MMCS assignment 3

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To prepare for natural disasters, a strategy to build warehouse for emergency goods in various locations is important. To solve the problem, we write an optimization model to deal with uncertainty and be able to cope with multiple goods.

To solve the problem, we introduce sets of road_connection(as binary data) as RCon(r1,r2), road capacity as Rca(r1,r2) and disaster probability as prob(d,r).
We need to explain here that r1 and r2 in formula are not the specific Region 1 and Region 2 in the specific problem, they are representatives of two non-specific regions (r1 for the region suffering disaster (we assume that at any specific time there is at most one disaster because the probability for two disasters together is not high), and r2 for other regions). For disaster, in region r, there would be d different cases of disaster, so we define them with d index to identify.
Region as r, the type of goods as g, warehouse type as t, warehouse capability as capacity(r,t), the percentage of road capacity reduced after a disaster as rrp(d,r1,r2), the percentage of storage capacity reduced after a disaster as srp(d,r), cost of buying one unit of goods as Cunit(g), cost of building one warehouse as Cbuild(t) and cost of transporting one unit of goods as Ctrans.

To simplify the invocation and calculation, we can rewrite the data in a new .dat document, some data relation is in it. Rewrite some matrixes and arrays to show the relationships. Take road connection as an example, R1 can connect to R2 and R4, then the first row of road_connection implies R1 can be write to [0 1 0 1]. Also, for the road_connection, we set that if a region is suffering disaster, it would not delivery goods to other regions (and its self-connection status is 0, too).

First of all, introduce a binary variable warehouse type,

$$\sum_{t \in T} warehouse(t, r) \leq 1 \quad for \ \ all \ \ r \in R$$

Which means the type of warehouse in a region only can be one type.

Then we obtain several constraints:

1. For goods transport:

$$\mathrm{Goods_{trans}(g, d, r1, r2)} \leq rrp(d, r1, r2) \times \mathrm{RCon(r1, r2)} \times \mathrm{Rca}(r1, r2) \quad for \ \ all \ \ d \in D, r1 \in R, r2 \in R, g \in G$$

$$\mathrm{Goods_{trans}(g, d, r1, r2)} \leq \mathrm{RCon(r1, r2)} * \mathrm{Goods_{storage}}(g, r2) \quad for \ \ all \ \ d \in D, r1 \in R, r2 \in R, g \in G$$

   $\mathrm{Goods_{storage}}(g, r)$ means the available storage amount in normal situation in other regions (without the effect of srp).
   So, the amount of goods transport between arbitrary two regions must less than or equal to the road capacity×the percentage of road capacity reduced after a disaster.
   Also, it must less than or equals to the storage amount in either one of two regions.

2. For goods storage:
   $\mathrm{Goods_{storage}}(g, r)$ means the storage in region r with g types of goods.

$$\sum_{g \in G} \mathrm{Goods}_{\mathrm{storage}}(\mathrm{g}, \mathrm{r}) \leq \sum_{t \in T} capacity(r, t) \quad for\ \ all\ \ r \in R$$

Which means the amount of stored goods must less than or equal to the warehouse capacity.

3. For goods demand:

$$Goods_{demand}(g, d, r1) \leq srp(r1, d) \times \mathrm{Goods}_{\mathrm{storage}}(\mathrm{g}, \mathrm{r1}) + \sum_{r2 \in R, g \in G} \mathrm{Goods}_{\mathrm{trans}}(\mathrm{g}, \mathrm{d}, \mathrm{r1}, \mathrm{r2})$$

$$for\ \ all\ \ g \in G, r1 \in R, d \in D$$

Which means the goods demand in a region should less than or equal to the local available amount after disaster (the storage within the capacity of warehouse will suffer a loss on average with the percentage of srp) plus the total amount of other regions will transport to.

Finally, we can obtain the cost of separate aspects then add them all.

1. For the cost of goods storage:

$$cost_{storage} = \sum_{g \in G, r \in R} Cunit(g) \times \mathrm{Goods}_{\mathrm{storage}}(\mathrm{r})$$

2. For the cost of building warehouse:

$$cost_{warehouse} = \sum_{r \in R, t \in T} Cbuild(t) \times warehouse(t, r)$$

3. For the cost of goods transport:

$$cost_{transport} = \sum_{r1 \in R, d \in D} (prob(r1, d) \times \sum_{g \in G, r1 \in R, r2 \in R} Ctrans \times \mathrm{Goods}_{\mathrm{trans}}(\mathrm{g}, \mathrm{d}, \mathrm{r1}, \mathrm{r2}))$$

We need to claim that the cost should be as low as possible, which is the objective function for the whole problem:

$$\min(cost_{storage} + cost_{warehouse} + cost_{transport})$$