Fundamentals of Operational Research Assignment 2

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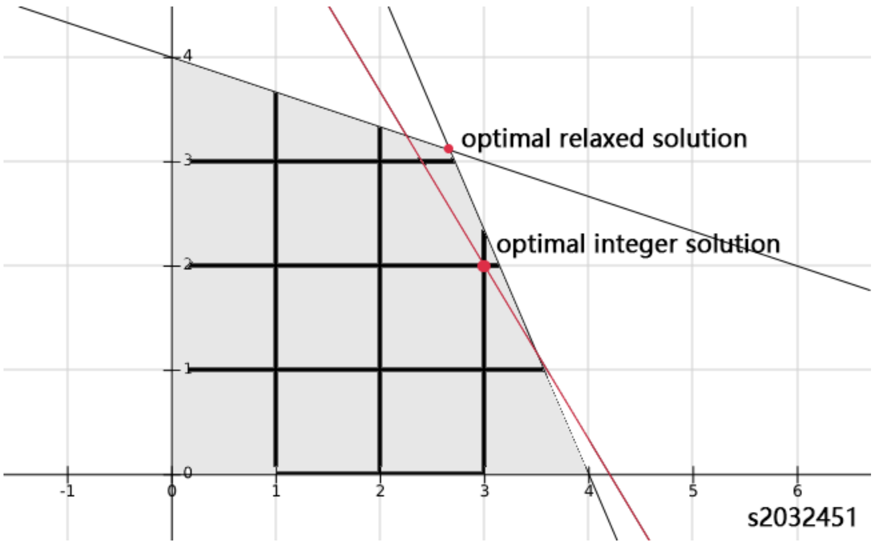
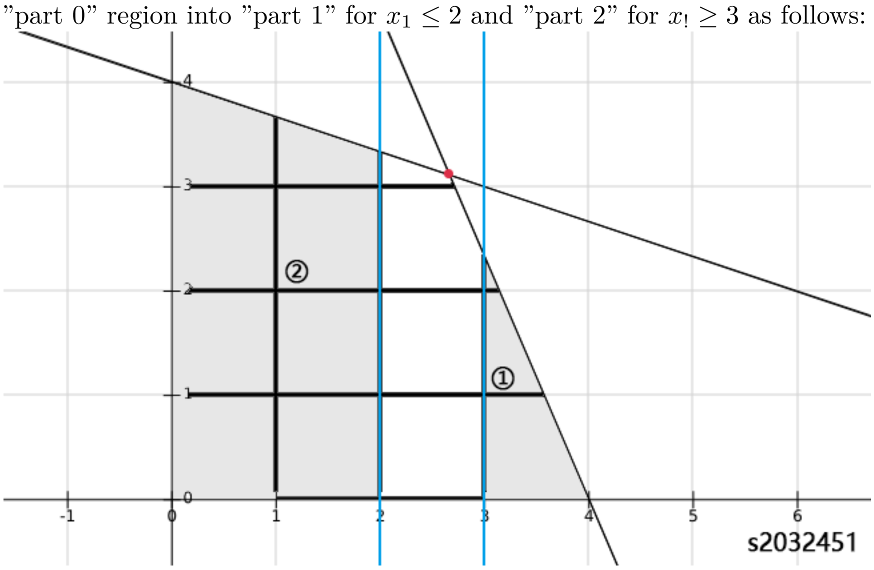
(1)

