Draft Solution :

Assignment 2

1)

(a) Given, $X \sim N(\mu, 6^2)$

$P(X \leq 10) = 0.65 = P(Z \leq \frac{10-\mu}{6})$

$P(X \geq 15) = 0.10 = P(Z \geq \frac{15-\mu}{6})$

We know, $Z = \frac{X-\mu}{6}$ So, $\frac{10-\mu}{6} = 0.385$ [Z-table]

$\boxed{\phi(z_1) = 0.65 \quad z_1 = 0.385}$

So, $10 - \mu = 0.385 6 \quad \ldots$ (i).

And $P(X \geq 15) = 0.10$ or $P(X < 15) = 0.90$.

or, $P(Z \leq z_1) = 0.90$

So, $\frac{15-\mu}{6} = 1.282$ [Z-table] $\rightarrow$ $\boxed{\phi(z_1) = 0.90 \quad z_1 = 1.282}$

$15 - \mu = 1.282 \quad \ldots$ (ii)

Solve, $\mu = 7.854 \quad 6 = 5.574$

$6^2 = 31.07$ Ans

(b) $P(6 < X < 10) = ?$

$X = 6, X = 10$

$\mu = 7.854, \sigma = 5.574$ [from 'a']

$z_1 = \dfrac{6 - 7.854}{5.574} = -0.33$

$z_2 = \dfrac{10 - 7.854}{5.574} = 0.39$

$\phi(-0.333)$

$= -\phi(0.333) + 1$

$= -0.6303 + 1$

$P(-0.33 < Z < 0.39) = P(Z < 0.39) - P(Z < 0.33)$

$= 0.6517 - 0.3707$

$= 0.281$

Ans.

(c) $P(X \geq x) = 0.34$

find x?

$P(X < x) = 1 - 0.34 = 0.66$ [z-score 0.41]

