Foreign Reserves, Fiscal Capacity, and Lender of Last Resort

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Abstract

Why do emerging markets accumulate foreign reserves for precautionary purposes while advanced economies do not? In this paper, I develop a three period model of an economy subject to aggregate liquidity shocks. My results show that, in contrast to advanced economies, developing countries accumulate reserves for precautionary purposes because they lack the sufficient fiscal capacity - ability to extract resources from its citizens - to provide liquidity during crises successfully. Moreover, I present empirical evidence consistent with this argument. This paper provides a novel rationale for reserves accumulation which, in contrast to the predominant view in the literature, explains endogenously why reserves accumulation as a self-insurance mechanism is mainly observed in developing countries. In terms of policy, it shows that overcoming original sin, avoiding sudden stop episodes, and being fiscally credible, without improving fiscal capacity, may not be sufficient to eliminate the need for foreign reserves.

Keywords: Foreign reserves, Fiscal capacity, Liquidity crises
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*Email: hm409economics.rutgers.edu. Preliminary - Please do not cite. I will greatly appreciate any comments and suggestions. This paper is the first outcome of a never ending exploration around the concept of liquidity and how do governments produce it. I am extremely grateful to Roberto Chang for his continuous support and guidance, specially, for the time he has spent discussing and revising the ideas in this paper. To Michael D. Bordo, I am indebted for his trust and mentorship. Finally, this paper is due greatly to the conversations about foreign reserves with Julio Carrillo during my internship at the BANXICO

1 Introduction

Since the collapse of the Bretton Woods System, monetary authorities across the world have increased dramatically their official holdings of foreign reserves. This build up has been driven by central banks in emerging and developing countries. According to the World Development Indicators dataset, while in 1970 both advanced economies and emerging countries held, on average, the equivalent of 5% of GDP in foreign reserves, that number, by number, had increased to about 10% in the former and close to 20% in the latter.

Central bankers accumulate foreign reserves, in part, as a self-insurance mechanism against balance of payments financing needs.[1] Moreover, empirical evidence supports the effectiveness of foreign reserves as an instrument against global shocks.[2]

Accumulating foreign reserves in emerging and developing countries wouldn't be puzzling, if it were not for the fact that it is costly to do so. As argued since Heller (1966), a country is foregoing a greater social return by investing resources in low-yield but highly liquid reserves and not investing in capital.[3] In addition, implementing this policy is also politically costly since people can push forward different uses for these resources.[4] However, the recent build up is a clear indication that monetary authorities consider that the benefits of holding reserves more than outweigh its costs.

Following Rodrik (2006), if the cost of foreign reserves accumulation is the price of admission to participate safely in financial globalization and given that mostly emerging countries incur in this cost, then two questions naturally follow:

- Why do some countries have to pay the price of safe admission while others do not?
- How is this related to a country's level of development?

[1]See Chamon et al. (2019) for a discussion of the precautionary motive, Ghosh et al. (2017) for an recent estimation of motives behind accumulation, and International Monetary Fund (2011) for the IMF's methodology to calculate reserves adequacy

[2]Frankel and Saravelos (2010) conclude that the stock of reserves is one of the best leading indicators in explaining crisis incidence across countries and crises

[3]Rodrik (2006) estimate this opportunity cost around 1% of GDP per year.

[4]For example, in 2021, national strike promoters in Colombia have stated that their demands could be financed with reserves. See here

In this paper, I argue that low income and emerging economies accumulate reserves precisely because, compared to advanced economies, they don't have a fully developed fiscal capacity[5] to implement ex-post liquidity provision policies successfully.

The intuition is simple. Under the precautionary motive, reserves are accumulated to be used during a crisis. However, this is at odds with Holmström and Tirole (2011) who argue that governments do not need to wastefully hoard liquidity since they can produce liquidity at any given moment by enforcing their power of taxation; in other words, by taking advantage of their fiscal capacity. Yet, as argued by Besley et al. (2013), the level of a country's fiscal capacity is a key feature of economic development. Consequently, governments in emerging countries have a lower fiscal capacity which for some may be insufficient to provide liquidity during crises.

Does this mean that these countries cannot provide liquidity to their economies ex-post? Not necessarily. Precisely, countries that want to be able to generate liquidity during crises accumulate foreign reserves to compensate for their lack of fiscal capacity.

To support this intuition, first, I provide empirical evidence consistent with the hypothesis that countries with lower fiscal capacity tend to have greater stocks of foreign reserves. More importantly, I show that fiscal capacity is empirically relevant even in the presence of other accumulation motives considered in the literature.

Second, I develop a three period theoretical model à la Farhi and Tirole (2012) to how fiscal capacity and foreign reserves compensate each other.[6] More formally, I study an economy that is inhabited by a continuum of banking entrepreneurs who invest in projects that yield high returns relative to the best outside option. Entrepreneurs finance projects with endowments and by issuing claims in international financial markets.

This economy is subject to aggregate shocks. If an aggregate shock materializes, which

[5] Understood as the government's ability to extract resources from its citizens as in Besley and Persson (2014)

[6] The literature that emerged from Holmstrom and Tirole's work on liquidity provision is a natural place to start to formally study why emerging countries accumulate reserves while advanced economies do not, since reserves are accumulated to address, precisely, liquidity shocks

hereinafter I denote as a crisis event, projects require additional investment to survive. I assume that the required reinvestment doesn't make projects insolvent. Projects suffer from limited pledgeability in this economy. That is, the value of the external claims that can be issued - what can be credibly *pledged or promised* to foreign investors - is less than the value of a project's expected total return.

This wedge between total and pledgeable income can be rationalized in many ways,[7] in this paper I explicitly model that entrepreneurs can runaway and default on their loans at no cost after investment and reinvestment decisions are made. Thus, a project's pledgeable income, in equilibrium, is equal to the maximum amount that can be promised to outside investors without driving entrepreneurs to runaway.[8]

By assuming limited pledgeability, the model departs from the Arrow and Debreu complete markets assumption. And, as a result, a crisis can cause solvent projects to shutdown when entrepreneurs are unable to attract sufficient liquidity to cover the reinvestment costs.

However, to prevent the possibility of shutting down if a crisis materializes, entrepreneurs can preemptively hoard liquidity to use if necessary. Yet, in a laissez-faire equilibrium, I find that they choose not to insure when the likelihood of the crisis is sufficiently low. This result is consistent with Holmström and Tirole (1998) who show that partial insurance is optimal since there is a trade-off between accumulating liquidity and initial investment scale. In other words, hoarding liquid assets that might up not being used is costly.

Holmström and Tirole (1998) also show that in environments with aggregate liquidity shocks such as this, government intervention is welfare improving if it acts as a *broker* between liquid and illiquid agents.[9] However, this result implicitly assumes that governments have a well developed fiscal capacity such that they have the ability to supply liquidity ex-post without incurring in wasteful liquidity hoarding.

[7] For example, resources diversion, moral hazard, adverse selection, or, in an open economy setting, the amount of tradable wealth produced.

[8] As argued by Holmström and Tirole (2011), assets backed up by pledgeable income, by definition, are non defaultable safe assets

[9] In the case of my model, between entrepreneurs and international financial markets

To see how a government's fiscal capacity affects its incentives to accumulate reserves, I compare the implementation of a lender of last resort policy (ex-post liquidity provision policy) by two different type of governments: a mature government versus an immature government. Both governments issue safe sovereign bonds in international financial markets backed up by their power to tax.

The difference between government types consists on their fiscal capacity. The mature has a sufficiently developed fiscal capacity such that it can extract resources from entrepreneurs regardless if they runaway or not, while the immature government can only collect resources when entrepreneurs do not runaway. I show that, in equilibrium, the difference in fiscal capacity implies that the total amount of resources that a mature government can collect equals projects total return while the immature government can only collect up to projects pledgeable income.

As a result, both governments can implement an ex-post policy that assures full-scale reinvestment of projects and avoids any potential liquidation or shutdown. The main difference, and the most important result of the paper, is that the immature government is forced to accumulate reserves ex-ante to do so while the mature government does not. Therefore, this model indicates that foreign reserves accumulation is an optimal response of a government that cares about providing liquidity ex-post but doesn't have a sufficiently developed fiscal capacity.

At the macro level, this result shows that aggregate liquidity supply, as argued by Calvo (2016), is bounded by an economy's pledgeable income and wealth. Thus, ex-post government intervention is successful as long as it expands the frontier of pledgeability and avoids *deflating liquidity*. A mature government does so by using its fiscal capacity to override the wedge between pledgeable and total income while the immature government accumulates reserves ex-ante to do so.

At the micro level, the intermediation role between international financial markets and entrepreneurs by a mature government allows projects to issue claims backed up by their total return, thus, their financing is no longer limited by pledgeable income. Meanwhile, with an immature government, entrepreneurs have access to a cheaper financing source that increases the relative value of pledgeable income just enough to allow for a full-scale continuation. To offer public loans with a less expensive rate than world markets, the immature government must accumulate reserves such that the burden of paying back sovereign bonds is distributed between entrepreneurs and reserves.

In the paper, I also show that ex-post public liquidity provision policies such as a lender of last resort lowers entrepreneurs' incentives to hoard liquidity themselves. In other words, private agents are willing to take more risks. This ex-ante moral hazard result is not new in the literature, and, in fact, it is also one of the greatest criticism to these type of policies.[10]

However, I also show that, under certain parameter values and only when the government is immature,[11] this ex-ante moral hazard is potentially preventable since an equilibrium with no private liquidity hoarding but positive foreign reserves accumulation coexists with an equilibrium with positive private liquidity hoarding but no foreign reserves accumulation. As in Farhi and Tirole (2012), multiplicity in equilibria is the result of strategic complementarities between entrepreneurs.

These complementarities are only present in the model under an immature government and not under a mature government. This result highlights that what connects entrepreneurs private insurance decisions is the cost of implementing a ex-post liquidity provision program. With no cost, a government can provide liquidity to one entrepreneur or to the continuum of entrepreneurs. However, when it is costly, a government only incurs in that cost if a sufficient share of entrepreneurs would require liquidity during a crisis.

Contribution to the literature. This paper contributes primarily to the literature by building an explicit bridge between foreign reserves accumulation for precautionary purposes and development. Aizenman and Lee (2007) was one of the first papers to empirically show

[10]See, for example, Bordo (1990) that describes it as one of the major critiques to the lender of last resort

[11]when the likelihood of a crisis is relatively high and the welfare cost of projects downsizing is sufficiently high

that precautionary motives explain an important share of the build up in foreign reserves by monetary authorities.[12]

Most of the extensive work on understanding why developing countries have accumulated a *war chest* of reserves argue that the demand for reserves comes from, at least, one of the following traits: currency mismatch, sudden stops and severe capital outflows. For example, Chang and Velasco (2001) place international illiquidity (currency mismatch) at the center of the financial fragility of emerging economies, thus, a lender of last resort that provides liquidity in foreign currency could alleviate the financial instability. But to do so, it needs to accumulate dollars ex-ante to provide dollars ex-post. Aizenman and Lee (2007) develop a model where the demand for reserves comes from a government's objective to stabilize output and consumption during sudden stops. And Obstfeld et al. (2010) argue that domestic financial risk comes from the possibility of domestic residents withdrawing their resources from the economy and buying assets abroad (capital outflows). In this environment, these authors conceive foreign reserves as a key policy tool to protect the domestic credit markets while limiting external currency depreciation.

In turn, a popular idea in the literature argues that currency mismatch, sudden stops and severe capital outflows episodes can ultimately traced back to developing economies suffering from *Original Sin*. This concept was conceived by Eichengreen et al. (2003) who define Original Sin as a country's inability to issue debt in domestic currency in international markets due to structural features of global financial markets. Consequently, as pointed out in International Monetary Fund (2011), reserve accumulation concerns are innocuous in advanced economies because they are reserve currency issuers or, if not, they can borrow in their own currency.

Such interpretation means that foreign reserves accumulation is due to factors that are exogenous to developing countries and are not related to their level of development. In contrast, I argue explicitly that a governments fiscal capacity is sufficient to explain why

[12] See Ghosh et al. (2017) for more recent estimations

low income and emerging economies hoard foreign reserves for precautionary purposes while advanced economies do not.

The model I develop is a real model and, as such, there is no currency mismatch nor there is a real exchange rate since there is only one good in the economy. Additionally, I assume that neither entrepreneurs nor the government loose access to international financial markets either to supply (capital flight) nor to demand resources (sudden stops).

Nevertheless, by only assuming a difference in fiscal capacity and a government that wants to act as a lender of last resort, the model produces a positive demand for foreign reserves in equilibrium. Consequently, I contribute to this literature by arguing that foreign reserves accumulation can be explained beyond currency mismatch, capital flight, and sudden stops.

In parallel, given that accumulating reserves is costly, a branch of this literature have put in effort to estimate the optimal level of foreign reserves holdings. Heller (1966) studied the optimal level for an economy facing trade shocks while Jeanne and Rancière (2011) estimate such level for an economy with the objective to insure against financial dry-ups in international markets and capital outflows. Similarly, Céspedes and Chang (2019) do so for a government that accumulates reserves to alleviate financial frictions.[13] In my paper, I don't study the optimal level of reserves. Instead, I focus on the necessary level of reserves that a government requires, given its fiscal capacity, to implement successfully ex-post liquidity provision policies.

Recently, some interesting work has emerged that relates foreign reserves accumulation with sovereign default. This body of literature has focused on analyzing optimal reserves holdings when governments can also choose the amount of sovereign debt. Alfaro and Kanczuk (2009) find that under this context the optimal level of reserves is zero. The reason is that reserves are a tool to smooth consumption during defaults, and, as such, reduces the opportunity cost of defaulting. As a result, the cost of debt is higher with positive levels of reserves, and the government chooses optimally to not accumulate. In contrast, Bianchi

[13] An interesting finding of this model is that the optimal level depends on the type of policy that is implemented during times of distress

et al. (2018) find that the optimal level of reserves is positive since reserves provide an insurance against rollover risk. For this result, it is key that the government issues long-term debt, otherwise, with one-period bonds, the model goes back to Alfaro and Kanczuk (2009)'s result. In my model, I assume that the government issues safe debt, thus, I eliminate any potential effect of reserves on sovereign's repayment decision. However, consistently with the data, the equilibrium with an immature government reproduces a state with positive levels of government debt and foreign reserves. The reason is that an immature government lacks the fiscal capacity to back up the necessary amount of debt, thus, reserves are useful for the government because it provides an additional real asset against to which issue its sovereign bonds. Once again, my results show that demand for reserves persist even if governments can issue safe government debt during crises.

Tavares (2018) in an modified canonical sovereign default model and Aizenman and Marion (2004) in a two period model where the government faces sovereign risk find that distortionary and costly tax collection increases the demand for reserves. The intuition behind is that reserves can be used to finance government expenditure during defaults instead of increasing tax collection which is costly. Thus, reserves and taxes are substitutes. In contrast, I assume that collecting resources is costless and non-distortionary. Instead, the total amount of taxes that can be collected by a government is limited by its fiscal capacity. Although this seems as a minor difference, it is not. Reserves in my model complement tax collection as sources to finance lender of last resort policies.

My paper is closest to Dominguez (2009) in this literature. This author test the hypothesis and finds evidence that part of the surge in foreign reserves holding is motivated by the goal to compensate for financial underdevelopment.[14] Hence, foreign reserves are not used as liquidity instrument per se but as instruments to increase pledgeability. The difference with my rationale is that Dominguez (2009) focuses on underdevelopment of the financial

[14]Foreign reserves are accumulated by sterilized intervention in the economy. Thus, as foreign currency is taken out circulation, an sterilized bond for the same value in local currency in put in circulation. This bond becomes an instrument that can be used to transfer liquidity across periods which become relevant when there is scarcity in these types of liquid instruments

sector while I focus on the underdevelopment of fiscal capacity.

Lastly, as discussed briefly previously, this paper also contributes to the research emanated from the work of Holmström and Tirole (1998). In my paper, by making explicit how pledgeable income is determined, I highlight that the effectiveness of government ex-post interventions doesn't rest solely on having the power to tax agents but on how is the economy being taxed. Governments maybe forced to accumulate market liquidity to circumvent lower development levels of that power of taxation. This is, to the best of my knowledge, the formalization of Tirole (2002) idea that increases in public debt may fail to increase aggregate liquidity if the expected tax incidence falls on the same agents that are illiquid.

The following section presents empirical evidence to support the argument of this paper. That is, I show that countries that have lower fiscal capacity tend to hoard greater stocks of foreign reserves. I follow that section with a description of the model and the solution of the laissez faire equilibrium. Then, the core of the paper is described with the presentation of the basic actions of a government and solving for the two different types of governments considered. I finish with a comparison between equilibria and some remarks about what lies ahead.

2 Foreign Reserves and Fiscal Capacity in the data

In this section, I take the main argument of this paper to the data.

Following Besley and Persson (2014), I measure a government's fiscal capacity using income tax revenue as a share of total tax revenue. These authors argue that collecting income tax requires major investments in enforcement and compliance mechanisms compared to other taxes such as tariffs. Both enforcement and compliance mechanisms ultimately improve a government's ability to extract resources from its citizens making income tax revenue an adequate proxy for fiscal capacity.

I collect data of foreign reserves, Gross Domestic Product (GDP), and tax revenue, total and income tax, between 1990 and 2018 for 206 countries from the World Development Indicators dataset.

Figure 1 plots country averages between 1990 and 2018 of foreign reserves holdings against total tax revenue (Panel 1a) and against fiscal capacity (Panel 1b).[15] While there is a weak positive correlation between foreign reserves and total tax revenue, Panel 1b shows that countries with a greater share of taxes collected through income tax tend to have lower levels of foreign reserves in line with the prediction of the model.

The naive approach depicted in Figure 1 provides some initial evidence that countries with lower fiscal capacity tend to accumulate larger stocks of foreign reserves. For a more rigorous exercise, I build on the previous work that empirically estimates the motives behind the accumulation of foreign reserves by emerging markets since 1990.[16]

The predominant empirical approach of this literature assumes that the stock of foreign reserves in year t held by country j ($\frac{FXR_{j,t}}{GDP_{j,t}}$) is a function of the exchange rate regime, the potential financing needs coming from the balance of payments (precautionary motives) and the mercantalist motive.[17] Additionally, to limit the potential endogeneity problems, most regressors are usually lagged one year except those that capture the exchange rate regime.

$$log\frac{FXR_{j,t}}{GDP_{j,t}} = \beta X_{j,t-1} + \alpha_0 log(\frac{TR_{j,t-1}}{GDP_{j,t-1}}) + \alpha_1 log(\frac{ITR_{j,t-1}}{TR_{j,t-1}}) + \varphi^t + \epsilon_{j,t} \tag{1}$$

To test whether fiscal capacity has any empirical power, I add to the predominant view model - captured by matrix $X_{j,t-1}$ in Equation 1, two additional regressors: i) the measure of fiscal capacity discussed previously (income tax revenue as share of total revenue - $\frac{ITR_{j,t-1}}{TR_{j,t-1}}$); ii) and total tax revenue ($\frac{TR_{j,t-1}}{GDP_{j,t-1}}$) to control for scale effects of tax collection that I don't want them to be captured by our measure of fiscal capacity.

Any support for the hypothesis in this paper depends on obtaining a negative estimate

[15]First, I take logs on foreign reserves (% of GDP), tax revenue (% of GDP), and income tax revenue (% of tax revenue), then I calculate the country average between 1990-2018 for each variable

[16]See, for example, Aizenman and Lee (2007), Obstfeld et al. (2010) and Ghosh et al. (2017)

[17]This motive understands foreign reserves accumulation as the by-product of a development strategy to explicitly undervalue the currency

for α_1. At this point, it is worth mentioning that the objective of this exercise is to evaluate whether there is a significant correlation between foreign reserves and fiscal capacity. The chosen approach doesn't have the power to establish causality, however, I use the theoretical model to provide an explanation of why such correlation can potentially exist.

Moreover, note that Equation 1 includes time fixed effects but not country fixed effects. The reason is that the empirical prediction of the model refers to a between countries and not within countries comparison since the level of fiscal capacity is exogenous. Given a lower fiscal capacity, an economy accumulates more reserves. The model doesn't provide any theory on how fiscal capacity evolves over time, and how that could be related with foreign reserves.

I follow Obstfeld et al. (2010) to determine the variables that are part of matrix $X_{j,t-1}$. These authors divide the different motives behind reserves hoarding considered by the literature into *models*. As argued by Heller (1966), reserves are useful because they can alleviate adjustment costs to external imbalances. Thus, the *traditional model* comprises variables that capture risks emanating from the current account. In this exercise,[18] I include imports of goods and services ($log(\frac{M}{GDP})$) and the three-year standard deviation of exports over GDP as a measure of volatility in receipts from the world. Additionally, I include the annual standard deviation of monthly exchange rate variation to capture the risk of what Heller (1966) called the expenditure-switching adjustment. Lastly, the traditional model includes the size of the economy ($log(GDP)$) to control for scale effects since the dependent variable is also in terms of GDP.

The second model captures financial stability risks. As argued by Obstfeld et al. (2010), both an internal (from deposits to currency) as well as an external drain (from domestic to foreign assets) can turn into a balance of payment crisis. Thus, this model includes the share of broad money in the economy and the external short term debt, again relative to the economy, to capture these two risks, respectively. Additionally, I include the Chinn-Ito

[18] Same as Ghosh et al. (2017)

Index (normalized) as a measure of a country's financial openness since where capital moves more freely it is more likely that a financial crisis turns into a balance of payment crisis. This group also includes dummy variables for a country being an high income as defined by the World Bank, and whether the country was implementing during the respective year a hard peg or a soft peg according to Ilzetzki et al. (2019) exchange rate regime index.[19]

The third group is the mercantalist model. Aizenman and Lee (2007) were one of the first to empirically consider the mercantalist strategy as a explanatory variable behind foreign reserves accumulation. A challenge with this motive is that there is no clear way how to measure a government's intent to undervalue their currency to promote their export sector. Aizenman and Lee (2007) use the *Penn Effect* while Ghosh et al. (2017) consider three different methodologies including non-public IMF assessments of currency undervaluation. Due to data availability, I follow Dominguez (2009) by measuring currency over-valuation equal to the ratio between the Parity Purchasing Power (PPP) conversion factor and the market exchange rate minus one. Hence, if this index is positive than the currency is considered to be over-valued and when it is negative it is undervalued.

The traditional model, the financial stability model and the mercantalist model comprise the existing predominant view in the literature on what drives foreign reserves accumulation, specially in emerging markets.