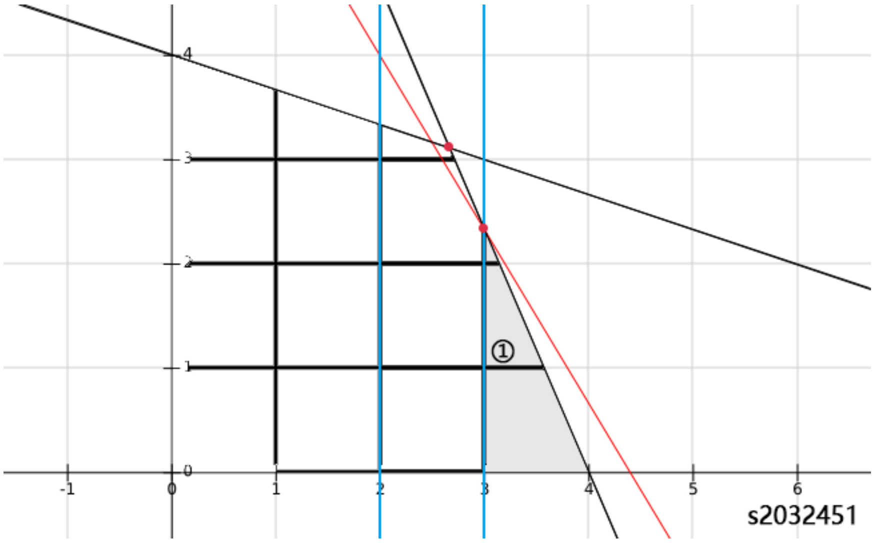


Due to the plot tool feature, I choose the shadow part to indicate the feasible range, hope this is still clear to view.
The horizontal axis represents x_1, and the vertical axis represents x_2.
As by this diagram, we could find the optimal solution to the LP relaxation at $x_1 = \frac{8}{3}, x_2 = \frac{28}{9}$, and the corresponding optimal value $F = \frac{68}{3}$.

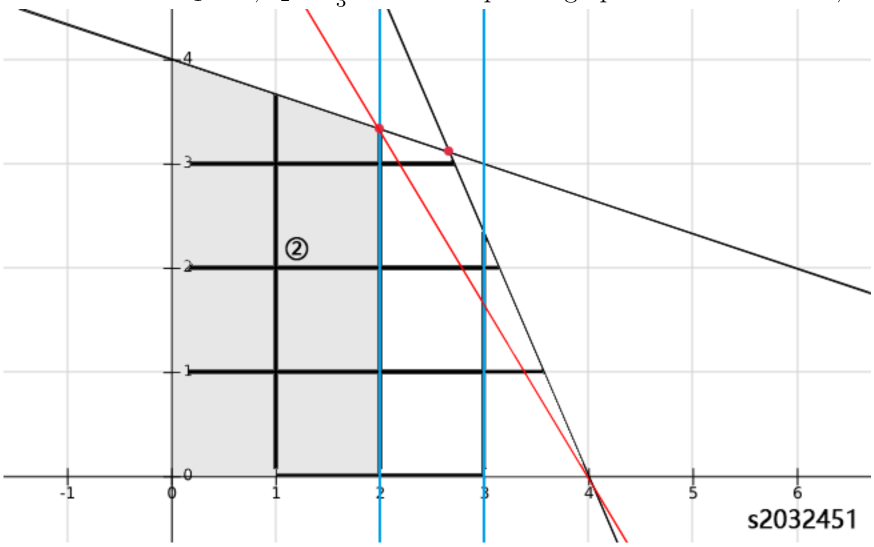
(2)
From (1) above, we already know that for the whole feasible solution range, the "part 0"'s optimal solution. (Note that here I mark the different dividing part of feasible solution in a slightly different way from the notes and slides, just for personal habit reason, hope this does not matter)
Now first we divede the "part 0" region into "part 1" for $x_1 \leq 2$ and "part 2" for $x_! \geq 3$ as follows:



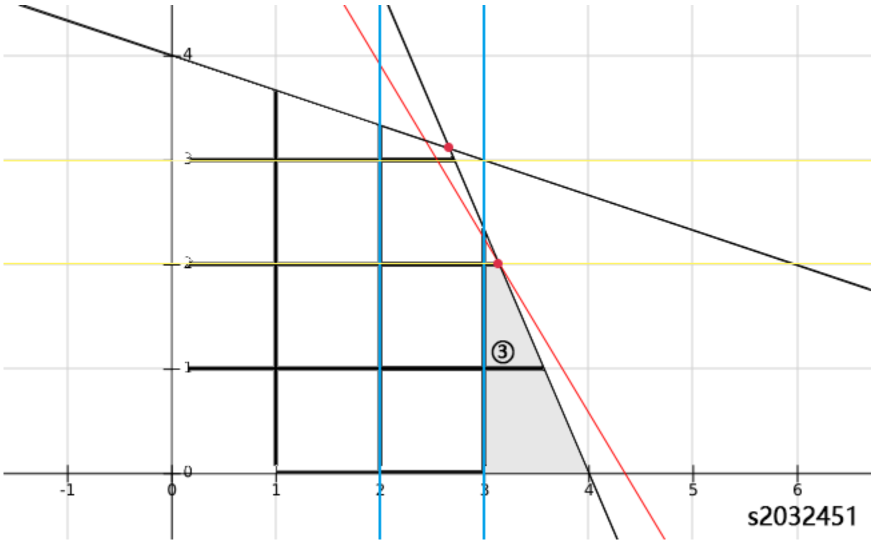
For "part 1", we have optimal solution $x_1 = 3, x_2 = \frac{7}{3}$ and corresponding optimal value $F = 22$, as follows:



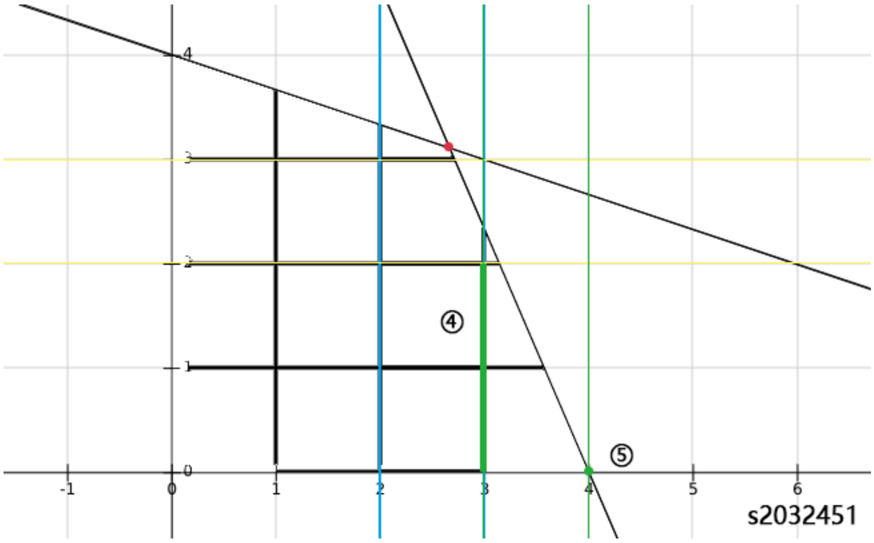
For "part 2", we have optimal solution $x_1 = 2, x_2 = \frac{10}{3}$ and corresponding optimal value $F = 20$, as follows:



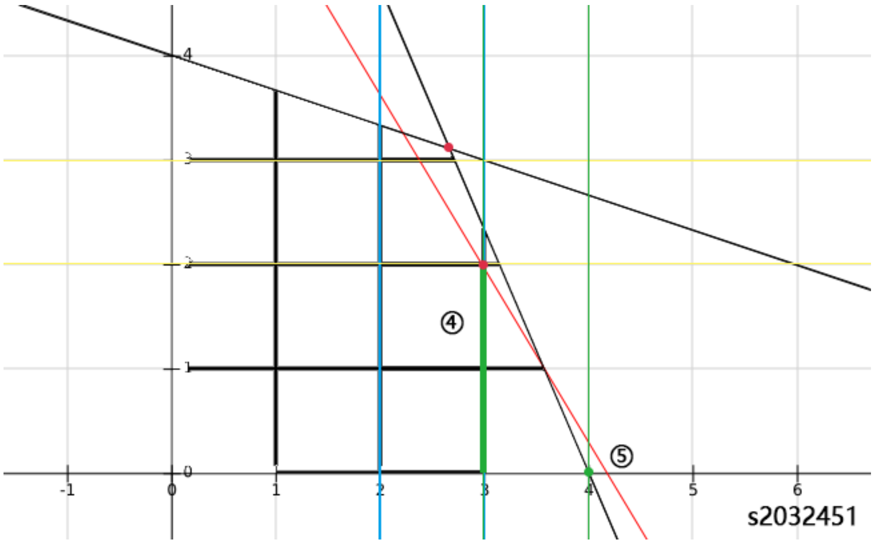
Now we consider to divide "part 1" first since it has a bigger optimal value.
We divide "part 1" as follows:



