C. MATH

22 April 2017

CSE 3101 : Mathematical Analysis for Computer Science

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(Intro)

Mathematical Analysis for C.S.

Part A — Concrete Math

Part B — Probability

Final Xm: (3 out of 4) ← Concrete Math; (2 out of 3) ← Probability

Reference Books

1. Concrete Mathematics : A Foundation for Computer Science (Latest Edition) [8^{th} or 9^{th}] — by Ronald L. Graham, Donald E. Knuth, Oren Patashnik

2. Introduction to Probability Models (Latest Edition) — by Sheldon M. Ross

Recurrent Problem

T_n : $\boxed{\text{Minimum}}$ no. of $\boxed{\text{moves}}$ to solve TOH with n disks.

$$T_n = T_{n-1} + 1 + T_{n-1}$$
$$= 2T_{n-1} + 1$$
$$= 2[2T_{n-2} + 1] + 1$$
$$= \ldots T_0$$

$T_1 = 1$

$T_2 = 3$

$T_3 = 7 \longrightarrow 3+1+3$

$$\boxed{T_n = 2^n - 1}$$

* For Tower of Hanoi, prove that $\boxed{T_n = 2^n - 1}$

→ Proof

→ Inductive Proof

Inductive proof : $\boxed{T_n = 2^n - 1}$

Basis : when $n=0$, no. of required moves $= 0$

$n=0$ $\boxed{T_0} = 2^0 - 1 = 1 - 1 = \boxed{0}$

So, $\boxed{T_n = 2^n - 1}$ holds for $n=0$

Hypothesis : Suppose, $\boxed{T_n = 2^n - 1}$ holds for -

$\boxed{n = n}$

$\therefore T_n = 2^n - 1$

Check whether it holds for $\boxed{n = n+1}$?

$$T_{n+1} \stackrel{?}{=} 2^{n+1} - 1$$

$$\Rightarrow T_{n+1} = T_n + 1 + T_n$$
$$= 2T_n + 1$$
$$= 2(2^n - 1) + 1$$
$$= 2^{n+1} - 2 + 1$$
$$= \boxed{2^{n+1} - 1}$$

Thus, holds for $n = n+1$.

23 April 2017

⊞ Double Tower of Hanoi (DTOH)

T_n : Minimum no of moves to solve DTOH problem with $\boxed{n \text{ pairs}}$ of Disks.

$$\boxed{T_0 = 0}$$

$$T_n = T_{n-1} + 2 + T_{n-1}$$

$$= \boxed{2T_{n-1} + 2}$$

$$= 2\left[2T_{n-2} + 2\right] + 2$$

$$= 2^2 T_{n-2} + 2^2 + 2$$

$$= \ldots\ldots$$

$$\therefore \boxed{T_n = 2\left[2^n - 1\right]}$$

Basis : $T_0 = 2\left(2^0 - 1\right) = 0$

* figure আঁকলে ভালো না আঁকলেও চলবে!

* Concrete Math এর বই এর exercise এ "Double & Triple Tower of Hanoi" আছে!

* "Tower of Hanoi" থেকে আসবে 1টি।

* "pdf lecture" এর "lec 1B" থেকে পড়ব।

Triple Tower of Hanoi (TTOH)

T_n : Minimum no. of moves to solve TTOH problem. with $\boxed{n \text{ triplets}}$ of Disks.

$T_0 = 0$

$$T_n = T_{n-1} + 3 + T_{n-1}$$
$$= \boxed{2T_{n-1} + 3}$$
$$= \cdots$$
$$= 3(2^n - 1)$$

Recurrent Problem

Lines in the plane problem :

(pizza cutting problem)

L_n = $\boxed{\text{maximum}}$ no. of regions of a circle defined by n straight lines

= maximum no. of slices of a pizza defined by

n straight cuts.

$L_0 = 1$

$L_n = L_{n-1} + n$ $\quad$ $n-1+1$

$L_4 = L_3 + 4$

$= L_{n-2} + (n-1) + n$

$= L_0 + 1 + 2 + 3 + \dots + n$

$= 1 + n(n+1)/2$

* justify করতে বললে ব্যাখ্যা করতে হবে which is given in the "pdf lec" as "yellow marked" lines !!

(Inductive Proof আমরাও পারব — xm এ)

Scanned Lecture - থেকে দেখব:

(v) n Zigs: $> \wedge <$
(2n সংখ্যক lines)

(N) ZigZag: Z N Σ (1টি ZigZag মানে 3টি straight lines)

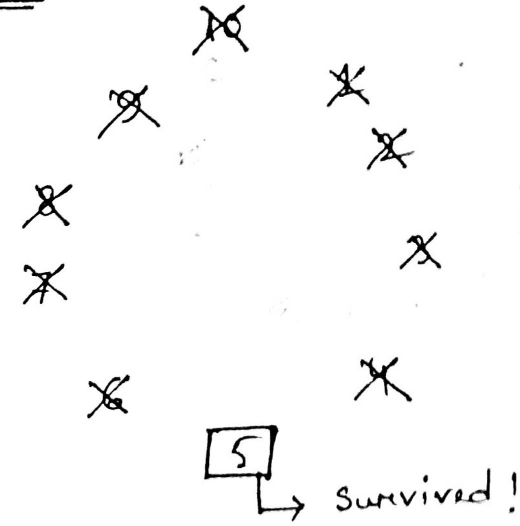
W Shapes

M Shapes

24 April 2017

[⌖] Josephus Problem :

(Recurrent Problem)



J(n) : Survivor's number
starting with n persons

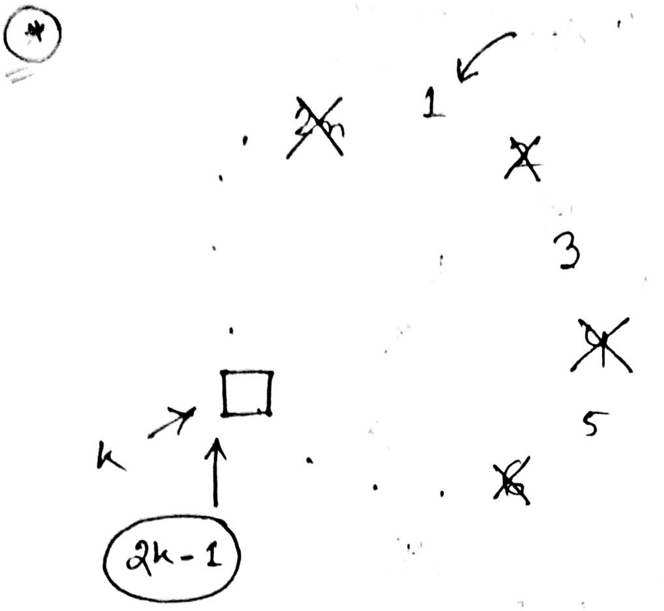
$J(10) = 5$

* xm এ logic চাইলে আলাদা করে দিতে হবে!

$J(1) = 1$

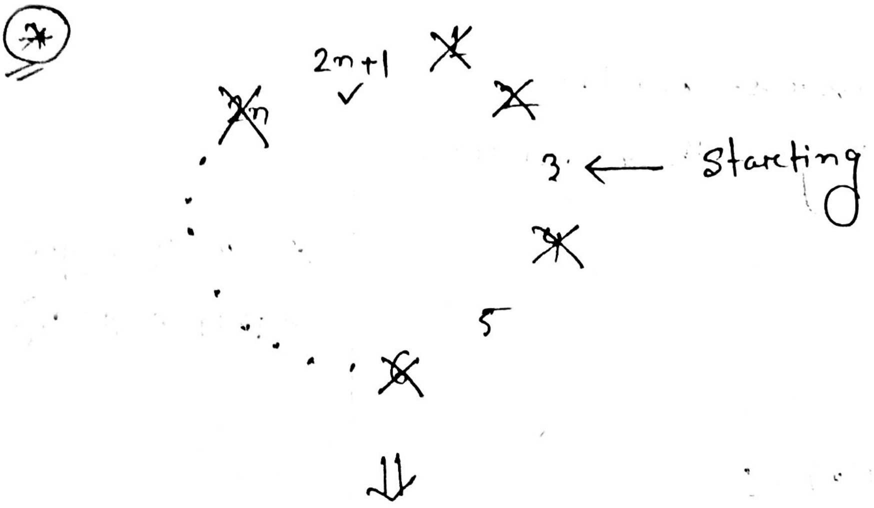
$J(2n) = 2J(n) - 1$

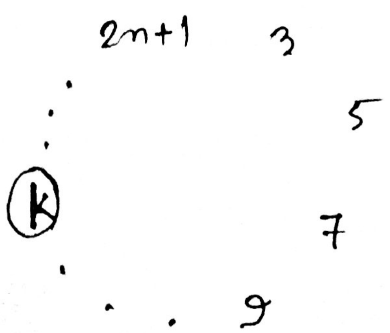
$J(2n+1) = 2J(n) + 1$



$J(1) = 1$

$J(2n) = 2k - 1$





[survivor cycle]

(7) no. of people 8 9 10

Survivor 1 3 5

$\downarrow$

$2l+1$

l = nearest power of 2

$J(n) = J(2^m + l)$

$= 2l+1$

$J(10) = J(2^3 + 2)$ [$l = 2$]

$= 2l+1$

$= 2(2)+1$

$= \boxed{5}$

$J(41) = J(2^5 + 9)$

$= 2(9)+1$

$= \boxed{19}$

$J(100) = J(2^6 + 36)$

$= 2(36)+1$

$= 73$

#: By Mathematical induction,

Prove that, $\boxed{J(n) = 2l + 1}$

where symbols have their usual meaning.