I include a model that captures the level of development of the financial sector. This is motivated by Dominguez (2009) who provides empirical evidence that countries with underdeveloped capital markets tend to accumulate more reserves.[20]. Dominguez (2009) tests her hypothesis by including the sum of domestic private credit creation and stock market capitalization (Domestic Financial Liabilities as % of GDP),[21] as a measure for financial development, together with the size of the external balance sheet broken down by

[19]Hard pegs are countries whose Ilzetzki et al. (2019) Fine index was less or equal to 9 or equal to 11, while soft pegs corresponds to categories 10 or 12.

[20]The reasoning is that foreign reserves are an instrument that can be used to offset financial constraints. See Céspedes and Chang (2019) on how reserves alleviate financial frictions

[21]These authors, in their paper, consider two other measures for financial development. However their results remain the same regardless of the measure.

private and public sectors.

This *financial development model* is a key group of regressors because a lender of last resort is by definition only necessary when there are no other lenders. Thus, in the reasoning presented in this paper, fiscal capacity becomes a relevant feature behind foreign reserves accumulation because financial markets are underdeveloped. Excluding the financial development model from this empirical exercise would cause the approach to suffer from endogeneity due to an omitted variable.

Lastly, one of the main arguments in this paper is that fiscal capacity is a relevant motive for foreign reserves hoarding even in an economy with no currency mismatch. I test this argument by including an *original sin model* in the empirical exercise.

Countries that suffer from original sin are more likely to experience an external currency mismatch which increases financial fragility[22]. Yet, as discussed in Hausmann and Panizza (2010), a government can offset this mismatch by accumulating foreign reserves. Thus, by including a measure for original sin,[23] I control for an economy's currency mismatch motive for the build up in foreign reserves.

I collected annual data for both advanced economies and low income and emerging economies between 1990-2018.[24] I exclude countries that had less than five observations available for the period of analysis as well as economies with a population lower than one million people. The final dataset is an unbalanced panel for 100 countries, of which 29 are advanced economies according to the IMF. Table 9.1 presents the summary statistics of the different variables.

First, I run Equation 1 without the Original Sin model. I do this because the original sin index is only available starting 2000. Table 9.2 presents the results for 6 different sample groups. The first column of each sample presents the results when I exclude the Fiscal Capacity model. The second column are the results when I include this model to the

[22]See Chang and Velasco (2001)

[23]This index ranges from zero to one and reflects country's fraction of international debt securities denominated in foreign currency

[24]Appendix describes the data, how variables were constructed and the original sources.

regression.

Columns I and II present the results for the whole sample. Note that the estimate for fiscal capacity (Income Tax revenue as share of total revenue) is significant with a negative sign as expected. Thus, countries with lower fiscal capacity tended to accumulate more reserves.

Columns III and IV are the results when only considering the emerging and developing countries, as classified by the IMF, in the dataset. Once again, the estimate of fiscal capacity is as expected.

Ghosh et al. (2017) point out that the motives behind foreign reserve accumulation could be shifting through time. To see this with regard to fiscal capacity, I chose as a breaking point the Global Financial Crisis (GFC).[25] Columns V and VI present the results for the period 1990-2007 while columns VII and VIII for 2010-2018. During both periods, the sign of fiscal capacity is negative but the estimate is only significant post GFC.

The dataset is an unbalanced panel due to the lack of data for some variables for some years. If there is an element in each country (i.e. quality of institutions) that explains this lack of data as well as fiscal capacity and foreign reserves accumulation, then the estimate of interest is biased. I identify 17 countries that have data for every year and I run Equation 1. The results (Columns IX and X) not only show that the estimate for fiscal capacity is negative and significant, but also that it is the double in magnitude.

Countries in the euro zone are part of a monetary union but not a fiscal union. That is, they share the same monetary authority but their fiscal capacity is idiosyncratic. In addition, most of the external debt of these countries is in euros which suggest that they don't experience currency mismatch, and, the national central banks of the system still have the prerogative to act as lenders of last resort. This context closely follows the assumptions of the model. Hence, it is an adequate sample to test whether fiscal capacity can explain the

[25] I select this point first because it put to the test the accumulation of foreign reserves as a self-insurance mechanism. And, second, because it is possible to consider that the policy response, such as the implementation of swap lines between reserve central banks and other central banks, could have modify the effect of fiscal capacity on reserves accumulation.

variance in levels of foreign reserves between countries.

Columns XI and XII present the results for nine euro zone countries between 1999 and 2018. These countries are part of the first group to put in motion the euro as a currency back in 1999.[26] For this sub-sample, estimates show that countries with lower fiscal capacity tended to have a greater stock of reserves in line with the results of the model developed in this paper.

Table 9.2 results support that fiscal capacity matters for foreign reserves accumulation. I now move to including original sin in the exercise Table 9.3 presents the estimates of these regressions.

Fiscal capacity is negatively correlated in most sub-samples with foreign reserves. Unlike the results that excluded original sin, fiscal capacity estimates are significant both before and after the GFC. However, the estimate in the regression with the balanced panel lost significance but the sign remains negative. Overall, the results still support that countries with lower fiscal capacity hold more foreign reserves even when controlling by original sin.

Note, as well, that the estimate for the original sin index is positive and significant in most regressions. Hence, countries with greater original sin have larger stocks of foreign reserves.

In terms of other variables, the results are robust regardless of including, or not, original sin. The results show a robust positive relationship between hard and soft pegs with foreign reserves stock. This is not surprising since reserves are necessary to implement less flexible exchange rate regimes.

Additionally, consistent with Obstfeld et al. (2010), there is evidence that financial stability concerns guide a share of the build up in foreign reserves as indicated by the positive and robust relationship between the dependent variable and broad money. Also consistent with these authors, my results show a negative sign for external short-term debt. This is opposite to the Guidotti-Greenspan rule that suggests that country should accumulate re-

[26]This initial group consisted of eleven countries, however, Spain is not part due to data availability and Luxembourg has a population lower than one million.

serves to cover any potential demand for repayment derived from a country's short term debt. Obstfeld et al. (2010) explain this finding to be consistent with the fact that countries accumulate reserves far in excess than short-term debt obligations.

Moreover, similar to Aizenman and Lee (2007) and Ghosh et al. (2017), I find support for the mercantalist motives. Countries with undervalued currency tend to have bigger stocks of reserves.

Lastly, I run a robustness check of Equation 1. Table 9.4 presents four different specifications for the model excluding original sin, and the same four for the model with original sin. Columns I and V is the main specification which includes year fixed effects (YE), columns II and VI includes country fixed effects (CFE), columns IV and VII includes both year and country fixed effects while columns IV and VIII is a cross-section regression where the observations are the panel average for each country.

Results show that the statistical significance of the correlation between fiscal capacity and foreign reserves is lost in the specifications with country fixed effects. This suggests that the within variation in foreign reserves is driven mainly by the predominant view variables. In addition, in line with Dominguez (2009), financial development appears significant and negatively correlated with foreign reserves with country fixed effects.

Interestingly, the estimate for original sin index changes sign with country fixed effects. This result is surprising since it is believed that lower currency mismatch leads to a lower need for foreign reserves. Taking stock, this section provides both naive and more formal evidence that countries with lower fiscal capacity accumulate more foreign reserves. This empirical evidence is robust to including both measures of original sin and other motives considered in the literature. The model developed in this paper suggests that the reason for this correlation is that governments want to act as lenders of last resort during crises but lack the ability to extract sufficient resources from their citizens. Consequently, they compensate their lower fiscal capacity by accumulating resources ex-ante.

3 Model

To study the role of foreign reserves in ex-post liquidity provision, I built a model à la Holmström and Tirole (1998) with an environment similar to Farhi and Tirole (2012).

Consider a three period small economy ($t = 0, 1, 2$) inhabited by two types of agents: entrepreneurs and the government. There is a continuum of entrepreneurs with population normalized to 1. Additionally, agents can trade, consume and invest the only perishable good existing in this economy. Moreover, both entrepreneurs and the government have access to the international capital market where they can issue one-period non-defaultable claims, either at date-0 or at date-1.

Foreign lenders that participate in this world capital market are deep pocketed and they are willing to lend resources to the small economy as long as they obtain the same expected return that they would get from lending at the world market equilibrium rate. Denote by γ_t the equilibrium return obtained in this international market between date-t and date-$t+1$. For simplicity, I normalize γ_t equal to 1 for both date-1 and date-2. The economy is assumed to be relatively small enough such that equilibrium returns of the international capital market are not affected by decisions of either entrepreneurs or the government.

Entrepreneurs are risk neutral agents that receive an initial endowment at date-0, denoted by A, of the only good in the economy. These agents do not receive further endowments in date-1 nor date-2 and are protected by limited liability.[27] Entrepreneurs can consume the initial endowment at date-0, they can lend it in the world capital market at the given rate, or they can use it to invest in a project.

Only entrepreneurs have access to this ***Project Technology*** (Figure 2). Following Farhi and Tirole (2012), this investment technology exhibits constant returns of scale. When i units of the perishable good are invested at date-0, it generates a safe cash flow of πi at date-1. If the project reaches date-2, it produces a total return of ρ_1 per unit of investment.

Date-2's project total return, however, is contingent on the realization of a negative

[27]Consumption levels cannot be negative in any period

aggregate shock to the economy. With probability $1-\alpha$, the economy experiences a *crisis* during date-1.[28] In such scenario, the project requires a reinvestment at date-1 to continue to date-2 and generate any return. If a reinvestment equal to j is made, the project continues to date-2 and produces a total return of $\rho_1 j$.[29] Whereas, if no reinvestment is made, the project is shutdown at date-1 and doesn't generate any return beyond the safe cash flow.

If the economy doesn't experience a crisis at date-1,[30] the project continues to date-2 without the need of any reinvestment. In that case, the project generates a total return of $\rho_1 i$. Note that, as Figure 2 shows, the realization of the aggregate shock occurs after the project produces the safe cash flow, thus, this source of income is not contingent to the economy's aggregate state.

As mentioned before, only entrepreneurs have access to these projects. However, these agents are also credit constrained since they only receive an initial endowment at date-0. Thus, this not only limits the initial investment scale but it also makes more challenging to finance any reinvestment if a crisis were to occur. Potentially, as long as projects are capable of generating the same expected return than the world capital market, an entrepreneur could sell claims to foreign lenders, backed up by the expected return of the project, to cover both initial investment and reinvestment levels.

Assumption 1 (Project's Expected High Return at Date-0)

1.1 $\rho_1 > 1 - \pi + (1-\alpha)$

1.2 $\alpha\rho_1 + \pi > 1$

Assumption 1 assures that projects produce returns that are attractive to entrepreneurs and foreign lenders. Assumption 1.1 indicates that projects' return is sufficient to cover for the expected value of investment.[31] Assumption 1.2, meanwhile, shows that the expected

[28] It is natural to assume that the probability of the good state is greater than the probability of a crisis ($\alpha > (1-\alpha)$)

[29] The reinvestment amount j cannot be greater than the initial investment scale i

[30] An event with probability α

[31] Initial investment plus the expected cost of reinvestment

return of the project, even under no expected reinvestment, is a more attractive investment option than lending in both periods in the world capital markets.

To close out with the description of the setup, the government in this environment is concerned about projects survival to date-2. Specifically, the government can collect taxes from entrepreneurs and issue government bonds in the world capital market. Collected resources are used to act as a *lender of last resort* to entrepreneurs providing another source of finance during a crisis. Following Bagehot's rule, the government implements an ex-post liquidity provision program to lend freely to illiquid but solvent agents with good collateral.[32]

As mentioned earlier, the setup of this model is similar to Farhi and Tirole (2012). These authors focus on ex-post liquidity provision policies involving, mainly, reductions in the economy's interest rate.[33] However, monetary independence is not always on the table for some countries, in particular developing countries.[34]

In this paper, I study a net borrower small open economy whose interest rate is determined directly by international financial markets. Moreover, I assume a government that doesn't manage the capital account. Instead, it can collect taxes and make transfers to the economy.

A second important difference with Farhi and Tirole (2012) is that these authors take as exogenous the amount that entrepreneurs can credibly promise to investors. Hence, they abstract from any potential effects that government programs have on the economy's pledgeability.

On the other hand, I explicitly model the amount that entrepreneurs can pledge to foreign investors. By doing so, I allow for different types of governments to have different effects on pledgeability. In the following section, I describe how I model pledgeability.

[32] See Bordo (1990) and Fischer (1999) for a discussion about Bagehot's rule.

[33] which they find is a more preferable policy tool than transfers if governments cannot adequately identify firms in distress from those that are not

[34] For example, Rey (2015) argue that the existence of a global financial cycle transformed the open macroeconomic trilemma into a "irreconcilable duo" where national monetary independence is only possible when a managed capital account; regardless of the exchange rate regime. Similarly, central banks in small open economies could be more driven to increase, not decrease, the policy rate during capital outflow episodes to prevent further outflows

3.1 Moral Hazard and Limited Pledgeability

The demand for liquidity results from the lack of synchronicity between revenues and outlays (Tirole, 2011). In this model, demand liquidity appears because investments must be made at date-0 and, potentially, at date-1, while returns are realized at date-1 and date-2.

Entrepreneurs cover liquidity needs either by using the liability side of its balance sheet (funding liquidity), or by using the asset side (market liquidity).[35] In this model, entrepreneurs tap on their funding liquidity by issuing claims at the world capital market (private funding liquidity) or by borrowing from the government (public liquidity). And as market liquidity, they can use their initial endowment at date-0, and, plausibly, any return they receive from the project or world capital markets at date-1.

Definition 1 (Date-2 Pledgeable Return)
Date-2 pledgeable return, denoted by ρ_0, is the maximum share per unit of investment that an entrepreneur can credibly promise outside investors

Any claim issued by an entrepreneur is backed up by a project's date-2's pledgeable return (Definition 1) and, if issued at date-0, also by the safe cash flow. With frictionless financial markets, a project's date-2 pledgeable return is equal to its total return (ρ_1), thus, an entrepreneur could always *finance as it goes* its liquidity needs. However, financial markets imperfections are common and limit the capacity to attract funding liquidity.

I introduce a friction to entrepreneurs' funding liquidity by assuming that they are subject to moral hazard. At date-2, an entrepreneur can run away with a fraction θ of the project's total output. If this happens, the remaining fraction $1-\theta$ is lost.

$$R_e j \geq \theta \rho_1 j \tag{2}$$

Foreign Lenders are aware of moral hazard, thus, they lend only to entrepreneurs that

[35] See Tirole (2011) for a further discussion on market and funding liquidity

promise to not run away. The credibility of this promise rests on the incentive compatibility constraint (Equation 2). When this constraint holds, the project's return that is allocated to the entrepreneur ($R_e j$) is sufficiently high such that it is in her benefit to not run away.

Limited pledgeability ($\rho_0 < \rho_1$) is a consequence of moral hazard in this model.[36] As long as θ takes positive values, satisfying Equation 2 assures limited pledgeability. Moreover, the greater moral hazard is, the greater is the gap between total and pledgeable return.

In turn, limited pledgeability prevents that every potential liquidity need can be financed solely by selling claims to foreign lenders. More precisely, although projects are capable of generating sufficient return to cover equilibrium returns (γ_t lower than ρ_1), entrepreneurs will not be able to attract foreign investors when γ_t is greater than ρ_0. For such set of γ_t, projects remain socially attractive,[37] but potentially shutdown due to the inability to produce sufficient credible liquidity.

Assumption 2 (Liquidity Constrained Projects)

1. $1 > \pi + \alpha\rho_0$

2. $\rho_1 > 1 > \rho_0$

3. $1 > \pi \geq 1 - \rho_0$

Assumption 2 holds through out this paper. This assumption assures that projects are credit constrained both at date-0, and at date-1 in case of a crisis ($1 > \rho_0$). Numeral 1 of Assumption 2 implies that expected pledgeable income of the project, followed by no reinvestment, is not enough to cover the opportunity cost of foreign lenders ($\gamma_0 = 1$). Otherwise, the scale of investment would be indeterminate since a project would produce sufficient pledgeable income per unit of investment to attract infinite amounts of funds from foreign lenders.

Numeral 2 assumes that projects are socially valuable in every state of nature because they offer a greater return than world capital markets ($\rho_1 > 1$) but they cannot self-finance

[36]See Holmström and Tirole (2011) for different ways to model an agency wedge between total and pledgeable return

[37]With a net present positive value

$(1 > \rho_0)$. In turn, Numeral 3 establishes that projects do not generate sufficient safe-cash flow, per unit of investment, for a positive reinvestment with solely market liquidity $(1 > \pi)$. Nevertheless, total safe cash flow together with the maximum amount of funding liquidity are, at least, just enough to finance a full-scale reinvestment $(\pi + \rho > 1)$.

Limited pledgeability is the symptom but moral hazard is the culprit. Any government program that seeks to alleviate liquidity shortages, ex-post, is effective either by solving moral hazard (increasing funding liquidity capabilities of the economy) or by circumventing it (increasing available market liquidity). In this paper, I show that governments that are unable to address moral hazard, choose optimally to accumulate foreign reserves ex-ante to implement successful ex-post liquidity provision policies.

4 Laissez Faire Model

When are government liquidity provision policies warranted in this environment? To answer this question, I start by solving the Laissez Faire equilibrium (LFE).

With no government, note that entrepreneurs are the only active decision makers in the model. Foreign lenders do not have a maximization problem, but, as described before, they are willing to lend to any entrepreneur as long as, the expected return is, at least, equal to the world capital market equilibrium rate and, the incentive compatibility constraint is satisfied.[38]

Consequently, entrepreneurs can offer a contract $C_0 = \{i,\ M_0,\ \phi_0,\ R_e^{NC},\ R_f^{NC},\ R_e^{C},\ R_f^{C}\}$ to foreign lenders at date-0 that stipulates the initial investment scale i, the amount of entrepreneur's market liquidity to be invested M_0 at date-0, the amount to borrow at date-0 ϕ_0 from investors, the expected payoffs for the entrepreneur and investors if the crisis doesn't happen, denoted by $(R_e^{NC},\ R_f^{NC})$, respectively, and the expected payoffs if the crisis is realized (R_e^{C}, R_f^{C}).

[38] In other words, private claims are safe.

After a crisis is materialized, entrepreneurs have the option to go to the world capital market a second time to acquire the necessary funds for reinvestment. This time, entrepreneurs offer foreign investors a contract $C_1 = \{j,\ M_1,\ \phi_1,\ R_e,\ R_f\}$ that signals the reinvestment amount j, the amount of entrepreneur's market liquidity to be invested at date-1 (M_1), the amount to borrow at date-1 from foreign investors (ϕ_1), and the allocation of the project's return between entrepreneurs (R_e) and investors (R_f). At this point, entrepreneurs take as given the initial investment scale i, and the amount of available resources for market liquidity x.[39] Entrepreneurs make a final decision at date-2 which is whether to run away with a fraction θ of the project's return, or to stay and follow through with foreign investors.

This is a three period model which can be solved through backward induction. I focus on the conditions for a Subgame Perfect Nash Equilibrium when entrepreneurs choose to not run away. The benefit of this type of equilibrium is that it produces time consistent outcomes.

Definition 2 (Laissez Faire Equilibrium (LFE))
A Laissez Faire Subgame Perfect Nash Equilibrium where entrepreneurs' don't run away is characterized by the following strategy profile

- *Date-2: Entrepreneurs' don't run away*
- *Date-1: C_1 maximizes the Entrepreneur's Date-1 Problem*
- *Date-0: C_0 maximizes the Entrepreneur's Date-0 Problem*

4.1 Date-2 Optimal Behavior

Proposition 1 states that date-2's pledgeable income in a LFE is equal to $\rho_0 = \rho_1(1-\theta)$. That is, if a project reaches date-2, the return per unit of investment for that period promised to foreign investors cannot be greater than ρ_0. Otherwise, the entrepreneur would be better off by running away. Additionally, I assume that the safe cash flow is fully pledgeable since the moral hazard opportunity for an entrepreneur rises at date-2.

[39]Note that x is equal to the payoff of date-1 following a crisis $R_e^C i$ plus any amount that wasn't invested in the project but lent in the world market valued at $(A - M_0)$ at date-1.

Proposition 1 (Pledgeable Return with no Government)
Let Date-2's pledgeable return, ρ_0, equal to $\rho_1(1-\theta)$
In the Laissez Faire Equilibrium, entrepreneurs will not run away as long as

- R_f^{NC} *is less or equal to* $\rho_0 + \pi$ *when no crisis materializes*
- R_f *is less or equal to* ρ_0 *when a crisis materializes*

Proof Let $\rho_0 := \rho_1(1-\theta)$ When a crisis doesn't materialize, the project's return $\rho_1 + \pi$ is distributed between $\{R_f^{NC}, R_e^{NC}\}$. For a given investment scale j, suppose that R_f^{NC} is less or equal to $\rho_0 + \pi$ but an entrepreneur chooses to run away. If so, by Equation 2, $\theta\rho_1 j > R_e^{NC} j$ must hold if an entrepreneur prefers to run away. However, this is a contradiction when $R_f^{NC} < \rho_0 + \pi$ and $\rho_1 + \pi = R_f^{NC} + R_e^{NC}$. Thus, an entrepreneur doesn't run away. When a crisis materializes, the project's date-2 return ρ_1 is distributed between $\{R_f, R_e\}$. For a given reinvestment scale j, suppose that R_f is less or equal to ρ_0 but an entrepreneur chooses to run away. If so, by Equation 2, $\theta\rho_1 j > R_e j$ must hold if an entrepreneur prefers to run away. However, this is a contradiction when $R_f < \rho_0$ and $\rho_1 = R_f + R_e$. Thus, an entrepreneur doesn't run away. ∎

4.2 Date-1 Optimal Behavior

As mentioned before, following a crisis, entrepreneurs could seek to finance any reinvestment by using a fraction of their C_0 payoff ($R_e^C i$), a fraction of any investments in the world market valued at $A - M_0$, or/and by selling new claims to foreign investors. At this point, both the initial investment scale i and $x = R_e^C i + (A - M_0)$ are taken as given since they are determined in C_0.

Entrepreneur's Date-1 Problem

$$\begin{aligned}
\underset{C_1}{\text{Maximize}}\ & jR_e + \left[x - M_1\right] && (OF_1)\\
\text{subject to: } & M_1 + \phi_1 j \geq j && (BC_1)\\
& R_f j \geq \phi_1 j && (PC_1)\\
& \rho_0 j \geq R_f j && (IC_1)\\
& \rho_1 j \geq R_e j + R_f j && (FC1_1)\\
& x \geq M_1 && (FC2_1)\\
& i \geq j && (FC3_1)\\
& j, M_1, \phi_1, R_e, R_f \geq 0 &&\\
& i,\ x,\ \rho_o,\ and\ \rho_1 \text{ are given} &&
\end{aligned}$$

The objective function of Entrepreneur's Date-1 Problem (OF_1) is the sum of two different terms. The first term is the expected total payoff for an entrepreneur from continuing the project to date-2, jR_e, and the second term refers to entrepreneurs option to lend the share of x that is not used to finance reinvestment in the world market with a expected return of 1.[40] Thus, the cost of using market liquidity for reinvestment is equal to the opportunity cost of not lending ($\gamma_1 = 1$).