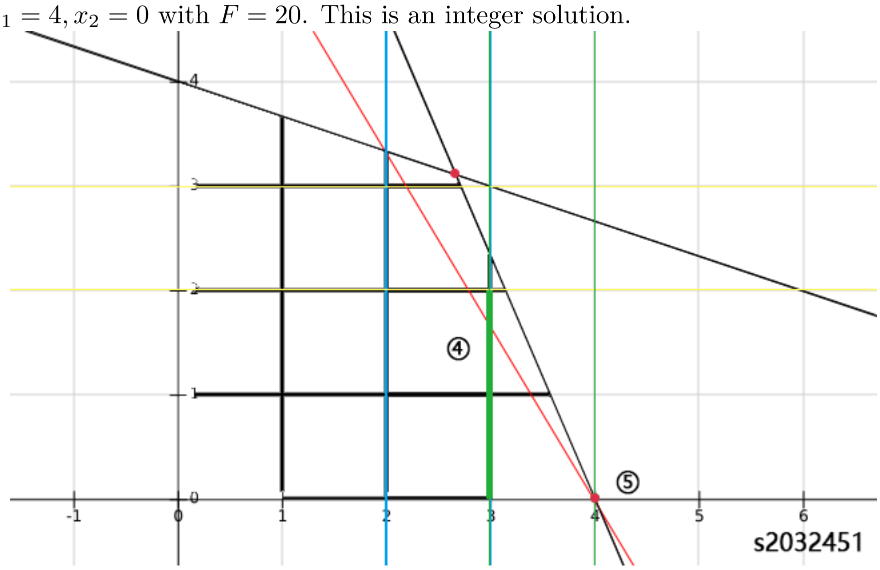
Since there is a part which is infeasible, marked as "part 3'" with $x_2 \geq 3$, we now only consider the "part 3". After calculation, we obtain its optimal solution with $x_1 = \frac{22}{7}, x_2 = 2$ and corresponding optimal value $F = \frac{152}{7}$. Since it is still not an integer solution, we divide "part 3" into "part 4" with $x_1 \leq 3$ and "part 5" with $x_1 \geq 4$.



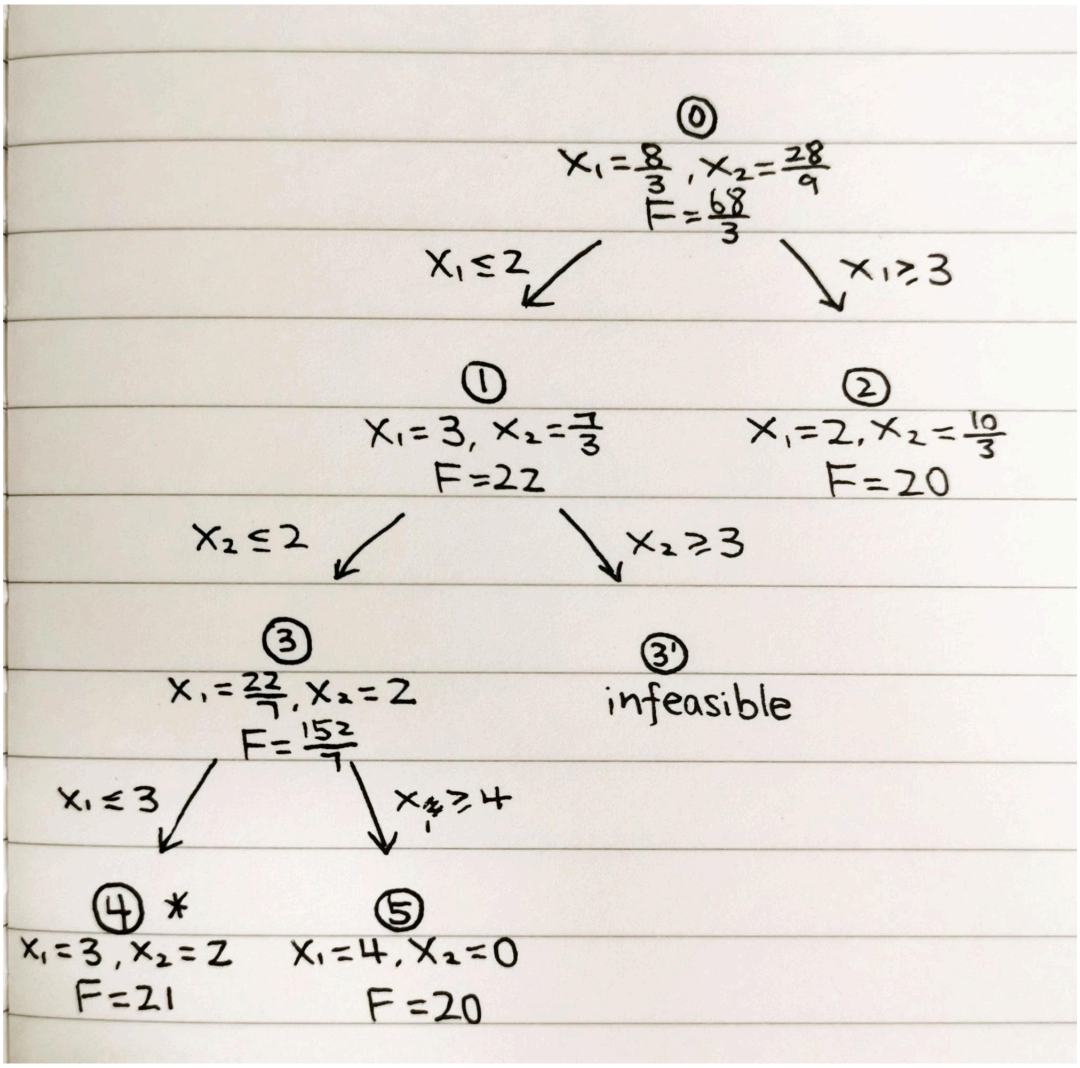
Note that we use the bold green line to identify the feasible solution region "part 4" and the bold green dot for "part 5". Now, for "part 4", from the following graph, we calculate the optimal solution $x_1 = 3, x_2 = 2$ with $F = 21$ (now it it an integer solution):