$\Rightarrow$ Follow Scanned Lecture!

* Josephus Problem এর জন্য scanned Lecture + pdf দুটোই দেখতে হবে [Very Important ***]

Q. Find the minimum three values of n such that person standing at the $n/3^{th}$ position survives.

* **Problems:** "Josephus-Add on.docx" file টা solve করব।

* "l" কে এক পার্শ্বে রাখব, বাকিগুলো অন্য পার্শ্বে where l must be integer.

* "m" always "2" এর power হিসেবে বসাবো!

Binary Property of the Josephus Problem :

((শুধু Derivation থাকতে পারে))

* Derive the Binary Property of the Josephus problem.

$\Rightarrow \quad n = 2^m + l$

$\downarrow \qquad \downarrow$

$n = 11$

$l = 3$

1011

0011

$\overset{11}{n} = 2^m + l$

$= 2^3 + \underline{\underline{3}}$

$3 \times 2 = 6$

011 $\longrightarrow$ 110

$\leftarrow$

* Left shift হয়ে শেষে হয়েছে 1^{st} bit টা !

* একে "Left Rotate" বলে !

* $J(100) = ?$

$J\left(\left(1100100\right)_2\right) = \left(1001001\right)$

$= 73$

$\underbrace{100 \quad 1001}_{73}$

* "Derivation + Binary Property" বের করা !

29 April 2017

Generalized Josephus Problem

$J(1) = \alpha$ [1 হলে]

$J(2n) = 2J(n) + \beta$ [জোড় হলে]

$J(2n+1) = 2J(n) + \gamma$ [বিজোড় হলে]

In the specific Josephus Problem

$\alpha = 1$

$\beta_0 = -1$

$\beta_1 = 1$

** Q. Solve the Generalized Josephus Recurrence/problem by Repertoire Method. (xm এ আসতে পারে)

→ Scanned Lecture এ detailed আছে।

Follow Lec 3 (Recurrence Problems)

$A(1) = 1$

$A(2n) = 2A(n)$

$A(2n+1) = 2A(n)$

$\boxed{n = 2^m + l} \Rightarrow \boxed{A(n) = 2^m}$

$B(n) = ?$

$C(n) = ?$

" যে Recurrence এ 2n এর অর্ধেকটি parameter হিসাবে ব্যবহৃত হয়, তাকে Josephus Recurrence বলে।"

$$\boxed{f(n) = 2^m \alpha + (2^m - l - 1)\beta + l\gamma}$$

* পরীক্ষায় যে কোনো Recurrence দিয়ে বলা থাকতে পারে —

" Solve the following Recurrence by Repertoire Method. "

$f(1) = 2$

$f(2n) = 2f(n) + 3 \rightarrow \beta$

$f(2n+1) = 2f(n) + 4 \rightarrow \gamma$

* α, β, γ বের করবো।
where $\gamma = -4$

* By using Repertoire Method বললে whole derivation করা লাগবে।

Q. Write the $\boxed{\text{Binary Property}}$ of the Generalized Josephus problem.

এই bit 1 হলে β_1 হবে

Ans $J((b_m b_{m-1} b_{m-2} \cdots b_1 b_0)_2)$

$= (\alpha \beta_{b_{m-1}} \beta_{b_{m-2}} \cdots \beta_{b_1} \beta_{b_0})_2$

$J(1) = \alpha$

$J(2n) = 2J(n) + \beta_0$

$= 2J(n) + \beta_1$

$\Rightarrow \alpha = 1$

$\beta_0 = -1, \ \beta_1 = 1$

$$n = (b_m\, b_{m-1} \ldots b_1\, b_0)_2$$

$$J(n) = ?$$

* $n = 100$

$$= (1100100)_2$$

$$\therefore J(n) = (\alpha\, \beta_1\, \beta_0\, \beta_0\, \beta_1\, \beta_0\, \beta_0)_2$$

$$= (1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; -1 \;\; -1)_2$$

$$= 1*2^6 + 1*2^5 - 1*2^4 - 1*2^3 + 1*2^2 - 1*2^1 - 1*2^0$$

$$= 73$$

30 April 2017

Q. What is the Radix based property of the Generalized Josephus problem?

$$f\left((b_m b_{m-1} \dots b_1 b_0)_d\right) = \left(\alpha_{b_m} \beta_{b_{m-1}} \beta_{b_{m-2}} \dots \beta_{b_1} \beta_{b_0}\right)_c$$

[The given recurrence is as follows:

$$f(j) = \alpha_j \qquad j = 1, \dots, (d-1)$$

$$f(dn+j) = c f(n) + \beta_j \qquad j = 0, 1, \dots, (d-1)$$

$* \; f(100) = ?$

$$\left.\begin{aligned} &= 10 f(33) - 2 \\ &= 10 \left[10 f(11) + 76\right] - 2 \end{aligned}\right\} \text{এইভাবে করবো না!}$$

$* \; f(19) = ?$

"Mathematical Analysis (Prepared by...).pdf" এর page - 26 follow করবো।

$(19)_{10} = (201)_3 \longrightarrow (\alpha_2 \beta_0 \beta_1)_{10}$

$= (5 \;\; 76 \;\; -2)_{10}$

$= 5 \times 10^2 + 76 \times 10^1 - 2 \times 10^0$

$= 1258$

$d = 3$

$c = 10$

$\alpha_2 = 5$

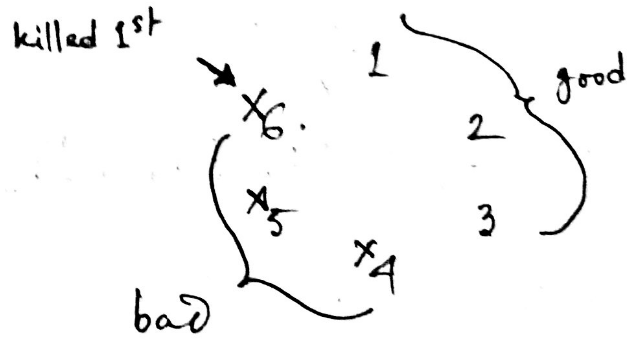
$\beta_0 = 76$

$\beta_1 = -2$

* Question ~ total recurrence টাই দেওয়া থাকবে।

Lec-3 (Recurrence Problem).pdf

21.



first $n/2$ good people

next $n/2$ bad people

[kill every m-th person]

Original Josephus ($m = 2$)

$m = ?$ $m = 60$

$4 * 5 * 6 = 120$

$\text{LCM}(4, 5, 6) = 60$

* LCM বা Multiple নিলে পাশাপাশি থাকা Bad person মারা পড়বে!

06 May 2017

Class Test - 1 [May. 27 (Saturday)]

* Chapter 1 (Concrete Math) → PDF lectures 1a, 1b, 2, 3
  Scanned Lectures (page 1 - 29)

Class Test - 2 [June 3 (Saturday)]

1 w → SUM

1 w → Probability Chapter 1

SUM

$$S_n = \underbrace{0+1+2+\cdots\cdots+(n-1)}_{S_{n-1}}+n$$

Recurrence:

$$\boxed{S_n = S_{n-1}+n} \Rightarrow S_n = S_{n-1} + \overset{\beta}{0} + \overset{\gamma}{1}\cdot n$$

$$\boxed{S_0 = 0}$$

114 Page

$\Rightarrow$ Concrete Math

General form:

$$\boxed{\begin{aligned} R_0 &= \alpha \\ R_n &= R_{n-1} + \beta + \gamma_n \end{aligned}}$$

Q. Solve by Repertoire Method –

$R_0 = \alpha$

$R_n = R_{n-1} + \beta + \gamma_n$ where, $\alpha = 0$, $\beta = 0$, $\gamma = 1$ $\rightarrow S_n$

$$R_n = A(n)\alpha + B(n)\beta + C(n)\gamma$$

↳ Follow Scanned Lecture

$\Rightarrow R_n = 1 \ldots \alpha, \beta, \gamma$ (3 eq) $\rightarrow A(n) = 1$

$R_n = n \ldots$ (3 eq) $\rightarrow B(n) = n$

$R_n = n^2 \ldots$ (3 eq) $\rightarrow C(n) = \frac{n(n+1)}{2}$

$$R_n = \alpha + n\beta + \frac{n(n+1)}{2}\gamma$$

* "SUM" Chapter এর 2nd, 3rd, 4th Page!