Constraint (BC_1) is the budget constraint where any reinvestment j is financed with market liquidity M_1 or funding liquidity $\phi_1 j$. This budget constraint highlights the different sources of liquidity available to entrepreneurs.

Foreign lenders lend to entrepreneurs,if their total payoff is, at least, equal to the total return obtained at the world capital market at date-1 but, at the same time, it is not greater than the project's total pledgeable return. These elements are captured by investors' participation constraint (PC_1) and by the incentive compatibility constraint (IC_1) derived from Proposition 1.

Lastly, date-1 problem is subject to 3 different feasibility constraints. First, project's total return is to be allocated between foreign investors and the entrepreneur ($FC1_1$), market liquidity used cannot be greater than what the entrepreneur has available at date-1 ($FC2_1$), and reinvestment cannot be greater than the initial investment scale of the project ($FC3_1$). This last feasibility constraint captures that projects reflect long-run investments that require occasional reinvestment (Farhi and Tirole, 2012).

Proposition 2 (Entrepreneur's Date-1 Problem Solution)
As long as Assumptions 1 and 2 hold, the optimal pecking order to cover reinvestment needs is

1. *Market Liquidity*
2. *Funding Liquidity*
3. *Partial liquidation*

Proof See Appendix ∎

[40] Of course, Entrepreneurs could also consume a fraction of x. However, since entrepreneurs are risk neutral, then it is not strictly optimal to consume rather than to lend at the world capital market

Proposition 2 depicts a optimal pecking order that an entrepreneur follows at date-1 to finance reinvestment. First, it is optimal for the entrepreneur to use all of its available resources at date-1 (x) as market liquidity since the return of the project is greater than these resources opportunity cost ($\rho_1 > 1$).

If market liquidity is not sufficient to finance a full-scale reinvestment, an entrepreneur issues claims to foreign investors up until reaching full-scale or running out of pledgeable income. Only if market liquidity and pledgeable income are not enough to cover a full-scale reinvestment, then the entrepreneur is forced to downsize the project (partial liquidation).

Corollary 1 (Need of Market Liquidity - LFE)
If x is equal to zero, then the optimal solution to Date-1 entrepreneur's problem is to shutdown the project ($j = 0$)

Proof See Proof of Proposition 2 ∎

It is worth underscoring that the entrepreneur needs to put in some market liquidity for any positive level of reinvestment to occur (Corollary 1). Thus, x requires to be positive if a project were to continue to date-2 after a crisis. This result highlights that funding liquidity by itself is not enough to assure any level of continuation. The reason is that the project's limited pledgeability implies that a loan from foreign investors, without any market liquidity, has a negative present value ($\rho_0 < 1$). Hence, entrepreneurs need to compensate for this negative return by investing some market liquidity to attract foreign investors.

4.3 Date-0 Optimal Behavior

At the initial period, entrepreneurs offer a C_0 contract to foreign lenders that maximizes their expected payoff due the two possible states in the economy. With probability α, a crisis doesn't materializes in which case, an entrepreneur's payoff is equal to the return of the project allocated to her (iR_e^{NC}), plus the return of the amount invested in the world market ($(A - M_0)$).[41]

[41] I assume that any share of the initial endowment that is not invested in the project, it is lend at the world capital market at equilibrium rates. In case of no crisis, at date-1, entrepreneur lends this share again.

With probability $(1-\alpha)$, a crisis materializes in which case the payoff is determined by function $f(x,i)$ which results from behaving optimally from date-1 onwards. This function is derived from Proposition 2.[42] Hence, at date-0, an entrepreneur is aware that a greater x provides insurance against the liquidity shock and allows for a higher payoff in case the crisis occurs.

Entrepreneur's Date-0 Problem

$$
\begin{aligned}
\underset{C_0}{\text{Maximize}}\ & \alpha\Big[R_e^{NC}i + (A - M_0)\Big] + (1-\alpha)f(x,i) && (OF_0)\\
\text{subject to: } & M_0 + \phi_0 i \geq i && (BC_0)\\
& \alpha R_f^{NC}i + (1-\alpha)R_f^C i \geq \gamma_0\phi_0 i && (PC_0)\\
& \rho_0 i + \pi i \geq R_f^{NC} i && (IC_0)\\
& \rho_1 i + \pi i \geq R_e^{NC} i + R_f^{NC} i && (FC1_0)\\
& \pi i \geq R_e^C i + R_f^C i && (FC2_0)\\
& A \geq M_0 && (FC3_0)\\
& x = R_e^C i + \gamma_0(A - M_0) && (ML)\\
& i, M_0, \phi_0, R_e^{NC}, R_f^{NC}, R_e^C, R_f^C \geq 0\\
& A,\ f(x,i),\ \rho_0,\ \pi,\ \mathit{and}\ \rho_1 \text{ are given}
\end{aligned}
$$

Constraint BC_0 is the budget constraint at date-0 for the investment scale. This constraint underscores the two different finance sources (market and funding liquidity). Constraint PC_0 guarantees that in any solution to the problem, foreign investors receive, at least, the same return as if they would lend those resources in the world capital market.

The incentive compatibility constraint (IC_0) only applies for payoffs allocated if a crisis doesn't materialize. The reason is that, in this case, projects continue directly to date-2 where entrepreneurs are subject to moral hazard.[43]

In this problem, there are three feasibility constraints. The first two signal that payoffs distributed between an entrepreneur and investors cannot be greater than the total return

Consuming this share is never strictly optimal since entrepreneurs are risk neutral.

[42]This function is presented explicitly in the Proof of Proposition 2 in the Appendix

[43]Note that this constraint guarantees that any solution is consistent with Proposition 1

generated by the project in each possible state of the economy. Meanwhile, feasibility constraint $FC3_0$ indicates that an entrepreneur cannot invest more than its initial endowment in the project.

Lastly, equation (ML) shows that there are two ways to accumulate market liquidity for date-1. First, by allocating some payoff from the project when a crisis happens ($R_e^C i$). And, second, by investing a share of the initial endowment in the world capital market instead of in the project. As it will be clear below, both ways affect negatively the investment scale. The former through what is called the equity multiplier and, the latter through a lower equity.

At this point, it is worth mentioning that the model assumes that entrepreneurs are indispensable to carry the project to fruition.[44] This characteristic allows entrepreneurs to decide what to do with the project following a crisis. Consequently, contract C_0 is designed such that payments contingent on a crisis are bounded by the projects safe cash flow ($FC2_0$).[45]

Proposition 3 (Entrepreneur's Date-0 Problem - Optimal Solution)
As long as Assumptions 1 and 2 hold, the solution to an entrepreneur's date-0 problem has the following characteristics:

1. *Entrepreneur's invest all their initial endowment in the project ($A = M_0$)*
2. *Foreign investors receive all pledgeable income in case of no crisis ($R_f^{NC} = \rho_0 + \pi,\ R_e^{NC} = \rho_1 - \rho_0$)*
3. *Investment scale is equal to $i = A\kappa(R_e^C)$ where $\kappa(R_e^C)$ is defined as the equity multiplier equal to $\frac{1}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C}$*
4. *For payoffs in case of a crisis, $R_f^C = \pi - R_e^C$ and $x = R_e^C i$ where*

$$R_e^C = \begin{cases} 0 & \text{if } 1-\alpha < \pi \\ 1-\rho_0 & \text{if } 1-\alpha \geq \pi \end{cases}$$

Proof See Appendix

[44] For example, whenever a crisis happens, foreign lenders cannot take over the project and continue without entrepreneurs

[45] This assumption implies that contracts are renegotiated if a crisis materializes

Proposition 3 characterizes the solution to an entrepreneur's date-0 problem. When projects provide sufficient returns (Assumption 1), it is optimal for an entrepreneur to maximize its leverage, that is to allocate to foreign lenders all pledgeable return contingent on a crisis not materializing.

The optimal solution to the problem at date-0 points out as well at the inherent trade-off between investment scale and insurance. Entrepreneurs can always choose to hoard sufficient liquidity (x) to continue in any state of the economy. That is, entrepreneurs could fully insurance against aggregate shocks. However, buying insurance is costly because it lowers the equity multiplier reducing the investment scale.

Accumulating x can be done either through R_e^C and/or by saving a share of the initial endowment ($A - M_0$). Proposition 3 states that, optimally, any hoarding of liquidity should be done solely through R_e^C. Thus, an entrepreneur invests all of its initial endowment in the project ($M_0 = A$).[46]

The intuition behind this result lies on the fact that payment R_e^C is contingent on a crisis happening, while saving a share of the endowment is not. Therefore, if a crisis doesn't materialize, a entrepreneur incurs in a waste of resources by not having invested all the endowment in the project. Subject to a positive level of x, investment scale is maximized by choosing $M_0 = A$ and $R_e^C i = x$.[47]

However, liquidity hoarding is not always the best option. Holmström and Tirole (1998) show that it is ex-ante optimal for entrepreneurs to partially insure against liquidity shocks. This model replicates this result because only if the probability of a crisis is relatively high ($1-\alpha \geq \pi$) entrepreneurs allocate a fraction of the safe cash flow to them as insurance against a crisis.[48] Otherwise, when the probability of the aggregate shock is relatively low ($1-\alpha < \pi$), entrepreneurs maximize the equity multiplier, thus investment scale, by promising the full safe cash flow to investors. No point on insuring against a crisis that is most likely never

[46]It is a common result in models à la Holmström and Tirole (1998), that the investment scale is equal to the initial endowment of the entrepreneur times the equity multiplier

[47]The cost of liquidity hoarding is minimized

[48]The same condition for liquidity hoarding can be found in Farhi and Tirole (2012)

going to happen.

4.4 Laissez Faire Equilibrum

Propositions 1, 2, and 3 characterize any potential Laissez Faire equilibrium of the model. Given the trade-off between investment scale and insurance, there are different possible Laissez Faire equilibria depending on the value of some parameters. In one equilibrium, as Corollary 2 shows, entrepreneurs choose to self-insure against crisis and accumulate just enough liquidity at date-0 to continue at full-scale at date-1 if necessary.[49]

However, there are certain parameter values where in equilibrium (Corollary 2), it is optimal for investors to not insure against crises (black swan events).[50] But if entrepreneurs do not hoard some liquidity at date-0, they are unable to continue at all if a crisis materializes. However, projects remain socially valuable given that total return is greater than the cost of liquidity ($\rho_1 > 1$). Thus, ex-post, entrepreneurs would have benefited from buying some insurance.

Corollary 2 (Laissez Faire Equilibria)
As long as Assumptions 1 and 2 hold, then

1. *When* $1-\alpha < \pi$ *(Black Swan Event), the laissez faire equilibrium consists of*
   - *Date 0:* $C_0 = \{i = \frac{A}{1-\pi-\alpha\rho_0},\ M_0 = A,\ \phi_0 = i - A,\ R_e^{NC} = \rho_1 - \rho_0,\ R_f^{NC} = \rho_0 + \pi,\ R_e^C = 0,\ R_f^C = \pi\}$
   - *Date 1:* $C_1 = \{j = 0,\ M_1 = 0,\ \phi_1 = 0,\ R_e j = 0,\ R_f j = 0\}$
   - *Date 2: Entrepreneurs' don't run away*
2. *When* $1-\alpha \geq \pi$*, the laissez faire equilibrium consists of*
   - *Date 0:* $C_0 = \{i = \frac{A}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C},\ M_0 = A,\ \phi_0 = i - A,\ R_e^{NC} = \rho_1 - \rho_0,\ R_f^{NC} = \rho_0 + \pi,\ R_e^C = 1-\rho_0,\ R_f^C = \pi - R_e^C\}$
   - *Date 1:* $C_1 = \{j = i,\ M_1 = R_e^C,\ \phi_1 = \rho_0,\ R_e j = (\rho_1 - \rho_0)i,\ R_f j = \rho_0 i\}$
   - *Date 2: Entrepreneurs' don't run away*

[49] When $1 - \alpha > \pi$

[50] An event is referred to as black swan if it is extremely rare, can potentially cause severe impact, and, the conclusion that is was quite obvious in hindsight. See Nassim Nicholas Taleb, The Black Swan: The Impact Of The Highly Improbable (Penguin, 2008)

Proof See Appendix

The Laissez Faire results suggest that public liquidity provision is most warranted after the realization of a black swan event. This is a scenario where the private sector by itself cannot generate the sufficient amount of liquidity to finance solvent but illiquid economic activity. Thus, the public sector can potentially play an important role.

5 The Government

In principle, a government has an advantage in producing liquid assets ex-post because of their unique power to tax citizens. More precisely, as argued by Holmström and Tirole (2011), this ability allows governments to transform future returns into pledgeable income. Thus, as long as the government is credible, governments can act as *brokers* between liquidity constrained borrowers and lenders.

However, limited pledgeability is a symptom, not the disease. Hence, government intervention is effective only if the regalian taxation power addresses the ultimate culprit; in case of this model, moral hazard. To see this, I introduce a government to the model to study the potential limits of ex-post liquidity provision programs.

The government's role in this model is to act as a lender of last resort by lending freely to solvent but illiquid firms. Thus, I consider a policy where a government provides financing to entrepreneurs in case of a crisis.

Consider the following lending scheme: At date-1, if a crisis is realized, an entrepreneur can ask the government for a fraction of the initial investment scale, τ, to finance potential reinvestments. In return, such entrepreneur promises to pay back $\hat{R}\tau i$ at date-2.[51]

Given that there is no initial heterogeneity between entrepreneurs in the model and they are subject to an aggregate shock, the total demand for loans T_1 is equal to the individual

[51] Note that the total amount transferred to this entrepreneur at date-1 is equal to τi where i is the initial investment scale of the project while the amount collected at date-2 is $\hat{R}\tau i$.

demand τi. Likewise, the total amount collected from entrepreneurs at date-2, T_2, is the same as the amount collected from each entrepreneur $\hat{R}\tau i$.

Similar to entrepreneurs, the government can hoard market liquidity to finance this program. At date-0, the government collects F_0 from entrepreneurs' initial endowment.[52] I assume that the government lends these resources at the world market both between date-0 and date-1, and, again, between date-1 and date-2. Thus, by date-2, the government has $F_2 = \gamma_0\gamma_1 F_0 = F_0$ worth of market liquidity.

Although it is not possible to separate between domestic and foreign goods in this model since there is only one type of good, I interpret this liquidity hoarding by a government equivalent to accumulating foreign reserves. The parallel between F_0 and foreign reserves is straight forward. First, these are resources in control of the government or a monetary authority and available for immediate use in each period. Second, both resources are claims of the economy on foreign markets, thus, they are assets on the external balance sheet. Third, as further discussed below, both imply a carry cost since these resources are invested in the technology with lower return ($\rho_1 > 1$). Finally, as in this model, foreign reserves are accumulated in part to meet any potential balance of payments financing need, ergo, to implement liquidity provision policies.[53]

If a crisis happens, a government can tap into its funding liquidity as well. I assume that, at date-1, the government can issue one-period sovereign bonds at the world market for an amount B to finance entrepreneurs' loan demand. These bonds need to offer a return of, at least, 1 at date-2 to be attractive to foreign lenders. Equation 3 and Equation 4 depict the government's budget constraints at date-1 and date-2, respectively.

$$B \geq T_1 \tag{3}$$

[52]Limited liability implies that $A - F_0 \geq 0$

[53]The Balance Payments Manual (BOPM6), paragraph 6.64, published by the IMF defines reserves assets as "external assets that are readily available to and controlled by monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets to affect the currency exchange rate, and for other related purposes"

$$F_2 + T_2 \geq B \tag{4}$$

The goal of the government with this ex-post liquidity provision program is to minimize the potential partial liquidation of projects if a crisis materializes. As argued above, to do so, the government sets two policy instruments. It collects F_0 at date-0 and it sets $\hat{R}$ at date-1.

I assume that the government issues safe bonds. Therefore, given τi and F_0, $\hat{R}$ such be set such that a government collects sufficient resources to pay back foreign lenders. Thus, setting $\hat{R}$ as indicated by response function $\hat{R}(\tau, F_0, i)$ assures that bonds are paid back at date-2.[54]

$$\hat{R}(\tau, F_0, i) = \begin{cases} 1 & \text{if } \tau = 0 \\ \\ max\{0, 1\left[1 - \frac{F_0}{\tau i}\right]\} & \text{if } \tau > 0 \end{cases} \tag{5}$$

In any equilibrium, by assumption of sovereign bonds safety, Equation 5 determines $\hat{R}$. Thus, F_0 remains to be determined. A greater accumulation of F_0 increases the probability of greater reinvestments by reducing the cost of public liquidity ($\hat{R}$) since a lower share of the repayment of government bonds falls to entrepreneurs. However, a greater F_0 also implies a deviation of resources from projects.

$$W(F_0, \hat{R}, x) = \psi F_0 \kappa(R_e^C) + (1 - \alpha)L(j) \tag{6}$$

The cost function $W(F_0, \hat{R}, x)$, Equation 6, captures these trade-offs faced by the government. Deviating a unit of initial endowment from projects implies giving up a net return, ψ, determined by the difference between a project's expected return and the return from

[54]This response function is derived by assuming that Equation 3 and Equation 4 are satisfied with equality. This assumption is consistent with a government whose only purpose is to minimize date-1 project downsizing. Hence, there is no point or value on setting $\hat{R}$ greater than the necessary to pay back foreign lenders

lending such unit at the world capital market[55] times a scale effect that arises in projects due to the equity multiplier. The first term of $W(F_0, \hat{R}, R_e^C)$ reflects this cost.

The second term of $W(F_0, \hat{R}, x)$ depicts, in a reduced form, the expected welfare losses due to partial liquidation of investments. The loss function L plausibly reflects, for example, losses due to rises in unemployment or increases in financial fragility. Note that a government values the continuation scale of projects, and not the utility of entrepreneurs. This approach follows Farhi and Tirole (2012) and the purpose is to underscore in the model potential negative spillover effects of downsizing on the economy.

Assumption 3 (Welfare Loss Function)
Define function $L : [0, i] \rightarrow R^+$ with the following characteristics;

1. *the argument is the level of reinvestment in the economy*
2. *Continuous and convex function*
3. *Non-increasing*
4. *bounded from below by zero when $L(i) = 0$*
5. *bounded from above by a positive constant $L(0) = K$*

Assumption 3 describes some characteristics of the loss function. Naturally, this function is bounded from below at zero when full-scale reinvestment is reached. No downsizing, no cost. Additionally, the function is convex to reflect that small levels of downsizing produce lower marginal costs than large levels of downsizing.

I also assume that the loss function is bounded from above by a positive constant when projects shutdown. If this were not the case, a government would always do whatever is necessary to prevent a shutdown, no matter the cost. Thus eliminating interesting equilibrium results. Although a plausible description for some economies,[56] most economies cannot afford to insure against all crises. The opportunity costs are too high.

[55] In the model, ψ is equal to $\rho_1 + \pi - 1 - (1 - \alpha)$ which, by Assumption 1, is strictly positive

[56] Remember the famous Mario Draghi speech "Whatever it Takes"

Taking stock the, in this economy, the government's problem at date-0 is to minimize $W(F_0, \hat{R}, R_e^C)$ by choosing F_0 subject to F_0 being less than A and the reaction function described in Equation 5.

Definition 3 (Equilibrium with Government (GE))
A Subgame Perfect Nash Equilibrium where entrepreneurs' don't run away and a Bagehotian Government implements a liquidity provision policy is characterized by the following strategy profiles

- *Date-0: C_0 maximizes the Entrepreneur's Date-0 Problem with Government, Government chooses F_0 that minimizes its Date-0 Problem*
- *Date-1: C_1 maximizes the Entrepreneur's Date-1 Problem with Government, Government's issue the necessary amount of bonds to satisfy C_1 and determines $\hat{R}$ according to Equation 5*
- *Date-2: Entrepreneurs' don't run away, governments pay off any outstanding sovereign bonds by using F_2 and resources collected from entrepreneurs*

At this point, some discussion about the ex-post liquidity provision policy allowed in the model is warranted. First, in present time, lending of last resort is a policy that is usually attributed to a monetary authority or central bank.[57] Yet, in the model, I abstract from any differences between a central bank and the government, and I assume that both fiscal and liquidity provision policies are chosen under the same roof.

Besides a loan scheme, there are other types of ex-post policies interventions that any government can implement.[58] The policy as it is described, due to the simplicity of the model, could be interpret as an capital injection, for example. However, the model is built around a liquidity friction, not a solvency friction.[59] Additionally, both the global financial crisis and the COVID-19 shock proved that credit facilities to private agents are a government go to policy tool to provide liquidity. Thus, a lender of last resort interpretation is more accurate since the point of the model is to study why foreign reserves are accumulated to provide liquidity.

[57] Bordo (1990) argues that historically this not necessarily the case. This type of policy needs to be implemented by a public authority where one candidate is the central bank

[58] i.e. a takeover, a bailout, purchase of legacy assets, etc.

[59] A lender of last resort is a policy designed to address this type of issues, while a capital injection or bailout are more designed to address solvency concerns

Similarly, there are other ways that a government could finance its liquidity provision program.[60] However, the goal is to depict a scenario where government intervention is optimal for small open economy. First, with an severe aggregate shock, it is natural to assume that resources are scarce within borders, and resources need to be collected from *outside*. And, second, the policy is designed to highlight the insurance brokerage role between foreign investors and domestic entrepreneurs that a government takes when accumulating reserves to provide liquidity. I argue that this role is possible and explained by a governments' power of taxation.

The efficiency of ex-post government intervention doesn't depend on whether bonds are safe or not, but on how does this liquidity provision policy affect moral hazard. Hence, in this model, results depend how does the collection of $\hat{R}\tau i$ affect entrepreneurs' incentives to run away.

I compare the equilibrium results of a government that collects $\hat{R}\tau i$ after an entrepreneur chooses to run away, or not, versus a government that collects before such choice. Although this seems as a mild difference, this comparison shows that, despite both governments issuing safe assets, the latter is only capable of implementing a successful liquidity provision program provided it hoarded market liquidity at date-0, while the latter does so without the need of any preemptive accumulation. Foreign reserves accumulation is an endogenous in this model.

6 Mature Government - Collection after runaway choice

The assumption of collecting the loan payment ($\hat{R}\tau i$) after the decision to runaway, or not, is made is a simplification of a government that is capable of reaching and enforcing its power of taxation at all times. Thus, this government is not subject to market frictions that

[60] The model could have included consumers within this economy that at date-1 would be taxed and, at date-2, would receive any resources collected from entrepreneurs. In this environment $\hat{R}$ would be set to compensate consumers for any dead-weight and welfare loss caused. The results of this version of the model shouldn't be much different given that the key of the paper lies on how governments collect resources from liquidity constrained agents, not on how a government finances its policy

cause limited pledgeability. For clarity, I denoted this type of government as the Mature Government (MG).

