underline{U}_1}{\underline{I}_1}$$

Using the notations from the parallel conection:

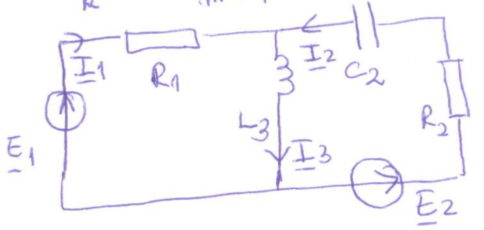
$$\begin{cases} \underline{U}_1 = \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_M \cdot \underline{I}_2 \mid : \underline{I}_1 & (1) \\ 0 = \underline{Z}_2 \cdot \underline{I}_2 + \underline{Z}_M \cdot \underline{I}_1 + \underline{Z}_L \cdot \underline{I}_2 & (2) \end{cases} \Rightarrow \begin{cases} (1)\ \underline{Z}_e = \frac{\underline{U}_1}{\underline{I}_1} = \underline{Z}_1 + \underline{Z}_M \cdot \frac{\underline{I}_2}{\underline{I}_1} \\ (2) \Rightarrow \frac{\underline{I}_2}{\underline{I}_1} = -\frac{\underline{Z}_M}{\underline{Z}_2 + \underline{Z}_L} \end{cases}$$

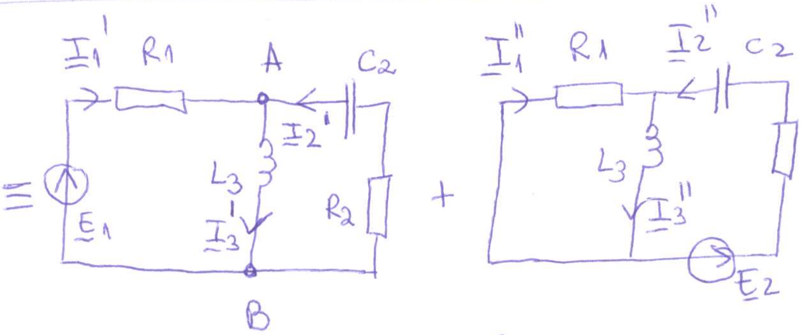
$$\Rightarrow \boxed{\underline{Z}_e = \underline{Z}_1 - \frac{\underline{Z}_M^2}{\underline{Z}_2 + \underline{Z}_L}}$$

3. Linear circuits analysis. Network theorems. (Analiza circuitelor el. liniare. Metode și teoreme de analiză).

1. The superposition method (Metoda superpoziției)

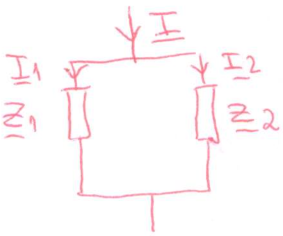
$$\underline{I}_k = \sum_{m=1}^{L} \underline{Y}_{km} \cdot \underline{E}_m$$





Reminder:

CURRENT DIVIDER



$$\underline{I}_1 = \underline{I} \cdot \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

$$\underline{I}_1' = \frac{\underline{E}_1}{R_1 + \dfrac{j\omega L_3 \cdot \left(R_2 + \frac{1}{j\omega C_2}\right)}{j\omega L_3 + R_2 + \frac{1}{j\omega C_2}}}$$

$$\underline{I}_3' = \underline{I}_1' \cdot \frac{R_2 + \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2} + j\omega L_3}$$

$$\underline{I}_2' = -\underline{I}_1' \cdot \frac{j\omega L_3}{j\omega L_3 + R_2 + \frac{1}{j\omega C_2}}$$

$$\underline{I}_2'' = \frac{\underline{E}_2}{R_2 + \frac{1}{j\omega C_2} + \frac{R_1 \cdot j\omega L_3}{R_1 + j\omega L_3}}$$