* সব এ sum থেকে প্রথমে Recurrence এ Convert করতে হবে or recurrence এর general form এ প্রকাশ করে নিয়ে then উল্লিখিত method দ্বারা solve করতে হতে পারে। (7 Marks এর Question থাকতে পারে সব এ)

* $$S_n = \sum_{k=0}^{n} (a+bk)$$

$$= \underbrace{a + (a+b) + (a+2b) + \ldots + (a+(n-1)b)}_{S_{n-1}} + (a+nb)$$

$$\boxed{S_n = S_{n-1} + a + nb}$$

$$\boxed{S_0 = a}$$

$$\boxed{R_n = \alpha + n\beta + \frac{n(n+1)}{2}\gamma}$$

$$\boxed{\begin{aligned} \alpha &= a \\ \beta &= a \\ \gamma &= b \end{aligned}}$$

$$\boxed{S_n = a(n+1) + \frac{n(n+1)}{2}b}$$

* Series নিয়ে Question হতে পারে।
* "Practice Question - 03 . docx" (page - 4)

* $3 + 10 + 17 + \dots + 703$

$$\sum_{k=0}^{n} a + bk = \sum_{k=0}^{100} 3 + 7k$$

$$S_n = S_{n-1} + a + bn$$

$$\boxed{S_n = a(n+1) + \frac{n(n+1)}{2} b}$$

$$\downarrow$$

$a = 3$

$b = 7$

$n = 100$

$S_{100} = ?$

$$\overset{a+nb}{\uparrow}$$

* $1+3+5+7+\dots+(201)$

$$S_{100} = \sum_{k=0}^{n} (2k+1) \longrightarrow \begin{matrix} a=1 \\ b=2 \end{matrix}$$

এই formula তে বসাবে!

$$S_n = 0+2+4+\dots+2n$$

$$= \sum_{k=0}^{n} 2k \longrightarrow \begin{matrix} a=0 \\ b=2 \end{matrix}$$

$$\boxed{S_{100} = a(n+1) + \frac{n(n+1)}{2} b}$$

$a = 3$

$b = 7$

$n = 100$

07 May 2017

Convert Recurrence into Sum

$T_0 = 0$

$T_n = 2T_{n-1} + 1$

$$\frac{T_n}{2^n} = \frac{T_{n-1}}{2^{n-1}} + \frac{1}{2^n}$$

$$\boxed{S_n = S_{n-1} + 2^{-n}}$$

2^{-n+1}

or, $\frac{1}{2^{n-1}}$

TOH এর Summation Factor

$$\boxed{S_n = S_{n-1} + 2^{-n}}$$

$$= S_{n-2} + 2^{-(n-1)} + 2^{-n}$$

$$= S_{n-3} + 2^{-(n-2)} + 2^{-(n-1)} + 2^{-n}$$

$$= \cancel{S_0}^{\,0} + 2^{-1} + \dots + 2^{-n}$$

$$= 2^{-1} + 2^{-2} + \dots + 2^{-n}$$

* "Scanned Lec" follow করুন।

$$= \sum_{k=1}^{n} 2^{-k}$$

$$\therefore \boxed{S_n = \sum_{k=1}^{n} 2^{-k}}$$

Q. Find the Summation factor for the following General Recurrence:-

$$a_n T_n = b_n T_{n-1} + c_n$$

$$S_n = \frac{a_{n-1} \dots a_2 a_1}{b_n \dots b_3 b_2}$$

Q. For the General recurrence, prove that -

$$T_n = \frac{1}{S_n a_n} \left(S_1 b_1 T_0 + \sum_{k=1}^{n} S_k c_k \right)$$

* xm ও formula এর Derivation আসবে না।

$$\boxed{\begin{aligned} T_0 &= 0 \\ T_n &= 2T_{n-1} + 1 \end{aligned}} \longrightarrow \begin{aligned} a_n &= 1 \\ b_n &= 2 \\ c_n &= 1 \end{aligned}$$

TOH Summation factor, $S_n = \dfrac{1 \dots 1.1}{2 \dots 2.2}$

$$= \frac{1}{2^{n-1}} = \boxed{2^{-n+1}}$$

Q. Write the Recurrence of TOH, find its Summation factor. Convert the recurrence into Sum using the Summation factor and find the value of T_n. Hence prove that, $T_n = 2^n - 1$.

** বিগত-4 Semester এর Question Solve করব।

Sol^n:

$$\sum_{1}^{n} 2^{-k+1}$$

$$= 2^0 + 2^{-1} + \dots + 2^{-n+1}$$

$$= 2^0 + 2^{-1} + \dots + 2^{-(n-1)}$$

$$= \boxed{\sum_{k=0}^{n-1} 2^{-k}}$$

08 May 2017

Quick Sort Recurrence :

$C_0 = 0$ recurrence

$$C_n = n+1 + \frac{2}{n}\sum_{k=0}^{n-1} C_k$$

$C_4 = C_0 C_1 C_2 C_3$

$C_5 = C_0 C_1 C_2 C_3 C_4$

* follow "mathematical analysis (prepared by waishy and Mishal) ~ 80 . pdf" (page-44)

$$* \quad T_n = \frac{1}{s_n a_n}\left[s_1 b_1 T_0 + \sum_{k=1}^{n} \overset{\frac{2}{k(k+1)}}{s_k} \, \overset{2k}{c_k} \right]$$

$a_n = n$

$b_n = n+1$

$c_n = 2n$

$$s_n = \frac{a_{n-1} \cdots a_2 a_1}{b_n \cdots b_3 b_2}$$

$$= \frac{(n-1) \cdots \cancel{3} \cdot 2 \cdot 1}{(n+1) \cdot n \cdot (n-1) \cdots 4 \cdot \underset{(2+1)}{\cancel{3}}}$$

$$\boxed{s_n = \frac{2}{n(n+1)}}$$

$S_k =$

$a_n T_n = b_n T_n + c_n$

$$\therefore C_n = 2(n+1) \sum_{k=1}^{n} \frac{1}{k+1}$$

Compare,

$nc_n = (n+1)\, c_{n-1} + 2n \;\big|\; a_n T_n = b_n T_{n-1} + c_n$

Q. Write the Quick Sort Recurrence, find its summation factor and prove that –

$$C_n = 2(n+1) \sum_{k=1}^{n} \frac{1}{k+1}$$

Harmonic Series এর সাথে মিল আছে।

$$C_n = 2(n+1) H_n - 2n$$

H_n = Harmonic Series upto n-th term

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$$

$$* \sum_{k=1}^{n} \frac{1}{k+1} \Rightarrow \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n+1}$$

$$= \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right) - 1 + \frac{1}{n+1}$$

$$= 2(n+1)\left[H_n - \frac{n+1-1}{n+1}\right]$$

$$= 2(n+1)\left[H_{n+1} - \frac{n}{n+1}\right]$$

$$= 2(n+1)H_n - 2n$$

* Universe এর সবচেয়ে রহস্যময় eqⁿ, " $e^{i\pi} + 1 = 0$ "

(জেনে রাখব, exam এ লাগবে না)

* Mathematical Analysis - 80.pdf

Index Replacement Technique :

$$S_n = \sum_{k=0}^{n} (a+bk)$$

$$S_n = \sum_{0\le k\le n} (a+bk)$$

After addition, [As index is same]

$$\sum_{0\le k\le n} (2a+bn) = (n+1)(2a+bn)$$

$$\therefore S_n = \frac{2a(n+1) + bn(n+1)}{2}$$

$$= a(n+1) + \frac{n(n+1)}{2} b$$

Q. $\sum_{0\le k\le n} k$, $\sum 2k$, $\sum 2k+1$ by index replacement

* Series নিয়ে Question হতে পারে।

(*) " $\sum_{k=0}^{n} (a+bk)$ " এ apply করবে।

13 May 2017

খুব সামান্য change
α

Perturbation Technique :

Q. Derive the formula for Perturbation Technique -

$$\boxed{S_n + a_{n+1} = a_0 + \sum_{k=0}^{n} a_{k+1}}$$

$a_0 \Rightarrow$ put $k=0$

$a_{k+1} \Rightarrow$ put $k=k+1$

$a_{n+1} \Rightarrow$ put $k=n+1$

what is a ?

