1

Date: 05/03/2

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

3rd Year, 1st Semester, Final Examination, Fall-2016

Course No: **CSE 3101** Course Title: **Mathematical Analysis for Computer Science**

Time: 3 hours

Full Marks: 70

[There are 7 (seven) questions carrying 14 marks each. **Answer any 3 (three) questions from Section A and any 2 (two) questions from Section B**]

[Marks allotted are indicated in the right margin]

SECTION – A

1.a) Formulate the recurrence for the Triple Tower of Hanoi (TTOH) problem. Find the closed form expression for the recurrence and prove its correctness by using mathematical induction. Then, find the minimum number of moves necessary to solve the TTOH problem with 12 disks. [5]

b) For the Josephus problem, formulate recurrence relation on $J(n)$ by establishing arguments on $J(2n)$ and $J(2n+1)$. From the solution pattern for small values of n, guess the general solution and prove its correctness by mathematical induction. [5]

c) Derive the binary property of Josephus problem. Also, compute the value of $J(35)$ by using the radix based property of the generalized recurrence. [4]

2.a) The average number of comparison steps C_n made by the quicksort algorithm to sort n items satisfies the following recurrence. [5]

$$C_0 = 0$$

$$C_n = (n+1) + \frac{2}{n}\sum_{k=0}^{n-1} C_k$$

Find the summation factor for the recurrence and prove that $C_n = 2(n+1)H_n - 2n$.

b) Derive the formula for Perturbation technique. Then, apply the perturbation method to find a closed form expression for the following sum: $S_n = \sum_{0 \le k \le n} k^3$ [5]

c) Write the following sum S_n into its general form and convert it into a recurrence. Then solve the recurrence to evaluate S_n. [4]

$$S_n = 25 + 35 + 45 + \dots\dots + 525$$

2

3.a) Write and prove the Addition formula for binomial coefficients. Also, prove that the Symmetry identity does not hold when both the upper and lower indices are negative. [5]

b) Prove or disprove that $\left\lfloor \sqrt{\lfloor x \rfloor} \right\rfloor = \left\lfloor \sqrt{x} \right\rfloor$. Also, find the closed form of the following series of binomial coefficients. [5]

(i) $\sum_{k=1}^{11} \binom{12}{k} \Big/ \binom{13}{k}$

(ii) $\binom{11}{2} + \binom{12}{2} + \dots\dots + \binom{24}{2}$

c) If n is an m-bit integer number, then prove that $m = 1 + \lfloor \lg(n) \rfloor$. [4]

4.a) Write an efficient algorithm to find the Least Common Multiple (LCM) of two integers. [3]

b) Prove the fundamental theorem of arithmetic. Also, find the value of $\varepsilon_2(128!)$. [3]

c) Prove or disprove that the Mersenne number is always prime. Also, draw the Stern-Brocot tree up to the 3rd level. [4]

d) Write an algorithm to generate the L-R sequence that locates a given fractional value in the Stern-Brocot tree. Also, demonstrate every step of the algorithm by locating the fraction $11/7$ in the Stern-Brocot tree. [4]

SECTION – B

5.a) Suppose an urn contains seven black balls and five white balls. We draw three balls from the urn without replacement. Assuming that each ball in the urn is equally likely to be drawn, what is the probability that all three balls are black? [3]

b) Find $E[X]$ where X is an exponential random variable with parameter λ. [3]

c) Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. [4]

i) What is the probability that at least one man selects his own hat?

ii) What is the probability that all of the three men select their own hats?

d) In a country, 60% of the adults are males. It is known that 10% of males in that country smoke cigar, whereas 5% of females smoke cigars. One adult is randomly selected for a survey who was smoking a cigar. Find the probability that the selected person is Male. [4]

6.a) Suppose that, whether or not it rains today depends on previous weather conditions through the last two days. If it has rained for the past two days, then it will rain tomorrow with probability 0.7. If it rained today but not yesterday, then it will rain tomorrow with probability 0.5. If it rained yesterday but not today, then it will rain tomorrow with probability 0.4. If it has not rained in the past two days, then it will rain tomorrow with probability 0.2. [5]

i) What is the probability that it will NOT rain on Thursday, given that it already rained on both Monday and Tuesday?

ii) What is the probability that it will rain on both Thursday and Wednesday, given that it rained neither on Monday nor on Tuesday?