6.1 Date-2 Optimal Behavior

Equation 7 depicts an entrepreneur's incentive compatibility constraint with a MG intervention. Note that collection of resources by the government doesn't affect this constraint, given that even if an entrepreneur runs away, it still needs to pay off its date-1 loan.

$$R_e j - \hat{R}\tau i \geq \rho_1 \theta j - \hat{R}\tau i \tag{7}$$

Proposition 8 shows that the return of the project that can be credibly promise to investors is the same as in the LFE. Government intervention doesn't reduce the amount a project can allocate to other financing sources, thus, there is no crowding out of pledgeability.

Proposition 4 (Pledgeable Return with a Mature Government)
Let Date-2's pledgeable return, ρ_0, equal to $\rho_1(1-\theta)$
In the Mature Government Equilibrium, entrepreneurs will not run away as long as

- R_f^{NC} *is less or equal to* $\rho_0 + \pi$ *when no crisis materializes*
- R_f *is less or equal to* ρ_0 *when a crisis materializes*

Proof Given that the public loan repayment affects both sides equally of the incentive compatibility, the proof of this proposition is the same as Proposition 1 proof. ∎

Since a Mature Government is not subject to the moral hazard constraint, the date-1 government loan is backed up by the total return of an entrepreneur at date-2.[61] Thus, unlike financing from foreign investors, an entrepreneur can used the total amount of its date-2 return to ask for a loan from the government. In other words, MG ex-post intervention eliminates the gap between total and pledgeable return.

[61] That is both the return from the project as well as any other return from investment activities or endowments

6.2 Date-1 Optimal Behavior

The objective function of the entrepreneur's Date-1 problem with a MG (OF_{MG1}) incorporates the potential repayment of the government loan into the problem. Hence, by adding $\hat{R}\tau i$ to OF_{MG1}, any solution to the problem satisfies entrepreneurs' limited liability. In other words, no agent will request a loan that would imply a negative consumption level at date-2. Consequently, by assuring the holding of limited liability, an entrepreneur always pays back the government.

Likewise, the budget constraint of this date-1 problem (BC_{MG1}) highlights that additional to market liquidity and private funding liquidity, an entrepreneur has access to public liquidity due to the implementation of the liquidity provision program. Plausibly, this new source of financing would allow an entrepreneur to reinvest, even without any market liquidity.

Entrepreneur's Date-1 Problem with a MG

$$\begin{aligned}
\underset{\tilde{C}_1}{\text{Maximize}}\ & jR_e + \left[x - M_1\right] - \hat{R}\tau i && (OF_{MG1})\\
\text{subject to: } & M_1 + \tau i + \phi_1 j \geq j && (BC_{MG1})\\
& R_f j \geq \phi_1 j && (PC_{MG1})\\
& \rho_0 j \geq R_f j && (IC_{MG1})\\
& \rho_1 j \geq R_e j + R_f j && (FC1_{MG1})\\
& x \geq M_1 && (FC2_{MG1})\\
& i \geq j && (FC3_{MBG1})\\
& j, M_1, \phi_1, R_e, R_f,\ \tau \geq 0 &&\\
& i, x,\ \rho_0,\ and\ \rho_1 \text{ are given} &&
\end{aligned}$$

Besides the aforementioned differences in the objective function and in the budget constraint, the other constraints of the date-1 problem are the same as the ones faced by an entrepreneur without a government. An entrepreneur maximizes this problem by choosing set $\tilde{C}_1 = \{j,\ M_1,\ \phi_1,\ R_e,\ R_f,\ \tau\}$ that determines the reinvestment amount (j), the amount of entrepreneur's market liquidity to be invested at date-1 (M_1), the amount to borrow at date-1 from foreign investors (ϕ_1), an allocation of the project's return between entrepreneurs (R_e) and investors (R_f), and, of course, the loan requested from the government (τi).

Proposition 5 (Entrepreneur's Date-1 Problem with MG Solution)
As long as Assumptions 1 and 2 hold, the optimal pecking order to cover reinvestment needs is

- *If* $\hat{R} \leq 1$
  1. *Public Liquidity*
- *If* $1 < \hat{R} \leq \frac{\rho_1 - \rho_0}{1-\rho_0}$
  1. *Market Liquidity*
  2. *Funding Liquidity*
  3. *Public Liquidity*
- *If* $\frac{\rho_1 - \rho_0}{1-\rho_0} < \hat{R}$
  1. *Market Liquidity*
  2. *Funding Liquidity*
  3. *Partial Liquidation*

with public liquidity demand function

$$\tau(\hat{R}, x) = \begin{cases} 1 & \text{if } 1 \geq \hat{R} \\ max\{0, \quad i(1-\rho_0) - x\} & \text{if } \frac{\rho_1-\rho_0}{1-\rho_0} \geq \hat{R} > 1 \\ 0 & \text{if } \hat{R} > 1\frac{\rho_1-\rho_0}{1-\rho_0} \end{cases}$$

Proof See Appendix ∎

Proposition 5 presents the optimal pecking order for reinvestment when a Mature Government offers a ex-post liquidity provision program. First, note that the pecking order depends on the cost of public liquidity. If public liquidity is relatively cheap ($\hat{R} \leq 1$), then a government loan by itself is more than enough to reach full-scale reinvestment. In this case, unlike without any government program, there is no need to hoard liquidity to achieve full scale continuation at date-1.

When public liquidity is more expensive than market and funding liquidity ($\hat{R} > 1$), an entrepreneur would only demand a government loan if both sources of liquidity are insufficient to reach full-scale reinvestment. Partial liquidation is only a viable choice if public liquidity is too expensive ($\hat{R} > \frac{\rho_1 - \rho_0}{1-\rho_0}$).

Corollary 3 (Guaranteed Full-Scale Reinvestment - Mature Government)
When $\hat{R} \leq \rho_1$, even if x is equal to zero, it is never optimal to downsize a project following a crisis

Proof To prove this corollary, note that, by Proposition 5, the optimal reinvestment scale is i for any $\hat{R}$ equal or lower than $\frac{\rho_1 - \rho_0}{1-\rho_0}$. See Appendix for the explicit reinvestment function. This is true, even for x equal to zero, because public liquidity bridges the gap of what is missing to reach full-scale. To finalize this proof, note that $\frac{\rho_1 - \rho_0}{1-\rho_0}$ is greater than ρ_1 since $\rho_1 > 1$. Thus, by Proposition 5, the optimal reinvestment scale is i for any $\hat{R}$ equal or lower than ρ_1. ∎

With a mature government, public intervention overcomes market frictions eliminating the reason for entrepreneurs to hoard liquidity. In other words, a project is no longer credit constrained when public liquidity is available. Thus, Corollary 3 shows that full-scale reinvestment is guaranteed when projects are solvent relative to the cost of public liquidity ($\rho_1 > \hat{R}$). Moreover, compared to a Laissez Faire world, market liquidity no longer becomes necessary at any moment.

6.3 Date-0 Optimal Behavior

At date-0, an entrepreneur's maximization problem is similar to the one with no government except for two aspects. The first difference is that she only has the endowment left after taxes ($A - F_0$) for market liquidity ($FC3_{G0}$). Hence, a government can affect the size of an investment scale by collecting more resources from entrepreneurs' at date-0. At the same time, it also diminishes the potential pool of resources available to hoard liquidity for date-1 (ML).[62]

The second difference is that the expected payoff function following a crisis, ($f(x, i, \hat{R})$), is also a function of the cost of public liquidity at date-1. Therefore, an entrepreneur's date-0

[62]Note that F_0 is not included in the objective function. The reason is that, if a crisis doesn't happen, the government rebates $F_2 = F_0$ at date-2 to entrepreneurs which cancels out what they collected at date-0. Hence $(A - F_0 - M_0) + F_0 \rightarrow (A - M_0)$

behavior is affected by their belief about the ex-post cost of public liquidity.[63]

Entrepreneur's Date-0 Problem with Government

$$
\begin{aligned}
&\underset{\tilde{C}_0}{\text{Maximize}}\ \alpha\left[R_e^{NC}i + (A - M_0)\right] + (1-\alpha)f(x,i,\hat{R}) && (OF_{G0})\\
&\text{subject to: } M_0 + \phi_0 i \geq i && (BC_{G0})\\
&\alpha R_f^{NC}i + (1-\alpha)R_f^{C}i \geq \gamma_0\phi_0 i && (PC_{G0})\\
&\rho_0 i + \pi i \geq R_f^{NC}i && (IC_{G0})\\
&\rho_1 i + \pi i \geq R_e^{NC}i + R_f^{NC}i && (FC1_{G0})\\
&\pi i \geq R_e^{C}i + R_f^{C}i && (FC2_{G0})\\
&A - F_0 \geq M_0 && (FC3_{G0})\\
&x = R_e^{C}i + (A - F_0 - M_0) && (ML)\\
&i, M_0, \phi_0, R_e^{NC}, R_f^{NC}, R_e^{C}, R_f^{C} \geq 0\\
&A,_0\, f(x,i,\hat{R}),\ \rho_0,\ \pi,\ \mathit{and}\ \rho_1 \text{ are given}
\end{aligned}
$$

Recall that, in the LFE, the incentives to hoard liquidity at date-0 are due to the inability to attract sufficient funding liquidity at date-1 as a result of limited pledgeability. However, these incentives disappear with a MG because public liquidity eliminates limited pledgeability.

As Proposition 6 shows, whenever the expected cost of public liquidity is less that q, an entrepreneur won't hoard any liquidity at date-0. In comparison with the LFE, this result stands for both black swan and non-black swan events.

Consequently, this model replicates the *"too big to fail conundrum"* where the possibility of future public liquidity provision creates a moral hazard problem at date-0 because private agents have less incentives to self-insure. Farhi and Tirole (2012) also find an equilibrium where bailout policies drive agents to hoard less liquidity.

Proposition 6 (Entrepreneur's Date-0 Problem - MG - Optimal Solution)
As long as Assumptions 1 and 2 hold, the solution to an entrepreneur's date-0 problem has the following characteristics:

1. *Entrepreneur's invest all their initial endowment in the project ($A - F_0 = M_0$)*

[63]See Appendix - Proof of Proposition 5 for the explicit function $f(x,i,\hat{R})$

2. *Foreign investors receive all pledgeable income in case of no crisis* $(R_f^{NC} = \rho_0 + \pi,\ R_e^{NC} = \rho_1 - \rho_0)$

3. *Investment scale is equal to* $i = (A - F_0)\kappa(R_e^C)$ *where* $\kappa(R_e^C)$ *is defined as the equity multiplier equal to* $\frac{1}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C}$

4. *If* $\hat{R} \leq 1$*, the optimal contract consists of* $R_f^C = \pi$ *and* $R_e^C = 0$

5. *If* $\hat{R} > \frac{\rho_1-\rho_0}{1-\rho_0}$*, follow Proposition 3 (For payoffs in case of a crisis)*

6. *If* $1 < \hat{R} \leq \frac{\rho_1-\rho_0}{1-\rho_0}$*, define* $q = \frac{\rho_1-\rho_0}{1-\pi-\alpha+(1-\rho_0)}$ *and with* $R_f^C = \pi - R_e^C$ *and* $x = R_e^C i$ *where*

$$R_e^C = \begin{cases} 0 & \text{if } 1-\alpha < \pi \\ & \text{if } 1-\alpha \geq \pi \\ 0 & \quad\quad \text{and } \hat{R} \leq q \\ 1-\rho_0 & \quad\quad \text{and } \hat{R} > q \end{cases}$$

Proof See Appendix

Having said that, entrepreneurs hoard liquidity themselves when the expected cost of public liquidity is greater than q[64] and the probability of crisis is relatively high.[65] If the cost of public funding is relatively too high, it becomes too expensive to demand a government loan equal to $i\big[1-\rho_0\big]$. Hence, it is optimal to accumulate some market liquidity to reduce the amount that needs to be asked from the government.

This result suggests that a penalty rate on lender of last resort loans could offset the *too big to fail conundrum*. As discussed by Bordo (1990) and Fischer (1999), a penalty rate is one of the key elements of a lender of last resort.

The potential cost with a penalty rate is that it could prevent any reinvestment if the crisis is a black swan. When the probability of a crisis is relatively low, entrepreneurs don't hoard any liquidity regardless of future public intervention or not.[66] Thus, without market liquidity, the potential penalty rate that could be set to eliminate date-0 moral hazard and,

[64] which is greater than 1
[65] $1 - \alpha > \pi$
[66] See Proposition 3

at the same time, allow full-scale reinvestment following black swan events would be one that lies between would be an q and ρ_1.[67]

At date-0, the government can accumulate liquidity to minimize Equation 6. A greater accumulation of reserves ex-ante, in principle, allows the government to set a lower interest rate loan at date-1 which would foster more reinvestment. A MG chooses optimally F_0 equal to zero as shown by Proposition 7.

The optimal behavior of a Mature Government at date-0 underscores that this government doesn't need to accumulate reserves because it is not subject to any market friction. Thus, the MG, due to its ability to enforce its power of taxation at any time, is an effective broker between lenders and entrepreneurs and increases the effective pledgeability of a project from ρ_0 to ρ_1.

Proposition 7 (Mature Government Date-0 Optimal Solution)
As long as Assumptions 1, 2 and 3 hold, An MG chooses F_0 equal to zero at date-0 to minimize its cost function

Proof Suppose there exists a feasible $\hat{F_0}$ such that $W(\hat{F_0}, \hat{R}, x)$ is less than $W(0, \hat{R}, x)$. Note that $\hat{F_0} > 0$, thus, by Equation 5, the interest rate implied by $\hat{F_0}$ is lower than the one implied by $F_0 = 0$. However, note that, for any positive demand of liquidity, $\hat{R}$ is less or equal to 1. Hence, regardless the government accumulates $\hat{F_0}$ or doesn't accumulate at all, by Proposition 5, τ is equal to 1 and, more importantly, a full-scale reinvestment is made. Thus, $L(i)$ is equal to zero. Therefore, $W(\hat{F_0}, R < 1, x)$ minus $W(0, 1, x)$ is equal to $\psi \hat{F_0} \kappa(R_e^C)$ which is strictly positive. Hence, $W(\hat{F_0}, R < 1, x) > W(0, 1, x)$ and I get a contradiction. ∎

6.4 Mature Government - Equilibirum (SPNE)

The SPNE with a Mature Government shows that a liquidity provision program is effective since a full-scale reinvestment happens whenever a crisis is materialized. More importantly, such effectiveness relies on the government's ability to enforce its power of taxation at any time, and, thus, overcome any potential market frictions. Consequently, an MG is not

[67] As the model is set up, the government is not bother by lower private liquidity hoarding. Hence, there are no incentives to set a $\hat{R}$ greater than what is strictly necessary to redeem sovereign bonds ($\gamma_1 = 1$). To study the appropriate level of a penalty rate, a different welfare function needs to be considered.

required to do any ex-ante liquidity hoarding to guarantee the success of an ex-post liquidity program.

With a MG, projects continuation happens even after a black swan event. Hence, in comparison to the LFE, public liquidity provision is effective facing situations when is most warranted. As suggested by Holmström and Tirole (2011), public liquidity has a higher value in low probability aggregate shocks.

However, this ability to provide a safety net to entrepreneurs' comes with the cost of lower private self-insurance. Incentives to hoard liquidity, sacrificing investment scale, diminish since a government intervention when necessary is always successful.

Corollary 4 (MBG Equilibrium)
As long as Assumptions 1, 2, and 3 hold, with a MG, a Subperfect Game Nash Equilibrium has the following characteristics

- *Date-0*
  - *Entrepreneur* $\tilde{C}_0 = \{i = \frac{A}{1-\pi-\alpha\rho_0},\ M_0 = A,\ \phi_0 = i - A,\ R_e^{NC} = \rho_1 - \rho_0,\ R_f^{NC} = \rho_0 + \pi,\ R_e^C = 0,\ R_f^C = \pi\}$
  - *Government* $\{F_0 = 0\}$
- *Date-1*
  - *Entrepreneur* $\tilde{C}_1 = \{j = i,\ M_1 = 0,\ \phi_1 = 0,\ R_e = \rho_1,\ R_f = 0,\ \tau i = i\}$
  - *Government* $\{B = i,\ \hat{R} = 1\}$
- *Date-2*
  - *Entrepreneurs don't runaway*
  - *Government collects* $\hat{R}i$ *to pay out sovereign bonds*

Proof Consider if no crisis happens. Then, at date-2, $\tilde{C}_0$ argues that R_e^{NC} is equal to $\rho_1 - \rho_0$ which is equal to $\theta\rho_1$, thus, not running away is a best response. Consider if a crisis happens, then $\tilde{C}_1$ establishes that R_e is equal to ρ_1 which is greater than $\theta\rho_1$, again, not running away is a best response. Moreover, the total payoff for the entrepreneur at date-2 is $i\big[\rho_1 - 1\big]$ which, by Assumption 1, is greater than zero, hence, limited liability is satisfied and the government credibly collects $\hat{R}i$. Government with these resources pays i to redeem government bonds. Bonds are safely paid back. At date-1, the government starts with F_0 equal to zero, therefore, for any τ, sets $\hat{R}$ equal to 1 by Equation 5. Entrepreneurs' best response to this cost of public liquidity is to demand $\tau i = i$, by Proposition 5, which establishes $\tilde{C}_1$ as the optimal solution. In return, since τi is equal to i, government's best response by Equation 5 and F_0 equal to zero is $\hat{R}$ equal to 1. At date-0, following Proposition 7, a government chooses F_0, and following Proposition 6, entrepreneurs maximize by choosing $\tilde{C_0}$ since $\hat{R}$ cannot be greater than 1 at date-1. $\blacksquare$

The *too big to fail conundrum* in the MG Equilibrium applies for events that are not black swans ($1 - \alpha > \pi$). That is, events where entrepreneurs' would optimally choose to self-insure will no longer do so because governments can rescue them ex-post.

Would a threat by a MG to not implement a liquidity provision program following a non-black swan event be credible? The answer is no. Entrepreneurs' are unable to reinvest at any scale if they don't hoard liquidity ex-ante (Proposition 2) without public assistance. Thus, if a government chooses to not implement a liquidity program ex-post, it would experience a welfare loss equal to $L(0)$. Given that a mature government doesn't need any prior liquidity hoarding to provide public liquidity ex-post, then, regardless if the event is a black swan or not, its best response is to provide liquidity to entrepreneurs that minimizes the cost function at its lowest level. In equilibrium, Mature Governments cannot escape the *too big to fail conundrum* because creating ex-post liquidity is relatively cost-less.

In contrast, immature governments can overcome the *too big to fail conundrum*. As I show below, governments that need to pre-commit for the success of an ex-post liquidity provision program by hoarding liquidity can credibly signal that they are not willing to assist entrepreneurs ex-post. Interestingly, it is their inability to produce easily liquidity ex-post that makes them credible.

7 Immature Government - Collection before runaway choice

An Immature Government (IG), similar to the MG, has access to world capital markets and can issue safe bonds following a crisis. However, unlike a MG, it cannot enforce its power of taxation at all times. Thus, the assumption in the model is that the collection of resources at date-2 by the government happens before entrepreneurs' decide to runaway or not.

An Immature Government is intended to capture developing countries where an important share of their economic activity is informal, and, by definition, outside the enforcement of a government. Not surprisingly, according to OECD data, tax revenue, as share of GDP, averaged above 30% for high income economies between 2000 and 2018, while, for the rest of the world, this figure was below 20%. Although these numbers might hide idiosyncratic fiscal preferences of each country, Besley and Persson (2014) show that low income countries typically have narrower tax bases than advanced economies, thus, lower enforcement power.

7.1 Date-2 Optimal Behavior

In this model, I assume that a IG collects resources at the project level. As Proposition **??** argues, this *minor* modification to the model implies that governments' are also subject to entrepreneurs' date-2 moral hazard. Public financing is limited by pledgeable income as well. Unlike a MG, an IG is unable to overcome market frictions ex-post.

Proposition 8 (Pledgeable Return with a Immature Government)
Let Date-2's pledgeable return, ρ_0, equal to $\rho_1(1-\theta)$
In a Immature Government Equilibrium, entrepreneurs will not run away as long as

- *$R_f^{NC} i$ is less or equal to $\rho_0 i + \pi i$ when no crisis materializes*
- *$R_f j$ is less or equal to $\rho_0 j - \hat{R}\tau i$ when a crisis materializes*

Proof Let $\rho_0 := \rho_1(1-\theta)$ When a crisis doesn't materialize, the project's return $\rho_1 + \pi$ is distributed between $\{R_f^{NC}, R_e^{NC}\}$. For a given investment scale j, suppose that R_f^{NC} is less or equal to $\rho_0 + \pi$ but an entrepreneur chooses to run away. If so, by Equation 2, $\theta\rho_1 j > R_e^{NC} j$ must hold. However, this is a contradiction when $R_f^{NC} < \rho_0 + \pi$ and $\rho_1 + \pi = R_f^{NC} + R_e^{NC}$. Thus, an entrepreneur doesn't run away. When a crisis materializes, the project's date-2 return $\rho_1 j - \hat{R}\tau i$ is distributed between $\{R_f, R_e\}$. For a given reinvestment scale j, suppose that $R_f j$ is less or equal to $\rho_0 j - \hat{R}\tau i$ but an entrepreneur chooses to run away. If so, by Equation 2, $\theta\rho_1 j > R_e j$ must hold. However, this is a contradiction when $R_f < \rho_0 j - \hat{R}\tau i$ and $\rho_1 - \hat{R}\tau i = R_f + R_e$. Thus, an entrepreneur doesn't run away. ∎

$$\rho_0 j \geq R_f j + \hat{R}\tau i \tag{8}$$

As a result, a project's pledgeable income is allocated between private funding and public liquidity (Equation 8). Given that pledgeable income is fixed, an increase in public liquidity *crowds out* private liquidity. However, an IG can still offer a financing option more attractive than funding liquidity since the government sets the price ($\hat{R}$) while foreign investors are price takers.