a is

$$\boxed{\text{find } \sum_{k=0}^{n} a_k}$$

Let,

$$S_n = \sum_{k=0}^{n} a_k$$

$$\sum_{k=0}^{n} \underbrace{a x^k}_{a_k}$$

$$a_0 = a x^0 = a$$

$$a_{k+1} = a x^{k+1}$$

$$\sum \underbrace{5k a^2 x^{k+1}}_{a_k}$$

$$a_0 = 5.0.a^2 x^{0+1}$$
$$= 0$$

$$a_{n+1} = 5(n+1) a^2 x^{n+2}$$

$$a_{k+1} = 5(k+1) a^2 x^{k+2}$$

Q. Find a closed form for $\sum_{0 \le k \le n} k^2$

$\sum 2^{-k}$ বাদ from syllabus.

$$S_n = \sum_{0 \le k \le n} a x^k$$

$$\Rightarrow S_n + a x^{n+1} = a x^0 + \sum_{k=0}^{n} a x^{k+1}$$

$$= a x^0 + x \sum_{k=0}^{n} a x^k$$

$$\boxed{S_n + a x^{n+1} = a + x S_n}$$

* একধাপ বেশি করে শুরু করব, তাহলে answer পাবো।

14 May 2017

Sheldon sir → Book

Probability Theory

Sample space (S) = set of All possible outcomes

Event, (E) = subset of S = Set of relevant outcomes.

$$P(E) = \frac{|E|}{|S|} = \frac{\text{size of set } E}{\text{size of set } S}$$

die → singular

dice → plural

fair coin = নিরপেক্ষ

unfair coin = নিরপেক্ষ নয়

E OR F = $E \cup F$

E AND F = $E \cap F$ (common in both sets)

→ EF ← represented as

(P.T.O.)

(খ) Conditional Probability

$$P(E|F) \quad \text{given}$$

$= P(E$, given that F has already happened$)$.

$$\boxed{P(E|F) = \frac{P(EF)}{P(F)}}$$

[subset ছোট হলে, Probability বাড়ে]

"given that" থাকলে probability বাড়ে।

Book Example

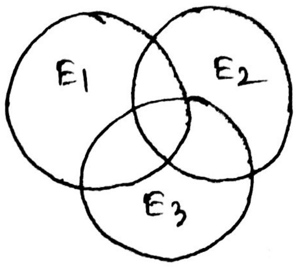
Example → (1.1 − 1.10)

(*) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$P(E|F) = \dfrac{P(EF)}{P(F)}$

$$\boxed{P(EF) = P(E) * P(F|E)}$$

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 E_2) - P(E_2 E_3) - P(E_3 E_1) + P(E_1 E_2 E_3)$$



$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{3} \times \frac{1}{2} - \frac{1}{3} \times \frac{1}{2} - \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1}$$

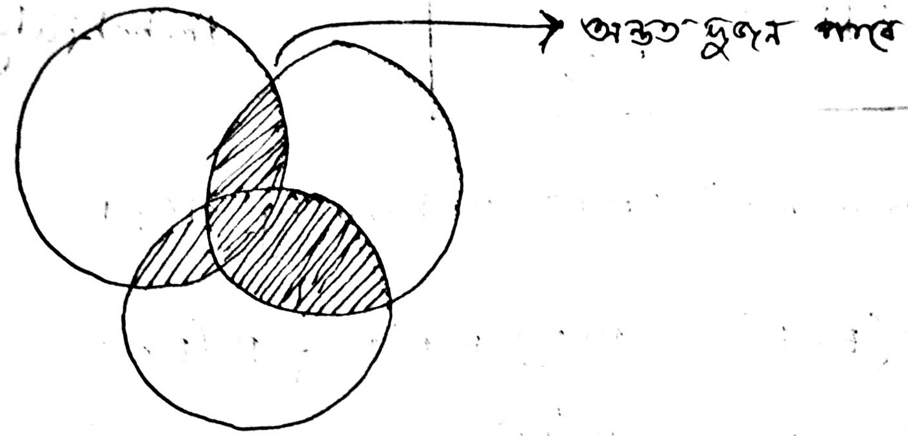
$$= 1 - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6} = 1 - \frac{1}{6} - \frac{1}{6} = 1 - \frac{1}{3} = \frac{2}{3}$$

$P(E_1 \cup E_2 \cup E_3)$ = Probability of getting hat atleast one person

$$P(E_1 \cup E_2 \cup E_3) = \frac{2}{3}$$

$\therefore$ Result is $= 1 - \frac{2}{3} = \frac{1}{3}$ (Ans).

(*)



তিনজনের মধ্যে দুজন hat পেয়েছে but 3^{rd} person পায়নি, তাহলে Probability $\boxed{\text{zero}}$.

15 May 2017

Bayes Theorem

$$E = EF \cup EF^c$$

$$P(E) = P(EF \cup EF^c)$$

$$= P(EF) + P(EF^c)$$

$P(EF^c) \rightarrow P(E|F^c) * P(F^c)$

$P(EF^c) \rightarrow P(F^c|E) * P(E)$

$P(EF) \rightarrow P(E|F) * P(F)$ $\quad$ $P(F|E) * P(E)$

* Bayes থেকে 1টি আসবেই xm এ।

Page – 122 (Scanned Lec)

125 page

Outcome of the toss is Head given. A white ball is selected?

$P(\omega|H) = \frac{2}{9}$

$P(w|H^c) = \frac{5}{11}$

$P(H) = \frac{1}{2}$

$P(H^c) = \frac{1}{2}$

$P(H|\omega) = ?$

2W 7B	5W 6B
↑ Head	↑ $Head^c$

$$P(H|\omega) = \frac{P(H\omega)}{P(\omega)}$$

$$= \frac{P(W|H) * P(H)}{P(W)}$$

$$= \frac{P(W|H) * P(H)}{P(W|H) * P(H) + P(W|H^c) * P(H^c)}$$

$$= \frac{22}{67} = \frac{\frac{2}{9} * \frac{1}{2}}{\frac{2}{9} * \frac{1}{2} + \frac{5}{11} * \frac{1}{2}}$$

$E = EF \cup EF^c$

$P(E) = P(EF) + P(EF^c)$

$P(E|F) * P(F)$

OR

$P(F|E) * P(E)$

1.13

$p(\text{Knows}) = p$

no. of MCQ options $= m$

$$p(\text{correct} \mid \text{knows}) = 1.0$$

$$p(\text{correct} \mid \text{knows}^c) = \frac{1}{m}$$

$$p(\text{knows} \mid \text{correct}) = ?$$

$$p(E \mid F) = \frac{p(EF)}{p(F)} = \frac{p(F \mid E) * p(E)}{p(F)}$$

$$p(\text{Knows} \mid \text{Correct}) = \frac{p(\text{knows } \& \text{ Correct})}{p(\text{correct})}$$

$$= \frac{p(\text{correct} \mid \text{knows}) * p(\text{knows})}{p(\text{Correct} \mid \text{Knows}) * p(\text{Knows}) + p(\text{Correct} \mid \text{Knows}^c) * p(\text{Knows}^c)}$$

$$= \frac{1.0 * p}{1.0 * p + \frac{1}{m} * (1-p)}$$

Laboratory Blood Test

E : test Result positive

D : Has disease

$P(E|D) = 0.95$

$P(E|D^c) = 0.01$

$P(D) = 0.005$

$P(D^c) = 1 - 0.005$

$= 0.995$

$P(D|E) = ?$

$$P(D|E) = \frac{P(DE)}{P(E)}$$

$$= \frac{P(E|D) * P(D)}{P(E|D) * P(D) + P(E|D^c) * P(D^c)}$$

* Scanned Lec এর exercise গুলো practise করব।

28 May 2017

Chapter - 3

n = integer

x: real $\left\langle \begin{array}{l} \text{integer} \\ \text{fraction} \end{array} \right.$

Integer functions

- floor $\lfloor x \rfloor = n$ → $\lfloor 2.6 \rfloor = 2$ $\quad \lfloor -1.2 \rfloor = \lceil -1.2 \rceil = (-$
  → $\lfloor 2 \rfloor = 2$ $\quad \lfloor 1.2 \rfloor = 1$
- ceiling $\lceil x \rceil = n$
- MOD $x \bmod y = x - y * \left\lfloor \frac{x}{y} \right\rfloor$

* ৭টি Rule আছে। পড়তে হবে।

$$\lfloor -x \rfloor = -\lceil x \rceil$$

$$\lceil -x \rceil = -\lfloor x \rfloor$$

* follow Scanned Lecture.

