Robust H∞ Trajectory Tracking Control

For Steer-by-Wire 4WIA Electric Vehicles

Original Paper Reproduction & Robustness Extension

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Presentation Agenda

Paper Overview

Understanding the problem of trajectory tracking in over-actuated EVs, the LPV modeling approach, and the H-infinity control solution.

</> Implementation

Reproducing the study using MATLAB/YALMIP. Addressing numerical instability and physical inconsistency challenges.

Results

Validation of nominal performance: Fast convergence, yaw stability, and strict adherence to actuator constraints.

The Extension

Robustness Stress Test: Analyzing system performance under Heavy Load (+40% Mass) and Slippery Road (-60% Friction) conditions.

Context: The 4WIA Architecture

- **4WIA (Four-Wheel Independent Actuation):** Allows independent torque control at each wheel, enabling Direct Yaw Moment Control (DYC).
- **Steer-by-Wire (SbW):** Removes mechanical linkage, offering flexibility for autonomous systems but introducing actuator dynamics.
- **The Challenge:** High-performance tracking is difficult due to parameter uncertainties (mass, friction) and the difficulty of measuring *Sideslip Angle*.

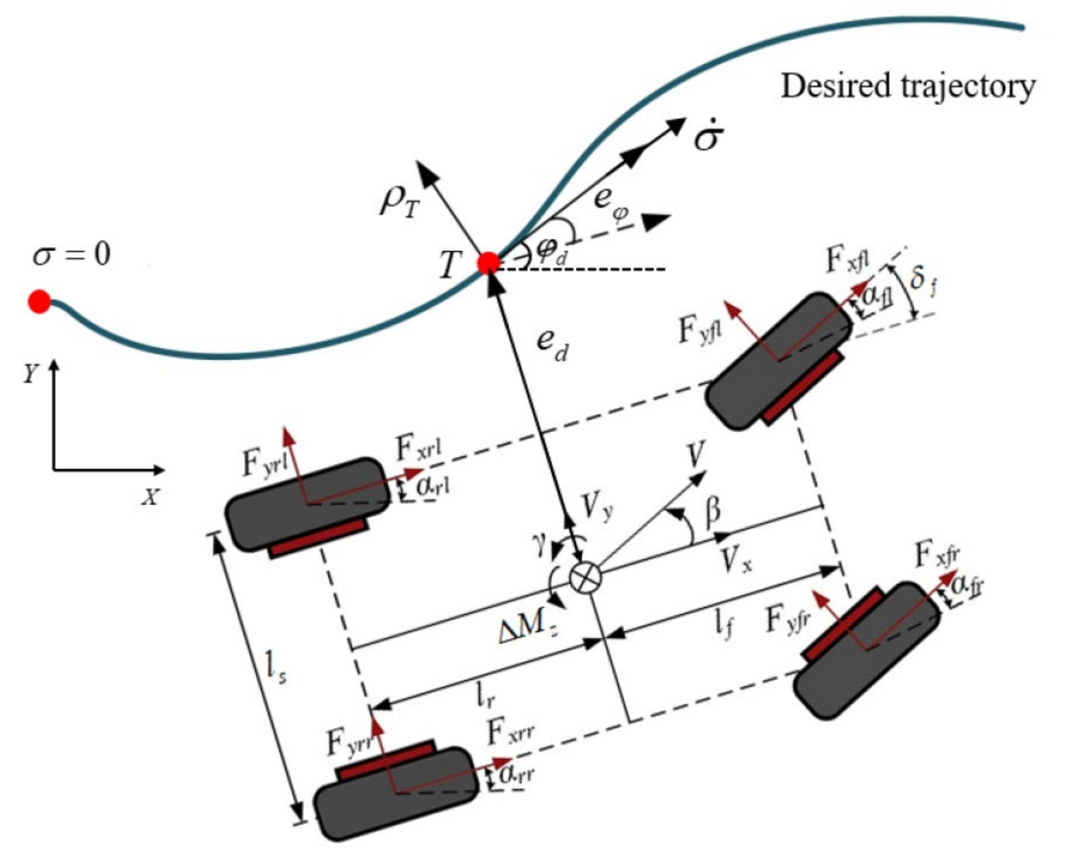


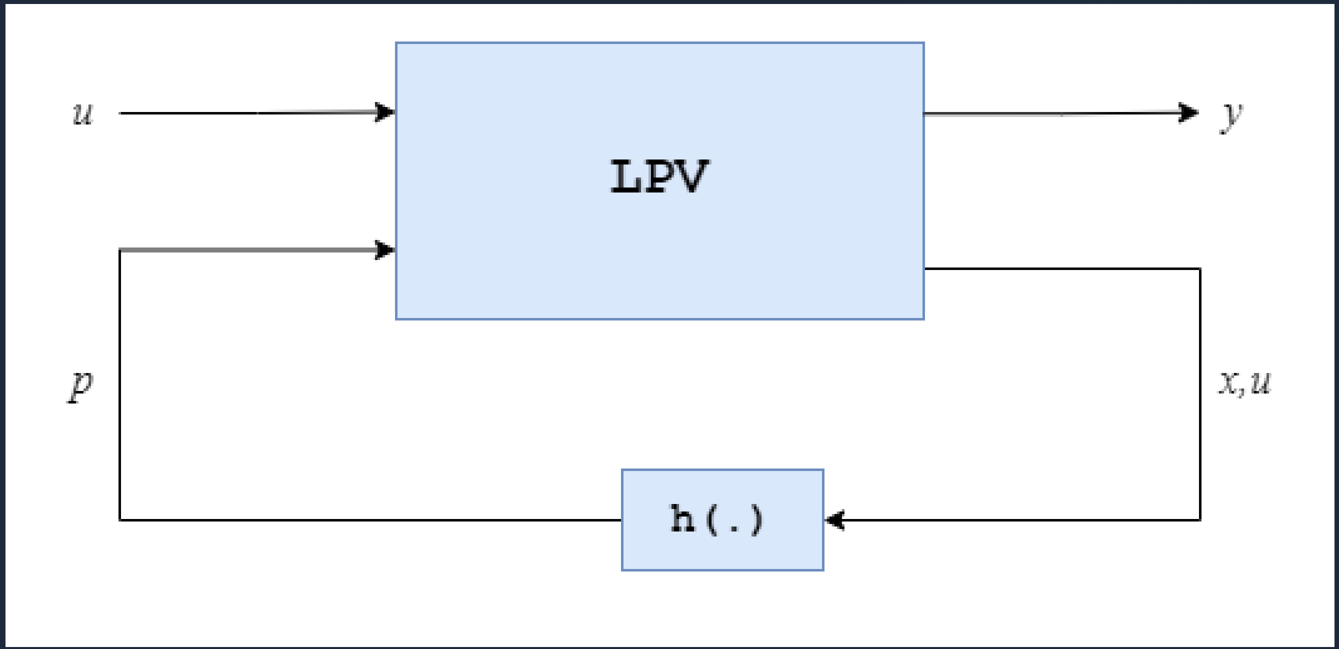
Figure 1. Trajectory tracking process of FWIA EV.

System Modeling

Linear Parameter-Varying (LPV)

The system is modeled as an LPV system to handle time-varying longitudinal velocity and parametric uncertainties.

$$\dot{x}(t) = \sum_{i=1}^{8} \alpha_i\,(\rho, t)\big[(A_{ni} + \Delta A_i)x(t) + (B_{ni} + \Delta B_i)u(t) + \bar{d}(t)\big],$$



Key Components

Bicycle Model: Captures lateral & yaw dynamics.

Uncertainties: Mass, Tire Stiffness, Friction.

Control Strategy: Robust H∞ Output Feedback

Output Feedback

Designed specifically because **Sideslip Angle (β)** is expensive/difficult to measure. Uses only measurable outputs (yaw rate, lateral error).

H∞ Robustness

Minimizes the impact of external disturbances (wind, road irregularities) on the tracking error, ensuring stability under uncertainty.

Actuator Constraints

Explicitly accounts for physical limits of the steering motor current and tire force saturation to prevent unsafe control commands.