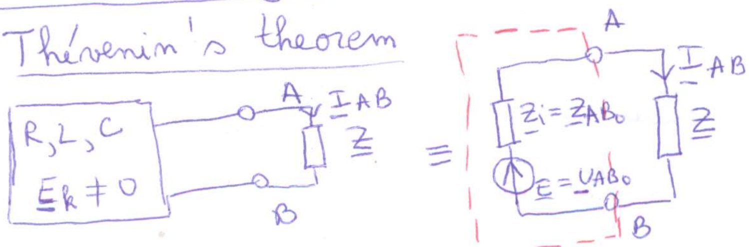
$$\underline{I}_3'' = \underline{I}_2'' \cdot \frac{R_1}{R_1 + j\omega L_3}$$

$$\underline{I}_1'' = -\underline{I}_2'' \cdot \frac{j\omega L_3}{R_1 + j\omega L_3}$$

Final computation: $\underline{I}_1 = \underline{I}_1' + \underline{I}_1''$; $\underline{I}_2 = \underline{I}_2' + \underline{I}_2''$; $\underline{I}_3 = \underline{I}_3' + \underline{I}_3''$

2. Circuits transformation

a) Thévenin's theorem



$$\boxed{\underline{I}_{AB} = \frac{\underline{U}_{AB_0}}{\underline{Z} + \underline{Z}_{AB_0}}}$$

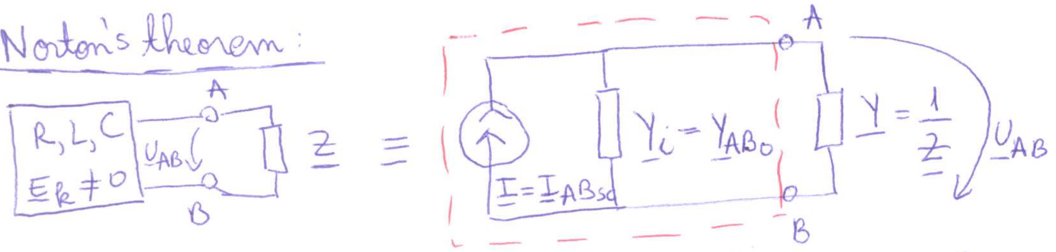
$\underline{U}_{AB_0}$ – the voltage between A and B when the circuit is opened between AB. (tensiunea de mers în gol între bornele A și B – circuitul este deschis între A și B)

$\underline{Z}_{AB_0}$ – the impedance of the internal circuit after setting all independent sources to zero (i.e. by replacing every independent voltage source by a short circuit and every independent current source by an open circuit) → impedanța circuitului intern pasivizat (sursele de tensiune se înlocuiesc cu un scurt circuit și sursele de curent se înlocuiesc cu o întrerupere)

Internal circuit: the part of the circuit inside 

Circuitul intern: partea de circuit încercuită

b) Norton's theorem:



$$\boxed{\underline{U}_{AB} = \frac{\underline{I}_{ABsc}}{\underline{Y} + \underline{Y}_{AB0}}}$$

$\underline{I}_{ABsc}$ – the short circuit current between AB (curentul de scurt circuit între bornele AB)

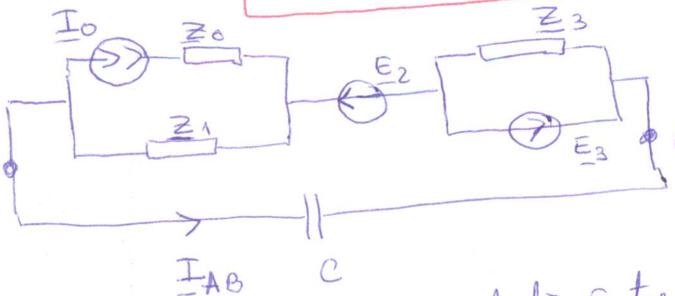
$\underline{Y}_{AB0}$ – the admittance of the internal circuit after setting all independent sources to zero (admitanța circuitului intern pasivizat)

$$\boxed{\underline{Y}_{AB0} = \frac{1}{\underline{Z}_{AB0}}}$$;

