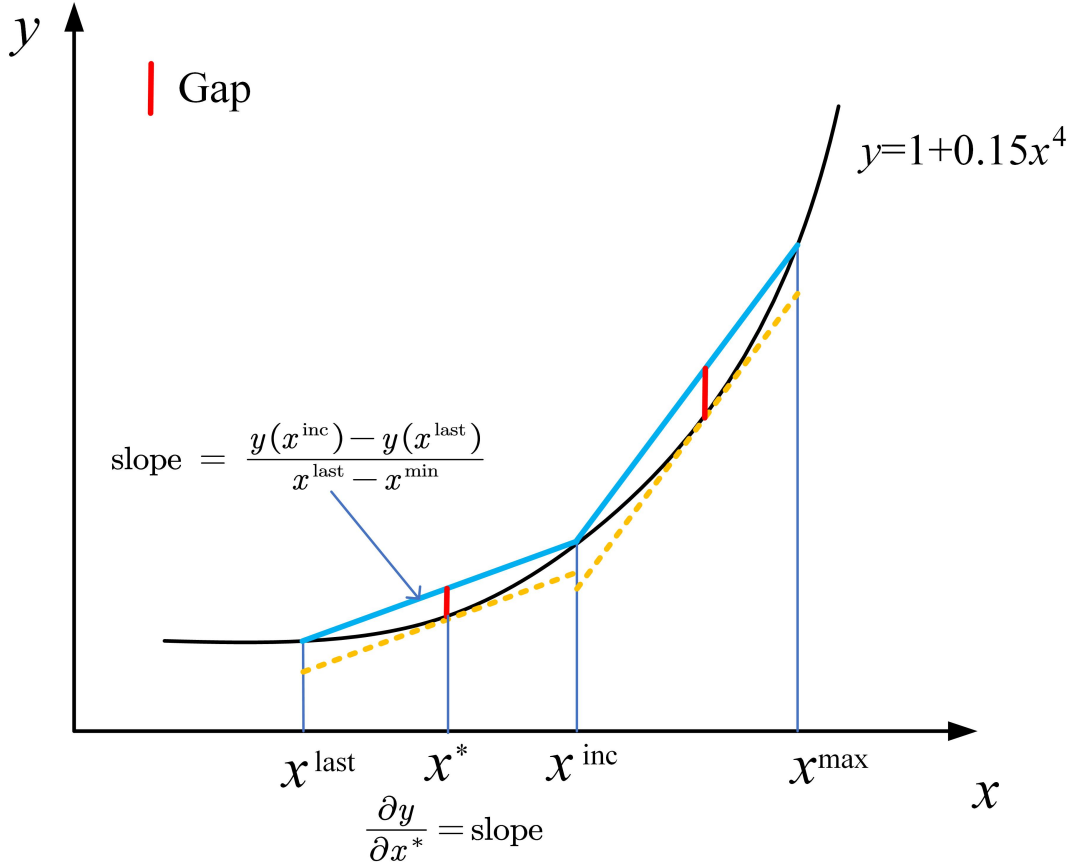The accuracy-aware adaptive piece-wise linearization (AAPWL) method adopted in **Case C** is depicted below for a better understanding.

The figure above represents the function y^1, with the maximum gap obtained at x^*. Therefore, we can determine the value of x^* as

$$y\prime\left(x^*\right) = \frac{y\left(x^{\mathrm{inc}}\right) - y\left(x^{\mathrm{last}}\right)}{x^{\mathrm{last}} - x^{\mathrm{min}}} \tag{1}$$

Besides, the approximation function $f(x)$ can be expressed as

$$\frac{f\left(x\right) - y\left(x^{\mathrm{last}}\right)}{x - x^{\mathrm{last}}} = \frac{y\left(x^{\mathrm{inc}}\right) - y\left(x^{\mathrm{last}}\right)}{x^{\mathrm{last}} - x^{\mathrm{min}}} \tag{2}$$

Thus, we can obtain the maximum gap as

$$gap = f\left(x^*\right) - y\left(x^*\right) \tag{3}$$

When satisfying the gap requirement, we aim to maximize x^{inc} to obtain the minimum number of segments. This leads to the following optimization problem

$$\begin{aligned} \mathcal{M} : \max \quad & x^{\mathrm{inc}} \\ s.t. \quad & f\left(x^*\right) - y\left(x^*\right) \leqslant \mathrm{gap} \\ & (1) - (2) \\ & x^{\mathrm{last}} \leqslant x^{\mathrm{inc}} \leqslant x^{\mathrm{max}} \end{aligned} \tag{4}$$

Therefore, we can adopt the following algorithm to obtain the breakpoints.

After obtaining the breakpoints, we can obtain the piece-wise linearization function $y(x)$ as

[1] Here the specific function y is just an example, any function can be adopted in the same way.

Algorithm 1: Accuracy-aware adaptive piece-wise linearization method

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Input: Lower and upper bound of variables: $x^{\min}$, $x^{\max}$, gap, function y.
Output: Set of breakpoints: Θ.

1   Initialize parameters: set iteration number $k = 1$, $x^{\text{last}} \leftarrow x^{\min}$, $\Theta = \Theta \cup x^{\text{last}}$;
2   **while** $x^{\text{inc}} < x^{\max}$ **do**
3 | $x^{\text{inc}} \leftarrow$ Solve $\mathcal{M}$;
4 | $x^{\text{last}} \leftarrow x^{\text{inc}}$;
5 | $\Theta = \Theta \cup x^{\text{last}}$;
6 | $k \leftarrow k + 1.$;
7   **end**
8   **if** $x^{\text{last}} \neq x^{\max}$ **then**
9 | $\Theta = \Theta \cup x^{\max}$;
10   **else**
11 | pass;
12   **end**

$$f(x) = \sum_{n=1}^{N} y(\theta_n)\pi_n$$

$$x = \sum_{n=1}^{N} \theta_n \pi_n$$

$$\pi_0 \leq v_0$$

$$\pi_n \leq v_{n-1} + v_n, for\ i = 1, 2, ..., N-1 \tag{5}$$

$$\pi_N \leq v_{N-1}$$

$$\sum_{n=0}^{N-1} v_n = 1, \quad \sum_{n=0}^{N} pi_n = 1$$

$$\text{where } v_i \in \{0, 1\}, \pi_n \geq 0,\ i = 0, 1, ..., N-1$$

Herein, the approximation through AAPWL method is performed.