7.2 Date-1 Optimal Behavior

Entrepreneur's Date-1 Problem with an IG

$$
\begin{aligned}
\underset{\tilde{C}_1}{\text{Maximize}}\ & jR_e + \left[x - M_1\right] && (OF_{IG1})\\
\text{subject to: } & M_1 + \tau i + \phi_1 j \geq j && (BC_{IG1})\\
& R_f j \geq \phi_1 j && (PC_{IG1})\\
& \rho_0 j \geq R_f j + \hat{R}\tau i && (IC_{IG1})\\
& \rho_1 j \geq R_e j + R_f j + \hat{R}\tau i && (FC1_{IG1})\\
& x \geq M_1 && (FC2_{IG1})\\
& i \geq j && (FC3_{IG1})\\
& j, M_1, \phi_1, R_e, R_f,\ \tau \geq 0 &&\\
& i,\ x,\ \rho_0,\ and\ \rho_1 \text{ are given} &&
\end{aligned}
$$

Unlike with a MB, an entrepreneur's problem at date-1 with IG includes the term $\hat{R}\tau i$ in both the incentive compatibility constraint (IC_{UBG1}), the first feasibility constraint ($FC1_{UBG1}$), but not in the objective function. These changes are the result of the government collecting its resources at date-2 before an entrepreneur chooses to run away or not. With a IG, public liquidity is constrained by pledgeable income instead of limited liability.

The optimal behavior of an entrepreneur at date-1 (Proposition 9) points out the competition for pledgeable income happening between private funding and public liquidity. Given the linearity of the problem, an entrepreneur exhausts pledgeable income in the less expensive option. Thus, when $\hat{R}$ is less than 1, the entrepreneur only taps into public liquidity while, when $\hat{R}$ is greater than 1,it is optimal to exhaust pledgeable income with only private liquidity.

Proposition 9 (Entrepreneur's Date-1 Problem with IG Solution)
As long as Assumptions 1 and 2 hold, the optimal pecking order to cover reinvestment needs is

- *If* $\hat{R} \leq \rho_0$
    1. *Public Liquidity*
- *If* $\rho_0 < \hat{R} \leq 1$
    1. *Public Liquidity complemented with Market Liquidity*
    2. *Partial Liquidation*
- *If* $1 < \hat{R}$
    1. *Market Liquidity*
    2. *Funding Liquidity*
    3. *Partial Liquidation*

with public liquidity demand function

$$\tau(\hat{R}, x)i = \begin{cases} i & \text{if } \rho_0 \geq \hat{R} \\ min\{\frac{\rho_0}{\hat{R}}i, \ \frac{\rho_0 x}{\hat{R}-\rho_0}\} & \text{if } 1 \geq \hat{R} > \rho_0 \\ 0 & \text{if } \hat{R} > 1 \end{cases}$$

Proof See Appendix ∎

Additionally, Proposition 9 indicates that when the marginal cost of public liquidity is, at the most, equal to the pledgeable return per unit of investment ρ_0, it is optimal for an entrepreneur to fully fund reinvestment with solely a government loan. This is true even if the entrepreneur has available market liquidity.

When $\hat{R}$ is below or equal to ρ_0, it is lower than the opportunity cost of market liquidity. Then, it is a cheaper reinvestment source. More importantly, it is possible to finance a full scale reinvestment given that one unit of public liquidity invested in the project has a non-negative net present value ($\rho_0 \geq \hat{R}$). This makes the project self-financing.

Corollary 5 (Need for Market Liquidity)
When $\hat{R} > \rho_0$*, a project shutdowns if* R_e^C *is equal to zero.*

Proof Suppose there is a solution such that $\tilde{C_1}$ has $j > 0$ and $R_e^C = 0$. If $\hat{R}$ is greater or equal than 1, then, it is optimal for the entrepreneur to exhaust pledgeable income with funding liquidity. However, as Proposition 2 shows, there is no solution with a positive reinvestment level and R_e^C equal to zero. Now, if $\hat{R}$ is less than 1, then it is optimal for an entrepreneur to run out of pledgeable income with only public liquidity. By the budget constraint (BC_{UBG1}), j is equal to τi with τ strictly positive since R_e^C and ϕ_1 are zero. Replacing j in the incentive compatibility constraint and simplifying sheds that any solution to the problem requires $\rho_0 \geq \hat{R}$ which is a contradiction. ∎

However, this is not the case when the cost of a government loan is greater than pledgeable income ($\hat{R} > \rho_0$) as proved by Corollary 5. In this scenario, the investment of one unit of public liquidity has a negative net present value ($\hat{R} > \rho_0$). Thus, this investment has a negative value for both the government and foreign investors. For any positive level of reinvestment, entrepreneurs need to have some skin in the game (market liquidity) to compensate other agents' investment.

If market liquidity is necessary, the question is how much? When $\hat{R} > 1$, an entrepreneur is indifferent between funding and market liquidity. Thus, it is optimal to invest all market liquidity first and complement with funding liquidity to reach, if possible, full-scale reinvestment.

When $\rho_0 \hat{R} \leq 1$, Proposition 9 states that it is optimal to demand the maximum amount possible from the government and complement with market liquidity to reach a full-scale reinvestment. That is, to invest the minimum amount possible of market liquidity so full-reinvestment is reached.

This optimal behavior is consistent with public liquidity being less expensive than the opportunity cost of market liquidity. Entrepreneurs incur in the cost of using market liquidity only as to attract other sources of finance and overcome Corollary 5's result. Thus, they incur in the minimum cost possible.

7.3 Date-0 Optimal Behavior

At date-0, an entrepreneur under IG faces the same problem as one under a MG with the only difference that the expected payoff function $f(x, i, \hat{R})$ is determined by Proposition

9. Again, the expectation about what is going to be the cost of public liquidity affects an entrepreneur's behavior at date-0.

Proposition 10 (Entrepreneur's Date-0 Problem - IG - Optimal Solution) *As long as Assumptions 1 and 2 hold, the solution to an entrepreneur's date-0 problem has the following characteristics:*

1. *Entrepreneur's invest all their initial endowment in the project ($A = M_0$)*
2. *Foreign investors receive all pledgeable income in case of no crisis ($R_f^{NC} = \rho_0 + \pi,\ R_e^{NC} = \rho_1 - \rho_0$)*
3. *Investment scale is equal to $i = A\kappa(R_e^C)$ where $\kappa(R_e^C)$ is defined as the equity multiplier equal to $\frac{1}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C}$*
4. *For payoffs in case of a crisis, define $z = \frac{\alpha\rho_0}{\alpha\rho_0+\pi-(1-\alpha)}$, $R_f^C = \pi - R_e^C$ and $x = R_e^C i$ where*

$$R_e^C = \begin{cases} 0 & \text{if } \hat{R} \leq \rho_0 & \\ & \text{if } \rho_0 < \hat{R} < 1 & \\ 1 - \frac{\rho_0}{\hat{R}} & & \text{and } 1-\alpha \geq \pi \\ 1 - \frac{\rho_0}{\hat{R}} & & \text{and } 1-\alpha < \pi \text{ and } \hat{R} \leq z \\ 0 & & \text{and } 1-\alpha < \pi \text{ and } \hat{R} > z \\ & \text{if } \hat{R} = 1 & \\ 1 - \frac{\rho_0}{\hat{R}} & & \text{and } 1-\alpha \geq \pi \\ 0 & & \text{and } 1-\alpha < \pi \\ \text{Proposition 3} & \text{if } \hat{R} > 1 & \end{cases}$$

Proof See Appendix

Proposition 10 suggests that an entrepreneur doesn't accumulate any liquidity at date-0 when it expects that the cost of public liquidity be at the most ρ_0. This result occurs regardless the economy is facing a black swan event or not, and it parallels the optimal behavior of an entrepreneur under a MG when $\hat{R} \leq 1$. Hence, similar to the problem under a MG, a *too big to fail conundrum* appears when entrepreneurs' expect a low cost of public liquidity provision.

When facing events that are not black swans, an entrepreneur finds optimal to accumulate some market liquidity at date-0 if it expects the cost of public liquidity to be greater than ρ_0.

This optimal behavior follows entrepreneurs' acknowledging Corollary 5; that is that some market liquidity needs to be invested to reach positive levels of reinvestment.

An IG is no different from foreign lenders since it is subject to entrepreneurs incentive compatibility constraint. However, unlike foreign lenders, an IG is a price setter and not a price taker. By setting $\hat{R}$ equal or lower than ρ_0, a UBG can still assure full scale reinvestment at date-1 with its ex-post liquidity provision policy just like an Mature Government.

Equation 5 establishes that, in any equilibrium, $\hat{R}$ can only be affected by a government through the hoarding of liquidity ex-ante (F_0). The reserves function (Definition 4) depicts the amount of reserves that, for a given R_e^C level, are necessary to reach a particular cost of public liquidity ($\hat{R}$). The value $\tilde{F}_0$ is the minimum amount of liquidity hoarding that assures an $\hat{R}$ equal to ρ_0.[68]

Definition 4 (Reserves Function) *Define function* $F(\hat{R}; R_e^C) : \big[0, 1\big] \to R$ *where*

$$F(\hat{R}; R_e^C) = \frac{(1-\hat{R})A\kappa(R_e^C)\tau(\hat{R}, R_e^C)}{1+(1-\hat{R})\kappa(R_e^C)\tau(\hat{R}, R_e^C)}$$

with $\tilde{F}_0$ *defined as the necessary amount of reserves to set* $\hat{R}$ *equal to* ρ_0

$$\tilde{F}_0 := F(\rho_0; R_e^C) = \frac{(1-\rho_0)A\kappa(R_e^C)}{1+(1-\rho_0)\kappa(R_e^C)}$$

An IG needs to accumulate $\tilde{F}_0$ at date-0 to set a $\hat{R}$ equal to ρ_0 in the event of a crisis at date-1. By doing so, the immature government guarantees the continuation of projects at full-scale. However, this comes at a marginal cost of $\psi\kappa(R_e^C)$. Thus, an IG faces a trade-off between the cost of hoarding liquidity and assuring the continuation of projects.

Note that the necessary amount of reserves accumulation ($\tilde{F}_0$) is a fraction of the initial size of the economy (endowment A). Hence, the necessary stock of reserves increases with the initial size of the economy. Moreover, the fraction that is hoarded is a function of the wedge between the cost of funding liquidity and pledgeable income $(1-\rho_0)$. This result underscores two elements: first, the broker role that a government is playing between foreign

[68] This function is derived from Equation 5 and the fact that initial investment scale is equal to $(A - F_0)\kappa(R_e^C)$

investors and entrepreneurs, and, second, the fact that reserves are compensating for the inability of the government to enforce its power of taxation at all times.

Additionally, market and public liquidity can be used together to finance any reinvestment. Therefore, any optimal behavior by the government takes into account the amount of liquidity hoarded by the private sector. Proposition 11 presents a government's best respond function at date-0 to private liquidity hoarding levels.

Proposition 11 (Immature Government Date-0 Optimal behavior)
As long as Assumptions 1, 2 and 3 hold, the optimal solution of an IG at date-0 is

- *When* $\psi \tilde{F}_0 \kappa(0) > (1-\alpha)L(0)$
  - F_0 *equal to zero If* $R_e^C = 0$
  - F_0 *equal to* ω *if* R_e^C *is positive*
- *When* $\psi \tilde{F}_0 \kappa(0) \leq (1-\alpha)L(0)$
  - F_0 *equal to* $\tilde{F}_0$ *If* $R_e^C = 0$
  - F_0 *equal to* ω *if* R_e^C *is positive*

  with $\omega \ \epsilon \ \big[0, K\big]$ *where*

  1. $z \ \epsilon \big]\rho_0, 1\big]$ *such that* $R_e^C + \frac{\rho_0}{z} = 1$
  2. K *be the result of the reserves function evaluated at* z $\{K = F(z; R_e^C)\}$
  3. $\omega \quad \epsilon \ argmax \ W(F_0, z, R_e^C)$

Proof See Appendix

For discussion purposes, Figure 3 follows Proposition 11 to provide an example of the government's optimal response (ω) to different levels of private liquidity hoarding.[69] The vertical axis signals ω as the share of K, that is, the optimal level of accumulation relative to the level of reserves that guarantees a full-scale reinvestment. The horizontal axis is the amount of liquidity hoarding by the private sector relative to the amount necessary to reach full-scale reinvestment without public liquidity ($\bar{x} = 1 - \rho_0 i$).

Three remarks can be made of an IG optimal response at date-0:

[69]Parameter values for example: $\rho_1 = 1.3$, $A = 1$, $\pi = max\{0, 1-\alpha-0.1\}$, $\theta = 0.53 \rightarrow = \rho_0 = 0.6$, and $\psi = 0.2$. The loss function is assumed to be a quadratic $(j-i)^2$

1. The lower the probability of a crisis $(1 - \alpha)$, the less likely that a government chooses to insure.
2. The government is playing a brokerage role between foreign investors and entrepreneurs
3. Some partial liquidation is also optimal

Remark #1 is a consequence of an IG being unable to overcome market frictions. Given that a government needs to accumulate reserves to implement an lender of last resort program, it cannot afford to do so in every state of nature (i.e. $\psi \tilde{F}_0 \kappa(0) > (1-\alpha)L(0) and R_e^C = 0$). Thus, there is no public insurance against crises that have a low probability of occurring. This doesn't happen to mature governments. A program of liquidity provision follows every and any crisis, precisely, because it is costless to implement.

Additionally, Proposition 11 shows the broker role that the government plays between foreign lenders and entrepreneurs (Remark #2). As private agents hoard liquidity, there is lower need for a government to play this role, thus, the lower is the need to accumulate reserves. Thus, ω is non increasing with respect to x.

Moreover, another consequence of a costly accumulation of reserves is that some partial liquidation is also optimal. When a government accumulates less than K,[70] it is allowing a for some downsizing of projects. This is optimal because one additional unit of reserves accumulate has a greater marginal cost than the marginal expected loss due to partial liquidation. This result is more likely to happen as the private sector hoards more liquidity.

7.4 Multiple SPNE Equilibria with a IG

The model with an Immature Government, for non-black swan events has, at least two equilibria; one with no hoarding of private liquidity and another with hoarding of private liquidity. Policy expectations at date-0 determine in which of these two potential equilibria this economy ends up.

[70] When ω lies in the interior of $[0, K]$

An equilibrium where entrepreneurs' don't accumulate any market liquidity at date-0 and the government accumulates sufficient reserves to implement a $\hat{R}$ equal to ρ_0 exists as long as $\psi\tilde{F}_0\kappa(0) \leq (1-\alpha)L(0)$. Thus, the only condition for this equilibrium is that a government considers that is worth incurring in the cost of hoarding liquidity in exchange of assuring the continuation of projects at full-scale. For example, the probability of the crisis cannot be too low.

Corollary 6 (No Private Hoarding - IG Equilibrium)
As long as Assumptions 1, 2 and 3 hold, then, if $\psi\tilde{F}_0\kappa(0) \leq (1-\alpha)L(0)$, a No Hoarding Subperfect Game Nash Equilibrium exists and has the following characteristics

- *Date-0*
  - *Entrepreneurs* $\tilde{C}_0 = \{i = \frac{A-\tilde{F}_0}{1-\pi-\alpha\rho_0},\ M_0 = A,\ \phi_0 = i - A,\ R_e^{NC} = \rho_1 - \rho_0,\ R_f^{NC} = \rho_0 + \pi,\ R_e^C = 0,\ R_f^C = \pi\}$
  - *Government* $\{F_0 = \tilde{F}_0\}$
- *Date-1*
  - *Entrepreneurs* $\tilde{C}_1 = \{j = i,\ M_1 = 0,\ \phi_1 = 0,\ R_e = \rho_1 - \rho_0,\ R_f = 0,\ \tau = 1\}$
  - *Government* $\{B = i,\ R = \rho_0\}$
- *Date-2*
  - *Entrepreneurs don't runaway*
  - *Government collects $\rho_0 i$ and together with $\tilde{F}_0$ uses these resources to pay out sovereign bonds*

Proof See Appendix

An equilibrium where the private sector insures against crises is also possible when an economy is not facing a black swan event (Corollary 7). Entrepreneurs' are aware that an IG needs to pre-commit for a successful liquidity provision program by accumulating reserves at date-0. Thus, if they expect that the government won't accumulate the sufficient amount of reserves, then, the optimal solution facing a non black swan event, as in the Laissez faire scenario, is to hoard themselves the liquidity necessary to reinvest at full-scale. In turn, when there is enough market liquidity hoarded by entrepreneurs' at date-0, governments find optimal to not accumulate any reserves.

Corollary 7 (Private Hoarding - IG Equilibrium)
As long as Assumptions 1, 2 and 3 hold, if $1-\alpha > \pi$, a Private Hoarding Subperfect Game Nash Equilibrium exists and has the following characteristics

- *Date-0*
  - *Entrepreneur* $\tilde{C}_0 = \{i = \frac{A}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C},\ M_0 = A,\ \phi_0 = i - A,\ R_e^{NC} = \rho_1 - \rho_0,\ R_f^{NC} = \rho_0 + \pi,\ R_e^C = 1 - \rho_0,\ R_f^C = \pi - R_e^C\}$
  - *Government* $\{F_0 = 0\}$
- *Date-1*
  - *Entrepreneurs* $\tilde{C}_1 = \{j = i,\ M_1 = R_e^C i,\ \phi_1 = 0,\ R_e = \rho_1 - \rho_0,\ R_f = 0,\ \tau = \rho_0\}$
  - *Government* $\{B = \rho_0 i,\ R = 1\}$
- *Date-2*
  - *Entrepreneurs don't runaway*
  - *Government collects $\rho_0 i$ that is enough to pay out sovereign bonds ($B = \rho_0 i$)*

Proof See Appendix

8 Analysis of Equilibria

Corollaries 2, 4, 6, and 7 the equilibria of interest of the model. In his section, I review these results from four different perspectives.

Success of Liquidity Provision Programs. The goal of a lender of last resort is to provide an economy with liquid instruments in order to minimize any potential downsizing or partial liquidation of solvent projects. The Laissez Faire equilibrium indicates that public liquidity provision is most warranted during black swan events. It is in this scenario that the private sector cannot generate by itself sufficient liquidity.

As long as the expected welfare loss is sufficiently large,[71] both type of governments are able to implement a liquidity provision policy that eliminates any possibility of downsizing ($i = j$). Thus, immature governments are able to address liquidity shocks as well.

[71] That is $\psi\tilde{F}_0\kappa(0) \leq (1-\alpha)L(0)$

However, IG's find it costly to implement lender of last resort policies. Therefore, for events with a extremely low probability of occurrence, regardless of the magnitude of the finite welfare loss, it is optimal to not insure. In contrast, MG provide insurance in any possible state.

This difference between IG and MG suggests that foreign reserves don't circumvent perfectly financial frictions. There are potential global shocks that, if they were to occur, IG's would be in a more vulnerable state than MG. This decoupling gives room to the existence of mechanisms of assistance from Mature Governments to Immature Governments during global shocks.[72]

Multiple Equilibria. The model with an immature government allows for multiple equilibria. Corollary 6 and Corollary 7 prove the existence of the two most extreme equilibria: one with private liquidity hoarding, and the other with public liquidity hoarding. However, a multiple equilibria is not possible under a mature government. Why is this the case?

As argued by Farhi and Tirole (2012), multiple equilibria occurs in these type of models because there are strategic complementarities between entrepreneurs self-insurance choices due to limited pledgeability, untargeted instruments, and time inconsistency on the policy side. These three elements are both present under a MG as well as under an IG.[73]

The difference between an IG and an MG setup that allows for multiple equilibria in one and not the other is that implementing a liquidity provision program for an IG is costly, whereas for an MG it is not. Because it is costly to accumulate reserves, the policy reaction function of an IG (Proposition 11) connects each entrepreneur's decision to hoard private liquidity with the decision of other entrepreneurs.

To see this, note that it is not optimal for one entrepreneur to hoard liquidity if others are not going to do so, because an IG government will not accumulate reserves and, therefore,

[72] Consider central bank swap lines, IMF's flexible credit lines.

[73] Limited pledgeability is at the heart of the model. The cost of public liquidity is the same for all, and, thus, it is an untargeted instrument in this sense. Time inconsistency from the government is assumed, hence, the definition of an equilibrium as the Subperfect Nash Equilibrium, that solves for time consistent strategies

cannot provide liquidity at date-1. In contrast, under a MG, regardless if others hoard liquidity or not, an entrepreneur can always ask for a transfer at date-1 if necessary, and an MG can provide it.

What Farhi and Tirole (2012) don't discuss is that an IG's strategy and the private sector strategy (entrepreneurs as a whole) at date-0 are strategic substitutes, which also are a characteristic of multiple equilibria. Given that the feasible amount of reinvestment is bounded by the initial scale, and it can be financed either with private liquidity or public liquidity, there is no need for both agents to buy insurance. Therefore, if the private sector expects an IG government to hoard liquidity, it will not do so, and viceversa. The only condition is that both type of agents find optimal to buy insurance, even if the other is not buying.[74]

Too Big to Fail Conundrum. I showed that a Mature Government cannot eliminate the *too big to fail conundrum* because it cannot make credible its promise to not intervene ex-post. However, Corollary 7 proves that this is not the case of an IG.

Despite allowing for multiple equilibria, an IG has the ability to drive the economy to its preferred equilibrium, which ever it is. To see the intuition behind this result, suppose for a moment that, at date-0, the government makes the decision of F_0 before entrepreneurs have to make any decision. Under an MG, entrepreneurs are indifferent to that decision because it can always provide liquidity at date-1. However, under an IG, if it chooses F_0 equal to zero, then, entrepreneurs know that it will not be able to provide liquidity, and they will choose to hoard liquidity if, under the parameter values, it is optimal to do so. Therefore, immature governments have outcomes without a *too big to fail conundrum* precisely because they find it costly to implement ex-post policies. Their choice of F_0 is a credible signal.

Pledgeability and Financial Frictions. In this model, the liquidity demand at date-1 rises for the necessity to reinvest in the project following a crisis. Under a Laissez Faire equilibrium, the reinvestment level can be covered using market liquidity and funding liq-

[74]This happens when facing a not black swan event, and the cost of accumulating reserves is lower than the expected cost of downsizing

uidity. These sources are bounded, in equilibrium, by x and total pledgeable income $\rho_0 j$ due to financial frictions (Equation 9).

$$j = M_1 + \phi_1 j \rightarrow j \leq x + \rho_0 j \tag{9}$$

How does the pledgeability frontier change with public liquidity provision? With government loans, entrepreneurs have an additional source of funding to cover reinvestment. In equilibrium, under a mature government, government loans are bounded by entrepreneurs limited liability which implies that $\hat{R}\tau i \leq jR_e + (X - M_1)$. Moreover, the SPNE equilibrium under a MG also indicates that $\hat{R} = 1$ and that $jR_e = \rho_1 j - \phi_1 j$. These equilibrium results can be sued to show that reinvestment levels are bounded by x plus the total return of projects $\rho_1 j$ (Equation **??**). Therefore, by overcoming financial frictions, an mature government expands the economy's pledgeability frontier from $x + \rho_0 j$ to $x + \rho_1 j$. Moreover, since $\rho_1 > 1$, then this new frontier is greater than i and full-scale reinvestment is possible.

$$j = M_1 + \phi_1 j + \tau i \rightarrow j \leq x + \rho_1 j \tag{10}$$

With a immature government, the analysis is a bit different. Recall that, in equilibrium under an IG, market liquidity is limited by x, funding liquidity is limited by $\rho_0 j - \hat{R}\tau i$ and that the government sets $\hat{R}$ by following Equation 5. Therefore, the pledgeability frontier is determined by $x + \rho_0 j + F_0$.

$$j = M_1 + \phi_1 j + \tau i \leq x + \rho_0 j - \hat{R}\tau i + \tau i \rightarrow j \leq x + \rho_0 j + F_0 \tag{11}$$

Equation 11 shows that an IG improves pledgeability in the economy, relative to a laissez faire scenario, only if it accumulates foreign reserves at date-0. The reason is that reserves allow an IG to offer public liquidity at a lower cost, that increases the relative value of a project's pledgeable income. Hence, even though immature governments are subject to financial frictions, reserves allows them to circumvent them and provide useful liquidity at date-1.