29 May 2017

$$m = \lfloor \lg \overset{n}{\overset{\uparrow}{100}} \rfloor + 1$$

$$\log_2 100 = \frac{\log 100}{\log 2}$$

$1000 = 2^3$

m - bits

$$\underbrace{100\ldots.0}_{m} = \boxed{2^{m-1}}$$

$$\underbrace{1111\ldots.1}_{m} = \boxed{2^{m-1}}$$

$$2^{m-1} \leq n \leq 2^m - 1$$

$$\boxed{2^{m-1} \leq n < 2^m}$$

$1111 = 2^4 - 1$

$111 = 2^3 - 1$

Prove or disprove that $\lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$

* Square Root ($\sqrt{\ }$) এমন একটি operation যার আগে Floor করার প্রয়োজন পড়ে না, পরে করলেই চলবে।

* যে কোনো Proof এ "Rule-1" লাগবেই।

* Side-note এ কেবল "Rule-1" লিখলেই চলবে না, বর্ণনা থাকতে হবে ঐ Rule এর minimum 1 বার।

"Scanned Lec" এর "Page-62" (follow).

$5 \bmod -3 = ?$

* $\boxed{x \bmod y = x - y * \left\lfloor \frac{x}{y} \right\rfloor}$ → Formula

$$\therefore 5 \bmod -3 = 5 - (-3) * \left\lfloor \frac{5}{-3} \right\rfloor$$
$$= 5 - (-3) * \left(- \left\lceil \frac{5}{3} \right\rceil\right)$$
$$= 5 - (-3)(-2)$$
$$= -1$$

→ * Property এর proof / Disproof আসতে পারে xm এ।

* Multiple number দিয়ে mod করা meaningless.

04 June 2017

Page 66 [Mathematical Analysis (Prepared by Washy & Mishal).pdf]

Example এ আছে

Number Theory

Divisibility $n \backslash m \Rightarrow n$ divides m

$m = nk$ for some integer $k \neq 0$

(Greatest Common Divisor)

* GCD ⇒ definition

LCM ⇒ definition

$\boxed{GCD(0, n) = n}$ ⇒ By definition

set নিয়েছে

* $\max \{ k \mid \underbrace{k \backslash m}$ and $\underbrace{k \backslash n} \}$

k, m এর গুণনীয়ক

k, n এর গুণনীয়ক

এরা both are connected with 'AND'

So, Common !!

Stein's Algorithm to find GCD

$$GCD(even, even) = 2 * GCD\left(\frac{even}{2}, \frac{even}{2}\right)$$

$$GCD(odd, even) = GCD\left(odd, \frac{even}{2}\right)$$

$$GCD(even, odd) = GCD\left(\frac{even}{2}, odd\right)$$

$$GCD(odd_{max}, odd_{min}) = GCD\left(\frac{odd_{max} - odd_{min}}{2}, odd_{min}\right)$$

Euclid's Algorithm:

$$GCD(u, v) = GCD(v \bmod u, u)$$

$$GCD(0, v) = v$$

15) 21 (1
15
6) 15 (2
12
3) 6 (2
6
0) 3 (

* '0' এবং অন্য যে কোনো number এর গ.সা.গু বের করা যায় না। তাই আমাদের step থেকে ঐ number টাই pick করা হয়।

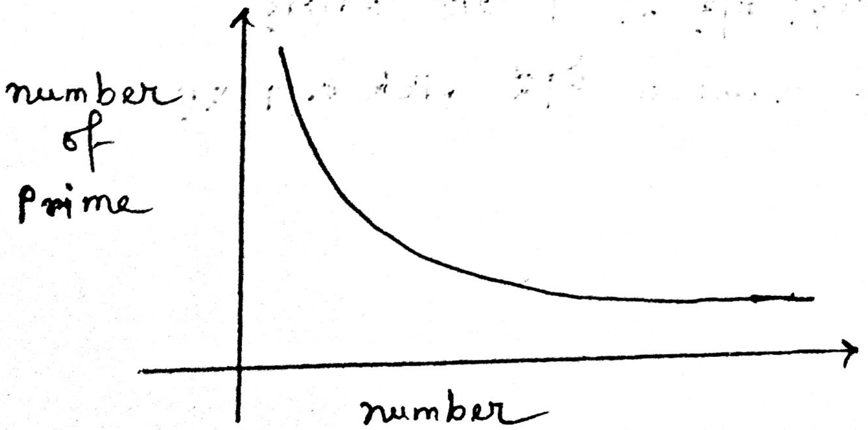
Q. Write an efficient algorithm to find GCD/LCM.

⇒ then you should follow "Stein's Algorithm to find GCD.

* LCM চাইলে Stein's Algorithm/ Euclid's Algorithm use করে GCD বের করে then,

$$LCM(m, n) = \frac{m * n}{GCD(m, n)}$$

* Prime Number, Composite Number ⇒ Definition
* প্রতিটি Composite number এর min একটি prime number থাকবে।
* দুটি Number এর গুণফল GCD ও LCM এর গুণফল এর সমান।
* Algorithm লিখতে বললে GCD এরটা লিখব।
* Any positive integer > 1 is either prime or product of primes.



Twin Prime

3 & 5, 5 & 7, 11 & 13.

Cousin Prime

3 & 7, 7 & 11, 19 & 23, 13 & 17

05 June 2017

* "Number Theory" এর জন্য Scanned Lecture + pdf both পড়তে হবে।

Sieve of Eratosthenes to find all prime number up to n.

* Algorithm & Diagram দেখাতে হবে।

Fibonacci Number:

Find the last digit.

After 60th fibonacci number the last digit of 61th number should repeat.

Q. What is the last digit of $f(1510)$?

$\Rightarrow$ Last digit of $f(1510)$

= Last digit of $f(1510 \% 60)$

= Last digit of $f(10)$ → 11^{th} fibonacci number

0	1	1	2	3	5	8	13	21	34	55
F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}

5 = Last digit

∴ Last digit of $f(1510)$ is = 5

(Ans)

Properties of Fibonacci Number

⇒ Fundamental theory of Arithmetic.

* Lecture-07.pdf → Every positive integer can be written uniquely. $\boxed{\text{Proof by Contradiction}}$

$n = P_1 P_2 \ldots\ldots P_m$ where, $P_1 \le P_2 \le \ldots \le P_m$

→ multiplication of prime numbers.

→ should be ordered in sorted.

→ GCD of two prime numbers is 1

→ We can write in linear combination,

$m, n, GCD(m, n)$

↓ Theory

always there exist a, b (integers) such that

$$\boxed{am + bn = GCD(m, n)}$$

$$\boxed{3} \cdot 2 + (-1) \cdot 4 = 2$$

$ap_1 + bq_1 = 1 = GCD(p_1, q_1)$ because p_1 & q_1 are prime numbers

Prime Exponent Representation of an Integer

$12 = \{2, 1, 0, 0, 0, \ldots\} \Rightarrow 12 = 2^2 \times 3^1 \times 5^0 \times 7^0 \times \ldots$

$18 = \{1, 2, 0, 0, 0, \ldots\} \Rightarrow 18 = 2^1 \times 3^2 \times 5^0 \times 7^0 \times \ldots$

* power গুলোকে একত্রে সেট এর মধ্যে লিখা যায়।

$\rightarrow GCD(12, 18) = 2^1 \times 3^1 \times 5^0 \times 7^0 \times \ldots = 6$

* গ.সা.গু. এর ক্ষেত্রে Power গুলোর মধ্যে Minimum নিবো।

$\rightarrow LCM(12, 18) = 2^2 \times 3^2 \times 5^0 \times 7^0 \times \ldots = 36$

(Max^m টা নিবো)

$\rightarrow 12 \times 18 = 2^3 \times 3^3 \times 5^0 \times 7^0 \times \ldots = 8 \times 27 = 216$

(P.T.O.)

** ☐ Euclid Number & Mersenne Number property
[দেখে নিতে হবে]

* Euclid Number always prime হবে না।

* Mersenne Number $= 2^{p}-1$ | Here, p = prime number $= 3, 5, 7$

10 June 2017

See Lecture after 15 July. 2017 !

12 June 2017

Factorial Factors (PDF Lecture 8)

$\varepsilon_p(n!)$ = The maximum power of prime p that divides $n!$

$$\varepsilon_2(10!) = \boxed{8}$$

$$\varepsilon_2(100!) = 97$$

$$\varepsilon_2(10!) = \left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{2^2} \right\rfloor + \left\lfloor \frac{10}{2^3} \right\rfloor + \left\lfloor \frac{10}{2^4} \right\rfloor + \left\lfloor \frac{10}{2^5} \right\rfloor + \dots$$

$$= 5 + 2 + 1 + 0 + 0 + \dots$$

$$= \boxed{8}$$

extra class

$$\mathcal{E}_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \cdots$$

* Prove that $\mathcal{E}_p(n!) \leq \frac{n}{p-1}$

Binary Property of Factorial factors

$$\boxed{\mathcal{E}_2(n!) = n - \nu_2(n)}$$

no. of 1's in n (binary)

→ Number Theory ⇒ PDF Lecture

* Find $\mathcal{E}_3(100!)$ ⇒ Direct formula Apply
* Prove that - $\mathcal{E}_2(10!) = 8$ } ভাগ করে করতে হবে
* Find $\mathcal{E}_2(100!)$ by using "Binary Property of Factorial factors".

