Fundamentals of Operational Research Assignment 1

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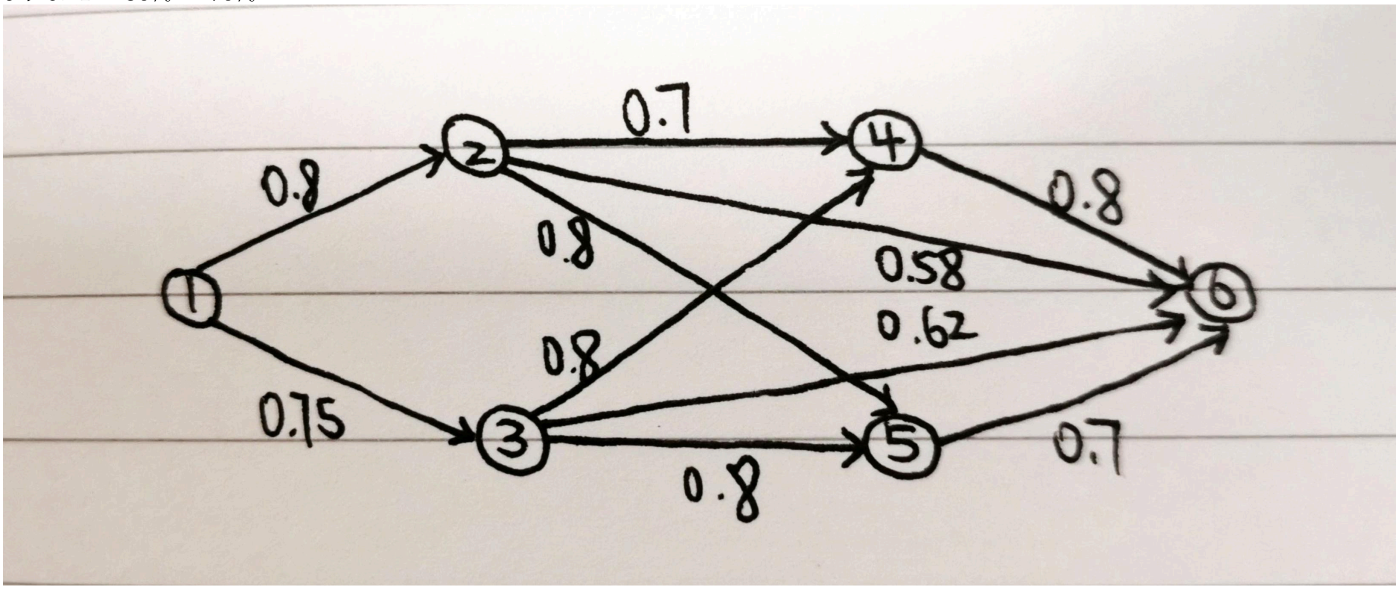
October 14, 2021

(1)
1->2: $1 - 20\% = 80\%$
1->3: $1 - 25\% = 75\%$
2->4: $1 - 30\% = 70\%$
2->5: $1 - 20\% = 80\%$
2->6: $1 - 42\% = 58\%$
3->4: $1 - 20\% = 80\%$
3->5: $1 - 20\% = 80\%$
3->6: $1 - 38\% = 62\%$
4->6: $1 - 20\% = 80\%$
5->6: $1 - 30\% = 70\%$



(2)
$p(1,2) + p(2,5) + p(5,6) = 80\% * 80\% * 70\% = 44.8\%$

(3)
Let:
$E = set of diedges in digraph$
$p_{i \to j}$ = probability of surviving from village i to j
$P(R)$ = accumulated probability of surviving on dipath R
$F_z(i)$ = maximum probability of surviving on a dipath from vertex i to vertex z
Let $\hat{R}_{j \to z}$ be the dipath from village j to z with maximum probability of surviving and $R_{j \to z}$ be any dipath from village j to z

Note that $p \in (0,1)$ and $P \in (0,1)$

Then:

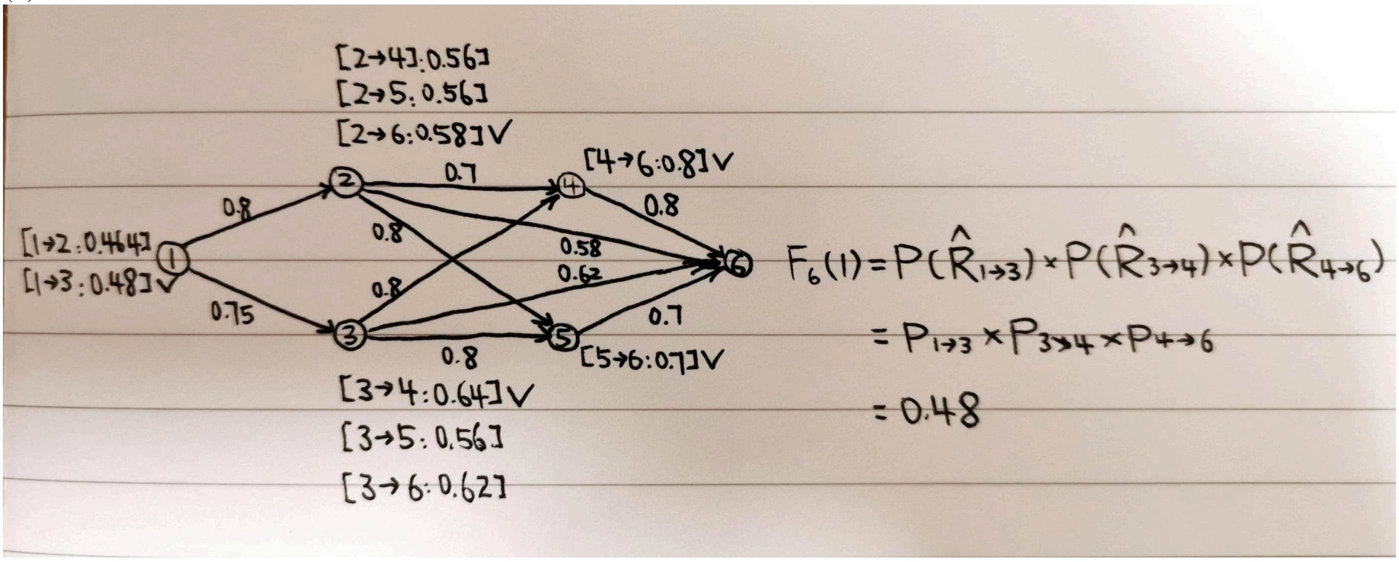
$$\begin{aligned} P(i \to j \& R_{j \to z}) &= p_{i \to j} * P(R_{j \to z}) \\ &\leq p_{i \to j} * P(\hat{R}_{j \to z}) \\ &= P(j \to j \& \hat{R}_{j \to z}) \end{aligned}$$

Hence:

$$
\begin{aligned}
\text{For } i \neq z, F_z(i) &= \max_{j:i\to j\in E} P(i \to j \& \hat{R}_{j\to z}) \\
&= \max_{j:i\to j\in E} (p_{i\to j} * P(\hat{R}_{j\to z})) \\
&= \max_{j:i\to j\in E} (p_{i\to j} * F_z(j)), \\
\text{and } F_z(z) &= 1.
\end{aligned}
$$

Above is the DP recurrence for the problem, and we need to solve it by finding $F_z(1)$

(4)



(5)

	Best Path
$F_6(6)$ (by definition) $= 1$	
$F_6(5) = F_6(6) * p_{5\to 6} = 1 * 0.7 = 0.7$	$5 \to 6$
$F_6(4) = F_6(6) * p_{4\to 6} = 1 * 0.8 = 0.8$	$\underline{4 \to 6}$
$F_6(3) = \max(F_6(4) * p_{3\to 4}, F_6(5) * p_{3\to 5}, F_6(6) * p_{3\to 6}) = \max(\underline{0.8 * 0.8}, 0.7 * 0.8, 1 * 0.62) = 0.64$	$\underline{3 \to 4}$
$F_6(2) = \max(F_6(4) * p_{2\to 4}, F_6(5) * p_{2\to 5}, \underline{F_6(6) * p_{2\to 6}}) = \max(0.8 * 0.7, 0.7 * 0.8, \underline{1 * 0.58}) = 0.58$	$2 \to 6$
$F_6(1) = \max(F_6(2) * p_{1\to 2}, F_6(3) * p_{1\to 3}) = \max(0.58 * 0.8, \underline{0.64 * 0.75}) = 0.48$	$\underline{1 \to 3}$

so the path from village 1 to 6 with highest probability of surviving is $1 \to 3 \to 4 \to 6$, and the probability is 48%