9 Final Remarks

In this paper, I provide a novel rationale for why emerging economies accumulate foreign reserves for liquidity provision purposes. I show that reserves accumulation is a potential equilibrium outcome for governments that lack the fiscal capacity to tax the private sector at any moment. To do so, I built a three period theoretical model to underscore the channels through which different levels of fiscal capacity affect incentives to accumulate foreign reserves.

Throughout the paper I argue that currency mismatch is not a necessary condition for governments to accumulate reserves, and that, fiscal capacity can explain a share of why countries do so. Therefore, an obvious extension to this model would be to include two different goods (tradable and non-tradable) to study how currency mismatch and fiscal capacity interact. Plausibly, the existence of different levels of fiscal capacity could even justify the surge of currency mismatches.

Moreover, the model has focused on a quite simplified version of a crisis. First of all, the probability of a crisis is exogenous when it has been shown empirically and also theoretically that the level of reserves diminishes the incidence and the likelihood of a crisis.[75] Likewise, the crisis modeled consists of only a required reinvestment, yet, there is no increase in the cost of private funding. A way forward would be to model more explicitly the behavior of foreign lenders and, therefore, see how fiscal capacity and reserves accumulation affects the probability and the cost of a crisis.

Finally, behind the main result of this paper lies the idea that foreign reserves, more than a liquidity provider instrument, are a policy tool that increases the pledgeable income of an economy. Thus, by accumulating reserves, a government compensates for an underdeveloped power of taxation by acquiring an asset that attracts necessary resources from world capital markets. In fact, in this model, foreign reserves do not need to be liquid assets nor do they need to be at the government's immediate disposal. The only necessary characteristic is that

[75]See Frankel and Saravelos (2010) and Céspedes and Chang (2019)

are safe and redeem after the crisis.

The idea that reserves are more than direct liquidity providers could be further explored in a dynamic model where a sequence of liquidity distress episodes are possible. In principle, since reserves increase the pledgeability frontier, then it would be optimal to issue government bonds backed up by reserves rather then selling them since in the future another crisis can happen, and you would need reserves to attract funding again. This could provide a rationale of why countries accumulate reserves for liquidity purposes but are reluctant to use them during distressed episodes.

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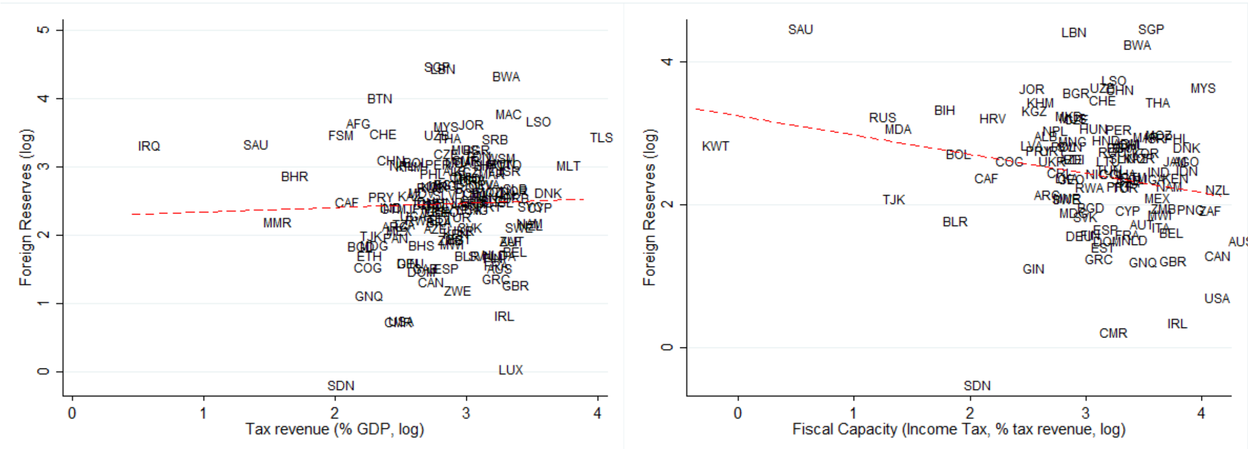
Tables and Figures



(a) Total Tax Revenue (b) Income Tax Revenue

Figure 1: Foreign Reserves and Fiscal Capacity
Country Average (1990 - 2018)

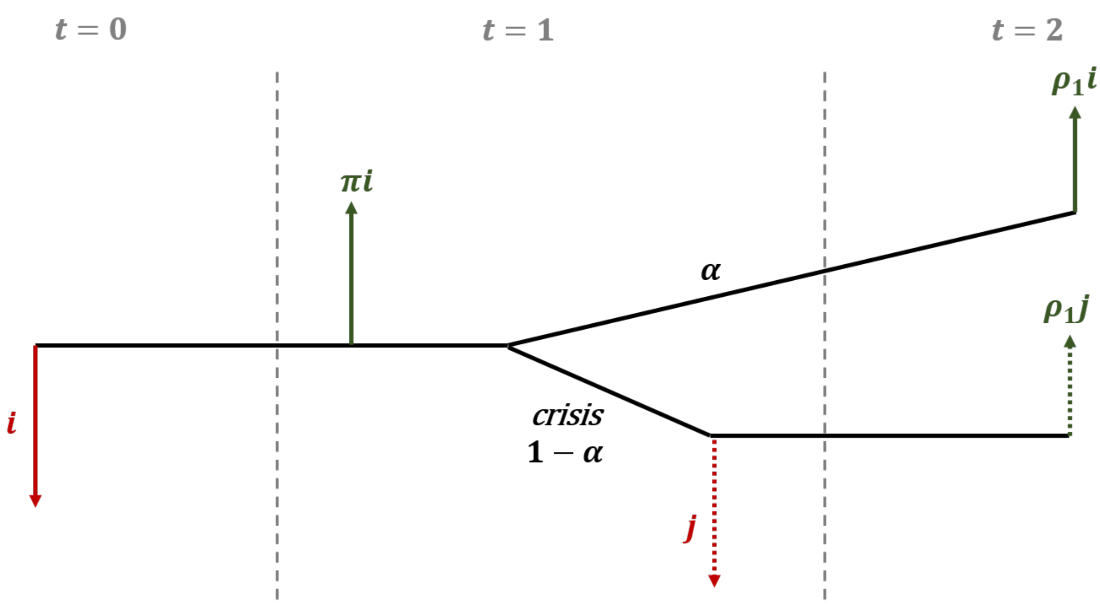


Figure 2: Project Technology - Timeline

Table 9.1: Summary statistics

	Mean	SD	Min	Max	N
Dependent Variable					
Foreign Reserves (% GDP, log)	2.5	1.0	-2.4	4.8	1,858
Fiscal Capacity Model					
Tax Revenue (% GDP, log)	2.7	0.4	-2.5	3.9	1,858
Income Tax Revenue (% TR, log)	3.0	0.7	-0.9	4.3	1,858
Traditional Model					
Imports (% GDP, log)	3.6	0.6	-2.3	5.3	1,858
Exports Vol. (log, 3-year sd)	0.1	0.1	0.0	2.9	1,858
Monthly ER Vol. (Annual sd)	1.8	75.1	0.0	3,238.3	1,858
GDP (log)	25.1	1.9	20.6	30.6	1,858
Financial Sta. Model					
Broad Money (% GDP, log)	4.0	0.6	2.1	5.6	1,858
Chinn Ito Index (0-1)	0.6	0.4	0.0	1.0	1,858
High Income dummy	0.4	0.5	0.0	1.0	1,858
Hard Peg dummy	0.7	0.5	0.0	1.0	1,858
Soft Peg dummy	0.1	0.3	0.0	1.0	1,858
Short Term Debt (% GDP, log)	-2.5	1.2	-7.6	1.3	1,858
Mercantalist Model					
Currency Overvaluation	-0.4	0.3	-0.9	0.6	1,858
Financial Dev. Model					
Domestic Financial Liab. (% GDP, log)	4.1	1.0	0.5	6.2	1,858
Private Foreign Liabilities (% GDP, log)	-2.3	1.2	-8.5	1.3	1,858
Public Foreign Liabilities (% GDP, log)	-4.0	1.6	-16.9	-0.3	1,858
Original Sin Model					
Original Sin Index (0-1)	0.8	0.3	0.0	1.0	1,180

Table 9.2: Foreign Reserves and Fiscal Capacity - OLS Regression

	Whole Sample		EME		Pre-GFC		Post GFC		Balanced Panel		Euro Area	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Imports (% GDP, log)	0.196	0.208	0.207**	0.174	0.201*	0.172	0.197	0.243	0.545**	0.636***	0.438	1.102**
	(0.157)	(0.176)	(0.103)	(0.116)	(0.110)	(0.129)	(0.296)	(0.290)	(0.248)	(0.198)	(0.360)	(0.333)
Exports Vol. (log, 3-year sd)	-0.181	-0.241	-0.112	-0.104	0.109	0.219	-0.154	-0.274	0.347	0.099	0.769	3.218
	(0.207)	(0.191)	(0.174)	(0.172)	(0.366)	(0.346)	(0.195)	(0.212)	(0.368)	(0.382)	(2.621)	(2.460)
Monthly ER Vol. (Annual sd)	-0.000***	-0.000***	-0.000***	-0.000***	-0.000***	-0.000***	-0.639	-0.348	0.556*	0.254	162.813	-350.968
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(2.506)	(2.140)	(0.282)	(0.296)	(587.797)	(552.104)
GDP (log)	-0.034	-0.023	0.015	0.030	-0.066*	-0.035	-0.003	-0.028	0.143***	0.268***	-0.031	0.223***
	(0.044)	(0.044)	(0.036)	(0.036)	(0.039)	(0.049)	(0.070)	(0.071)	(0.049)	(0.071)	(0.075)	(0.053)
Broad Money (% GDP, log)	0.609***	0.605***	0.814***	0.813***	0.555***	0.559***	0.668**	0.654**	0.135	0.114	-0.396***	-0.430***
	(0.190)	(0.183)	(0.160)	(0.158)	(0.167)	(0.162)	(0.297)	(0.284)	(0.255)	(0.214)	(0.116)	(0.123)
Chinn Ito Index (0-1)	0.156	0.171	0.326*	0.346**	0.064	0.096	0.339	0.403*	0.472	0.622	-0.496	0.908
	(0.167)	(0.173)	(0.166)	(0.169)	(0.187)	(0.199)	(0.213)	(0.225)	(0.368)	(0.383)	(2.041)	(1.999)
High Income dummy	-0.030	-0.107	0.172	0.088	0.331**	0.258	-0.495	-0.656**	0.415**	0.296**		
	(0.193)	(0.185)	(0.154)	(0.131)	(0.161)	(0.170)	(0.351)	(0.308)	(0.183)	(0.138)		
Hard Peg dummy	0.528***	0.462***	0.313***	0.316***	0.508***	0.461***	0.553***	0.506***	0.561**	0.413*	-2.931	7.817
	(0.122)	(0.124)	(0.112)	(0.099)	(0.134)	(0.139)	(0.189)	(0.182)	(0.222)	(0.210)	(11.769)	(11.164)
Soft Peg dummy	0.672***	0.642***	0.306***	0.337***	0.461***	0.446***	0.815***	0.793***	0.589**	0.496**		
	(0.130)	(0.135)	(0.111)	(0.107)	(0.129)	(0.130)	(0.212)	(0.214)	(0.261)	(0.229)		
Short Term Debt (% GDP, log)	-0.345**	-0.361**	-0.304*	-0.304	-0.172	-0.165	-0.563**	-0.598**	-0.163	-0.151	0.177	0.248
	(0.154)	(0.153)	(0.180)	(0.186)	(0.107)	(0.102)	(0.225)	(0.231)	(0.175)	(0.165)	(0.350)	(0.292)
Currency Overvaluation	-1.182**	-1.052**	-0.444	-0.459	-1.659***	-1.701***	-0.582	-0.189	-1.323**	-1.576**	-0.773**	-1.960***
	(0.454)	(0.466)	(0.380)	(0.367)	(0.419)	(0.434)	(0.603)	(0.547)	(0.574)	(0.544)	(0.307)	(0.365)
Domestic Financial Liab. (% GDP, log)	0.067	0.093	-0.111	-0.107	0.058	0.055	0.125	0.218	0.011	0.031	0.661**	0.347
	(0.110)	(0.110)	(0.086)	(0.084)	(0.089)	(0.090)	(0.198)	(0.203)	(0.177)	(0.174)	(0.234)	(0.319)
Private Foreign Liabilities (% GDP, log)	0.327**	0.341***	0.323*	0.321	0.265***	0.269***	0.415**	0.449**	0.247	0.245	-0.813	-0.850**
	(0.125)	(0.125)	(0.193)	(0.202)	(0.088)	(0.088)	(0.198)	(0.206)	(0.158)	(0.182)	(0.471)	(0.281)
Public Foreign Liabilities (% GDP, log)	-0.037	-0.037	-0.016	-0.016	-0.079	-0.089	0.041	0.057	0.097	0.053	0.511***	0.344***
	(0.053)	(0.052)	(0.030)	(0.031)	(0.058)	(0.062)	(0.059)	(0.061)	(0.060)	(0.056)	(0.138)	(0.058)
Tax Revenue (% GDP, log)		-0.004		0.143		0.203		-0.335		0.430		1.869***
		(0.204)		(0.145)		(0.179)		(0.295)		(0.272)		(0.484)
Income Tax Revenue (% TR, log)		-0.171**		-0.152**		-0.175		-0.195**		-0.467**		-1.446**
		(0.082)		(0.062)		(0.107)		(0.088)		(0.210)		(0.474)
Observations	1858	1858	1290	1290	918	918	779	779	476	476	170	170
R^2	0.39	0.40	0.52	0.53	0.46	0.47	0.35	0.39	0.66	0.69	0.74	0.82
Countries	100	100	71	71	94	94	94	94	17	17	9	9

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. Observations clustered by country. Time fixed effects are not reported but are included in every regression.

Table 9.3: Foreign Reserves, Fiscal Capacity and Original Sin - OLS Regression

	Whole Sample		EME		Pre-GFC		Post GFC		Balanced Panel		Euro Area	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Imports (% GDP, log)	0.215	0.239	0.314**	0.332**	0.091	0.129	0.161	0.200	0.469**	0.469**	0.440	1.271**
	(0.220)	(0.217)	(0.133)	(0.145)	(0.224)	(0.212)	(0.246)	(0.242)	(0.217)	(0.192)	(0.440)	(0.414)
Exports Vol. (log, 3-year sd)	0.609	0.379	0.741*	0.581	0.959*	1.025**	1.031	0.145	0.733	0.637	-0.185	1.977
	(0.613)	(0.588)	(0.435)	(0.463)	(0.497)	(0.514)	(1.075)	(0.978)	(0.532)	(0.590)	(2.007)	(2.171)
Monthly ER Vol. (Annual sd)	-0.974	-0.996	-1.650*	-1.511*	-1.799	-1.842	0.180	0.896	-1.681	-1.949		
	(1.415)	(1.152)	(0.907)	(0.797)	(1.378)	(1.296)	(2.257)	(1.611)	(1.819)	(1.779)		
GDP (log)	0.007	-0.020	0.045	0.024	-0.120*	-0.116	0.022	-0.034	0.144*	0.192*	-0.024	0.247***
	(0.050)	(0.048)	(0.034)	(0.033)	(0.064)	(0.074)	(0.056)	(0.058)	(0.074)	(0.104)	(0.086)	(0.065)
Broad Money (% GDP, log)	0.483**	0.474**	0.642***	0.621***	0.368*	0.360*	0.570**	0.562**	-0.032	0.041	-0.407**	-0.550**
	(0.206)	(0.193)	(0.174)	(0.164)	(0.196)	(0.185)	(0.240)	(0.228)	(0.358)	(0.314)	(0.138)	(0.171)
Chinn Ito Index (0-1)	0.044	0.029	0.223*	0.193	-0.294	-0.280	0.384*	0.371*	0.409	0.550	-0.813	0.685
	(0.180)	(0.187)	(0.127)	(0.127)	(0.238)	(0.256)	(0.197)	(0.208)	(0.337)	(0.398)	(1.723)	(1.631)
High Income dummy	0.032	-0.055	0.119	0.022	0.387*	0.339	-0.225	-0.382	0.211	0.112		
	(0.238)	(0.212)	(0.176)	(0.148)	(0.217)	(0.226)	(0.335)	(0.288)	(0.255)	(0.252)		
Hard Peg dummy	0.678***	0.567***	0.415**	0.392***	0.920***	0.766***	0.508**	0.421**	1.133***	0.936**	0.210	0.510**
	(0.166)	(0.164)	(0.172)	(0.141)	(0.177)	(0.181)	(0.210)	(0.197)	(0.275)	(0.385)	(0.238)	(0.200)
Soft Peg dummy	0.744***	0.681***	0.333*	0.337**	0.772***	0.670***	0.642***	0.590**	1.217***	1.067**		
	(0.176)	(0.172)	(0.175)	(0.152)	(0.206)	(0.208)	(0.237)	(0.227)	(0.274)	(0.370)		
Short Term Debt (% GDP, log)	-0.102	-0.132	0.037	0.015	-0.258	-0.317**	0.021	0.003	-0.152	-0.179	0.117	0.307
	(0.135)	(0.128)	(0.101)	(0.093)	(0.158)	(0.143)	(0.160)	(0.158)	(0.164)	(0.156)	(0.310)	(0.218)
Currency Overvaluation	-0.925**	-0.640	-0.229	-0.079	-1.230**	-1.043**	-0.787	-0.338	-0.854	-0.877	-0.799**	-2.404***
	(0.412)	(0.405)	(0.495)	(0.524)	(0.497)	(0.497)	(0.530)	(0.467)	(0.612)	(0.630)	(0.259)	(0.492)
Domestic Financial Liab. (% GDP, log)	0.161	0.230	-0.081	-0.023	0.308**	0.360**	0.198	0.287	0.132	0.172	0.675**	0.227
	(0.137)	(0.139)	(0.102)	(0.097)	(0.129)	(0.140)	(0.173)	(0.178)	(0.174)	(0.179)	(0.237)	(0.231)
Private Foreign Liabilities (% GDP, log)	0.032	0.049	-0.012	-0.002	0.238	0.295*	-0.142	-0.150	0.239	0.267	-0.788	-0.974**
	(0.141)	(0.133)	(0.121)	(0.105)	(0.180)	(0.154)	(0.160)	(0.160)	(0.173)	(0.179)	(0.463)	(0.311)
Public Foreign Liabilities (% GDP, log)	0.074	0.088	0.033	0.055	-0.085	-0.086	0.189**	0.213**	-0.001	-0.009	0.489***	0.210
	(0.080)	(0.080)	(0.042)	(0.042)	(0.112)	(0.113)	(0.089)	(0.089)	(0.057)	(0.066)	(0.133)	(0.125)
Original Sin Index (0-1)	1.721***	1.649***	-1.217**	-1.216***	1.071**	1.069**	2.172***	2.031***	-2.559***	-2.704***	0.067	-0.026
	(0.339)	(0.329)	(0.555)	(0.430)	(0.457)	(0.456)	(0.353)	(0.353)	(0.751)	(0.682)	(0.655)	(0.899)
Tax Revenue (% GDP, log)		-0.230		-0.159		-0.162		-0.400		-0.061		2.162***
		(0.231)		(0.186)		(0.257)		(0.261)		(0.184)		(0.502)
Income Tax Revenue (% TR, log)		-0.220**		-0.154**		-0.222**		-0.214**		-0.312		-1.629***
		(0.085)		(0.074)		(0.103)		(0.084)		(0.312)		(0.365)
Observations	1180	1180	708	708	397	397	656	656	301	301	162	162
R^2	0.51	0.53	0.54	0.57	0.61	0.62	0.48	0.53	0.72	0.74	0.75	0.84
Countries	86	86	57	57	69	69	82	82	17	17	9	9

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. Observations clustered by country. Time fixed effects are not reported but are included in every regression.

