Să se arate că $f:[a,b]\to\mathbb{R}$ este convexă dacă și numai dacă $\forall\, x_i$, $i=1,2,3$, $x_i\in[a,b]$, $x_i\neq x_j$ $\forall\, i\neq j$, are loc inegalitatea:

$$[x_0, x_1, x_3 \,;\, f] \geq 0$$

<u>Rezolvare</u>:

Fără a restrânge generalitatea, presupunem că:

$$a\leq x_1 < x_2 < x_3 \leq b$$

$\Rightarrow \exists\, t\in(0,1)$ a.î. $x_2=(1-t)x_1+tx_3$

$$[x_1,x_2,x_3\,;\,f]\geq 0 \iff \frac{[x_2,x_3\,;\,f]-[x_1,x_2\,;\,f]}{x_3-x_1}\geq 0$$

$$\iff \frac{\dfrac{f(x_3)-f(x_2)}{x_3-x_2}-\dfrac{f(x_2)-f(x_1)}{x_2-x_1}}{x_3-x_1}\geq 0$$

$$\iff \frac{(x_2-x_1)f(x_3)+(x_3-x_2)f(x_1)-(x_3-x_1)f(x_2)}{(x_3-x_1)(x_3-x_2)(x_2-x_1)}\geq 0$$

$$\iff \frac{[(1-t)x_1+tx_3-x_1]\cdot f(x_3)+[x_3-(1-t)x_1-tx_3]\cdot f(x_1)-(x_3-x_1)\cdot f(x_2)}{(x_3-x_1)[x_3-(1-t)x_1-tx_3][(1-t)x_1+tx_3-x_1]}\geq 0$$

$$\iff \frac{(1-t)(x_3-x_1)\cdot f(x_1)+t(x_3-x_1)\cdot f(x_3)-(x_3-x_1)f((1-t)x_1+tx_3)}{(1-t)\cdot t\cdot(x_3-x_1)^3}\geq 0$$

$$\iff \frac{(1-t)f(x_1)+t\cdot f(x_3)-f((1-t)x_1+tx_3)}{(1-t)\cdot t\cdot(x_3-x_1)^2}\geq 0$$

$\iff (1-t)\,f(x_1)+t\cdot f(x_3)\geq f((1-t)x_1+tx_3)$

$\iff f$ convexă (x_1,x_2,x_3 au fost aleși arbitrar)