* 100 এর Binary 1100100
  ↳ 1 আছে ৩টি

$$\mathcal{E}_2(100!) = 1000 - \underset{}{3} \longrightarrow V_2(n)$$

$$= 97$$

Prove,

$$\mathcal{E}_p(n!) \le \frac{n}{p} + \frac{n}{p^2} + \frac{n}{p^3} + \cdots$$

$$= \frac{n}{p}\left(1 + \frac{1}{p} + \frac{1}{p^2} + \cdots\right)$$

* Most Important ⇒ Number Theory ("Lecture-9" [pdf])

Q. What is <u>Stern–Brocot Tree?</u>

⇒ Stern–Brocot Tree is a data structure to produce all fractions $\frac{m}{n}$ such that m and n are $\underbrace{\text{relative prime}}$ (i.e. $m \perp n$)

↳ example $\frac{2}{3}$ $\frac{3}{2}$ $\frac{3}{8}$

$\frac{5}{3}$ $\frac{3}{5}$ $\frac{7}{8}$ $\frac{7}{9}$

not $\frac{6}{4}$ ✗ $\frac{3}{2}$ ✓

$\Rightarrow$ denoted by $m \perp n$

$\gcd(m, n) = 1$

The fraction $\frac{m}{n}$ or, $\frac{n}{m}$ are irreducible.

Left   Right

$$\frac{m}{n} \qquad\qquad \frac{m'}{n'}$$

$$\frac{m+m'}{n+n'}$$

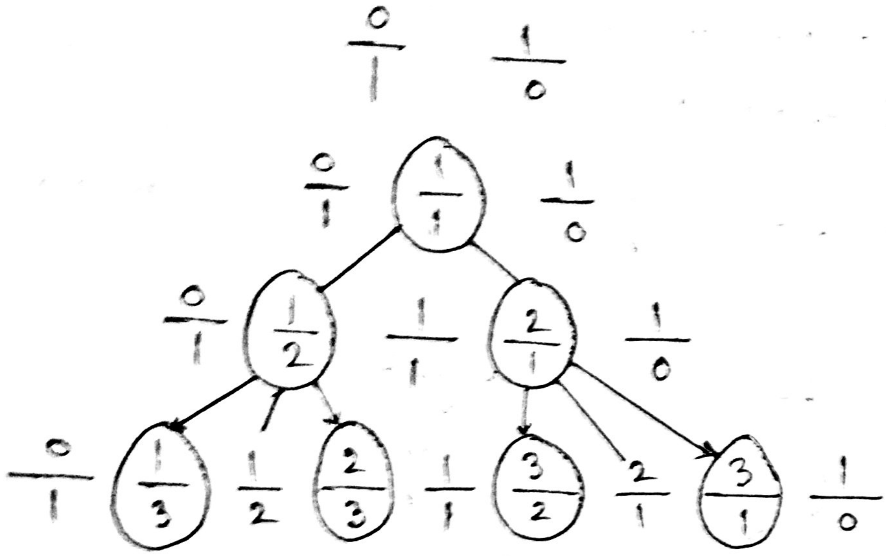
$$\frac{0}{1} \qquad\qquad \frac{1}{0}$$

$$\frac{0+1}{1+0} = \boxed{\frac{1}{1}}$$

$$\frac{0+1}{1+1} = \frac{1}{2} \qquad\qquad \frac{1+1}{1+0} = \boxed{\frac{2}{1}}$$

$$\frac{0+1}{1+2} = \frac{1}{3} \qquad \frac{1+1}{2+1} = \frac{2}{3}$$

$$\frac{1+2}{1+1} = \frac{3}{2} \qquad \frac{2+1}{1+0} = \frac{3}{1}$$



<u>08 July 2017</u>

<u>Quiz-3 [Next Saturday]</u>

→ Integer Function (scanned lec)

→ Number Theory (Scanned + pdf w imp)

→ Binomial Co-efficient (Scanned + Practice Ques.)

* $a \perp b \Rightarrow a, b$ relative prime.

* $f(LRRL) = ?$ $\left(\frac{5}{7}\right)$

* $f(LRRLLRRL) = ?$

কোনো Matrix থেকে fraction পাওয়ার way ⇒

$$\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \Rightarrow \frac{\text{row 2 sum}}{\text{row 1 sum}} = \frac{5}{7} \longrightarrow \text{fractional Answer}$$

Q. In which level of the SB tree the fraction $\frac{22}{15}$ exist?

Q. Find the LR sequence for the fraction $\frac{22}{15}$ in SB tree?

LRRL $\longrightarrow$ $4+1 = 5^{th}$ level

09 July 2017

Chapter - 5

Binomial Co-efficient [1টি Proof আসবেই]

$$^{n}C_{k} = \binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{1.2.\dots.k} = \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k)!}$$

[n choose k]

Upper Index ↑, lower Index ↓

[r choose k]

$$\binom{r}{k} = \begin{cases} \text{formula} & r<0 \\ \text{formula} & \\ 0 & k<0 \\ 0 & k>r \\ 1 & k=0 \end{cases}$$

[r can be negative]

$\binom{-7}{3}$ ⇒ সেক্ষেত্রে $k>r$ হবে না, formula বসবে না!

$$= \frac{(-7)(-8)(-9)}{1.2.3}$$

* Write & Prove

Symmetry Identity $\rightarrow \binom{r}{k} = \binom{r}{r-k}$ proof

ex. $\binom{10}{7}\binom{10}{3}$

Absorption Identity $\rightarrow \binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}$

Addition Rule

Others

Symmetry identity upper index negative value এর জন্য কাজ করে না। (Proof একটা দিয়ে দেখলেই হবে)

* Prove or disprove that Symmetry identity holds wh

i) $r<0$ [upper index negative] $r<0$ and $k=0$ [upper index negative, lower index 0] $\Rightarrow$ (Scanned 1st Part)

ii) $r<0$ and $k<0$ [upper index negative, lower index negative] (Scanned 3rd Part)

iii) $r<0$ and $k>0$ [upper index negative, lower index positive] (Scanned 2nd Part)

Absorption Identity:

$$\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1} \quad \Big| \quad ex// \binom{5}{3} = \frac{5}{3}\binom{4}{2}$$

* Prove that, $(r-k)\binom{r}{k} = r\binom{r-1}{k}$ |

Addition Formula:

$$\binom{r}{k} = \binom{r-1}{k} + \binom{r-1}{k-1} \quad \Big| \quad ex// \binom{7}{3} = \binom{6}{3} + \binom{6}{2}$$

* Prove করতে বললে prove করে আনতে হবে!

10 July 2017

$$\binom{5}{2}+\binom{6}{2}+\binom{7}{2}+\dots$$

$$\sum_{k=0}^{n}\binom{k}{m}=\binom{n+1}{m+1}\qquad \boxed{m \text{ fixed}}$$

Summation formula

$$\binom{5}{2}+\binom{6}{3}+\binom{7}{4}+\dots$$

$$\sum_{k=0}^{n}\binom{n+k}{k}=\binom{x+n+1}{n}$$

$$\sum_{k=0}^{m}\frac{\binom{m}{k}}{\binom{n}{k}}=\frac{n+1}{n+1-m}\qquad (m<n)$$

Practise Question

Q. Compute the value of the following series of binomial ...

$$* \quad \sum_{k=1}^{20}\binom{12}{k}\Big/\binom{15}{k}=\sum_{k=1}^{12}\binom{12}{k}\Big/\binom{15}{k}+\underbrace{\sum_{k=13}^{20}\binom{12}{k}\Big/\binom{15}{k}}_{=0}$$

$$= \sum_{k=0}^{12} \binom{\overset{m}{12}}{k} \Big/ \binom{\overset{n}{15}}{k} - \frac{\binom{12}{0}}{\binom{15}{0}}$$

$$= \frac{15+1}{15+1-12} - \frac{1}{1} = \frac{16}{4} - 1 = \boxed{3}$$

(*) $\binom{20}{0} + \binom{\overset{20+1}{21}}{1} + \binom{\overset{20+2}{22}}{2} + \dots + \binom{\overset{20+40}{60}}{40}$ $\quad \Big| \; r = 20, \; n = 40$

$$= \binom{r+n+1}{n} = \binom{20+40+1}{40} = \binom{61}{40}$$

(*) $\binom{23}{3} + \binom{24}{4} + \binom{25}{5} + \dots + \binom{60}{40}$

$$= \binom{20}{0} + \binom{21}{1} + \binom{22}{2} + \binom{23}{3} + \dots + \binom{60}{40} - \dots$$

$$= \text{formula} - 1 - 21 - \frac{22 \times 21}{1 \times 2}$$

(*) $\binom{0}{5} + \binom{1}{5} + \binom{2}{5} + \dots + \binom{60}{5} \leftarrow$ fixed