b) Explain the memoryless property of a probability distribution. Suppose, the number of road accidents each day in Dhaka city has a Poisson distribution with mean=3. Calculate the probability that there is NO more than one accident on a particular day in Dhaka city. [5]

c) Max and Patty decide to flip two fair coins. If they get at least one head from a flip, then Max is declared winner on that flip. Otherwise, Patty is declared winner. After each flip, the winner receives 10$ from the looser. If Max starts with 40$ and Patty with 110$, then what is the probability that Max will wipe Patty out? [4]

7.a) For a single server exponential queuing system, find the values of P_n, L, L_Q, W and W_Q, where the symbols have their usual meanings. [7]

b) For the shoeshine shop model, it is given that $\lambda = 1$, $\mu_1 = 1$ and $\mu_2 = 2$. Calculate the following quantities for this system. [7]

i) The proportion of potential customers who can't enter the system.

ii) The probability that both the chairs are occupied.

iii) The average number of customers in the system.

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Date: 03/10/2016

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

3rd Year, 1st Semester, Final Examination, Spring-2016

Course No: **CSE 3101** Course Title: **Mathematical Analysis for Computer Science**

Time: 3 hours Full Marks: 70

[There are 7 (seven) questions carrying 14 marks each. **Answer any 3 (three) questions from Section A and any 2 (two) questions from Section B**]

[Marks allotted are indicated in the right margin]

SECTION – A

1.a) For the Josephus problem, formulate the recurrence relation on $J(n)$ after establishing arguments on $J(2n)$ and $J(2n+1)$. From the solution pattern for small values of n, guess the general solution and prove its correctness using mathematical induction. [5]

b) Formulate the recurrence for the minimum number of moves required to solve the Triple Tower of Hanoi (TTOH) problem. Derive the closed form expression for the recurrence. Then, prove the correctness of the closed form expression by using mathematical induction. Also, find the minimum number of moves necessary to solve the TTOH problem with 12 disks. [5]

c) Find the maximum number of regions that can be obtained from n number of intersecting V-shapes in the plane. [4]

2.a) Derive summation factor for the following recurrence, convert the recurrence into a sum and then, evaluate the sum to find the closed form value of T_n. [5]

$$T_0 = 0$$
$$T_n = 2T_{n-1} + 1$$

b) Derive the formula for Perturbation technique. Then, apply the perturbation formula to find a closed form expression for the following sum $S_n = \sum_{0 \le k \le n} k^2$ [5]

c) Determine the closed form expression of the following sum by using the index replacement technique. [4]

$$T_n = \sum_{0 \le k \le n} (2+3k)$$

3.a) Prove the Addition formula and Absorption identity for binomial coefficients. [5]

b) Prove or disprove that (x mod ny) mod y = x mod y, where n is an integer and $y \neq 0$. [5]

c) Find closed form of the following sum of binomial coefficients: $\sum_{2 \le k \le 10} \binom{11}{k} \Big/ \binom{12}{k}$ [4]

5

4.a) Write an efficient algorithm to find the Greatest Common Divisor of two integers. Also, prove or disprove that Mersenne number is always prime. [5]

b) If m/n and m'/n' are consecutive fractions at any stage of construction of the Stern-Brocot tree, then prove than $m'n - mn' = 1$ by mathematical induction. [5]

c) Calculate the value of $\varepsilon_2(130!)$. Find the fractional value in the Stern-Brocot tree for the following sequence — RLRL by using matrix multiplications. [4]

SECTION – B

5.a) Write the Bayes' theorem on conditional probability. Then, consider two urns — the first urn contains three white and seven black balls while the second urn contains seven white and three black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was Head or Tail, respectively. Find out the conditional probability that the outcome of the toss was Head given that a black ball was selected. [5]

b) What do you understand by sample space and random variable? Ebon can either take a course in computers or in chemistry. If Ebon takes the computer course, then he will receive an A+ grade with probability = 0.75. If he takes the chemistry course then he will receive an A+ grade with probability = 0.4. Ebon decides to base his decision on the flip of a fair coin. What is the probability that Ebon will get an A+ in chemistry? [5]

c) Define Poisson random variable. Suppose, the number of accidents occurring on a highway each day is a Poisson random variable with mean = 3. What is the probability that no less than two accidents will occur today? [4]

6.a) Write the Chapman-Kolmogorov equation. On any given day, Zarin is either cheerful (C), so-so (S) or glum (G). She will be cheerful tomorrow if she is C, S or G today with respective probabilities 0.5, 0.4, 0.1. She will be glum tomorrow if she is C, S or G today with probabilities 0.2, 0.3, 0.5, respectively. Given that Zarin is Glum on Friday, what is the probability that she will NOT be Glum on Sunday? [5]

b) Yaseen and Shishir decide to flip three fair coins. If they get at least two heads from a flip, then Yaseen is declared winner on that flip. Otherwise, Shishir is declared winner. After each flip, the winner receives 10$ from the looser. If Yaseen starts with 100$ and Shishir with 50$, then what is the probability that Shishir will wipe Yaseen out? [5]