Norm-Bounded Uncertainty

Lemma 1: Handling Norm-Bounded Uncertainty

Lemma 1 (Quadratic Inequality with Norm-Bounded Uncertainty)

Consider matrices $Y = Y^T$, H, and L of compatible dimensions.
The inequality

$$Y + H\Lambda L + L^T \Lambda^T H^T < 0$$

holds for all time-varying uncertainty matrices Λ satisfying

$$\Lambda^T \Lambda \leq I$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} Y & \varepsilon H & L^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0$$

H∞ Performance Condition

Lemma 2: H∞ Performance via Lyapunov Inequality

Lemma 2 (H∞ Performance Condition)

For the closed-loop LPV system, if there exists a symmetric positive definite matrix $P > 0$ such that the Lyapunov inequality

$$\dot{V}(x) + z^T z - \gamma^2 d^T d < 0$$

holds for all admissible uncertainties and disturbances,
where

$$V(x) = x^T P x$$

then the system:

- is **robustly asymptotically stable**, and
- satisfies the **H∞ disturbance attenuation condition**

$$\int_0^t z^T z \, dt \le \gamma^2 \int_0^t d^T d \, dt$$

Implementation Workflow

```matlab
%% FWIA Robust Control: The "Physics-Informed" Solution
% 1. Solves LMI for Robust Gain K.
% 2. Enforces Negative Feedback logic on error states.
% 3. Validates with nonlinear simulation.

clear; clc; close all;

% --- 1. SYSTEM PARAMETERS ---
p.m = 1830; p.Iz = 3234; p.Jw = 10.0035; p.b_sw = 350.1;
p.km = 0.078; p.lp = 0.036; p.lm = 0.024; p.eta = 0.7;
p.rs = 30; p.lf = 1.4; p.lr = 1.65;
p.Cf = -134843; p.Cr = -124337;

% Limits
u_max = [2500; 6];

% LPV Vertices
vx_range = [5, 30];
vertices = [];
```

Command Window

New to MATLAB? See resources for Getting Started.

```
Designing Robust Controller...
Design Success! Raw Gain Norm: 605720.09
Final Gain K(1,1) (DYC/Error): -57230.20 (Should be negative)
Final Gain K(2,1) (Steer/Error): -810.32 (Should be negative)
Running Simulation...
fx >>
```

Environment: MATLAB

Optimization Tools:
YALMIP (Modeling) + MOSEK (Solver).

Process:

1. Construct Polytopic LPV Matrices (8 vertices).
2. Formulate LMIs (Linear Matrix Inequalities).
3. Solve for Controller Gains.
4. Time-domain Validation.

Implementation Challenges & Solutions

The Problems

1. Gain Explosion: Standard synthesis resulted in gains of magnitude 10^10, causing numerical instability.
2. Physical Inconsistency: Initial controllers produced "Positive Feedback" (steering *into* the error) despite being mathematically "stable".