Table 9.4: Foreign Reserves and Fiscal Capacity - Robustness Check

	Main Specification (MS)				MS with Original Sin			
	(1) YE	(2) CFE	(3) YE+CFE	(4) Between	(5) YE	(6) CFE	(7) YE+CFE	(8) Between
Imports (% GDP, log)	0.208	0.136***	0.084*	0.463*	0.239	0.097	-0.098	0.481*
	(0.176)	(0.048)	(0.049)	(0.273)	(0.217)	(0.087)	(0.105)	(0.267)
Exports Vol. (log, 3-year sd)	-0.241	-0.233*	-0.249*	2.414	0.379	-0.139	-0.332	6.895**
	(0.191)	(0.132)	(0.132)	(1.664)	(0.588)	(0.242)	(0.246)	(2.711)
Monthly ER Vol. (Annual sd)	-0.000***	-0.000	-0.000	-0.006**	-0.996	1.514***	0.570	-0.006***
	(0.000)	(0.000)	(0.000)	(0.002)	(1.152)	(0.563)	(0.612)	(0.002)
GDP (log)	-0.023	0.382***	0.123*	0.065	-0.020	0.162***	-0.109	0.077
	(0.044)	(0.035)	(0.066)	(0.062)	(0.048)	(0.045)	(0.080)	(0.063)
Broad Money (% GDP, log)	0.605***	0.305***	0.260***	0.687**	0.474**	0.067	-0.023	0.732***
	(0.183)	(0.062)	(0.065)	(0.262)	(0.193)	(0.087)	(0.093)	(0.193)
Chinn Ito Index (0-1)	0.171	-0.056	-0.150**	0.475	0.029	0.165*	0.165*	0.413
	(0.173)	(0.064)	(0.066)	(0.293)	(0.187)	(0.094)	(0.093)	(0.310)
High Income dummy	-0.107			-0.540**	-0.055			-0.232
	(0.185)			(0.247)	(0.212)			(0.202)
Hard Peg dummy	0.462***	0.168***	0.208***	0.401	0.567***	-0.011	-0.021	0.423
	(0.124)	(0.037)	(0.056)	(0.323)	(0.164)	(0.039)	(0.072)	(0.266)
Soft Peg dummy	0.642***	0.155***	0.182***	0.867**	0.681***	0.021	0.007	0.714***
	(0.135)	(0.049)	(0.062)	(0.345)	(0.172)	(0.054)	(0.082)	(0.268)
Short Term Debt (% GDP, log)	-0.361**	-0.118***	-0.128***	-0.866***	-0.132	-0.038	-0.044	-0.497*
	(0.153)	(0.034)	(0.034)	(0.277)	(0.128)	(0.044)	(0.044)	(0.266)
Currency Overvaluation	-1.052**	-1.282***	-0.958***	-0.242	-0.640	-0.865***	-0.816***	-0.171
	(0.466)	(0.142)	(0.182)	(0.611)	(0.405)	(0.149)	(0.178)	(0.524)
Domestic Financial Liab. (% GDP, log)	0.093	-0.153***	-0.125***	0.129	0.230	-0.099*	-0.029	0.121
	(0.110)	(0.039)	(0.040)	(0.173)	(0.139)	(0.052)	(0.054)	(0.135)
Private Foreign Liabilities (% GDP, log)	0.341***	0.158***	0.153***	0.636***	0.049	0.162***	0.182***	0.338
	(0.125)	(0.030)	(0.030)	(0.232)	(0.133)	(0.048)	(0.048)	(0.261)
Public Foreign Liabilities (% GDP, log)	-0.037	-0.055***	-0.058***	0.042	0.088	0.059**	0.061**	0.098
	(0.052)	(0.013)	(0.013)	(0.090)	(0.080)	(0.025)	(0.025)	(0.099)
Original Sin Index (0-1)					1.649***	-1.060***	-0.879***	1.528***
					(0.329)	(0.194)	(0.195)	(0.354)
Tax Revenue (% GDP, log)	-0.004	0.320***	0.331***	0.032	-0.230	0.503***	0.564***	-0.112
	(0.204)	(0.080)	(0.080)	(0.275)	(0.231)	(0.119)	(0.118)	(0.269)
Income Tax Revenue (% TR, log)	-0.171**	-0.011	-0.035	-0.322***	-0.220**	0.053	0.037	-0.394***
	(0.082)	(0.042)	(0.042)	(0.115)	(0.085)	(0.065)	(0.064)	(0.090)
Observations	1858	1858	1858	100	1180	1180	1180	86
R^2	0.40	0.21	0.24	0.55	0.53	0.11	0.17	0.65

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. YE=Year Fixed Effects, CFE=Country Fixed Effects, and Between refers to results of running regressions across panel averages for each country.

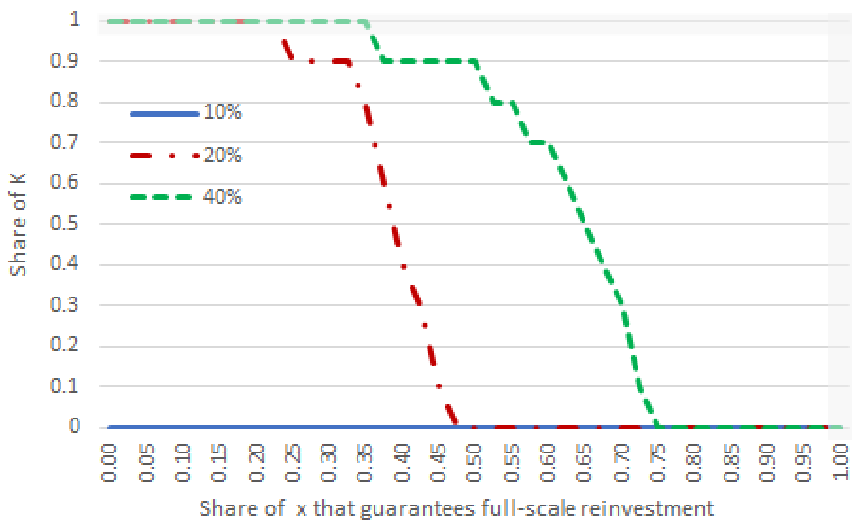


Figure 3: Reserves Accumulation - Optimal IG response Date-0 - by probability of crisis

Appendices

A Proofs

A.1 Proposition 2

First, note that a project shutdown ($C_1 = \{j = 0, M_1 = 0, \phi_1 = 0, R_f = 0, R_e = 0)$ is a feasible solution to the problem. Now, I focus for solutions with positive levels of reinvestment ($j^* > 0$). In any solution of this type, given that it is a zero sum game between entrepreneurs and foreign investors, PC_1 binds because an entrepreneur would allocate just enough to a foreign investor for that person to participate. Additionally, BC_1, and $FC1_1$ bind as well because, otherwise, an entrepreneur would be wasting funding resources and/or would be leaving a some of the project's return on the table.

Now, suppose there is a solution to the problem where M_1 is equal to zero. By BC_1, ϕ_1 is equal to one for any positive level of reinvestment. Thus, by PC_1 and IC_1, $\rho_0 \geq 1$ which is a contradiction because $\rho_0 < 1$. Therefore, there is no solution with positive reinvestment where no market liquidity is invested by an entrepreneur.

Could partial liquidation be optimal when there is enough market and funding liquidity for full-scale reinvestment ($x + \rho_0 \geq i$)? The answer is no. To see this, suppose that there is a C_1^* with $j^* < i$ that maximizes date-1 problem. By rearranging constraints and the objective function, the payoff of such solution is equal to $j^*\big[\rho_1 - 1\big] + x$. Let's consider two other candidate solutions. The first is a candidate where $M_1 = i$ and $\phi_1 = 0$. This candidate solution is feasible when $x \geq i$ and has a payoff equal to $i\big[\rho_1 - 1\big] + x$. Since $i > j^*$ and $\rho_1 > 1$, then the payoff of the feasible candidate solution is greater than the solution with partial liquidation. Now, consider a second candidate solution where $x = M_1$ and $\phi_1 = 1 - \frac{x}{i}$. Thus is a solution that is feasible since $x + \rho_0 \geq i$ and consists of exhausting market liquidity and using funding liquidity to reach full-scale reinvestment. The payoff of this solution is equal to $i\big[\rho_1 - (1 - \frac{x}{i})\big]$ that, by rearranging, is equal to $i\big[\rho_1 - 1\big] + x$. Again, for the same reasons as before, this payoff is strictly greater than $j^*\big[\rho_1 - 1\big] + x$. Partial liquidation is not an option when full-scale reinvestment is feasible.

Define υ as the minimum amount of market liquidity to reach a full-scale reinvestment. That is, from BC_1, $\upsilon = i\big[1 - \rho_0\big]$. This is the minimum amount that an entrepreneur needs to invest because it can reach full-scale by exhausting funding liquidity up to $\rho_0 i$ which is the maximum available amount due to the incentive compatibility constraint. If $x \geq \upsilon$, then a full-scale reinvestment is reached. Define set $\Gamma := \{(M_1, \phi_1)|\ M_1\ \epsilon\ \big[\upsilon, x\big],\ \phi_1 \leq \rho_0,\ and\ \frac{M_1}{1-\phi_1} = i\}$, so, any pair (M_1, ϕ_1) that belongs to set Γ reaches full-scale reinvestment. Note that, by using the constraints, the payoff function of an entrepreneur, when $x \geq \upsilon$, can be written as $i\big[\rho_1 - 1\big] + x$. Thus, any pair of set Γ reaches the same value in the objective function. The intuition behind this result is that once M_1 is greater or equal to υ, an entrepreneur is indifferent because the marginal cost of funding liquidity is equal to the opportunity cost of market liquidity. Given that an entrepreneur is indifferent between pairs of this set, one optimal pecking order is where it exhausts market liquidity available first, and then moves to funding liquidity to reach full-scale reinvestment as suggested by Proposition 2.

What happens when full-scale reinvestment is not possible ($x < i\big[1-\rho_0\big]$)? Note that exhausting both market liquidity and funding liquidity implies a payoff equal to $\frac{\rho_1-\rho_0}{1-\rho_0}x$. Note that since $\rho_1 > 1$ then the ratio is greater than 1, thus, this payoff is greater than x which is the payoff if the entrepreneur chooses to shutdown. Finally, following the same reasoning as why an entrepreneur delays partial liquidation as much as possible, then it is never optimal to invest less than total market and funding liquidity even if a downsizing is required.

$$j(x) = \begin{cases} i & \text{if } i\big[1-\rho_0\big] \le x \\ \frac{x}{1-\rho_0} & \text{if } 0 < x < i\big[1-\rho_0\big] \\ 0 & \text{if } x = 0 \end{cases}$$

$$f(x,i) = \begin{cases} i\big[\rho_1 - 1\big] + x & \text{if } i\big[1-\rho_0\big] \le x \\ \big[\rho_1-\rho_0\big]\frac{x}{1-\rho_0} & \text{if } 0 < x < i\big[1-\rho_0\big] \\ 0 & \text{if } x = 0 \end{cases}$$

A.2 Proposition 3

First, note that not investing in the program is a feasible solution. Now, any optimal solution to an entrepreneur's date-0 problem with positive investment has BC_0, $FC1_0$, and $FC2_0$ binding because, otherwise, the entrepreneur would be either wasting resources or leaving some on the table. Additionally, recall that the allocation of payoffs between the entrepreneur and foreign investors is a zero sum game, thus, it is optimal to allocate investors just enough such that they participate. Thus, PC_0 binds as well. Given that these equations bind, the problem can be rewritten as

$$\begin{aligned} \underset{\{R_f^{NC}, M_0, R_e^C\}}{\text{Maximize}} \;\; & \alpha\big[\rho_1 + \pi - R_f^{NC}\big]i + (1-\alpha)f(x,i) + \alpha(A - M_0) \\ \text{subject to: } & i = M_0\kappa(R_f^{NC}, R_e^C) \\ & \kappa(R_f^{NC}, R_e^C) = \frac{1}{1-(1-\alpha)\pi - \alpha R_f^{NC} + (1-\alpha)R_e^C} \\ & x = R_e^C i + (A - M_0) \\ & \rho_o i + \pi i \ge R_f^{NC} i \\ & \pi i \ge R_e^C i \\ & A \ge M_0 \\ & i, M_0, R_f^{NC}, R_e^C \ge 0 \\ & A,\; f(x,i),\; \rho_0,\; \pi,\; \textit{and } \rho_1 \text{ are given} \end{aligned}$$

To start, I show that it is optimal to allocate as much as possible to foreign investors when there is no crisis ($R_f^{NC} = \rho_0 + \pi$). Choose a feasible pair (M_0, R_e^C). The first order condition for R_f^{NC}, presented below, shows that allocating one unit to entrepreneur when there is no crisis increases the investment scale times the expected payoff but it reduces an entrepreneurs payoff per unit of investment if no crisis happens. Since

$R_e^C \leq \pi$, Assumption 1, both $\frac{\partial f}{\partial i}(x,i)$ and $\frac{\partial i(R_f^{NC}, R_e^C, M_0)}{\partial R_f^{NC}}$ are non-negative, then this F.O.C is positive for any possible pair of (M_0, R_e^C). Thus, an entrepreneur optimizes by choosing the maximum value for R_f^{NC} $(\rho_0 + \pi)$.

$$\frac{\partial i(R_f^{NC}, R_e^C, M_0)}{\partial R_f^{NC}}\big[\alpha(\rho_1 + \pi - R_f^{NC}) + (1-\alpha)\frac{\partial f}{\partial i}(x,i)\big] - \alpha i$$

For the rest of the proof, I assume that this first result holds. Thus, by letting R_f^{NC} be equal to $\rho_0 + \pi$, the equity multiplier is equal to $\kappa(\rho_0 + \pi, R_e^C) = \kappa(R_e^C) = \frac{1}{1-\pi-\alpha\rho_0+(1-\alpha)R_e^C}$.

Turning to the optimal choice of R_e^C and M_0. Choosing the levels for these variables implies choosing x. By selecting a positive x, an entrepreneur is sacrificing investment scale in order to increase reinvesting levels if a crisis happens. Assuming that an entrepreneur wants to hoard a positive level of market liquidity for date-1, denoted by $\bar{x}$, what is the combination of R_e^C and M_0 that minimizes the cost (i.e. maximizes the investment scale). Note that to reach $x = \bar{x}$, using equation ML, then $M_0 = \frac{A-\bar{x}}{1-R_e^C\kappa(R_e^C)}$. By replacing this last equation in i, it is possible to derive the level of investment when $x = \bar{x}$. In this scenario, investment scale is equal to $\frac{A-\bar{x}}{1-\pi-\alpha\rho_0-\alpha R_e^C}$. The derivative of this equation with respect to R_e^C is positive, thus, the cost of hoarding liquidity is minimized when is done only through R_e^C and not M_0. The intuition is simple. By hoarding liquidity through lower M_0, an entrepreneur is sacrificing scale even if a crisis doesn't happen. While, by accumulating liquidity through R_e^C, it is contingent on the realization of a crisis. Thus, no waste of resources. This result argues that it is optimal for M_0 to be A and the decision of liquidity hoarding be left only to R_e^C. Therefore $x = R_e^C A\kappa(R_e^C)$. Now, the F.O.C of the problem relative to R_e^C is equal to

$$\frac{\partial \kappa}{\partial R_e^C}(R_e^C)A\alpha\big[\rho_1 - \rho_0\big] + (1-\alpha)\frac{\partial f}{\partial R_e^C}(R_e^C A\kappa(R_e^C), A\kappa(R_e^C))$$

Given Assumption 1, numeral 1, this F.O.C is decreasing for values R_e^C greater or equal than $1-\rho_0$, thus, an entrepreneur never chooses hoarding liquidity beyond $1-\rho_0$. The intuition is straightforward. Hoarding liquidity has the cost of sacrificing investment scale, thus, it is never optimal to hoard more liquidity than what you need for full scale reinvestment complemented with funding liquidity. For values R_e^C less or equal than $1-\rho_0$, the sign of the F.O.C can be simplified to the sign of $\big[(1-\pi-\alpha\rho_0) - \alpha + \alpha\rho_0\big]$. If $\alpha + \pi > 1$, then the F.O.C is negative, thus, an entrepreneur chooses $R_e^C = 0$. If $\alpha + \pi \leq 1$, this F.O.C is increasing and the optimal R_e^C is equal to $1-\rho_0$. ∎

A.3 Corollary 2

I prove this equilibrium showing that an entrepreneur has no incentive to deviate. Start considering when a crisis doesn't happen. In this scenario, foreign investors get π at date-1 and get ρ_0 at date-2 according to C_0. Since this payoff is equal to the pledgeable income of the project, an entrepreneur would no benefit from running away since it would not increase its payoff (Proposition 1). Now, what about when a crisis happens, If the crisis is a black swan $(\alpha + \pi > 1)$, the project is shutdown at date-1, thus, there is nothing to runaway with. If the crisis is not a black swan, C_1 establishes that the date-2 payoff for a foreign investor is equal to $\rho_0 i$ which is, again, equal to the pledgeable income of the project. Hence, an entrepreneur would no benefit from running away since it would not increase its payoff (Proposition 1). At date-1, the LFE establishes two scenarios. Start with the black swan event. Given C_0, since x is equal to zero, an entrepreneur's only feasible option is to shutdown as suggested by Corollary 1. If the event is not a black swan event, C_1 suggests a full-scale reinvestment. An entrepreneur could choose to shutdown the program and keep $i\big[1-\rho_0\big]$ for herself. However, Proposition 2 shows that if $x = i\big[1-\rho_0\big]$, the optimal choice is to reinvest. At date-0, Proposition 3 shows that given the strategy profile at date-2 and date-1, the optimal choice is C_0. No benefit on deviating. ∎

A.4 Proposition 5

First, note that a project shutdown ($C_1 = \{j = 0, M_1 = 0, \phi_1 = 0, R_f = 0, R_e = 0, \tau = 0$) is a feasible solution to the problem. Now, I focus on solutions with positive levels of reinvestment ($j^* > 0$). In any solution of this type, given that it is a zero sum game between entrepreneurs and foreign investors, PC_{MG1} binds because an entrepreneur would allocate just enough to a foreign investor for that person to participate. Additionally, BC_{MG1}, and $FC1_{MG1}$ bind as well because, otherwise, an entrepreneur would be wasting funding resources and/or would be leaving a some of the project's return on the table. Thus, by replacing with these equations, the objective function can be rewritten as

$$\frac{M_1 + \tau i}{1-\phi_1}\big[\rho_1 - 1\big] + \big[1-\hat{R}\big]\tau i + x$$

Suppose $\hat{R} \leq 1$. Proposition 5 states that $C_1 = \{j = i, M_1 = 0, \phi_1 = 0, R_f = 0, R_e = \rho_1, \tau = 1\}$ is the optimal solution to the problem with a payoff of $i\big[\rho_1 - 1\big] + \big[1-\hat{R}\big]i + x$. First, it is straight forward to see that C_1 is feasible by replacing values in constraints. I show by contradiction that C_1 is the optimal solution. Suppose that there exists a feasible $\tilde{C}_1$ that generates a greater payoff than C_1. Thus, the payoff of $\tilde{C}_1$ is equal to $\tilde{j}\big[\rho_1 - 1\big] + \big[1-\hat{R}\big]\tilde{\tau}i + x$ which, by assumption, should be greater than C_1's payoff. Since $\tilde{C}_1$ is feasible, then $\tilde{j} \leq i$ and $\tilde{\tau} \leq 1$. Given that $\rho_1 > 1$, $1 \geq \hat{R}$, and x is non-negative, then $\tilde{j}\big[\rho_1 - 1\big] + \big[1-\hat{R}\big]\tilde{\tau}i + x \leq i\big[\rho_1 - 1\big] + \big[1-\hat{R}\big]i + x$. This contradiction shows that C_1 is the optimal answer when $\hat{R} \leq 1$.
Suppose $\hat{R} > 1$. In this scenario, one unit of public liquidity is more expensive than market and funding liquidity. Thus, this condition brings the entrepreneur back to a scenario without government. Therefore, if $x + \rho_0 i \geq i$, then as Proposition 2 shows, entrepreneurs reach full scale reinvestment with no need to tap into public liquidity. The question is what is optimal for the entrepreneur to do if $x + \rho_0 i < i$. One option is to only finance reinvestment with market and funding liquidity and obtain payoff $\frac{x}{1-\rho_0}\big[\rho_1 - \rho_0\big]$, or reach full-scale reinvestment tapping into public liquidity and obtain payoff $i\big[\rho_1 - \rho_0\big] - \hat{R}\big[(1-\rho_0)i - x\big]$. Which payoff is greater depends on the value of $\hat{R}$. I prove Proposition by contradiction. Suppose $\hat{R} \leq \frac{\rho_1 - \rho_0}{1-\rho_0}$ and that the entrepreneur prefers to downsize. If so, then the difference $i\big[\rho_1 - \rho_0\big] - \hat{R}\big[(1-\rho_0)i - x\big] - \frac{x}{1-\rho_0}\big[\rho_1 - \rho_0\big]$ is negative. This difference can be written as $\big[(1-\rho_0)i - x\big]\big[\frac{\rho_1-\rho_0}{1-\rho_0} - \hat{R}\big]$ Note that the left term of this difference is positive since

$x + i(1-\rho_0)$ is less than i and the right term is also positive since $\hat{R} \leq \frac{\rho_1-\rho_0}{1-\rho_0}$. Thus, this difference is positive which is a contradiction. Then, in this scenario, it is optimal to reach full-scale reinvestment by tapping to public liquidity than to downsize. When $\hat{R} > \frac{\rho_1-\rho_0}{1-\rho_0}$, partial liquidation is preferred and the proof follows the same argument than before but assuming that the difference is positive, then reaching the same equation that given the level assumed for $\hat{R}$ is negative, and a contradiction is obtained. Below the reinvestment function and the payoff function implied by choosing optimally at Date-1.

$$j(\hat{R}, x) = \begin{cases} i & \text{if } 1 \geq \hat{R} \\ i & \text{if } \frac{\rho_1-\rho_0}{1-\rho_0} \geq \hat{R} > 1 \\ min\{i, \frac{x}{1-\rho_0}\} & \text{if } \hat{R} > \frac{\rho_1-\rho_0}{1-\rho_0} \end{cases}$$

$$f(x, i, \hat{R}) = \begin{cases} i\big[\rho_1 - \hat{R}\big] + x & \text{if } 1 \geq \hat{R} \\ i\big[\rho_1 - \rho_0\big] - \hat{R}\big[(1-\rho_0)i - x\big] & \text{if } \frac{\rho_1-\rho_0}{1-\rho_0} \geq \hat{R} > 1 \\ min\{i[\rho_1-\rho_0], \frac{x}{1-\rho_0}\big[\rho_1 - \rho_0\big]\} & \text{if } \hat{R} > \frac{\rho_1-\rho_0}{1-\rho_0} \end{cases}$$

A.5 Proposition 6

First, note that not investing in the project is a feasible solution. Now, any optimal solution to an entrepreneur's date-0 problem with positive investment has BC_{G0}, $FC1_{G0}$, and $FC2_{G0}$ binding because, otherwise, the entrepreneur would be either wasting resources or leaving some on the table. Additionally, recall that the allocation of payoffs between the entrepreneur and foreign investors is a zero sum game, thus, it is optimal to allocate investors just enough such that they participate. Thus, PC_{G0} binds as well. Given that these equations bind, the problem can be rewritten as

$$\begin{aligned} \underset{\{R_f^{NC}, M_0, R_e^C\}}{\text{Maximize}} \quad & \alpha\big[\rho_1 + \pi - R_f^{NC}\big]i + (1-\alpha)f(x, i, \hat{R}) + \alpha(A - M_0) \\ \text{subject to: } & i = M_0\kappa(R_f^{NC}, R_e^C) \\ & \kappa(R_f^{NC}, R_e^C) = \frac{1}{1-(1-\alpha)\pi - \alpha R_f^{NC} + (1-\alpha)R_e^C} \\ & x = R_e^C i + (A - F_0 - M_0) \\ & \rho_o i + \pi i \geq R_f^{NC} i \\ & \pi i \geq R_e^C i \\ & A - F_0 \geq M_0 \\ & i, M_0, R_f^{NC}, R_e^C \geq 0 \\ & A,\ f(x, i, \hat{R}),\ \rho_0,\ \pi,\ \textit{and } \rho_1 \text{ are given} \end{aligned}$$

Follow Proof of Proposition 3 for numerals 1, 2, and 3. Thus, only R_e^C is left to determine. Below I present the F.O.C for this variable given results 1, 2, and 3.

$$\frac{\partial \kappa}{\partial R_e^C}(R_e^C)(A-F_0)\alpha\big[\rho_1-\rho_0\big]+(1-\alpha)\frac{\partial f}{\partial R_e^C}(x,i,\hat{R})$$