3

c) Define binomial random variable. Also, find $E[X]$ where X is a binomial distributed random variable with parameters n and p. [4]

7.a) What are the applications of queuing theory? For an $M/M/1$ queue, prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$, where the symbols have their usual meaning. [5]

b) For the shoeshine shop model, it is given that $\lambda = 1$, $\mu_1 = 1$ and $\mu_2 = 2$. Calculate the following quantities. [5]

i) The fraction of time both the chairs are empty.

ii) The probability that either or both chairs are occupied.

iii) The proportion of potential customers that can't enter the system.

c) Suppose that an airplane engine will fail, when in flight, with probability $(1-p)$ independently from engine to engine. The airplane can make a successful flight if at least 50 percent of its engines remain operative. Find the value of p for which a four-engine plane will be preferable to a two-engine plane? [4]

(7)

Date: 21/03/16

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

3rd Year, 1st Semester, Final Examination, Fall-2015

Course No: **CSE 3101** Course Title: **Mathematical Analysis for Computer Science**

Time: 3 hours Full Marks: 70

[There are 7 (seven) questions carrying 14 marks each. **Answer any 3 (three) questions from Section A and any 2 (two) questions from Section B**]

[Marks allotted are indicated in the right margin]

SECTION - A

1.a) Formulate the recurrence for the minimum number of moves required to solve the Double Tower of Hanoi (DTOH) problem. Derive the closed form expression for the recurrence. Then, find the minimum number of moves necessary to solve the DTOH problem with 10 disks. [5]

b) Given a set of $(n-1)$ lines in the plane, explain how adding a new line can create n new regions. Also, explain the case when it would create fewer than n regions. Then, find the maximum number of regions that can be obtained from n number of intersecting straight lines in the plane. [5]

c) Suppose, there are n people standing around a circle. From them, every second remaining person is being executed, until only one person remains, who is then released to survive. Find the minimum value of n such that the person standing at the $n/3$-th position finally survives, given that $n \geq 150$. [4]

2.a) The average number of comparison steps C_n made by the quicksort algorithm to sort n items satisfies the following recurrence. Find the summation factor for the recurrence and prove that $C_n = 2(n+1)H_n - 2n$. [5]

$$C_0 = 0$$

$$C_n = (n+1) + \frac{2}{n}\sum_{k=0}^{n-1} C_k$$

b) Derive the formula for Perturbation technique. Then, apply the perturbation method to find a closed form expression for the following sum: $S_n = \sum_{0 \leq k \leq n} k.2^k$ [5]

c) Write the following sum S_n into its general form and convert it into a recurrence. Then solve the recurrence to evaluate S_n. [4]

$$S_n = 10 + 25 + 40 + \dots\dots\dots + 610$$

(8)

3.a) Write down the addition formula for binomial coefficients. Also, Prove that: [5]

$$\sum_{k \leq n} \binom{m+k}{k} = \binom{m+n+1}{n} \quad \text{for } m+k \geq 0$$

b) Prove or disprove that $\left\lfloor \sqrt{\lfloor x \rfloor} \right\rfloor = \left\lfloor \sqrt{x} \right\rfloor$. Also, find the closed form of the following sum of binomial coefficients: $\sum_{k=1}^{15} \binom{12}{k} \Big/ \binom{20}{k}$ [5]

c) If n is an m-bit integer number, then prove that $m = 1 + \lfloor \lg(n) \rfloor$. [4]

4.a) Write down the Sieve of Eratosthenes algorithm to find all prime numbers up to the integer n. [3]

b) Write an efficient algorithm to find the Greatest Common Divisor of two integers. [3]

c) What do you understand by $m \backslash n$ and $m \perp n$? Give an inductive proof of the following property of the Stern-Brocot tree — if m/n and m'/n' are consecutive fractions at any stage of the construction, then $m'n - mn' = 1$. [4]

d) Write an algorithm to generate the L-R sequence that locates a given fractional value in the Stern-Brocot tree. Also, demonstrate every step of the algorithm by locating the fraction 13/8 in the Stern-Brocot tree. [4]

