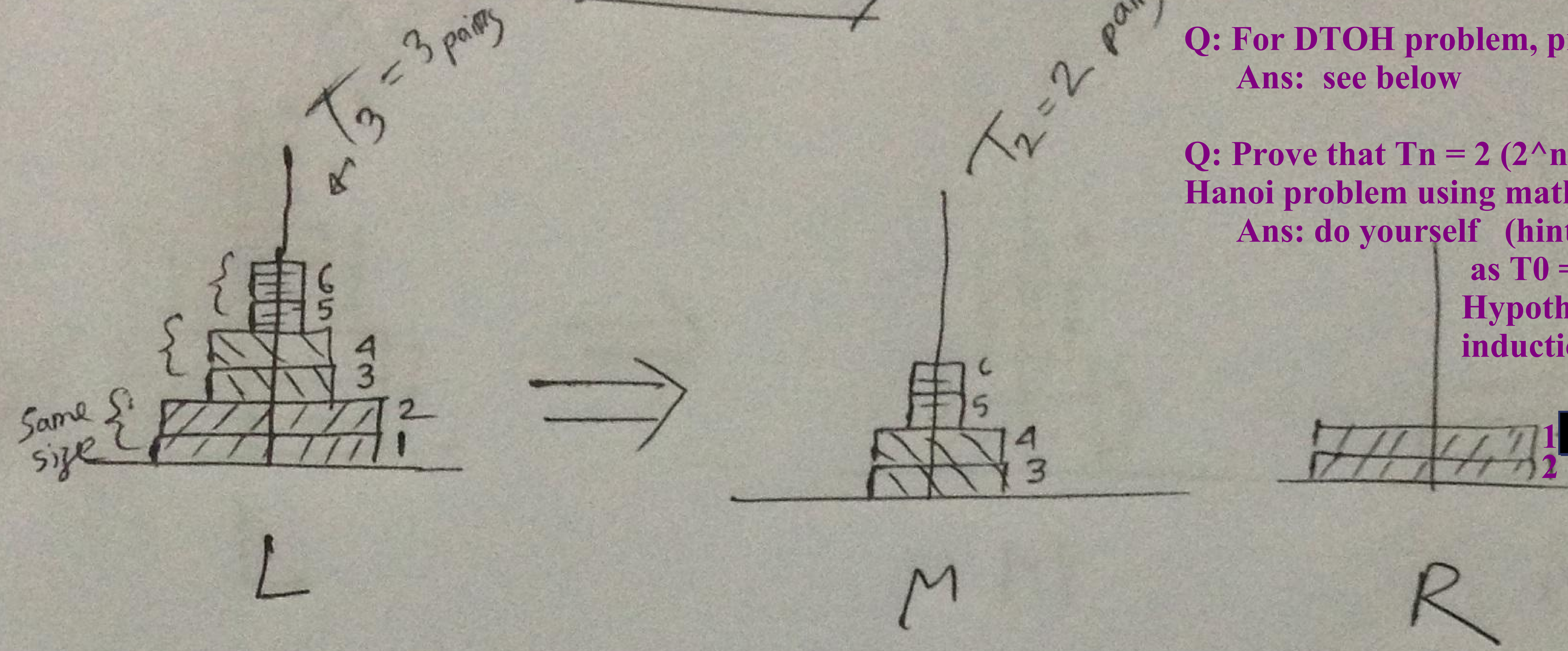
Double Tower of Hanoi

in DTOH, the Tn = minimum no. of moves to solve the DTOH problem with n pairs of disks (2n disks). Each two consecutive disks have equal size, so either one can be placed above the other one of its equal size

Q: For DTOH problem, prove that Tn = 2 (2^n - 1)
Ans: see below

Q: Prove that Tn = 2 (2^n - 1) for the Double Towr of Hanoi problem using mathematical induction on n.
Ans: do yourself (hint: basis: T0 = 0 obvious,
as T0 = 2 (2^0 - 1) = 2 (1-1) = 0
Hypothesis: T_{n-1} = 2*(2^n-1 - 1)
induction: Tn = $2T_{n-1}$ + 2
= 2 [2*(2^n-1 - 1)] + 2
= 2 [2^n - 2] + 2
= 2 [2^n - 1]



Move Top n-1 pairs from L to M	*Move Bottom Two disks (Largest Two) from L to R (destination)*	*Now, Again Move the n-1 pairs from M to R (destination)*

$$T_n = T_{n-1} + 2 + T_{n-1} = \boxed{2 \cdot T_{n-1} + 2}$$

$$T_n = 2(T_{n-1}) + 2 \quad = 2\,(2T_{n-2} + 2) + 2$$

$$= 2^2 T_{n-2} + 2^2 + 2$$

$$= 2^3 T_{n-3} + 2^3 + 2^2 + 2$$

$$= \dots\dots$$

$$= 2^n T_{n-n} + 2^n + 2^{n-1} + \dots + 2^1$$

$$= 2^n \cdot 0 + [2 + 2^2 + \dots + 2^n]$$

$$= 2[2^0 + 2^1 + \dots + 2^{n-1}] = 2\left(\frac{2^{n-1+1} - 1}{2-1}\right) = 2^{n+1} - 2$$