$n = 60, \; m = 5$

→ missing থাকলে যোগ করে বিয়োগ করে দিব।

$$= \binom{61}{6}$$

(*) $\binom{11}{5} + \binom{12}{5} + \binom{13}{5} + \dots + \binom{60}{5}$

$$= \binom{0}{5} + \binom{1}{5} + \dots + \binom{60}{5} - \left[\binom{0}{5} + \binom{1}{5} + \dots + \binom{10}{5}\right]$$

* Quiz - 3 [Saturday এর পরিবর্তে Monday (17-07-17) হবে]

15 July 2017

Random Variable → Discrete R.V. (dice, coin)
Random Variable → Continuous R.V. (infinite domain)

crete R.V. ← Probability Mass Function (PMF) → $p(\,)$

Probability

$$p(a) = P\{X = a\}$$

$$p(1) = \frac{1}{6} \qquad p(2) = \frac{1}{6}$$

dice throw

ontinuous R.V. ← Probability Density Function (PDF) → $f(\,)$

$$\boxed{P\{a \le x \le b\} = \int_a^b f(x)\,dx}$$

$$\boxed{\int_{-\infty}^{+\infty} f(x)\,dx = 1}$$

ogle

Cumulative Distribution Function (CDF) → $F(\,)$

$$F(a) = P\{X \le a\} = P\{-\infty \le X \le a\}$$

$$= \int_{-\infty}^{a} f(x)\,dx$$

$F(+\infty) = 1$ | $F(-\infty) = 0$

II) Binomial R.V.

$$PMF = p(i) = \binom{n}{i} p^i (1-p)^{n-i}$$ [Probability to get i successes from n trials]

$$p(i) = p\{x = i\} =$$

eg. i heads from n throws of coin

at least one $x \geq 1$

at most one $x \leq 1 = x = 0 + x = 1$

No less than one $x \geq 1$

No more than one $x \leq 1$

$$p\{x=0\} + p\{x=1\}$$

$$\binom{4}{0}(0.1)^0(0.9)^{4-0} +$$

$$\binom{4}{1}(0.1)^1(0.9)^{4-1}$$

$p\{$ at most one defective $\}$

(Revisited)

10 June 2017

Review on Integer Functions :

→ Ceiling

→ Floor

→ Mod

* Proofs : See Carefully!

Number Theory

* Must see both Scanned & PDF lectures (7, 8, 9)

* PDF Lectures → 7, 8, 9 **

Stern Brocot Tree

Quiz - 3 [After Mid Vacation] (Sat)

* Integer Function
* Number Theory
* Binomial Co-efficients

16 July 2017

Important => Stern Brockot tree

* GCD ~~and~~ Efficient Algorithm

Q. What do you understand by Binomial / Bernoulli / Geometric / Poisson RV ?

represents no. of trials to get the first success. $p(n)$

represents no. of success in 1 random trial / experiment $p(0), p(1)$

represents no. of successes in n random trials $p(i)$

* failure = $(1-p)$

* poisson R.V. x — discrete R.V.

given => parameter λ

$E[x] = \text{mean}(x) = \lambda$

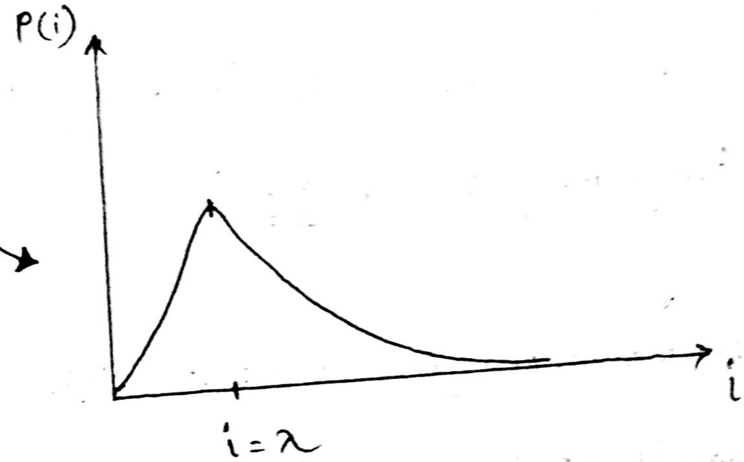
PMF => $P(i) = P\{x = i\} = e^{-\lambda} \frac{\lambda^i}{i!}$

Exponential R.V. — Continuous R.V. Y parameter λ

$$\text{mean}(Y) = E[Y] = \boxed{\frac{1}{\lambda}}$$

PMF =

$$\boxed{e^{-\lambda} \frac{\lambda^i}{i!}}$$



x : poisson distributed R.V.

$$\ast \quad P\{x = i\} = e^{-\lambda} \frac{\lambda^i}{i!}$$

$$P\{x = 0\} = e^{-\lambda} \frac{\lambda^0}{0!}$$

$$\checkmark \; P\{x = 1\} = e^{-\lambda} \frac{\lambda^1}{1!}$$

$$\checkmark \; P\{x = 2\} = e^{-\lambda} \frac{\lambda^2}{2!}$$

$\vdots$

$$P\{x \geq 1\} = 1 - P\{x = 0\} = 1 - e^{-\lambda} \frac{\lambda^0}{0!}$$

$$= 1 - e^{-\lambda}$$

$$= 1 - e^{-1}$$

$$= 0.633$$

* at least 2 accident $\longrightarrow$ $x \geq 2 \rightarrow 1 - P\{x=0\} - P\{x$

at most 2 accident $\longrightarrow$ $x = 0, 1, 2$

No less than 2 accident

No more than 2 accident $\rightarrow x \leq 2 \rightarrow P\{x=0\} + P\{x=1\} + P\{x=2\}$

* Example - 7 বাদ !

(Lec - 14)

Continuous R.V. $\rightarrow$ Uniform R.V.

Continuous R.V. $\rightarrow$ Exponential R.V.

Probability Density function দেখব!

(Example দেখব Uniform এর)

22 July 2017

(Imp) [Scanned Lec লাগবে না]
** Lec 16, 17 → Markov Chain

Lec 13, 14, 15 ⇒ R.V.

Example-1, 2

* Previous Multiple state এর উপর Next state depend করলে Markov Chain apply করা যাবে না!

⊞ Transition Probability Matrix (Row sum = 1)

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* Example - 1
* Example - 2

$p \Rightarrow$ Probability of Correctly Data Transmission

* Example - 3

Glum না থাকলে,

$1 \rightarrow$ (Cheerful + so-so)

* খেয়াল করবে whathere Row's or Column's value is given!

Also এই Problem → Column wise

Q. What are the applications of Markov Chain?

(Random walk Model, Gambling Model)

Chapman – Kolmogorov Equation :

Intermediate probability গুণ করতে হবে!

Example – 4 :

$(P^4)_{00}$

Sat Rain   Wed Rain

P ↪ Sat থেকে Sun

P^2 ↪ Sat থেকে Mon

* আজকের State গত 2 দিনের উপর dependent হলে, গত 2 দিনেরটা মিলে State বানাবো!