SECTION – B

5.a) An airplane can make a successful flight to its destination if at least 50% of its engines remain operative. If the probability of failure of an airplane engine is 0.25 during each flight, then what is the probability that a four engine air plane can make a successful flight to the destination? [3]

b) Find the expected value of X, where X is the outcome when we roll a fair dice. [3]

c) Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. [4]

i) What is the probability that at least one man selects his own hat?

ii) What is the probability that all of the three men select their own hats?

d) Write down the Bayes' theorem on conditional probability. Also, apply the Bayes' formula to solve the following problem — A laboratory blood test is 80 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 5 percent of the healthy persons tested. If only 3 percent of the population actually has the disease, then what is the probability that a person has the disease given that his test result is positive? [4]

(5)

6.a) Suppose that, whether or not it rains today depends on previous weather conditions through the last two days. If it has rained for the past two days, then it will rain tomorrow with probability 0.7. If it rained today but not yesterday, then it will rain tomorrow with probability 0.5. If it rained yesterday but not today, then it will rain tomorrow with probability 0.4. If it has not rained in the past two days, then it will rain tomorrow with probability 0.2. [5

i) What is the probability that it will rain on Thursday, but not on Wednesday, given that it already rained on both Monday and Tuesday?

ii) What is the probability that it will not rain on Thursday given that it rained neither on Monday nor on Tuesday?

b) What do you understand by the memoryless property of a probability distribution? Suppose that the amount of time a customer spends in a bank is exponentially distributed with mean = 15 minutes. Then, what is the probability that a customer will spend more than 15 minutes in the bank? What is the probability that a customer will spend more than 1 hour in the bank given that she is still in the bank after 30 minutes? [5

c) Barry and Garry decide to throw two fair dice. If the sum of outcomes of two dice is greater than 10, then Barry is declared winner on that throw. Otherwise, Garry is declared winner. After each throw, the winner receives \$25 from the looser. If Barry starts with \$100 and Garry with \$25, then what is the probability that Garry will wipe Barry out? [4

7.a) What do you understand by $M/M/k$ and $M/G/1$ queuing models? For a single server exponential queuing system, prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$. [5

b) For the shoeshine shop model, it is given that $\lambda = 1$, $\mu_1 = 1$ and $\mu_2 = 2$. Calculate the following quantities for this system. [5

i) The proportion of potential customers who can't enter the system.

ii) The mean number of customers in the system.

c) Define geometric random variable. Suppose, the number of typographical errors on a single page of a book has a poison distribution with mean = 2. Calculate the probability that there are at least two errors on a certain page. [4

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(11)

(10)

Date: 20/10/15

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

3rd Year, 1st Semester, Final Examination, Spring 2015

Course No: **CSE 3101** Course Title: **Mathematical Analysis for Computer Science**

Time: 3 hours Full Marks: 70

There are 7 (seven) questions carrying 14 marks each.

Answer any 3 (three) questions from Section A and any 2 (two) questions from Section B

[Marks allotted are indicated in the right margin]

SECTION: A

1. a) Define the Josephus problem. Derive the recurrence relation for the Josephus problem. From the solution pattern for small values of n for the above problem, guess the general solution, and prove its correctness using induction. [6]

b) Consider the following object 'Z' on the plane. [4]

Z

Derive an equation for Z_n, where Z_n is the maximum number of regions obtainable by n number of intersecting Z's on the plane.

c) Suppose we have a recurrence like : [4]

$f(j) = \alpha_j$, for $1 \le j < d$;
$f(dn+j) = c f(n) + \beta_j$, for $0 \le j < d$ and $n \ge 1$;

Solution of the above recurrence relation is given below:

$f((b_m b_{m-1} \dots b_1 b_0)_d) = (\alpha_{b_m} \beta_{b_{m-1}} \beta_{b_{m-2}} \dots \beta_{b_1} \beta_{b_0})_c$

Here, $d = 3$, $c = 5$, $\alpha_j = 2j + 1$, $\beta_j = 3j - 1$

Write down the equations and compute f(40).

2. a) How can we reduce a recurrence into a sum? Show it using 'Tower of Hanoi' problem. [6]

b) $S_n = \sum_{k=0}^{n} (a + bk)$ where a=2, b=3. Calculate S_{90}. [3]

(11)

c) Determine a closed form expression for $\sum_{0\le k\le n} k\cdot 2^k$ using perturbation method. [5]

3. a) Prove Addition Formula and Symmetry Identity. [6]

b) Prove or disprove that (p mod nq) mod q = p mod q; where n is integer. [4]

c) Show that $\binom{r}{m}\binom{m}{k}=\binom{r}{k}\binom{r-k}{m-k}$ [4]

