

Derivatives of Trig Function Examples

Example 1.) $\frac{d}{d\theta} 4 \sin 3\theta$

or $\frac{d}{d\theta} 4 \sin \theta = 4 \cos \theta$ apply chain rule $\frac{d}{d\theta} 4 \sin 3\theta$ apply chain rule

$u(\theta) = 3\theta$ $\frac{df}{d\theta} = \frac{df}{du} \cdot \frac{du}{d\theta} = 4 \cos u \cdot 3 = 12 \cos u = 12 \cos 3\theta$

$f(u) = 4 \sin u$

Example 2.) $\frac{d}{d\theta} 2 \cos^3 \theta$ $\frac{df}{du} \cdot \frac{du}{d\theta} = -\sin \theta (6u^2) = -\sin \theta (6 \cos^2 \theta) = -6 \sin \theta \cos^2 \theta$

$u = \cos^3 \theta$ $\frac{df}{d\theta} = \frac{df}{du} \cdot \frac{du}{d\theta} = -2 \sin u \cdot 3\theta^2 = -2 \sin^3 \theta (3\theta^2) = -6 \sin^3 \theta$

$f \rightarrow 2 \cos u$

Example 3.) $\frac{d}{d\theta} \sec \theta \tan \theta$

$(\sec \theta)'(\tan \theta) + (\tan \theta)'(\sec \theta) = (\sec \theta \tan \theta)(\tan \theta) + \sec^2 \theta (\sec \theta) = \tan^2 \theta \sec \theta + \sec^3 \theta$

Example 4.) $\frac{d}{d\theta} \left(\frac{1 + \sin \theta}{\cos^2 \theta} \right)$

$\frac{(1 + \sin \theta)'(\cos^2 \theta) - (\cos^2 \theta)'(1 + \sin \theta)}{(\cos^2 \theta)^2} = \frac{\cos^2 \theta - (-\sin \theta)(1 + \sin \theta)}{\cos^4 \theta} = \frac{\cos^2 \theta + \sin \theta + \sin^2 \theta}{\cos^4 \theta} = \frac{1 + \sin \theta}{\cos^2 \theta}$

Example 5.) $\frac{d}{dx} 4 \tan^{-1}(x^3)$

$u(x) = x^3$

$f(u) = \frac{4}{\tan u}$

$\frac{df}{dx} \cdot \frac{du}{dx} = \frac{4}{\sec^2 u} \cdot 3x^2 = \frac{4}{1+u^2} (3x^2) = \frac{4}{1+x^2} (3x^2) = \frac{12x^2}{1+x^2}$

Example 6.) $\frac{d}{dx} \sin^{-1} x \cos^{-1} x$

$\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ for $|x| < 1$

$\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$ for $|x| < 1$

$(\sin^{-1} x)' \cos^{-1} x + (\cos^{-1} x)' (\sin^{-1} x)$

$= \frac{1}{\sqrt{1-x^2} \cdot \cos x} - \frac{1}{\sqrt{1-x^2} \cdot \sin x}$

$= \frac{1}{\sqrt{1-x^2}} \left(\frac{1}{\cos x} - \frac{1}{\sin x} \right)$