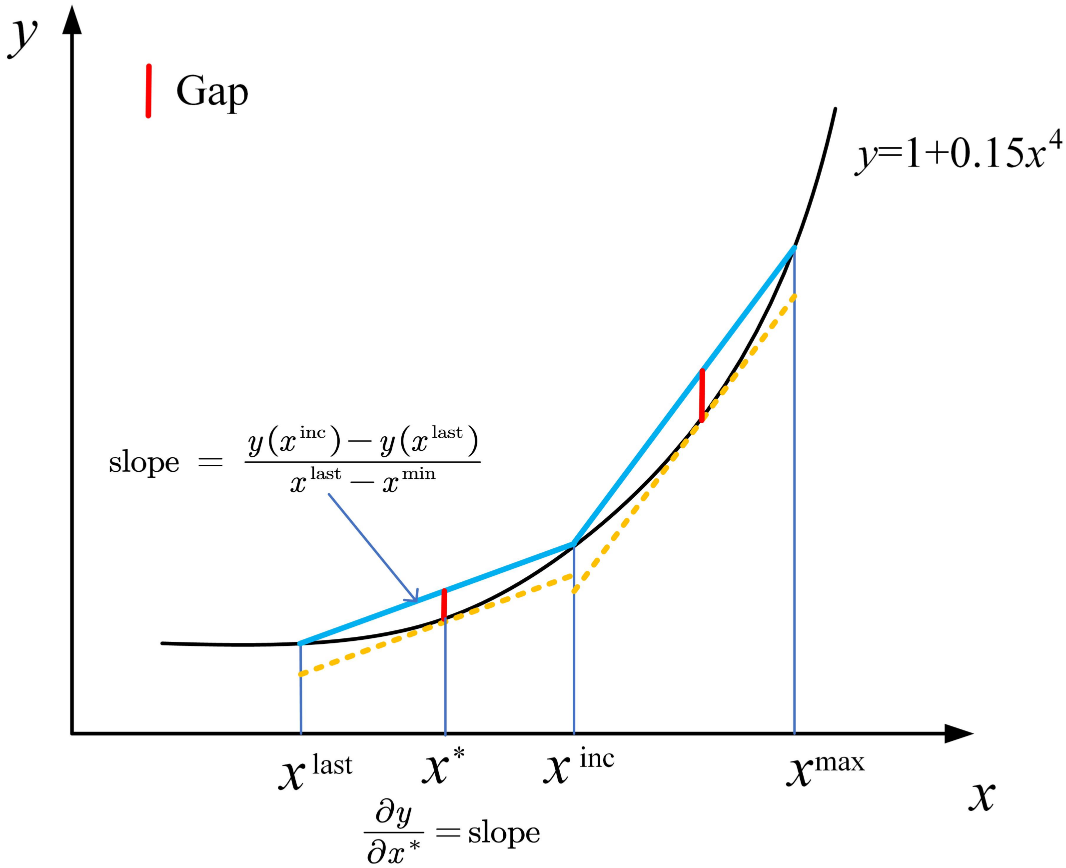
The accuracy-aware adaptive piece-wise linearization (AAPWL) method adopted in **Case C** is depicted below for a better understanding.



The figure above represents the function y[1], with the maximum gap obtained at x^*. Therefore, we can determine the value of x^* as

$$y\prime\left(x^*\right) = \frac{y\left(x^{\text{inc}}\right) - y\left(x^{\text{last}}\right)}{x^{\text{last}} - x^{\text{min}}} \tag{1}$$

Besides, the approximation function $f(x)$ can be expressed as

$$\frac{f\left(x\right) - y\left(x^{\text{last}}\right)}{x - x^{\text{last}}} = \frac{y\left(x^{\text{inc}}\right) - y\left(x^{\text{last}}\right)}{x^{\text{last}} - x^{\text{min}}} \tag{2}$$

Thus, we can obtain the maximum gap as

$$gap \;=\; f\left(x^*\right) - y\left(x^*\right) \tag{3}$$

When satisfying the gap requirement, we aim to maximize x^{inc} to obtain the minimum number of segments. This leads to the following optimization problem

$$\begin{aligned} \mathcal{M}: \max \quad & x^{\text{inc}} \\ s.t. \quad & f\left(x^*\right) - y\left(x^*\right) \leqslant \text{gap} \\ & (1) - (2) \\ & x^{\text{last}} \leqslant x^{\text{inc}} \leqslant x^{\text{max}} \end{aligned} \tag{4}$$

Therefore, we can adopt the following algorithm to obtain the breakpoints.

After obtaining the breakpoints, we can obtain the piece-wise linearization function $y(x)$ as

[1] Here the specific function y is just an example, any function can be adopted in the same way.

Algorithm 1: Accuracy-aware adaptive piece-wise linearization method

Input: Lower and upper bound of variables: $x^{\min}$, $x^{\max}$, gap, function y.
Output: Set of breakpoints: Θ.

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Initialize parameters: set iteration number k = 1, x^last ← x^min, Θ = Θ ∪ x^last;
while x^inc < x^max do
    x^inc ← Solve M;
    x^last ← x^inc;
    Θ = Θ ∪ x^last;
    k ← k + 1.;
end
if x^last ≠ x^max then
    Θ = Θ ∪ x^max;
else
    pass;
end
```

$$
\begin{aligned}
f(x) &= \sum_{n=1}^{N} y(\theta_n)\pi_n \\
x &= \sum_{n=1}^{N} \theta_n \pi_n \\
\pi_0 &\leq v_0 \\
\pi_n &\leq v_{n-1} + v_n, for\ i = 1, 2, ..., N-1 \\
\pi_N &\leq v_{N-1} \\
&\sum_{n=0}^{N-1} v_n = 1, \quad \sum_{n=0}^{N} \pi_n = 1 \\
&\text{where } v_i \in \{0, 1\}, \pi_n \geq 0,\ i = 0, 1, ..., N-1
\end{aligned}
\tag{5}
$$

Herein, the approximation through AAPWL method is performed.