4. a) Prove that every composite number has a prime divisor. Evaluate $\varepsilon_2(120!)$ [6]

b) Write down Stein's algorithm to find gcd(u,v). Also, calculate gcd(60,90) using it. [5]

c) Draw Stern-Brocot tree upto level 4. [3]

$S_n = S_0 + \sum a^k$

SECTION: B

5. a) Define the following terms: [4]

i. Binomial random variable

ii. Expectation of a random variable

b) The probability to die in a car accident in a 24 hour period is one in a million. The probability to die in a car accident at night is one in two millions. At night there is 30% traffic. You hear that a relative of yours died in a car accident. What is the probability that the accident took place at night? [5]

c) At the Express House Delivery Service, providing high quality service to customers is the top priority of the management. The company guarantees a refund of all charges if a package it is delivering does not arrive at its destination by the specified time. It is known from past data that despite all efforts, 2% of the packages mailed through this company do not arrive at their destinations within the specified time. Suppose a corporation mails 10 packages through Express House Delivery Service on a certain day. Find the probability that at most one of these 10 packages will not arrive at its destination within the specified time. [5]

6. a) Suppose that an airplane engine will fail, when in flight, with probability 1− P independently from engine to engine; suppose that the air plane will make a successful flight if at least 50% of its engines remain operative. For what values of P is a four engine plane preferable to a two engine plane? [6]

b) A company needs to hire a new director of advertising. It has decided to try to hire person A or B, who are assistant advertising directors for its major competitor. To decide between A and B, the company does research on the campaigns managed by either A or B (no campaign is managed by both), and finds that A is in charge of twice as many campaigns as B. Also, A's campaigns have satisfactory results 3 out of 4 times, while B's campaigns have satisfactory [8]

(12)

results 2 out of 5 times. Suppose one of the campaigns (managed by A or B) is selected randomly.

(i) Find the probability that A is in charge of the selected campaign and that it produces satisfactory results.

(ii) Find the probability that B is in charge of the selected campaign and that it produces satisfactory results.

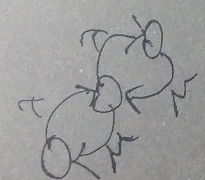
(iii) What is the probability that the selected campaign is satisfactory?

(iv) What is the probability that the selected campaign is unsatisfactory?

7. a) What is queuing theory? Write down some applications of queuing theory. [3]

b) Suppose that, in a barber shop, customers arrive at a rate of one per 18 minutes and the service time for each customer is at a rate of one service per 10 minutes. Find the probability of having 6 customers (on an average) in the system. Also, find the average number of customers in the waiting queue and the average amount of time a customer spends in the system. [4]

c) In a single server exponential queuing system find the value of the following parameters: L_Q and W_Q, where the symbols have their usual meanings. [7]



$L = \lambda W$

$L_Q = \lambda W_Q$

(13)

Date: 19.04.15

Ahsanullah University of Science and Technology
Department of Computer Science and Engineering
Final Examination, Fall-2014
3rd Year, 1st Semester
Course No: CSE 3101
Course Title: Mathematical Analysis for Computer Science

Full Marks: 70 — Time: 3 hours

There are TEN questions, answer any SEVEN. The figures in the margin indicate full marks.

1)

a) Find the closed form of the following recurrence:

$$T_0 = 1$$
$$nT_n = 4T_{n-1} + 6$$

(4)

b) Suppose we have a recurrence like :

$f(j) = \alpha_j$, for $1 \le j < d$;

$f(dn + j) = c\,f(n) + \beta_j$, for $0 \le j < d$ and $n \ge 1$;

Solution of the above recurrence relation is given below:

$$f((b_m\, b_{m-1} \ldots b_1\, b_0)_d) = (\alpha_{b_m}\, \beta_{b_{m-1}}\, \beta_{b_{m-2}} \ldots \beta_{b_1}\, \beta_{b_0})_c$$

The Equation is as follows:

$f(1) = 54$, $f(2) = 5$

$f(3n) = 5f(n) + 8$, $f(3n+1) = 5f(n) - 4$, $f(3n+2) = 5f(n) + 7$

Compute $f(37)$ (6)

2)

a) Solve the equation using perturbation technique: (6)

$$S_n = \sum_{0 \le k \le n} k x^k$$

b) Find the sum of the following recurrence: (4)

$$T_0 = 1$$
$$T_n = 2T_{n-1} + 1$$

3)

a) Prove or Disprove:

$k\backslash m$ and $k\backslash n \Leftrightarrow k\backslash \gcd(m,n)$ (2)

b) Define Relative Primality. If a path to a node from root in a Stern Brocot Tree is LLRR, Find the fraction defined on that node? (1+4)

c) How many times 90! will be divided by 3 i.e.; the highest power 3 can be raised so that it divides 90!? (3)

(14)

4)

a) A slow but persistent worm, W starts at starts at one end of a 1 meter long rubber band and crawls 1 centimeter per minute towards the other end. At the end of each minute, an equally persistent keeper of the band, K stretches the band by 1 meter. Find the condition for the Worm for reaching the other end of the rubber band? (6)

b) Prove the following rules: (4)

i) $x < n \Leftrightarrow \text{floor}(x) < n$

ii) $n < x \Leftrightarrow n < \text{ceil}(x)$

5) In a Roulette Wheel Game with 1000 slots numbered 1 to 1000. If the number **n that comes up on a spin is divisible by the floor of its cube** then it is a winner and the house pays \$5 and otherwise it's a loser and the player pays \$1. So, Find out how many games have to be won to return with some money on average? **(10)**

6)

a) Prove $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(FG) - P(EG) + P(EFG)$ (3)

b) Suppose an urn contains seven black balls and five white balls. We draw two balls from the urn without replacement. Assuming that each ball in the urn is equally likely to be drawn, what is the probability that both balls are black? (3)

c) A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive? (4)

7)

a) If X is uniformly distributed over (0,10), calculate the probability that

i) $X > 7$ ii) $2 < X < 6$ (4)

b) Suppose the amount of time one spends in a bank is exponentially distributed with mean ten minutes, that is $\lambda = 1/10$. What is the probability that a customer will spend more than fifteen minutes in the bank? What is the probability that a customer will spend more than fifteen minutes in the bank given that she is still in the bank after ten minutes? (6)

8)

a) Suppose that an airplane engine will fail, when in flight, with probability 1-p independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what value of *p* is a four-engine plane preferable to a two-engine plane? (5)

15

b) Suppose that the number of errors on a single page of a journal has a poison distribution with parameter value 1.2. Calculate the probability that there is at least one error on a certain page? 0.699 (2)

c) Given that, p(0) =0.2, p(1) = 0.5, p(2) =0.3, Calculate $E[X^3]$ ~? (3)

2.9

9) a) On any given day Gary is cheerful (*C*), so-so (*S*) or glum (*G*). If he is cheerful today, then he will be C, S, or G tomorrow with respective probabilities 0.4, 0.3, 0.3. If he is felling so-so today, then he will be *C*, *S* or *G* tomorrow with probabilities 0.2, 0.4, 0.4 respectively. If he is glum today, then he will be *C*, *S* or *G* tomorrow with probabilities 0.2, 0.3, 0.5. Draw the transition matrix. (3)

b) Suppose that, whether or not, it rains today depends on previous weather conditions through the last two days. Specially suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday? 0.61 (7)

10) a) Suppose Max and Patty decide to flip pennies; the one coming closest to the wall wins. Patty, being the better player, has a probability 0.6 of winning on each flip.
(i) If Patty starts with five pennies and Max with ten, then what is the probability that patty will wipe Max out? 0.87
(ii) What happens if Patty starts with ten and Max with twenty? 0.983 (7)

b) Find the recurrent and transient states out of the following transition matrix on the last page. 4 0,1,2,3 (3)

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

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Date - 03.09.1[illegible]

Ahsanullah University of Science and Technology
Department of Computer Science and Engineering
3^{rd} year, 1^{st} semester, Final Examination (Spring, 2013)
Course No. : CSE-301, Course Title : Mathematical Analysis for Computer Science
Time : 3 Hours | Full Marks : 70

[Direction: There are 10 (ten) questions. Answer any 7 (seven) taking at least 3 (three) from each section. Marks allotted are shown on the right margin.]

Section – A

1. a) Show that we require $2^n - 1$ moves for n disks, to solve the Tower of Hanoi problem. 6

b) Suppose we have a recurrence like : 4

$$f(j) = \alpha_j; \quad \text{for } 1 \le j < d;$$
$$f(dn+j) = cf(n) + \beta_j; \quad \text{for } 0 \le j < d \text{ and } n \ge 1;$$

Solution of the above recurrence relation is given below:

$$f((b_m b_{m-1} \cdots b_1 b_0)_d) = (\alpha_{b_m} \beta_{b_{m-1}} \beta_{b_{m-2}} \cdots \beta_{b_1} \beta_{b_0})_c$$

Here, d = 3, c = 10, $\alpha_1 = 34$, $\alpha_2 = 5$, $\beta_0 = 76$; $\beta_1 = -2$; $\beta_2 = 8$

Find out $f(19)$ using the above solution.

