Order $n = 3$ problems

Problem 1

Let $x_0 = 0 < x_1 = 0.5 < x_2 = 1$ be the partition of $[0, 1]$, and consider the following problem:

$$\partial_t q - \partial_{xxx} q = 0 \tag{PDE}$$

$$q(x, 0) = f(x) \tag{IC}$$

$$Uq = \sum_{l=1}^{2} R_l \begin{bmatrix} q_l^{(0)}(x_{l-1}) \\ q_l^{(1)}(x_{l-1}) \\ q_l^{(2)}(x_{l-1}) \end{bmatrix} + N_l \begin{bmatrix} q_l^{(0)}(x_l) \\ q_l^{(1)}(x_l) \\ q_l^{(2)}(x_l) \end{bmatrix} = \vec{0}, \tag{BC}$$

where

$$R_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix};$$

$$N_1 = \begin{bmatrix} 0 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad N_2 = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \quad N_3 = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -2 \\ 0 & 1 & -2 \end{bmatrix}.$$

Problem 2

Let $x_0 = 0 < x_1 = 0.2 < x_2 = 1$ be the partition of $[0, 1]$, and consider the following problem:

$$\partial_t q - \partial_{xxx} q = 0 \tag{PDE}$$

$$q(x, 0) = f(x) \tag{IC}$$

$$Uq = \sum_{l=1}^{2} R_l \begin{bmatrix} q_l^{(0)}(x_{l-1}) \\ q_l^{(1)}(x_{l-1}) \\ q_l^{(2)}(x_{l-1}) \end{bmatrix} + N_l \begin{bmatrix} q_l^{(0)}(x_l) \\ q_l^{(1)}(x_l) \\ q_l^{(2)}(x_l) \end{bmatrix} = \vec{0}, \tag{BC}$$

where

$$R_1 = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad R_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -2 & 0 & 0 \end{bmatrix};$$

$$N_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad N_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix} \quad N_3 = \begin{bmatrix} 0 & -2 & 0 \\ 3 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

Order $n = 2$ problems

Problem 3

Let $x_0 = 0 < x_1 = 0.5 < x_2 = 1$ be the partition of $[0, 1]$, and consider the following problem:

$$\begin{aligned} \partial_t q - i\partial_{xx} q &= 0 && \text{(PDE)} \\ q(x,0) &= f(x) && \text{(IC)} \\ Uq = \sum_{l=1}^{2} R_l \begin{bmatrix} q_l^{(0)}(x_{l-1}) \\ q_l^{(1)}(x_{l-1}) \end{bmatrix} + N_l \begin{bmatrix} q_l^{(0)}(x_l) \\ q_l^{(1)}(x_l) \end{bmatrix} &= \vec{0}, && \text{(BC)} \end{aligned}$$

where

$$R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \quad N_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \quad N_2 = \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix}.$$

Problem 4

Let $x_0 = 0 < x_1 = 0.25 < x_2 = 0.5 < x_3 = 1$ be the partition of $[0, 1]$, and consider the following problem:

$$\begin{aligned} \partial_t q - i\partial_{xx} q &= 0 && \text{(PDE)} \\ q(x,0) &= f(x) && \text{(IC)} \\ Uq = \sum_{l=1}^{3} R_l \begin{bmatrix} q_l^{(0)}(x_{l-1}) \\ q_l^{(1)}(x_{l-1}) \end{bmatrix} + N_l \begin{bmatrix} q_l^{(0)}(x_l) \\ q_l^{(1)}(x_l) \end{bmatrix} &= \vec{0}, && \text{(BC)} \end{aligned}$$

where

$$R_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad R_3 = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \quad R_4 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix};$$
$$N_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad N_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad N_3 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \quad N_4 = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}.$$

Problem 5

Let $x_0 = 0 < x_1 = 0.25 < x_2 = 0.5 < x_3 = 1$ be the partition of $[0, 1]$, and consider the following problem:

$$\begin{aligned} \partial_t q - i\partial_{xx} q &= 0 && \text{(PDE)} \\ q(x,0) &= f(x) && \text{(IC)} \\ Uq = \sum_{l=1}^{3} R_l \begin{bmatrix} q_l^{(0)}(x_{l-1}) \\ q_l^{(1)}(x_{l-1}) \end{bmatrix} + N_l \begin{bmatrix} q_l^{(0)}(x_l) \\ q_l^{(1)}(x_l) \end{bmatrix} &= \vec{0}, && \text{(BC)} \end{aligned}$$

where

$$R_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix} \quad R_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R_4 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix};$$

$$N_1 = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \quad N_2 = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} \quad N_3 = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \quad N_4 = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}.$$