In Norton's theorem, if $\underline{Z} = 0 \Rightarrow \underline{I}_{AB} = \underline{I}_{ABsc} = \frac{\underline{U}_{AB0}}{\underline{Z}_{AB0}}$

$$\Rightarrow \boxed{\underline{Z}_{AB0} = \frac{\underline{U}_{AB0}}{\underline{I}_{ABsc}}}$$

Example 1:



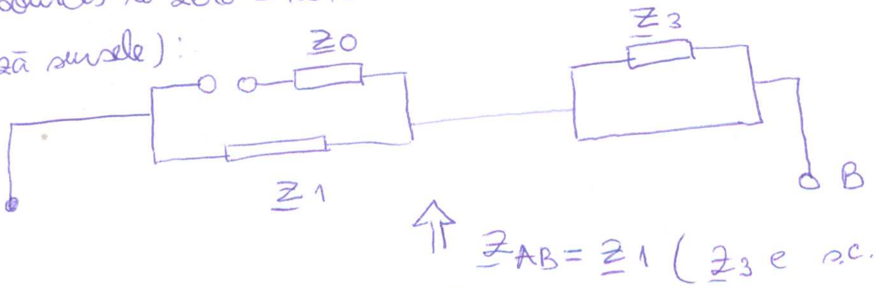
Find the current $\underline{I}_{AB}$ using Thévenin theorem. (să se det. $\underline{I}_{AB}$ cu t. Thévenin)

$$\underline{I}_{AB} = \frac{\underline{U}_{AB0}}{\underline{Z}_{AB0} + \underline{Z}}$$

First we find $\underline{Z}$ (impedanța conectată între A și B):

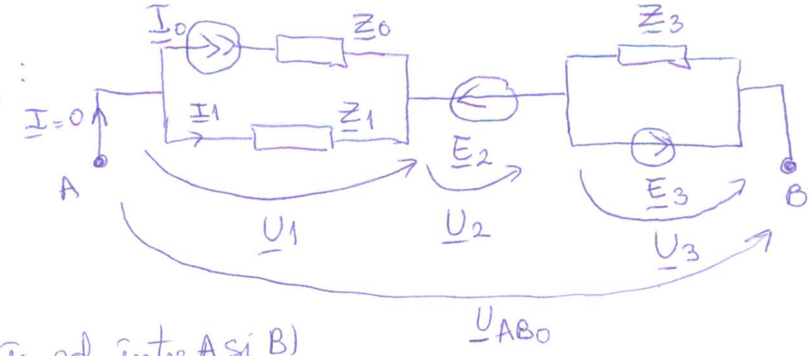
$$\underline{Z} = \frac{1}{j\omega C}$$

Next we find $\underline{Z}_{AB0}$ (we draw the circuit without the branch AB and set all sources to zero – se redesenează circuitul fără latura AB și se pasivizează sursele):



$\Uparrow$ $\underline{Z}_{AB} = \underline{Z}_1$ ($\underline{Z}_3$ e s.c.)

Finally, we compute $\underline{U}_{AB0}$:



$\underline{U}_{AB0} = \underline{U}_1 + \underline{U}_2 + \underline{U}_3$

$\underline{U}_2 = \underline{E}_2$; $\underline{U}_3 = -\underline{E}_3$

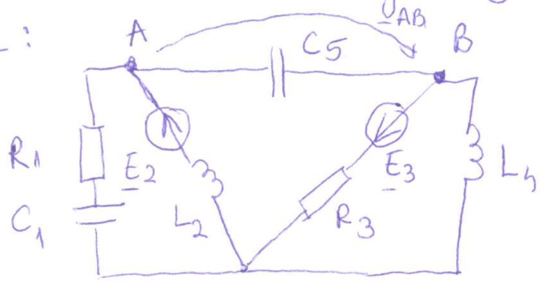
$\underline{I} = 0$ (circuitul merge în gol între A și B)

$\Rightarrow \underline{I} = \underline{I}_0 + \underline{I}_1 \Rightarrow \underline{I}_1 = -\underline{I}_0 \Rightarrow \underline{U}_1 = \underline{Z}_1 \underline{I}_1 = -\underline{Z}_1 \underline{I}_0$

$\Rightarrow \underline{U}_{AB0} = -\underline{Z}_1 \cdot \underline{I}_0 + \underline{E}_2 - \underline{E}_3$. Applying Thévenin we get $\underline{I}_{AB}$!