For "part 5", we have $x_1 = 4, x_2 = 0$ with $F = 20$. This is an integer solution.



The total process is as following tree plot:



Note that, for "part 4", its optimal value is $F = 21$ under integer optimal solution, which is higher than the upper bound of optimal value of "part 2", which is $F = 20$, so there is no any more need for us to consider the case under "part 2" then. Overall, the integer optimal solution is $x_1 = 3, x_2 = 2$ with optimal value $F = 21$.

(3)
As now we need to find the "second best integer solution", we need to find out the best two optimal value f_1 and f_2 ($f_1 > f_2$) with corresponding integer solutions $x*$ and x^{**}.
Since booth x^* and x^{**} are integer solution, the minimum difference between f_1 and f_2 would be ϵ whereas $f_2 \le f_1 - \epsilon$ (so in this problem ϵ could be set as 1 since all the coefficients are integer and solutions are integer, too). (Note that this is not the unique value set of ϵ in additional constraint $F \le F^* - \epsilon$, as long as we set it to be small enough)
Now as for the modified "B&B Algorithm" in notes, we have following modification (considering a MAXIMUM problem here):
1. In the beginning definition, we define both $f_1 = -\infty$ and $f_2 = -\infty$, and $f_1 > f_2$ (or in detail, $f_2 \le f_1 - \epsilon$).
2. In Line 7, it is modified as "if problem is infeasible or $LP(P_i) \le f_2$ then".
3. In line 10, it is modified as "set $f_1 = max\{f_1, LP(P_i)\}$. If this is the best integer solution found so far ($f_1 \ge LP(P_i)$): remember it as x^*, and introduce an additional constraint $P^* : F \le LP(P_i) - \epsilon$ in which ϵ is a small enough value, and find the optimal integer solution within $P \cap P^*$, and compare to choose the bigger one between its corresponding f_2^* with existing f_2. But if $f_1 \ge LP(P_i)$, we then set $f_2 = max\{f_2, LP(P_i)\}$, and if it is the second best integer solution found so far, remember it as x^{**}".

(4)
Introducing the binary variable δ that takes value 1 when $8x_1 - 6x_2 - 7 > 0$. This can be modelled with the constraint:

$$8x_1 - 6x_2 - 7 \le M\delta$$

As the maximal value for $8x_1 - 6x_2 - 7$ over the LP feasible region is 25 (occuring at $x_1 = 4, x_2 = 0$), $M = 25$ is the tightest possible M.
Then modify the objective function as $maxF = 5x_1 + 3x_2 - 3\delta$.