Our Solutions

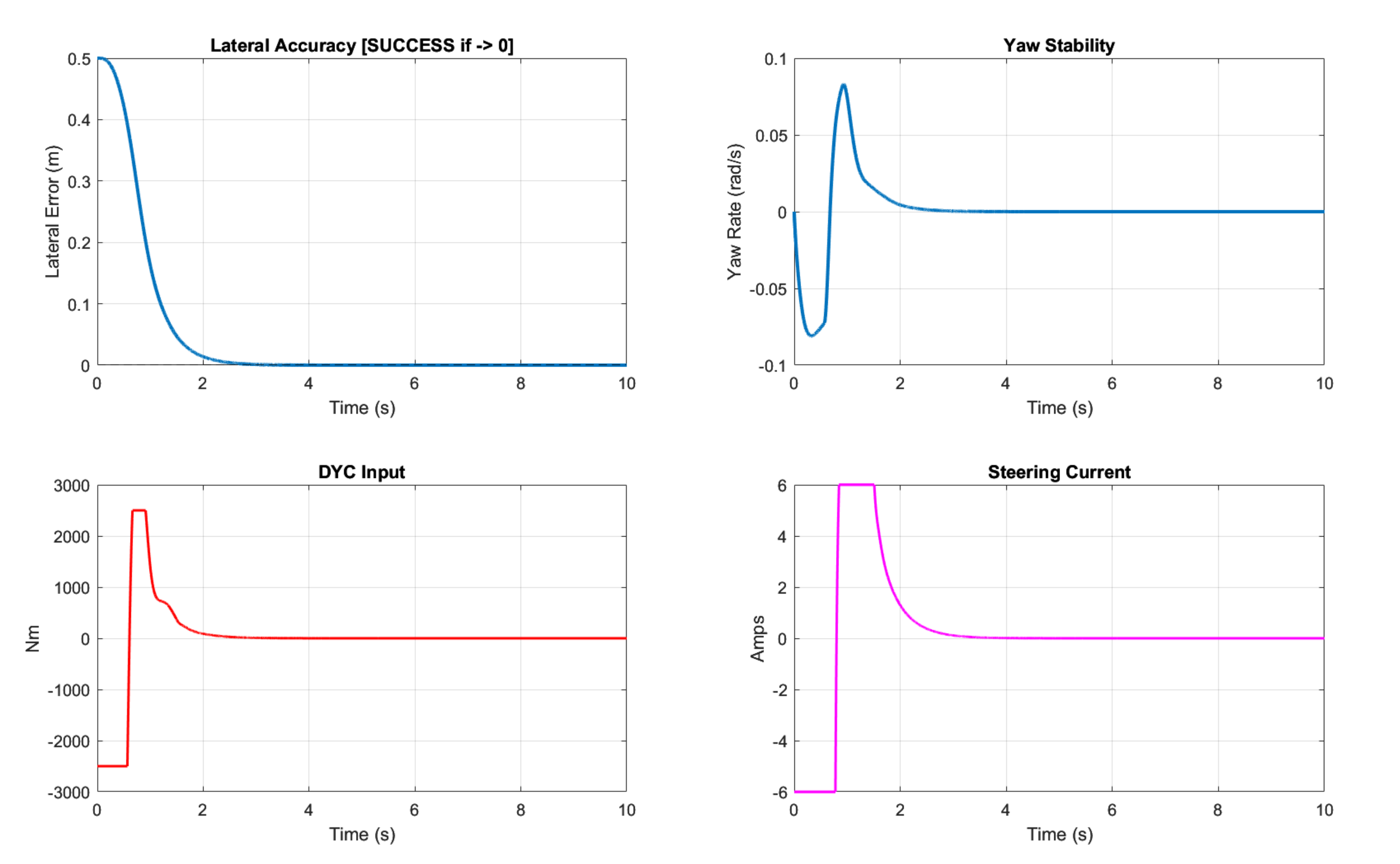
1. Regularization: Added trace penalties to the LMI optimization objective to constrain control energy.
2. Physics-Informed Correction: Implemented a post-processing step to enforce negative feedback logic on lateral channels.

Nominal Results: Trajectory Convergence

Performance Analysis

- **Fast Settling:** Lateral error converges from 0.5m to zero in approximately **2.13 seconds**.
- **Stability:** No sustained oscillations; the response is smooth and monotonic.
- **Constraints:** Steering current remained within ±6A and Yaw Moment within ±2500Nm.