2. a) How can we transform a sum into a recurrence? Evaluate $\sum_{k=0}^{n}(a+bk)$ using your driven formula. 6

b) Find out the summation $\sum_{0 \le k \le n} k \cdot 2^k$ using perturbation method. 4

3. a) Prove the fundamental theory of mathematics. 6

b) What is Euclid number? Write down first 5 Euclid numbers. 4

4. a) Find out the fraction corresponding to LRRLRL from the Stern-Brocot tree. 6

b) Find out the maximum value of n where 100! is divisible by 3^n. 4

5. a) Write down and prove Addition Formula and Symmetry Identity. 6

b) A slow but persistent worm, W starts at one end of a meter-long rubber band and crawls one centimeter per minute toward the other end. At the end of each minute and equally persistent keeper of the band, *K*, whose sole purpose in life is to frustrate *W*, stretches it one meter. Thus after one minute of crawling, *W* is 1 centimeter from the start and 99 from the finish; then *K* stretches it one meter. During the stretching operation *W* maintains his relative position 1% from the start and 99% from the finish; so *W* is now 2cm from starting point and 198cm from the goal. After *W* crawls for another minute the score is 3cm traveled and 197cm to go; but K stretches and distance become 4.5cm and 295.5cm and so on. Does the worm ever reach the finish? (We are assuming an infinite longevity for *K* and *W*, an infinitely elasticity of the band and an infinitely tiny worm). 4

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Section - B

6. a) Define the Binomial Random Variable. Prove that for Binomial Random Variable $\sum p(i)=1$. Find out the expectation for Binomial Random Variable. 6

b) Suppose that an airplane engine will fail, when in flight, with probability $1-p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what value of p is a four-engine plane preferable to a two-engine plane? 4

7. a) Suppose that the chance of rain tomorrow depends on previous conditions only through whether or not it is raining today and not on past weather conditions. Suppose also that if it rains today, then it will rain tomorrow with probability 0.7; and if it does not rain today, then it will rain tomorrow with probability 0.4. Calculate the probability that it will rain four days from today given that it does not rain today.. 6

b) Consider two urns. The first contains two white and seven black balls and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected? 4

8. a) Consider a gambler who at each play of the game has probability p of winning one unit and probability $q=(1-p)$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability, that starting with i units, the gambler's fortune will reach N before reaching 0? 6

b) Suppose Max and Patty decide to flip pennies; the one coming closest to the wall wins. Patty, being the better player, has a probability 0.6 of winning on each flip. 4
(a) If Patty starts with five pennies and Max with ten, then what is the probability that patty will wipe Maxout?
(b) What if Patty starts with ten and Max with twenty?

9. a) For M/M/1 queues with finite capacity prove that, $P_n = \dfrac{\left[\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)\right]}{\left[1-\left(\frac{\lambda}{\mu}\right)^{N+1}\right]}$ 6

b) Find out the average number of customer in the system (L) for M/M/1 queues with finite capacity. 4

10. a) Consider a shoeshine shop considering two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus for instance, a potential customer might enter even if there is a customer in chair 2). If we suppose that potential customers arrive at rate λ and that the service time for the two chairs are independent and have respective 6

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exponential rates of μ_1 and μ_2 then
(i) What proportion of potential customers enters the system?
(ii) What is the mean number of customers in the system?
(iii) What is the average amount of time that an entering customer spends in the system?

b) If $\lambda=1$, $\mu_1=1$ and $\mu_2=2$ then calculate L, L_Q, W, W_Q.

☺ Good Luck ☺

$\frac{4}{37} + \frac{12}{37}$

$\frac{8}{37}$

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Date – 17/2/13

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

3rd year, 1st semester, Final Examination (Fall, 2012)

Course No.: CSE-301, Course Title: Mathematical Analysis for Computer Science

Time: 3 Hours Full Marks: 70

[Direction: There are 10 (ten) questions. Answer any 7 (seven) taking at least 3 (three) from each section. Marks allotted are shown on the right margin.]

Section – A

1. a) Find out recurrence relation for the Josephus problem (where every 2nd person is eliminated). 6

   b) Suppose we have a recurrence like: 4

   $f(j)=\alpha_j$; for $1 \le j < d$;

   $f(dn+j)=cf(n)+\beta_j$; for $0 \le j < d$ and $n \ge 1$;

   Solution of the above recurrence relation is given below:

   $$f((b_m b_{m-1} \cdots b_1 b_0)_d) = (\alpha_{b_m} \beta_{b_{m-1}} \beta_{b_{m-2}} \cdots \beta_{b_1} \beta_{b_0})_c$$

   Here, $d=3$, $c=10$, $\alpha_1=5$, $\alpha_2=10$, $\beta_0=15$, $\beta_1=-20$, $\beta_2=25$

   Find out $f(30)$ using the above solution.