Homework (temă): Find $\underline{U}_{AB}$ using Norton!

Example 2:

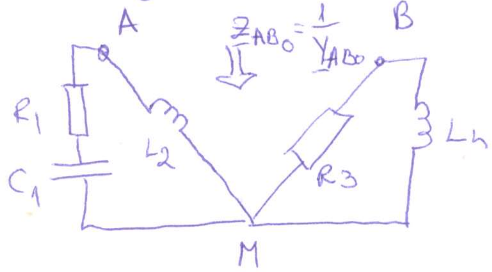


$$\underline{U}_{AB} = \frac{\underline{I}_{ABsc}}{\underline{Y} + \underline{Y}_{AB_0}}$$

Find $\underline{U}_{AB}$ using Norton's theorem (să se det. U_{AB} folosind t. Norton)

First we find $\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{\frac{1}{j\omega C_5}} = j\omega C_5$

Next we find $\underline{Y}_{AB_0}$



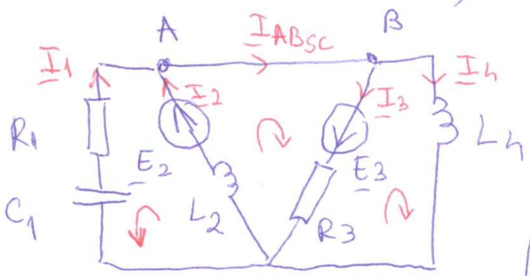
$$\underline{Z}_{AB_0} = \underline{Z}_{AM} + \underline{Z}_{MB}$$

$$\underline{Z}_{AM} = \frac{j\omega L_2 \cdot \left(R_1 + \frac{1}{j\omega C_1}\right)}{j\omega L_2 + R_1 + \frac{1}{j\omega C_1}}$$

$$\underline{Z}_{MB} = \frac{R_3 \cdot j\omega L_4}{R_3 + j\omega L_4}$$

$$\Rightarrow \underline{Y}_{AB_0} = \frac{1}{\underline{Z}_{AB_0}}$$

Finally, we compute $\underline{I}_{AB_{SC}}$ (we replace the branch AB with a short circuit – latura AB se înlocuiește cu un scurt circuit):



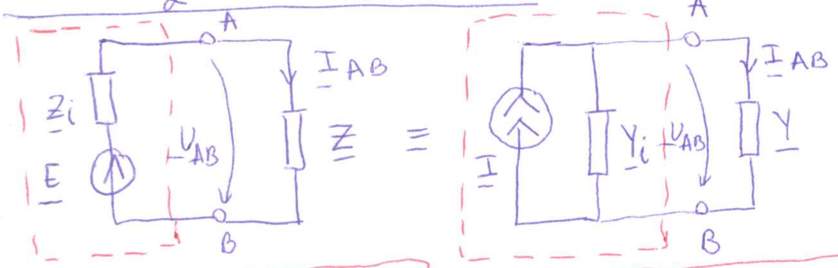
$$\begin{cases} \underline{I}_1 + \underline{I}_2 = \underline{I}_{ABsc} \quad (KCL = TKI) \\ \underline{I}_{AB_{sc}} = \underline{I}_3 + \underline{I}_4 \quad (KCL = TKI) \\ \underline{E}_2 = j\omega L_2 \cdot \underline{I}_2 - \left(R_1 + \frac{1}{j\omega C_1}\right) \cdot \underline{I}_1 \quad (KVL = TKII) \\ \underline{E}_2 + \underline{E}_3 = j\omega L_2 \cdot \underline{I}_2 + R_3 \cdot \underline{I}_3 \quad (KVL = TKII) \\ -\underline{E}_3 = j\omega L_4 \cdot \underline{I}_4 - R_3 \cdot \underline{I}_3 \quad (KVL = TKII) \end{cases}$$

Sistem de 5 ec. cu 5 necunoscute (curenții). Se rezolvă (dacă avem date numerice) => $\underline{I}_{ABsc}$

Temă (Homework): Find $\underline{I}_{AB}$ using Thévenin!

