Fundamentals of Optimization Homework 1

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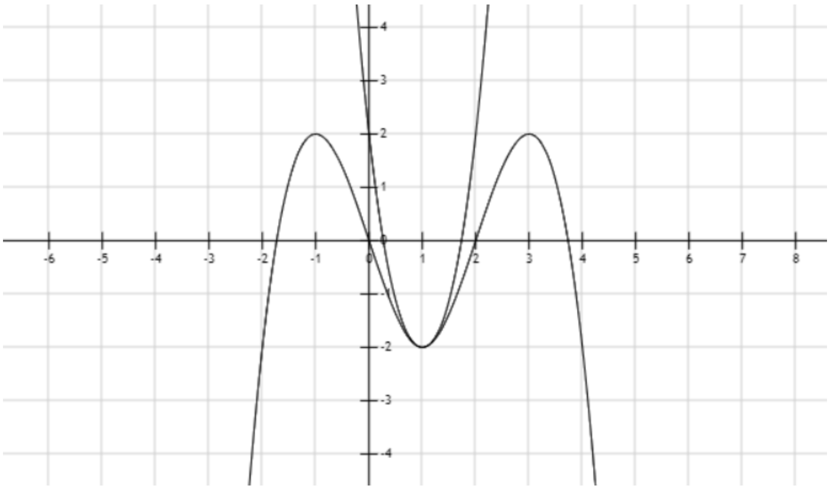
(5.1)
Give a counterexample here (to prove that the proposition is wrong)

$$f_1 = x^3 - 3x$$
$$f_2 = -(x-2)^3 + 3(x-2)$$



both of them are non-convex functions while their epigraphs' intersection is a convex set.

(5.2)
1. f is convex function -> $a >= 0$
because f is convex, for x1, x2 (in domain), $\lambda \in [0,1]$

$$f(\lambda x_1+(1-\lambda)x_2) = a(\lambda x_1+(1-\lambda)x_2)^2+b(\lambda x_1+(1-\lambda)x_2)+c <= \lambda a x_1^2+(1-\lambda)a x_2^2+b(\lambda x_1+(1-\lambda)x_2)+c = \lambda f(x_1)+(1-\lambda)f(x_2)$$

since from the deature of quadratic function, we know $(\lambda x_1 + (1-\lambda)x_2)^2 <= \lambda x_1^2 + (1-\lambda)x_2^2$. so only when $a >= 0$, the inequality is tenable.
2. $a >= 0$ -> f is convex function
when $a > 0$, consider the feature of quadratic function and the inequality function above, we can prove that f is convex.
when $a = 0$, $f(x) = bx + c$, and either $b \neq 0$ (linear function) or $b = 0, f(x) = c$ (constant function), f(x) is always convex.
In this case, the proposition is right.

(6.1)
for any $x_1, x_2 \in L_a(f)$, due to the feature that $if x \neq y, then f(x) \neq f(y)$, we know $x_1 = x_2$ so that $f(x_1) = f(x_2) = a$
in this case, for $\lambda \in [0,1]$, we have $f(\lambda x_1 + (1-\lambda)x_2) = \lambda f(x_1) + (1-\lambda)f(x_2)$, so we prove that the level set for an injective function is convex set

(6.2)
we consider a counterexample that f is a constant function, like $f = 0$. linear function's level set is convex, and it's the same for our constant function f.
now we cannot say our $f = 0$ is a one-to-one fuction, since for any x in domain, f(x) will be the same value, 0. so this proposition is not true.