Să se calculeze diferența divizată:

$$\left[0,1,\dots,n;\ \frac{x}{(x+1)(x+2)^2}\right]$$

Rezolvare:

Vom folosi descompunerea

$$\frac{x}{(x+1)(x+2)^2} = \frac{1}{x+2} + \frac{2}{(x+2)^2} - \frac{1}{x+1}$$

și faptul că

$$\left[x_0, x_1, \dots, x_n;\ \frac{1}{x+a}\right] = \frac{(-1)^n}{\prod_{i=0}^{n}(x_i+a)}$$

și

$$\left[x_0, x_1, \dots, x_n;\ \frac{1}{(x+a)^2}\right] = (-1)^n \frac{\sum_{i=0}^{n}\frac{1}{x_i+a}}{\prod_{i=0}^{n}(x_i+a)}$$

Obținem

$$\left[0,1,\dots,n;\ \frac{1}{x+2}\right] = (-1)^n \cdot \frac{1}{\prod_{i=0}^{n}(i+2)} = \frac{(-1)^n}{(n+2)!}$$

$$\left[0,1,\dots,n;\ \frac{1}{x+1}\right] = (-1)^n \cdot \frac{1}{\prod_{i=0}^{n}(i+1)} = \frac{(-1)^n}{(n+1)!}$$

$$\left[0,1,\dots,n;\ \frac{1}{(x+2)^2}\right] = (-1)^n \frac{\sum_{i=0}^{n}\frac{1}{i+2}}{\prod_{i=0}^{n}(i+2)} = \frac{(-1)^n}{(n+2)!}\sum_{i=0}^{n}\frac{1}{i+2}$$

$$\Rightarrow \left[0,1,\dots,n;\ \frac{x}{(x+1)(x+2)^2}\right] = \left[0,1,\dots,n;\ \frac{1}{x+2}\right] + 2\left[0,1,\dots,n;\ \frac{1}{(x+2)^2}\right] -$$

$$-\left[0,1,\dots,n;\ \frac{1}{x+1}\right] =$$

$$= \frac{(-1)^n}{(n+2)!} + 2\frac{(-1)^n}{(n+2)!}\sum_{i=0}^{n}\frac{1}{i+2} - \frac{(-1)^n}{(n+1)!} \quad \underline{\underline{i+2=k}}$$

$$= (-1)^n\left(\frac{1}{(n+2)!} - \frac{1}{(n+1)!}\right) + 2\cdot\frac{(-1)^n}{(n+2)!}\sum_{k=2}^{n+2}\frac{1}{k} =$$

$$= \frac{(-1)^n}{(n+2)!}\left(2\cdot\sum_{k=2}^{n+2}\frac{1}{k} - n - 1\right)$$