2. a) Find out the summation $S_n=\sum_{k=0}^{n} k3^k$ using perturbation method. Also find out the value of S_3 from the derived formula. 6

   b) Find out the maximum number L_n of regions defined by n lines in the plane. Also evaluate L_{10} by using this equation. 4

3. a) Prove that there are infinitely many primes and find out 6
   (i) The 5th Euclid number.
   (ii) The maximum power of 7 divides 105!

   b) Write down the Farey series F_N for N= 1,2,3,4,5,6,7 and 8. 4

4. a) Find out from the Stern-Brocot tree 6
   (i) The fraction corresponding to RRLLRL.
   (ii) The position of the fraction $\frac{7}{19}$ using existing algorithm.

   b) Prove for Stern-Brocot tree $m'n-mn'=1$. 4

5. a) Given n cards and a table, find out the largest possible overhang by stacking the cards over the table's edge. 6

   b) Prove that, 4

   (i) $\binom{r}{k}=\frac{r}{k}\binom{r-1}{k-1}$

   (ii) $\binom{r}{k}=\binom{r-1}{k}+\binom{r-1}{k-1}$

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Section - B

6. a) Define the Bernoulli Random Variable. Prove that for Bernoulli Random Variable $\sum p(i)=1$. Find out the expectation for Possion Random Variable. 4

   b) If the number of accidents occurring on a highway each day is a Poisson Random Variable with parameter $\lambda=3$, what is the probability that at least two accidents occur today? 3

   c) It is known that all items produced by a certain machine will be defective with probability 0.3, independently of each other. What is the probability that in a sample of four items, at most one will be defective? 3

7. a) On any given day Antora is cheerful (C), so-so (S), or glum (G). If she is cheerful today, then she will be C, S or G tomorrow with respective probabilities 0.6, 0.3, 0.1. If she is feeling so-so today, then she will be C, S or G tomorrow with probabilities 0.3, 0.5, 0.2. If she is glum today then she will be C, S, or G tomorrow with probabilities 0.1, 0.2, 0.7. Model this situation as a Markov chain and calculate the probability that Antora will cheerful today given that she was glum 3 days ago. 4

   b) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.9; If it rained today but not yesterday, then it will rain tomorrow with probability 0.8; If it rained yesterday but not today, then it will rain tomorrow with probability 0.4; If it has not rained in the past two days, then it will rain tomorrow with probability 0.3. Model this as Markov chain. 3

   c) Consider question 7(b). Given that it rained on Saturday and Sunday, what is the probability that it will rain on Tuesday? 3

8. a) Consider a gambler who at each play of the game has probability p of winning one unit and probability $q=(1-p)$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability, that starting with i units, the gambler's fortune will reach N before reaching 0? 5

   b) Suppose Robiul and Porosh decide to flip coins; they decided to play until anyone become without any pennies. Porosh, being the better player, has a probability 0.55 of winning on each flip. If Robiul starts with 25 coins and Porosh with 15, then what is the probability that Porosh will wipe Robiul out? 2

   c) Consider two urns. The first contains two white and seven black balls. The second contains five white and six black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected? 3

P(H|W) = P(

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9. a) For M/M/1 queues with infinite capacity prove that, $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$ 5

b) Suppose that customer arrive at a Poisson rate of one per every 10 minutes and that the service time is exponential at a rate of one service per 8 minutes. Find out L, L_Q, W and W_Q. 2

c) What is M/M/1 Queue? Prove by a counter example that in queuing models, $(a_n = d_n)$, but P_n is not necessarily equal to a_n and d_n. 3

10. a) Consider a shoeshine shop considering two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus for instance, a potential customer might enter even if there is a customer in chair 2). If we suppose that potential customers arrive at rate λ and that the service time for the two chairs are independent and have respective exponential rates of μ_1 and μ_2 then (i) What proportion of potential customers enters the system? (ii) What is the mean number of customers in the system? (iii) What is the average amount of time that an entering customer spends in the system? 6

b) If $\lambda = 1$, $\mu_1 = 2$ and $\mu_2 = 3$ then calculate L, L_Q, W, W_Q. 4

☺ Good Luck ☺