First, assume that $\hat{R}\leq 1$. With the cost of public liquidity at this level, Assumption 1 guarantees that the F.0.C is decreasing for all values of R_e^C, thus, the optimal answer is for R_e^C to be zero.
Consider $\frac{\rho_1-\rho_0}{1-\rho_0}\geq \hat{R}>1$. Start by assuming that this F.O.C is positive. Then, by using that $\frac{\rho_1-\rho_0}{1-\rho_0}\geq \hat{R}$, this F.O.C is bounded from above by $A\kappa(R^{eC})^2\frac{\rho_1-\rho_0}{1-\rho_0}(1-\pi-\alpha)$. Therefore, if $1-\pi-\alpha<0$, this is a negative value and I get a contradiction. In this case, R_e^C is chosen to be zero. Continue to consider $\frac{\rho_1-\rho_0}{1-\rho_0}\geq \hat{R}>1$ but now I assume that $1-\pi-\alpha\geq 0$. Define $q=\frac{\rho_1-\rho_0}{1-\pi-\alpha+(1-\rho_0)}$. Below I rewrite the F.O.C under these assumptions.

$$(A-F_0)\kappa(R_e^C)^2\Big[\hat{R}(1-\pi-\alpha)-(\rho_1-\rho_0)+\hat{R}(1-\rho_0)\Big]$$

As $\hat{R}\to 1$, this F.O.C is negative due to numeral 1 of Assumption 1. As $\hat{R}\to\frac{\rho_1-\rho_0}{1-\rho_0}$, the F.O.C is non-negative because $1-\pi-\alpha\geq 0$. Therefore, since this F.O.C is continuous for $\hat{R}$, then, by the intermediate value theorem, there exists a c between 1 and $\frac{\rho_1-\rho_0}{1-\rho_0}$ such that the F.O.C valued at c is equal to zero. Note that q is in fact such c. Therefore, for $\hat{R}\leq q$, this F.O.C is negative and then the optimal R_e^C is zero, and for $\hat{R}>q$, the F.O.C is positive and the optimal R_e^C is equal to $1-\rho_0$.
For $\hat{R}>\frac{\rho_1-\rho_0}{1-\rho_0}$, follow the proof of Proposition 3. ∎

A.6 Proposition 9

First, note that a project shutdown ($C_1=\{j=0,M_1=0,\phi_1=0,R_f=0,R_e=0,\tau=0$) is a feasible solution to the problem. Now, I focus on solutions with positive levels of reinvestment ($j^*>0$). In any solution of this type, given that it is a zero sum game between entrepreneurs and foreign investors, PC_{IG1} binds because an entrepreneur would allocate just enough to a foreign investor for that person to participate. Additionally, BC_{IG1}, and $FC1_{IG1}$ bind as well because, otherwise, an entrepreneur would be wasting funding resources and/or would be leaving a some of the project's return on the table. Thus, by replacing with these equations, the objective function can be rewritten as

$$\frac{M_1+\tau i}{1-\phi_1}\big[\rho_1-1\big]+\big[1-\hat{R}\big]\tau i+x$$

Suppose $\hat{R}\leq\rho_0$. Proposition 9 states that $C_1=\{j=i,M_1=0,\phi_1=0,R_f=0,R_e=\rho_1,\tau i=i\}$ is the optimal solution to the problem with a payoff of $i\big[\rho_1-1\big]+\big[1-\hat{R}\big]i+x$. First, it is straight forward to see that C_1 is feasible by replacing values in constraints, specially the incentive compatibility constraint (IC_{IG1}) since $\hat{R}\leq\rho_0$. I show by contradiction that C_1 is the optimal solution. Suppose that there exists a feasible $\tilde{C_1}$ that generates a greater payoff than C_1. Thus, the payoff of $\tilde{C_1}$ is equal to $\tilde{j}\big[\rho_1-1\big]+\big[1-\hat{R}\big]\tilde{\tau}i+x$ which, by assumption, should be greater than C_1's payoff. Since $\tilde{C_1}$ is feasible, then $\tilde{j}\leq i$ and $\tilde{\tau}i\leq i$.

Given that $\rho_1 > 1$, $1 \geq \rho_0 \to 1 \geq \hat{R}$, and x is non-negative, then $\tilde{j}\big[\rho_1 - 1\big] + \big[1 - \hat{R}\big]\tilde{\tau}i + x \leq i\big[\rho_1 - 1\big] + \big[1 - \hat{R}\big]i + x$. This contradiction shows that C_1 is the optimal answer when $\hat{R} \leq \rho_0$.

Suppose that $\rho_0 < \hat{R} \leq 1$. Proposition 9 states that the optimal pecking order is first market liquidity up to $min\{x, i\big[1 - \frac{\rho_0}{\hat{R}}\big]\}$, then public liquidity equal to $min\{\frac{\rho_0}{\hat{R}}i, \frac{x}{\hat{R}-\rho_0}\rho_0\}$. And if $x < i\big[1 - \frac{\rho_0}{\hat{R}}\big]$, then downsizing is optimal. First, I show that funding liquidity does not belong in this pecking order. With an IG, public liquidity and funding liquidity compete for pledgeable income. Since $\hat{R} \leq 1$, an entrepreneur obtains more resources to finance reinvestment by exhausting pledgeable income with public liquidity. To see this, by (IC_{IG1}), the maximum total amount of public liquidity that can be asked for is $\frac{\rho_0}{\hat{R}}j$ which is greater or equal than the maximum amount total amount of funding liquidity ($\rho_0 j$). Any solution has $\phi_1 j = 0$ while $\hat{R} < 1$ and when $\hat{R} = 1$, an entrepreneur is indifferent between funding and public liquidity. For simplicity, I assume that when $\hat{R} = 1$, the entrepreneur chooses public liquidity. To see that what Proposition 9 states is optimal, consider there is a $\{\hat{j}, \hat{M}_1, \hat{\tau}i, \hat{\phi}_1\}$ that is feasible and produces a greater payoff. Since this potential set is feasible, then it must be true that $\hat{j} = \hat{M}_1 + \hat{\tau}i + \hat{\phi}_1\hat{j}$, $x \geq \hat{M}_1$, and $\rho_0\hat{j} \geq \hat{R}\hat{\tau}i + \hat{\phi}_1\hat{j}$. First, suppose that $x\big[1 - \frac{\rho_0}{\hat{R}}\big]$, thus the optimal solution according to the proposition consists of $\{j = i, M_1 = i\big[1 - \frac{\rho_0}{\hat{R}}\big], \tau i = \frac{\rho_0}{\hat{R}}i, \phi_1 = 0\}$. Note that these values are feasible. Now, I prove by contradiction that it is optimal. Suppose that the feasible candidate solution generates a greater payoff. Then $0 > \big[i - \hat{j}\big]\big[\rho_1 - 1\big] + \big[1 - \hat{R}\big]\big[\frac{\rho_0}{\hat{R}}i - \hat{\tau}i\big]$ must hold. However, this can only be true if the second adding term is negative since feasibility implies $i \geq \hat{j}$. Thus, it must be true that $\rho_0 i < \hat{R}\hat{\tau}i$. But, this is a contradiction because $\hat{R}\hat{\tau}i \leq \rho_0\hat{j}$ by feasibility, implying that $i < \hat{j}$ which is not possible. Now suppose that $x < i\big[1 - \frac{\rho_0}{\hat{R}}\big]$, thus the optimal solution according to the proposition consists of $\{j = \frac{\hat{R}x}{\hat{R}-\rho_0}, M_1 = x, \tau i = \frac{\rho_0 x}{\hat{R}-\rho_0}, \phi_1 = 0\}$. Note that these values are feasible. Now, I prove by contradiction that it is optimal. Suppose that the feasible candidate solution generates a greater payoff. Then $0 > \big[\frac{\hat{R}x}{\hat{R}-\rho_0} - \hat{j}\big]\big[\rho_1 - 1\big] + \big[1 - \hat{R}\big]\big[\frac{\rho_0 x}{\hat{R}-\rho_0} - \hat{\tau}i\big]$ must hold. However, both adding terms are non negative so this is not possible. I show that this is true one term at a time. The sign of the first adding term depends on the sign of $\big[\hat{R}x - (\hat{R} - \rho_0)\hat{j}\big]$. Using the budget constraint and the incentive compatibility constraint of the candidate solution, it can be derived that this is greater or equal to $\hat{R}\big[x - \hat{M}_1\big] + \hat{\phi}_1\hat{j}\big[1 - \hat{R}\big]$ that is non-negative. Therefore, the first adding term is non-negative. The sign of the second adding term depends on the sign of $\rho_0 x - (\hat{R} - \rho_0)\hat{\tau}i$. Using the budget constraint and the incentive compatibility constraint of the candidate solution, it can be derived that this is greater or equal to $\rho_0\big[x - \hat{M}_1\big] + \hat{\phi}_1\hat{j}\big[1 - \rho_0\big]$ that is non-negative. Therefore, there isn't a feasible solution that produces a greater payoff than what is established by Proposition 9.

Suppose $\hat{R} > 1$. Since an entrepreneur exhausts pledgeable income on the least expensive source, when $\hat{R} > 1$, public liquidity is too expensive, thus, any optimal solution has $\tau i = 0$. Hence, this scenario is the same as the one with no government and the proof follows the proof of Proposition 2. ∎

$$
j(\hat{R},x)=\begin{cases} i & \text{if } \hat{R}\leq\rho_0 \\ & \text{if } \rho_0<\hat{R}\leq 1 \\ i & \qquad\qquad \text{and } x\geq i\big[1-\frac{\rho_0}{\hat{R}}\big] \\ \frac{\hat{R}x}{\hat{R}-\rho_0} & \qquad\qquad \text{and } x< i\big[1-\frac{\rho_0}{\hat{R}}\big] \\ & \text{if } 1<\hat{R} \\ i & \qquad\qquad \text{and } x\geq i\big[1-\rho_0\big] \\ \frac{x}{1-\rho_0} & \qquad\qquad \text{and } x< i\big[1-\rho_0\big] \end{cases}
$$

$$
f(x,i,\hat{R})=\begin{cases} i\big[\rho_1-\hat{R}\big]+x & \text{if } \hat{R}\leq\rho_0 \\ & \text{if } \rho_0<\hat{R}\leq 1 \\ i\big[\rho_1-\rho_0\big]+\big[x-i(1-\frac{\rho_0}{\hat{R}})\big] & \qquad\qquad \text{and } x\geq i\big[1-\frac{\rho_0}{\hat{R}}\big] \\ \frac{\hat{R}x}{\hat{R}-\rho_0}\big[\rho_1-\rho_0\big] & \qquad\qquad \text{and } x< i\big[1-\frac{\rho_0}{\hat{R}}\big] \\ & \text{if } 1<\hat{R} \\ i\big[\rho_1-1\big]+x & \qquad\qquad \text{and } x\geq i\big[1-\rho_0\big] \\ \frac{x}{1-\rho_0}\big[\rho_1-\rho_0\big] & \qquad\qquad \text{and } x< i\big[1-\rho_0\big] \end{cases}
$$

A.7 Proposition 10

First, note that not investing in the project is a feasible solution. Now, any optimal solution to an entrepreneur's date-0 problem with positive investment has BC_{G0}, $FC1_{G0}$, and $FC2_{G0}$ binding because, otherwise, the entrepreneur would be either wasting resources or leaving some on the table. Additionally, recall that the allocation of payoffs between the entrepreneur and foreign investors is a zero sum game, thus, it is optimal to allocate investors just enough such that they participate. Thus, PC_{G0} binds as well. Given that these equations bind, the problem can be rewritten as

$$
\begin{aligned}
\underset{\{R_f^{NC},M_0,R_e^C\}}{\text{Maximize}}\ & \alpha\big[\rho_1+\pi-R_f^{NC}\big]i+(1-\alpha)f(x,i,\hat{R})+\alpha(A-M_0) \\
\text{subject to: } & i=M_0\kappa(R_f^{NC},R_e^C) \\
& \kappa(R_f^{NC},R_e^C)=\frac{1}{1-(1-\alpha)\pi-\alpha R_f^{NC}+(1-\alpha)R_e^C} \\
& x=R_e^C i+\gamma_0(A-F_0-M_0) \\
& \rho_o i+\pi i\geq R_f^{NC} i \\
& \pi i\geq R_e^C i \\
& A-F_0\geq M_0 \\
& i,M_0,R_f^{NC},R_e^C\geq 0 \\
& A,\ f(x,i,\hat{R}),\ \rho_0,\ \pi,\ and\ \rho_1 \text{ are given}
\end{aligned}
$$

Follow Proof of Proposition 3 for numerals 1, 2, and 3. Thus, only R_e^C is left to determine.

Below I present the F.O.C for this variable given results 1, 2, and 3.

$$\frac{\partial \kappa}{\partial R_e^C}(R_e^C)A\alpha\big[\rho_1-\rho_0\big]+(1-\alpha)\frac{\partial f}{\partial R_e^C}(x,i,\hat{R})$$

First, assume that $\hat{R}\le\rho_0$. With the cost of public liquidity at this level, Assumption 1 guarantees that the F.0.C is decreasing for all values of R_e^C, thus, the optimal answer is for R_e^C to be zero. To see this, note that the sign of the F.O.C depends on the sign of $(1-\pi-\alpha\rho_0)+\alpha\rho_0+(1-\alpha)\hat{R}-\rho_1$ which less or equal to $-\big[((1-\alpha)(1-\hat{R})\big]$ that is non-positive.

Consider $1\ge\hat{R}>\rho_0$. First, I show that accumulating more than $1-\frac{\rho_0}{\hat{R}}$ is not optimal. To see this, note that the sign of the F.O.C. when $x\big[1-\frac{\rho_0}{\hat{R}}\big]$ depends on the sign of $(1-\pi-\alpha\rho_0)+(1-\alpha)(\hat{R}-\rho_0)\frac{1}{\hat{R}}+\rho_0-\rho_1$ which, by using Assumption 1, is bounded from above by $-\big[(1-\alpha)(1-\hat{R})\frac{\rho_0}{\hat{R}}\big]$ that is non-positive. Now, I focus on values of x between zero and $i\big[1-\frac{\rho_0}{\hat{R}}\big]$ Start by assuming that this F.O.C is negative. Then, by using that $1\ge\hat{R}$, this F.O.C is bounded from below by $(A-F_0)(1-\alpha)\kappa(R^{eC})^2\hat{R}\frac{\rho_1-\rho_0}{\hat{R}-\rho_0}\bigg[1-\pi-\alpha+\alpha\rho_0\frac{1-\hat{R}}{\hat{R}}\bigg]$. Therefore, if $1-\pi-\alpha\ge 0$, this is a positive value and I get a contradiction. In this case, R_e^C is chosen to be $1-\frac{\rho_0}{\hat{R}}$.

Continue to consider $1\ge\hat{R}>\rho_0$ but now I assume that $1-\pi-\alpha<0$. Define $c=\frac{\alpha\rho_0}{\alpha\rho_0+\pi-(1-\alpha)}$. Below I rewrite the F.O.C under these assumptions.

$$(A-F_0)(1-\alpha)\kappa(R^{eC})^2\hat{R}\frac{\rho_1-\rho_0}{\hat{R}-\rho_0}\bigg[1-\pi-\alpha+\alpha\rho_0\frac{1-\hat{R}}{\hat{R}}\bigg]$$

As $\hat{R}\to 1$, this F.O.C is negative because $1<\pi+\alpha$. As $\hat{R}\to\rho_0$, the F.O.C is positive because by Assumption 2, $1-\pi-\alpha\rho_0$ is positive. Therefore, since this F.O.C is continuous for $\hat{R}$ between feasible values, then, by the intermediate value theorem, there exists a c between ρ_0 and 1 such that the F.O.C valued at c is equal to zero. Note that q is in fact such c. Therefore, for $\hat{R}\le q$, this F.O.C is positive and then the optimal R_e^C is $1-\frac{\rho_0}{\hat{R}}$, and for $\hat{R}>q$, the F.O.C is negative and the optimal R_e^C is equal to zero.

For $\hat{R}>1$, follow the proof of Proposition 3. ∎

A.8 Proposition 11

When R_e^C is zero, reinvestment can only happen if $\hat{R}$ is less or equal than ρ_0. Then, accumulating $\tilde{F}_0$ is enough to assure full-scale reinvestment. Accumulating more is not optimal because reinvestment is already at its maximum level but a government would continue to deviate more resources from entrepreneurs. Moreover, note that $\tilde{F}_0$ is strictly less than A, thus, it is feasible. Accumulating $\tilde{F}_0$ produces a cost function equal to $W(\tilde{F}_0,\rho_0,0)=\psi\tilde{F}_0\kappa(0)$. Accumulating anything less than $\tilde{F}_0$ produces a $\hat{R}$ greater than ρ_0, that, as shown by Corollary 5, forces projects to shutdown. Hence, it is better to not accumulate $F_0=0$ than to accumulate anything below $\tilde{F}_0$. When a government doesn't accumulate resources, $W(0,1,0)=(1-\alpha)L(0)$. Therefore,the optimal solution results from determining which is lower between $(1-\alpha)L(0)$ and $\psi\tilde{F}_0\kappa(0)$. If R_e^C is positive, define z as the cost of public liquidity that would allow entrepreneurs to continue at full-scale by using both market and public liquidity. An entrepreneur cannot solely reinvest at full-scale with only market liquidity since R_e^C is constrained by π which, by assumption, is lower than 1. If z is greater than 1, then an entrepreneur could reinvest at full-scale tapping on funding liquidity instead of public liquidity. Thus, z is between ρ_0 and 1. By Function $F(\hat{R}; R_e^C)$, K is the necessary amount of liquidity that a government need to hoard to implement z at date-1. This level of accumulation allows for a full-scale reinvestment, thus, hoarding more liquidity increases the cost function without reaping the benefits of more reinvestment. Therefore, the optimal solution to Date-0 is equal or less than K. By feasibility, the government cannot accumulate a negative level of reserves. Thus, any optimal solution is greater or equal to zero. Hence, the set of feasible solutions is bounded, thus, compact. Also this set is not empty since zero is always possible. Then, since the loss function L is continuous, then, $W(F_0, \rho_0, R_e^C)$ is also continuous. By Weierstrass Theorem, $W(F_0, \rho_0, R_e^C)$ reaches reaches its minimum between 0 and F and ω exists and it is between zero and K. ∎

A.9 Corollary 6

Consider if no crisis happens. Then, at date-2, $\tilde{C}_0$ argues that R_e^{NC} is equal to $\rho_1 - \rho_0$ which is equal to $\theta\rho_1$, thus, entrepreneurs' are indifferent from running away or not. Not running away is a best response. Consider if a crisis happens, then $\tilde{C}_1$ establishes that R_e is equal to $\rho_1 - \rho_0$ which is equal to $\theta\rho_1$, thus, an entrepreneur's best response is to not runaway. The remainder of the project's total return $\rho_0 i$ is credibly collected by the government and used to cover the redemption of sovereign bonds worth i, together with $\tilde{F}_0$. Bonds are safely paid back. At date-1, the government starts with F_0 equal to, therefore, for any τ, sets $\hat{R}$ equal to ρ_0 by Equation 5. Entrepreneurs' best response to this cost of public liquidity, by Proposition 5, is to set $\tau = 1$ which establishes $\tilde{C}_1$ as the optimal solution. In return, since τ is equal to 1, government's best response by Equation 5 and F_0 equal to $\tilde{F}_0$ is $\hat{R}$ equal to ρ_0. At date-0, suppose entrepreneurs expect that $\hat{R}$ to be equal to ρ_0. Thus, by Proposition 9, they choose optimally R_e^C equal to zero. As a consequence, as long as $\psi\tilde{F}_0\kappa(0) \leq (1-\alpha)L(0)$, a government's best response is to accumulate $\tilde{F}_0$, which, in turn, allows to set $\hat{R}$ equal to ρ_0 at date-1. Now, suppose that the government expects R_e^C equal to zero. Then, as long as $\psi\tilde{F}_0\kappa(0) \leq (1-\alpha)L(0)$, a government's best response is to accumulate $\tilde{F}_0$, which, in turn, allows to set $\hat{R}$ equal to ρ_0 at date-1. Therefore, at date-1, entrepreneur's will demand τ equal to 1, and its bets response is to choose R_e^C equal to zero at date-0. ∎

A.10 Corollary 7

Consider if no crisis happens. Then, at date-2, $\tilde{C}_0$ argues that R_e^{NC} is equal to $\rho_1 - \rho_0$ which is equal to $\theta\rho_1$, thus, entrepreneurs' are indifferent from running away or not. Not running away is a best response. Consider if a crisis happens, then $\tilde{C}_1$ establishes that R_e is equal to $\rho_1 - \rho_0$ which is equal to $\theta\rho_1$, thus, an entrepreneur's best response is to not runaway. The remainder of the project's total return $\rho_0 i$ is collected by the government who uses these resources to redeem sovereign bonds valued at $\rho_0 i$. At date-1, the government starts with F_0 equal to zero, therefore, for any τ, sets $\hat{R}$ equal to 1 by Equation 5. Entrepreneurs' best response to this cost of public liquidity, by Proposition 5, is to set $\tau = \rho_0$ which establishes $\tilde{C}_1$ as the optimal solution. In return, since τ is equal to ρ_0, but F_0 equal to 0, government's best response by Equation 5 is $\hat{R}$ equal to 1. Entrepreneurs start date-1 with $x = i\big[1 - \rho_0\big]$ and since $\hat{R} = 1$, Proposition 9 indicates that is optimal for an entrepreneur to ask for a government loan worth $\rho_0 i$ and invest all x in the reinvestment. Thus, $i\big[1 - \rho_0\big] + \rho_i \to i$ which means a full-scale reinvestment as suggested by $\tilde{C}_1$. At date-0, suppose entrepreneurs expect that $\hat{R}$ to be equal to 1. Thus, by Proposition 9 and $1 > +\pi$, they choose optimally R_e^C equal to ρ_0. As a consequence, a government defines z which in this case is equal to 1 since $1 - \rho_0 + \frac{\rho_0}{z} = 1$. Thus, by Equation 4, K is equal to zero. That is, the amount that needs to be accumulated to reach an $\hat{R}$ equal to 1 is zero. Since the government's optimal choice ω belongs to the close interval $\big[0, K\big]$ and $K = 0$, then ω has to be equal to zero. Suppose that the government expects R_e^C equal to $1 - \rho_0$. Then, by Proposition 11 and discussed before, a government's best response is for $F_0 = 0$, which, in turn, allows to set $\hat{R}$ equal to 1 at date-1. Therefore, at date-1, entrepreneur's will demand τ equal to ρ_0, and its bets response is to choose R_e^C equal to $1 - \rho$ at date-0. ∎