

Assignment 2

CIS 410/510: Selected Topics on Optimization

Note: Please exhibit your intermediate steps of derivation. Only showing the results will not receive any credits.

Problem 1 (8 points) For the (primal) linear program below, where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, d_3, e_1, e_2, e_3$ are non-zero constants, derive (1) its dual linear program, and (2) the KKT conditions for the dual linear program (note: the KKT conditions that are requested here are for the “dual” program, not for the “primal” program).

$$\begin{aligned} \max_{x_1 \geq 0, x_2 \geq 0, x_3 \leq 0} \quad & a_1 x_1 + a_2 x_2 + a_3 x_3 \\ \text{s. t.} \quad & b_1 x_1 + b_2 x_2 + b_3 x_3 \leq e_1, \\ & c_1 x_1 + c_2 x_2 \geq e_2, \\ & d_3 x_3 = e_3. \end{aligned}$$

Problem 2 (5 points) For the following problem, where $a_{ij}, b_{ij}, c_j, d_i, i = 1, 2, \dots, m, j = 1, 2, \dots, n$ are positive constants, derive its KKT conditions.

$$\begin{aligned} \min \quad & \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n b_{ij} ((x_{ij} + 1) \ln(x_{ij} + 1) - x_{ij}) \\ \text{s. t.} \quad & \sum_{i=1}^m x_{ij} \geq c_j, j = 1, 2, \dots, n, \\ & \sum_{i=1}^m \sum_{j=1}^n x_{ij} + d_i \geq \sum_{j=1}^n x_{ij} + \sum_{j=1}^n c_j, i = 1, 2, \dots, m, \\ & x_{ij} \leq 0, i = 1, 2, \dots, m, j = 1, 2, \dots, n. \end{aligned}$$

Problem 3 (2 points) Give a specific problem as an example to show that, if “Slater’s condition” is not satisfied, the KKT conditions are no longer the necessary conditions to characterize the (local) optimal solutions; or, alternatively, the (local) optimal solutions can violate the KKT conditions. Please (1) write your problem formulation and (2) explain why the KKT conditions are not necessary any more, or explain which KKT condition is violated and how.