Solution

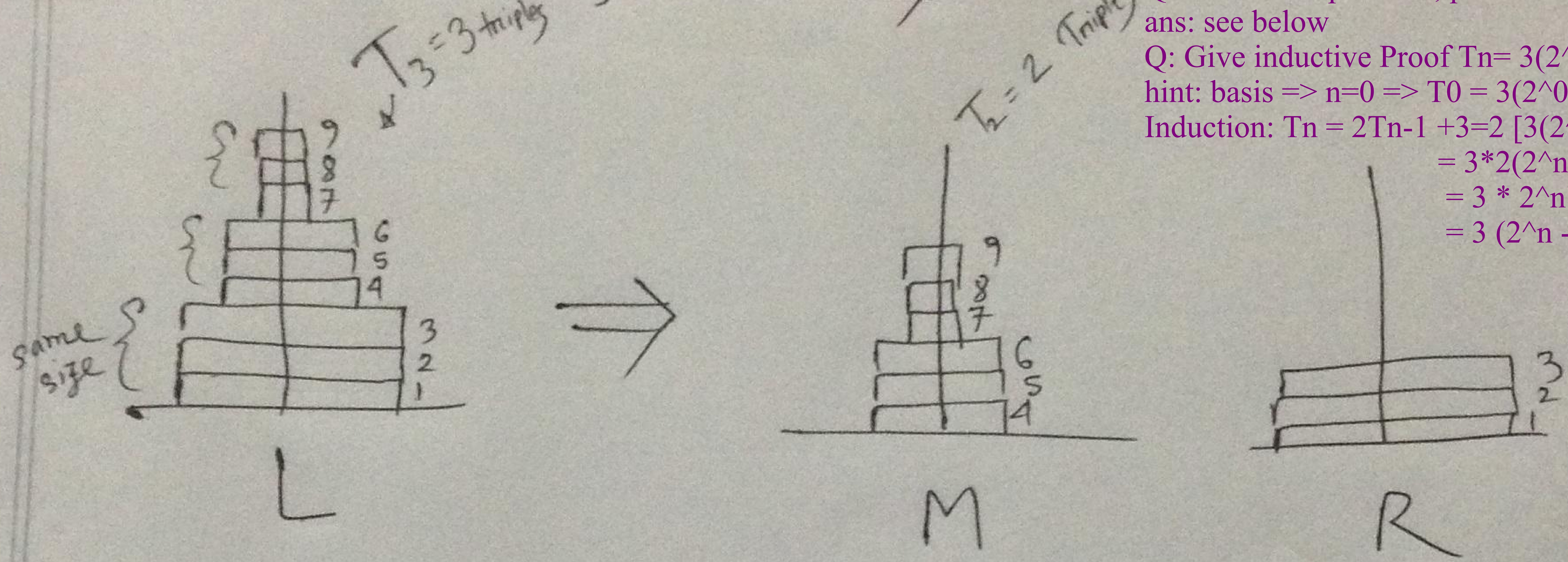
$T_n = 2^{n+1} - 2$

= 2 (2^n - 1)

Triple Tower of Hanoi

in TTOH, the Tn = minimum no. of moves to solve the DTOH problem with n triplets of disks (3n disks). Each three consecutive disks have equal size, so either one can be placed above the other one/two of equal size

Q: For TTOH problem, prove Tn = 3 (2^n - 1)
ans: see below
Q: Give inductive Proof Tn= 3(2^n-1) for TTOH
hint: basis => n=0 => T0 = 3(2^0-1) = 0 obvious
Induction: Tn = 2Tn-1 +3=2 [3(2^n-1 - 1)] + 3
= 3*2(2^n-1) - 6 + 3
= 3 * 2^n - 3
= 3 (2^n - 1) proved :)



Move Top n-1 triplets from L to M | ***Move Bottom Three disks from L to R (destination)*** | ***Move Remaining n-1 triplets from M to R (destination)***

$$T_n = T_{n-1} + 3 + T_{n-1} = \boxed{2T_{n-1} + 3}$$

$$= 2\,(2T_{n-2} + 3) + 3$$

$$= 2^2 T_{n-2} + 2.3 + 3$$

$$= 2^3 T_{n-3} + 2^2.3 + 2.3 + 2^0.3$$

$$= \ldots\ldots$$

$$= 2^n \cancel{T_{n-n}}^{\,0} + 2^{n-1}.3 + 2^{n-2}.3 + \ldots + 2^1.3 + 2^0.3$$

$$= 3\left[2^0 + 2^1 + \ldots + 2^{n-1}\right]$$

$$= 3.\left(\frac{2^{n-1+1} - 1}{2-1}\right) = 3.2^n - 3$$

$$= 3\left(2^n - 1\right)$$

Solution

Solution

$$T_n = 3\left(2^n - 1\right)$$