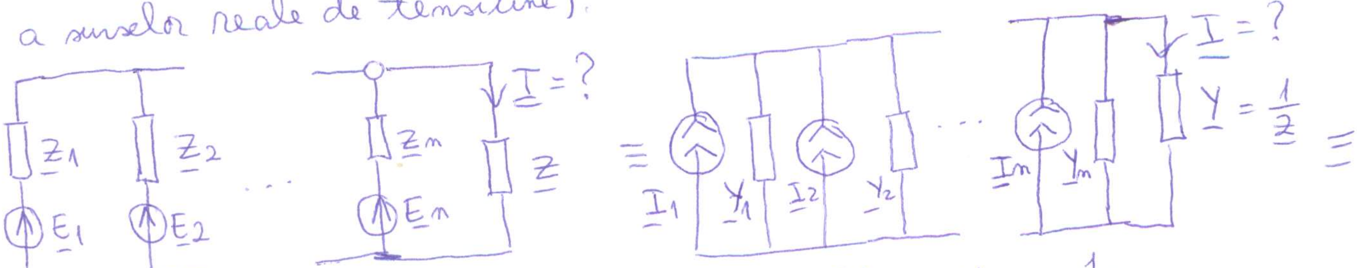
Observație: Puteți determina $\underline{U}_{AB_0}$ sau $\underline{I}_{ABsc}$ (mărimea care e mai ușor de găsit din circuit) și se poate aplica formula: $\underline{Z}_{AB_0} = \frac{\underline{U}_{AB_0}}{\underline{I}_{ABsc}}$ și determina cealaltă mărime. Pe $\underline{Z}_{AB_0}$ trebuie să-l calculați și la Norton și la Thévenin oricum.

c) The equivalence theorem (Teorema echivalenței)

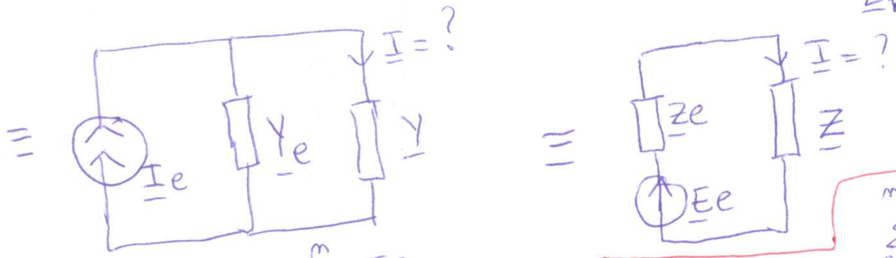


$$\boxed{\underline{E} = \frac{\underline{I}}{\underline{Y}_i};\ \underline{Z}_i = \frac{1}{\underline{Y}_i}} \qquad \boxed{\underline{I} = \frac{\underline{E}}{\underline{Z}_i};\ \underline{Y}_i = \frac{1}{\underline{Z}_i}}$$

Example: The parallel connection of real voltage sources (conexiunea paralel a surselor reale de tensiune).



$$\underline{I}_k = \frac{\underline{E}_k}{\underline{Z}_k};\ \underline{Y}_k = \frac{1}{\underline{Z}_k}$$



$$\Rightarrow \boxed{\underline{I} = \frac{\underline{E}e}{\underline{Z} + \underline{Z}e}}$$

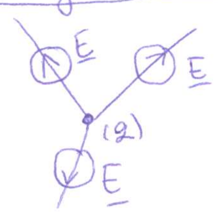
$$\underline{I}_e = \sum_{k=1}^{m} \underline{I}_k = \sum_{k=1}^{m} \frac{\underline{E}_k}{\underline{Z}_k}$$

