

Adding the first $i - 1$ of these equations yields, $P_i - P_1 = P_1 \frac{q}{p} + \frac{q}{p} + \dots + \frac{q}{p}$

$$P_i - P_1 = 1 - \frac{q}{p} + \frac{q}{p} + \dots + \frac{q}{p}$$

If $\frac{q}{p} < 1$, then $P_i - P_1 = \frac{1 - \left(\frac{q}{p}\right)^i}{1 - \frac{q}{p}}$, $1 + x + x^2 + \dots + x^{n-1} = \frac{1 - x^n}{1 - x}$

If $\frac{q}{p} > 1$, then $P_i - P_1 = \frac{1 - \left(\frac{q}{p}\right)^i}{1 - \frac{q}{p}}$

Thus, $P_i = \frac{1 - \left(\frac{q}{p}\right)^i}{1 - \frac{q}{p}} P_1$, if $\frac{q}{p} < 1$

$$iP_1, \text{ if } \frac{q}{p} > 1$$

Putting $i = N$ in equation (2), we get, $P_N = \frac{1 - \left(\frac{q}{p}\right)^N}{1 - \frac{q}{p}} P_1$, if $\frac{q}{p} < 1$

$$NP_1, \text{ if } \frac{q}{p} > 1$$

Now, using the fact that $P_N = 1$ in equation (3), we obtain that

If $\frac{q}{p} < 1$, then $P_N = \frac{1 - \left(\frac{q}{p}\right)^N}{1 - \frac{q}{p}} P_1$

$$P_1 = \frac{1 - \left(\frac{q}{p}\right)^N}{1 - \frac{q}{p}}$$

If $\frac{q}{p} > 1$, then $1 = NP_1$

$$P_1 = \frac{1}{N}$$

