Fundamentals of Optimization Homework 2

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(4.1)

$$\begin{aligned}\min_{s.t.} & -2x_1 + x_5 - 3x_3^- - 2x_4^+ + 2x_4^- + 0x_5 + 0x_6 \\ & 2x_1 + x_5 - x_3^- - x_4^+ + x_4^- = 1 \\ & x_1 + 2x_5 - 3x_3^- - x_4^+ + x_4^- + x_5 = -2 \\ & x_1 - 2x_5 + x_3^- + 2x_4^+ - 2x_4^- + x_6 = 4 \\ & x_1 \geq 0, x_5 \geq 0, x_3^- \geq 0, x_4^+ \geq 0, x_4^- \geq 0, x_5 \geq 0, x_6 \geq 0\end{aligned}$$

(5.1)
$P \in P_1$, and from definition 6.4 we know P_1 is bounded, so P is bounded, too. In this case, the bounded polyhedron is polytope

(5.2)
counterexample:
$P_1 = x_1 \geq -1, x_1 \leq 1$
$P_2 = x_2 \geq -1, x_2 \leq 1$
then $P \in P_1 \cup P_2$ is a square (bounded) so it is a polytope but neither of P_1, P_2 is polytope actually
In other word, several 'unbounded' polyhedron can bound into a polytope with constraints increasing (adding extra polhedron with extra constrains, so that the original unbouned constraints may become bounded)

(6.1)
1. prove "P does not contain a vertex -> rank(A)<n"
Since P does not contain a vertex, it contains a 'line'. Suppose $\hat{x} \in P$, $\exists d->$ non-zero, $\lambda \in R$, $\hat{x} + \lambda d \in P$ into $Ax \leq b$, we have $Ad = 0 \& d$ is non-zero, so that A could not be full-rank then
2. prove "rank(A)<n -> P does not contain a vertex"
If rank(A)<n, then A is not a full column rank matrix, and from proposition 13.1, there would be no basic solution, so no vertex any more