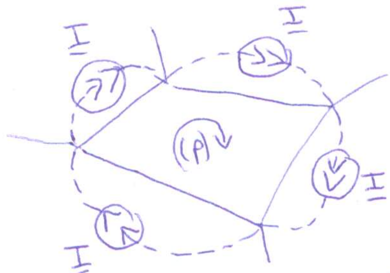
$$\underline{Y}_e = \sum_{k=1}^{m} \underline{Y}_k = \sum_{k=1}^{m} \frac{1}{\underline{Z}_k}$$

$$\boxed{\underline{E}e = \frac{\underline{I}e}{\underline{Y}e} = \frac{\sum_{k=1}^{m} \frac{\underline{E}_k}{\underline{Z}_k}}{\sum_{k=1}^{m} \frac{1}{\underline{Z}_k}}}$$

$$\boxed{\underline{Z}e = \frac{1}{\underline{Y}e} = \frac{1}{\sum_{k=1}^{n} \frac{1}{\underline{Z}_k}}}$$

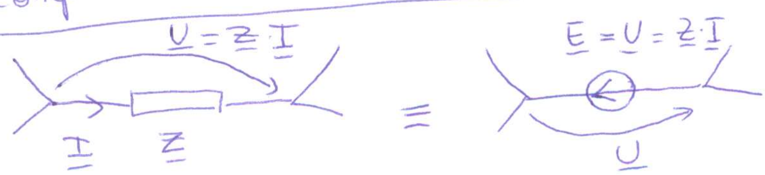
d) Vaschy's theorem



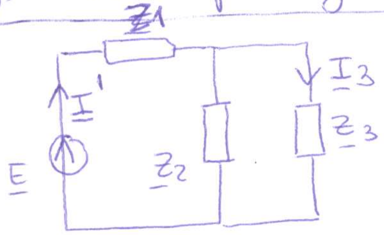


By adding these ideal voltage sources or current sources, the currents of the circuit remain unchanged (adăugând surse de tensiune sau de curent ideale, ca în figură, curenții circuitului nu se schimbă).

e) The compensation theorem (teorema compensației)



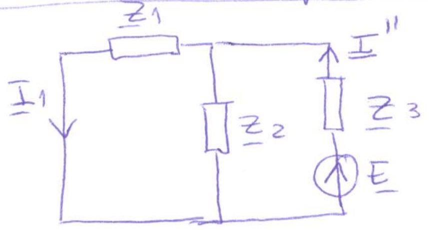
f) The reciprocity theorem (teorema reciprocității)



(fig 1)

$$\Downarrow$$

$$\boxed{I_3 = I_1}$$



(fig 2)

TRUE FOR A CIRCUIT WITH A SINGLE SOURCE!

The current flowing in the impedance Z_3, given by the voltage source E situated in branch 1 is the same with the current flowing in the branch 1 if the voltage source has been moved in the branch 3. (Curentul printr-o latură „m" a unui circuit electric liniar, dat de o sursă situată în altă latură „k" (fără să mai existe și alte surse în circuit) este egal cu curentul pe care l-ar produce în latura „k" aceeași sursă mutată în latura „m").

PROOF

(fig 1) $Z_2 \parallel Z_3$

$$\Rightarrow I' = \frac{E}{Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3}} = \frac{E\,(Z_2+Z_3)}{Z_1Z_2+Z_1Z_3+Z_2Z_3}$$

$$I_3 = I' \cdot \frac{Z_2}{Z_2+Z_3} = \frac{E \cdot Z_2}{Z_1Z_2+Z_1Z_3+Z_2Z_3}$$

(fig 2) $Z_1 \parallel Z_2$

$$\Rightarrow I'' = \frac{E}{Z_3 + \frac{Z_1 Z_2}{Z_1+Z_2}} = \frac{E\,(Z_1+Z_2)}{Z_1Z_3+Z_2Z_3+Z_1Z_2}$$

$$\Rightarrow I_1 = I'' \cdot \frac{Z_2}{Z_1+Z_2} = \frac{E \cdot Z_2}{Z_1Z_3+Z_2Z_3+Z_1Z_2}$$

Formula divizorului de curent
(current divider)