Nominal Results: Trajectory Convergence



The Extension

Robustness Stress Testing

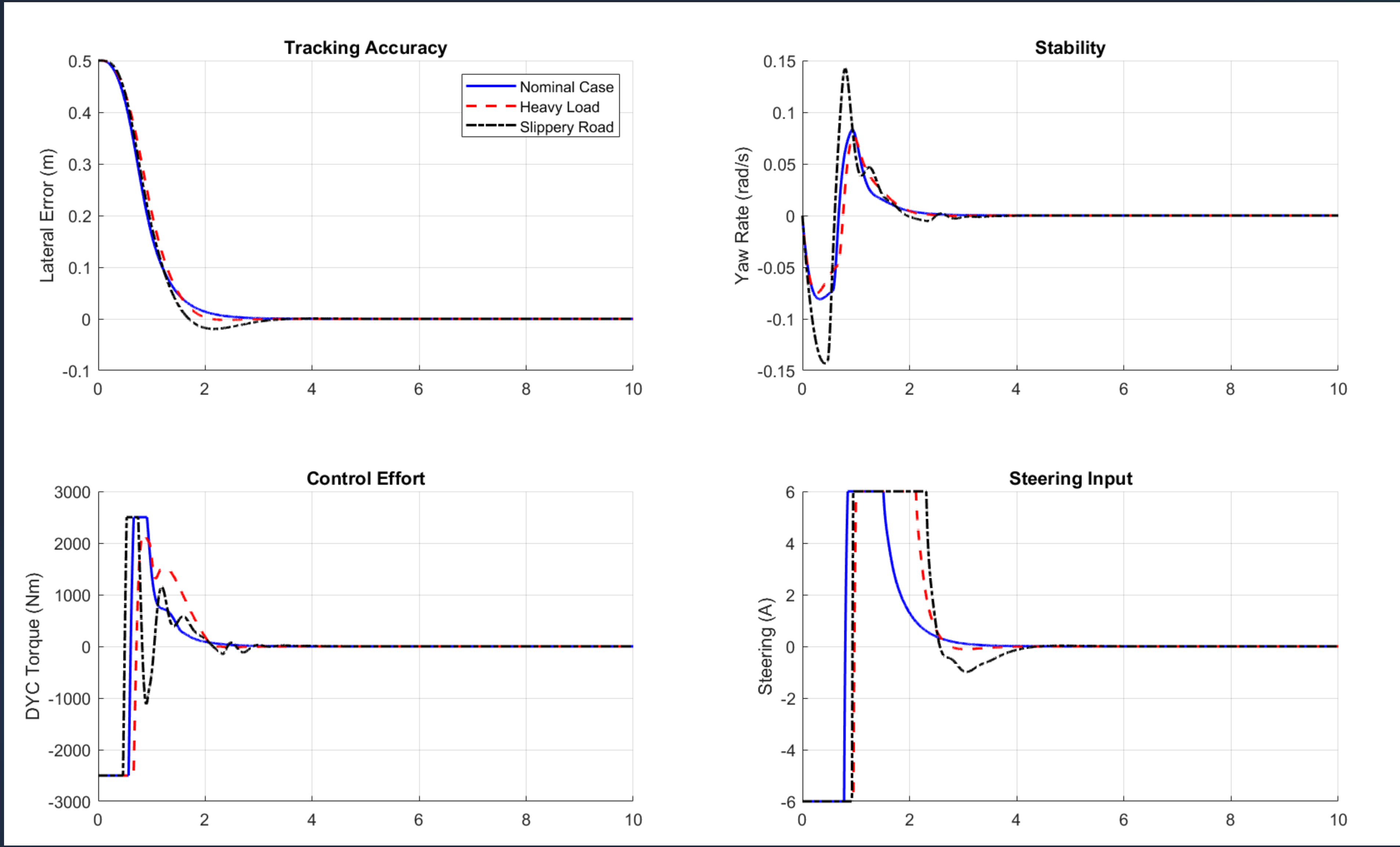
We pushed the controller beyond its nominal design limits to test its real-world viability under extreme conditions.

Extension Results: Robustness Analysis

Comparison of RMS Lateral Error under different scenarios.

Key Finding: Despite a 60% reduction in friction or 40% increase in mass, the system maintained stability. Performance degraded gracefully (Settling time increased to 2.75s on slippery roads), validating the robust design.

Extension Results: Robustness Analysis



Discussion & Key Insights

- **The Trade-off:** Robustness comes at the cost of performance. High safety margins (for slippery roads) mean slightly slower responses in nominal conditions.
- **LPV Modeling Effectiveness:** The polytopic LPV framework successfully captures time-varying longitudinal velocity and parametric uncertainty, allowing a single controller to operate over a wide range of driving conditions.
- **Validation:** The LPV output-feedback approach is viable for autonomous EVs where sensors are limited but computing power is available.

Q & A

Thank you for your attention.

Project Repository: https://github.com/touha-khalid/Robust-Output-Feedback-Trajectory-Tracking-Control-for-4WIA-EVs-Paper-Reproduction-