→ $(P^2)_{00}$

$(P^2)_{01}$

23 July 2017

Lec 17 : Gambler's Ruin Problem

i : starting no. of coins

N : Total No. of coins (player 1 + player 2)

$\boxed{P_i :}$ Probability of ultimate win (obtaining all N coins)

starting from i coins

$P_{i,i+1} = p$ (Probability of win in next gamble)

$P_{i,i-1} = 1-p = \boxed{q}$ (Probability of Lose in next gamble)

$P_{0,0} = 1$

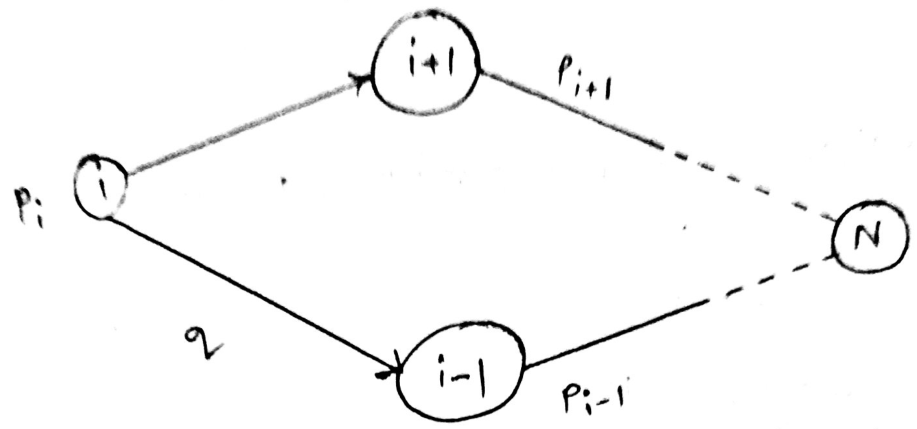
$P_{N,N} = 1$

P_{19}

P_1

$P_{5,4}$

$P_{5,6} = p$



$$P_i = \begin{cases} \dfrac{1-\left(\frac{q}{p}\right)^i}{1-\left(\frac{q}{p}\right)^N} & ; \text{ if } q \neq p \\ \dfrac{i}{N} & ; \text{ if } p = q \end{cases}$$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

$$E[x] = \sum_{\substack{\text{for all} \\ x}} x \cdot p(x)$$

x : Discrete RV

$$E[x] = \int_{\substack{\text{for all} \\ x}} x\, f(x)\, dx$$

$$E[x] = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \boxed{3.5}$$

24 July 2017

Expectation: (Average Random Variable)

Example - 11

$X = \text{die}1 + \text{die}2$

$X: 2, 3, \ldots\ldots, 12$

$\frac{1}{36}, \frac{2}{36}, \ldots\ldots \frac{1}{36}$

$E[x] = 2 * \frac{1}{36} + 3 * \frac{2}{36} + \ldots\ldots$

Example - 2.1 (PDF Book Ross)

Find $E[x]$ =
mean (x) =

- ~~X~~ Bernoulli R.V.
- $\boxed{\text{Binomial R.V.}}$
- $\boxed{\text{Geometric R.V.}}$
- Poisson R.V.
- Uniform R.V.
- ** $\boxed{\text{Exponential R.V.}}$

90% probability

(6টির যেকোনোটাই xm এ আসবে!)

Example - 12, 13 ***

Example - 14 (Three Star)

Example - 15, 16

* $$E[x] = \sum_{i=0}^{n} i * p(i)$$

$$= \sum_{i=0}^{n} i * nC_i \, p^i (1-p)^{n-i}$$

$$\vdots$$

$$= \boxed{np}$$

$$\sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-k}$$

* Poisson Random Variable,

$$p(i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

$$E[x] = \sum_{i=0}^{\infty} i * p(i)$$

Continuous R.V. → uniform, exponential

$$\int_{-\infty}^{\infty} x \cdot f(x) \, dx$$

$$= \int_{\infty}^{\beta} x \cdot \frac{1}{\beta - \alpha} dx = \frac{\left[\frac{x^2}{2} \right]_{\alpha}^{\beta}}{\beta - \alpha}$$

***** Example - 17 (v.v. Imp)

Uniform R.V. $\Rightarrow f(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha \le x \le \beta \\ 0, & \text{otherwise} \end{cases}$

Quiz-3 [Last week এর saturday]

(*) Lec-15: Expectation R.V

Lec-16 : } Markov
Lec-17 : }

[PDF + PDF Book]

[pdf Book থেকে Math এর Comment দেখব।]

29 July, 2017

Lec-18

* V.V. Imp (1টি না 1টিই আসবেই)

Arrival Rate:

Arrival rate : λ → Poisson distributed
Service rate : μ → Exponentially distributed

} Probabilistic

$M/M/1$ Queue $\xrightarrow{}$ Memory less (Service time exponen... distributed, exponential distribution is Memoryless)

$\downarrow$

Markovian

$M/M/1$

$M/G/1$ $\longrightarrow$ Service Time $\Rightarrow$ any general distribution

$M/M/K$ - - - > K Servers

$M/G/K$ - - -

For single server exponential,

Queueing system

find P_n

L

L_Q = Avg length of Queue

W

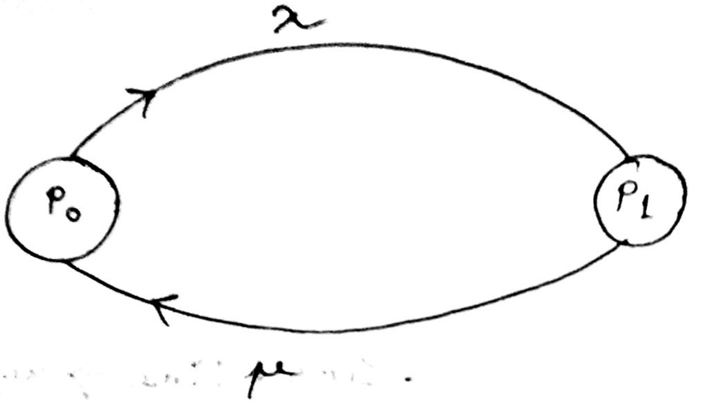
W_Q = Avg waiting time in Queue

$W = W_Q +$ Service time

$= W_Q + \frac{1}{\mu}$

Queue State Transition Diagram.

Service rate — departure rate



outgoing edge দ্বারা গুণ হয়।

$\lambda \times P_0 = \mu \times P_1$

$$\lambda P_{n-1} + \mu P_{n+1} = \mu P_n + \lambda P_n$$

$$\Rightarrow P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$P\{\text{system is idle}\}$

$= P_0$

$n = 0$
$\lambda \Rightarrow$ 'Value দেওয়া থাকবে'
$\mu \Rightarrow$ 'Value দেওয়া থাকবে'

L = average number of customers

$\longrightarrow$ number of customer

$E[n] = \sum n P_n$

$0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 + \dots$

$$\boxed{\omega * \lambda = L}$$

on average

ω → 10 min waiting time

λ → 5/min

$$L = \frac{\lambda}{\mu - \lambda}$$

30 July 2017

Lecture – 19 (বাদ)

Lecture – 20

Shoeshine shop Model

→ No queue

2 Chair

((কোনো waiting থাকবে না))

Arrival rate, λ

Service rate (chair 1) μ_1

Service rate (chair 2) μ_2

State Transition Diagram for the Shoeshine shop,

$$\mu_2 P_{01} = \lambda P_{00}$$

$$\lambda P_{00} + \mu_2 P_{11} = \mu_1 P_{10}$$

* Examples (দেখব !

* Probability that the customer will not enter the system?

⇒ Expected value of customer → E [no. of customers in the system]

→ $\Sigma X . P(x)$

$$0 . P_{00} + 1 . (P_{01} + P_{10}) + 2 (P_{11} + P_{\cdot 1})$$

Q. What is the average waiting time?

$$\omega = \frac{L}{\lambda a} = \frac{L}{\lambda (P_{00} + P_{01})}$$

λ_a = effective arrival (Who will enter into the system

Lecture - 14

Exponential Random Variable এর property :-

1) Memoryless Property of Exponential R.V.

$$P\{X > s + t \mid X > t\} = P\{X > s\}$$

* Proof ⇒ Less Imp. (আসতে পারে)

* For Exponential R.V., prove that,

$$P\{X > x\} = e^{-\lambda x}$$

$$P\{X > x\} = 1 - P\{X \leq x\} \quad \text{(Proof (পড়বে))}$$

$$= 1 - F(x)$$

$$= 1 - \int_{-\infty}^{x} f(x)\, dx \qquad f(x) \to \lambda e^{-\lambda x}$$

* Example - 9, 10 দেখবে ।

$$\lambda = \frac{1}{\text{Mean}} = \frac{1}{10}$$

* Example - 9 Complete করবে ।

31 July 2017

(Question Pattern)

Set - A (Concrete Math)

Answer 3 out of 4 Questions !

Chapter - 1 : Recurrent Problems → 1 Q

Chapter - 2 : Sum → $\boxed{0.8 - 1 \text{ Q}}$

Number Theory → 1 Q

Integer Function + Binomial Co-efficient → $\boxed{1 - 1.2 \text{ Q}}$

(P.T.O.)

Set-B (Probability)

Answer 2 out of 3 Questions !

- Chapter 1 + Random Variable $\longrightarrow$ 1Q

  $\approx 60\% / 50\%$ $\approx 40\% / 50\%$

- Markov Chain
  $\Downarrow$
  (PDF Lec 16 + 17) + R.V. $\longrightarrow$ 1Q

  $\approx 80\%$ $\approx 20\%$

- Queuing Theory + R.V. $\longrightarrow$ 1Q

  (PDF Lec 18, 20) $\approx 20\%$

  $\approx 80\%$

** Last 3 year এর Question solve করব।
(Last 5 Semester)

* Bayes এর Math exercise থেকে অবশ্যই আসবে/ Quiz এর 2টা Math !