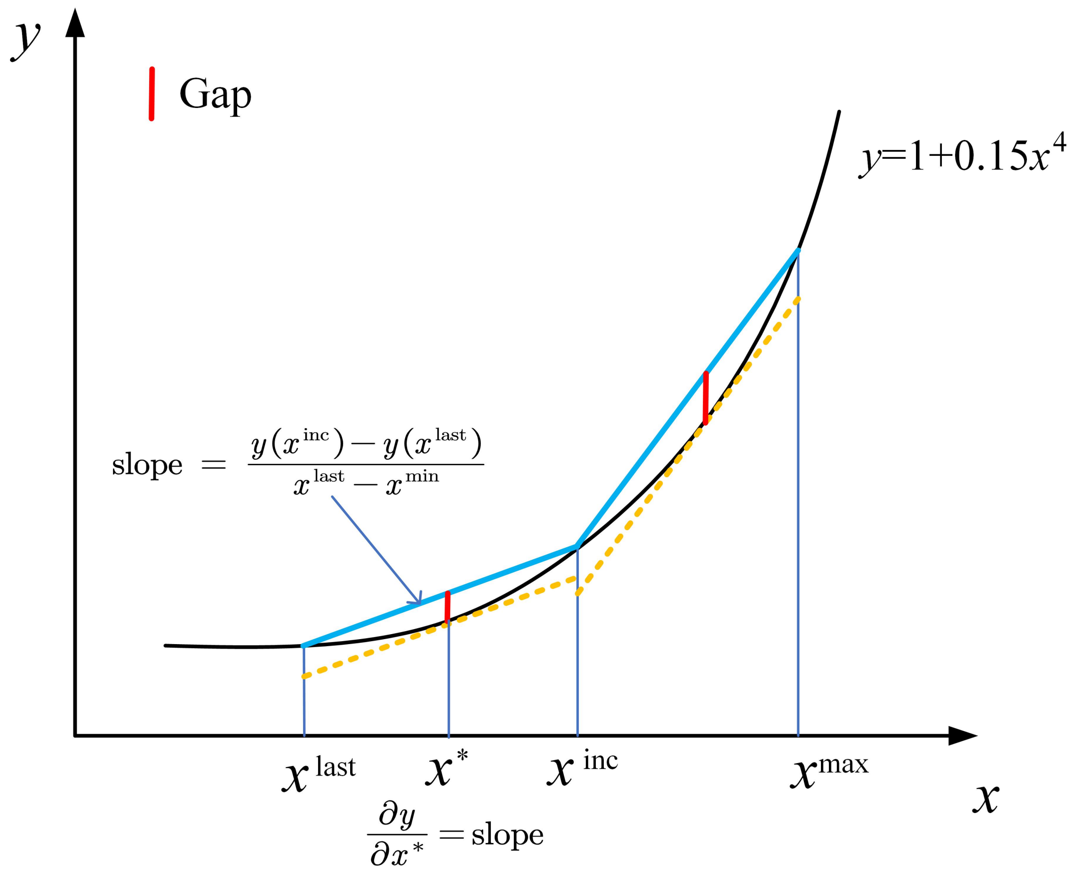
The accuracy-aware adaptive piece-wise linearization (AAPWL) method adopted in **Case C** is depicted below for a better understanding.



The figure above represents the function y[1], with the maximum gap obtained at x^*. Therefore, we can determine the value of x^* as

$$y\prime\left(x^*\right)=\frac{y\left(x^{\text{inc}}\right)-y\left(x^{\text{last}}\right)}{x^{\text{last}}-x^{\text{min}}} \tag{1}$$

Besides, the approximation function $f(x)$ can be expressed as

$$\frac{f\left(x\right)-y\left(x^{\text{last}}\right)}{x-x^{\text{last}}}=\frac{y\left(x^{\text{inc}}\right)-y\left(x^{\text{last}}\right)}{x^{\text{last}}-x^{\text{min}}} \tag{2}$$

Thus, we can obtain the maximum gap as

$$gap\ =\ f\left(x^*\right)-y\left(x^*\right) \tag{3}$$

When satisfying the gap requirement, we aim to maximize x^{inc} to obtain the minimum number of segments. This leads to the following optimization problem

$$\begin{aligned}\mathcal{M}:\max\quad & x^{\text{inc}}\\ s.t.\quad & f\left(x^*\right)-y\left(x^*\right)\leqslant \text{gap}\\ & (1)-(2)\\ & x^{\text{last}}\leqslant x^{\text{inc}}\leqslant x^{\text{max}}\end{aligned} \tag{4}$$

Therefore, we can adopt the following algorithm to obtain the breakpoints.

After obtaining the breakpoints, we can obtain the piece-wise linearization function $y(x)$ as

[1]Here the specific function y is just an example, any function can be adopted in the same way.

Algorithm 1: Accuracy-aware adaptive piece-wise linearization method

Input: Lower and upper bound of variables: $x^{\min}$, $x^{\max}$, gap, function y.
Output: Set of breakpoints: Θ.

Initialize parameters: set iteration number $k = 1$, $x^{\text{last}} \leftarrow x^{\min}$, $\Theta = \Theta \cup x^{\text{last}}$;
while $x^{\text{inc}} < x^{\max}$ **do**
    $x^{\text{inc}} \leftarrow$ Solve $\mathcal{M}$;
    $x^{\text{last}} \leftarrow x^{\text{inc}}$;
    $\Theta = \Theta \cup x^{\text{last}}$;
    $k \leftarrow k + 1.$;
end
if $x^{\text{last}} \neq x^{\max}$ **then**
    $\Theta = \Theta \cup x^{\max}$;
else
    pass;
end

$$
\begin{aligned}
f(x) &= \sum_{n=1}^{N} y(\theta_n)\pi_n \\
x &= \sum_{n=1}^{N} \theta_n \pi_n \\
\pi_0 &\leq v_0 \\
\pi_n &\leq v_{n-1} + v_n, for\ i = 1, 2, ..., N-1 \\
\pi_N &\leq v_{N-1} \\
&\sum_{n=0}^{N-1} v_n = 1, \quad \sum_{n=0}^{N} \pi_n = 1 \\
&\text{where } v_i \in \{0, 1\}, \pi_n \geq 0,\ i = 0, 1, ..., N-1
\end{aligned}
\tag{5}
$$

Herein, the approximation through AAPWL method is performed.