$\phi(z) = 0.66$

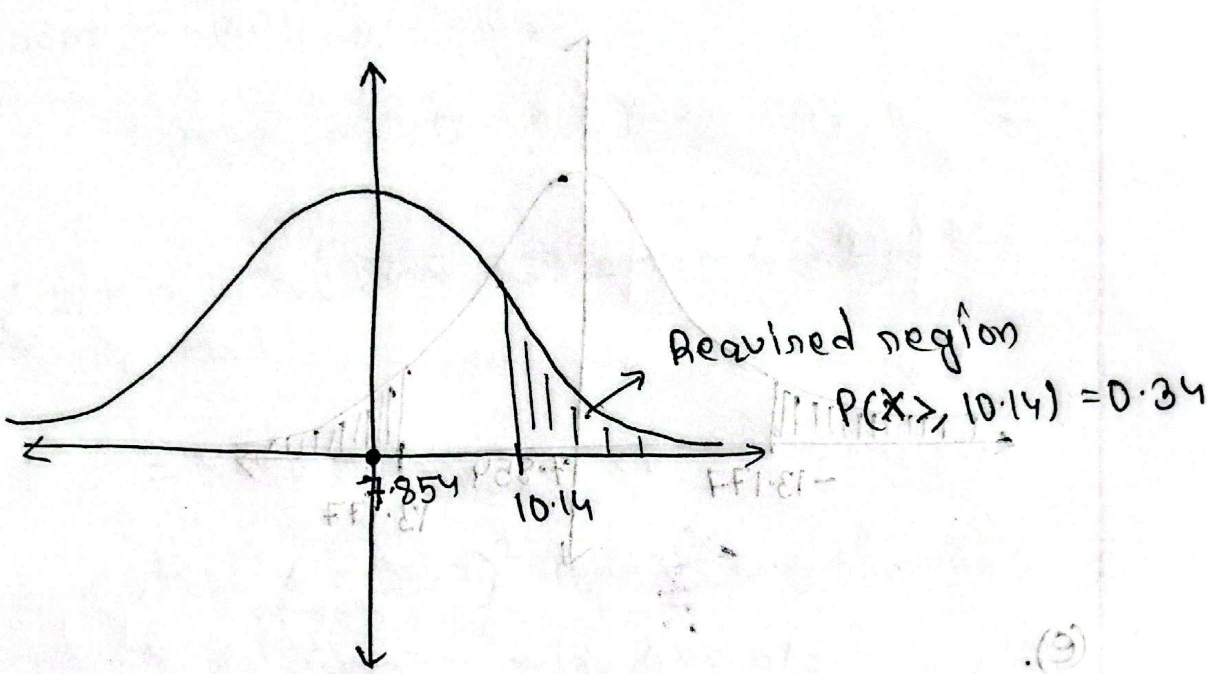
$z = 0.412$

$x = \mu + z\sigma$

$= 7.854 + (0.41 \times 5.574)$

$= 10.14$

Ans



(d). $P(|X| \geq x) = 0.34$

$P(|X| \leq x) = 1 - 0.34 = 0.66$

$P(-x < X < x) = 0.66$

$\Phi\left(\frac{x-\mu}{6}\right) - \Phi\left(\frac{-(x-\mu)}{6}\right) = 0.66$

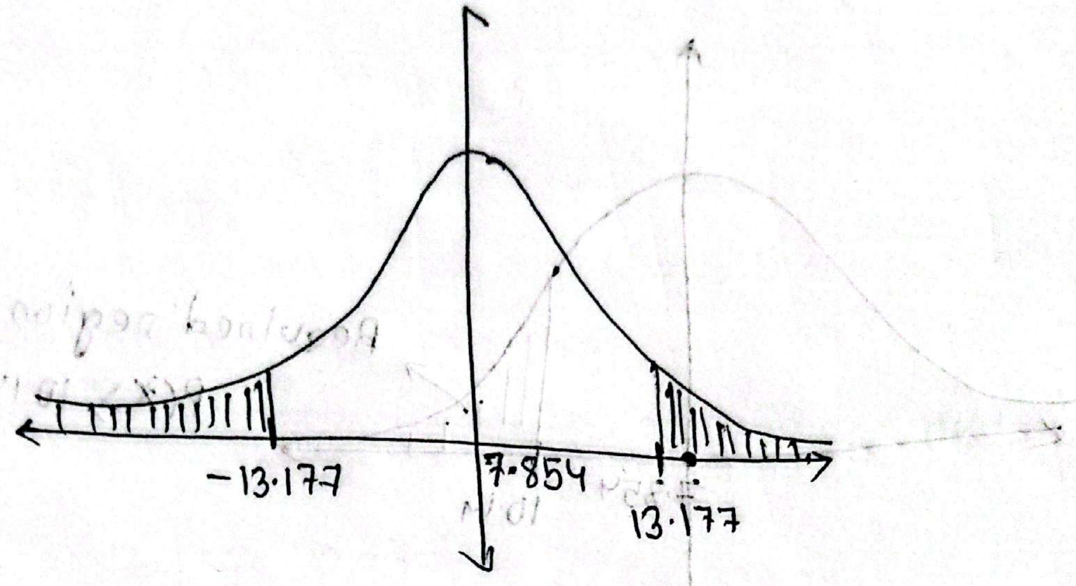
$2\Phi\left(\frac{x-\mu}{6}\right) - 1 = 0.66$

$\Phi\left(\frac{x - 7.854}{5.574}\right) = 0.83$

$x = 7.854 + \{\Phi^{-1}(0.83) \cdot \times 5.574\}$

$= 13.177$

An



(e).

$P(z < z_1) = 0.25$

$z_1 \approx -0.675$

$P(z < z_3) = 0.75$

$z_3 = +0.675$

$z = \frac{x - \mu}{\sigma}$

$x = \mu + z\sigma$

$Q_1 = -0.675 \times 5.584 + 7.854$

$= 4.092$

$Q_3 = 0.675 \times 5.584 + 7.854$

$= 11.616$

$IQR = Q_3 - Q_1 = 7.524$ Ans

② From given data point,

Sample $n = 30$,

a). $\sum x = \sum_{i=1}^{30} (25.7 + 29.3 + \cdots 46.2)$

$= 1122.6$ Ans

$\sum x^2 = \sum_{i=1}^{n} \{(25.7)^2 + (29.3)^2 + \cdots + (46.2)^2\}$

$= 42646.64$ Ans

$\sum (x+5) = \sum_{i=1}^{n} x + \sum_{i=1}^{n} 5$

$= 1122.6 + (30 \times 5) = 1272.6$ Ans

$\bar{x}$, Mean $= \frac{1122.6}{30} = 37.42$

$\sum (x-\bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$

$= 42646.464 - \frac{(1122.6)^2}{30}$

$= 639.188$ Ans

(b)

Sample mean ($\bar{x}$): 37.42 [from 'a']

Median,

For $n=30$, median is average of 15th 16th value after ordering data:

$$\text{Median} = \frac{37.5+37.9}{2} = 37.7$$

Ans

20th Percentile:

$i = p(n+1) = 0.2 \times 31 = 6.2^{th}$ [Betw 6-7th]

So, $P_{20} = 32.8 + 0.2(33.8-32.8)$

$= 33$

Ans

77th Percentile:

$i = p(n+1) = 0.77 \times 31 = 23.87^{th}$ [Betw 23-24th]

$P_{77} = 40.7 + 0.87(41.1-40.7) = \cancel{40.7}$

$= 41.048$

A

Sorted data:

25.7, 29.3, 29.6, 32.1, 32.6, 32.8, 33.8,
34.2, 34.4, 35.7, 36.3, 36.3, 36.3, 36.4,
37.5, 37.9, 39.1, 39.7, 39.8, 40.0, 40.1,
40.4, 40.7, 41.1, 41.3, 42.1, 42.7, 44.1,
44.4, 46.2.

(c). Variace $(s^2) = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{639.18}{29} = 22.041$

Standard deviation, $s = \sqrt{22.041} = 4.695$

$IQR = Q_3 - Q_1$

$= 40.8 - 34.1$

$= 6.7$

Ans

$Q_3 = 0.75 \times 31) = 23.25^{th}$ term

$Q_3 = 40.7 + (0.25 \times 0.4)$

$= 40.8$

$Q_1{}^{th}$ term $= 0.25 \times 31$

$= 7.7^{th}$ term

$Q_1 = 33.8 + 0.75 \times 0.4$

$= 34.1$

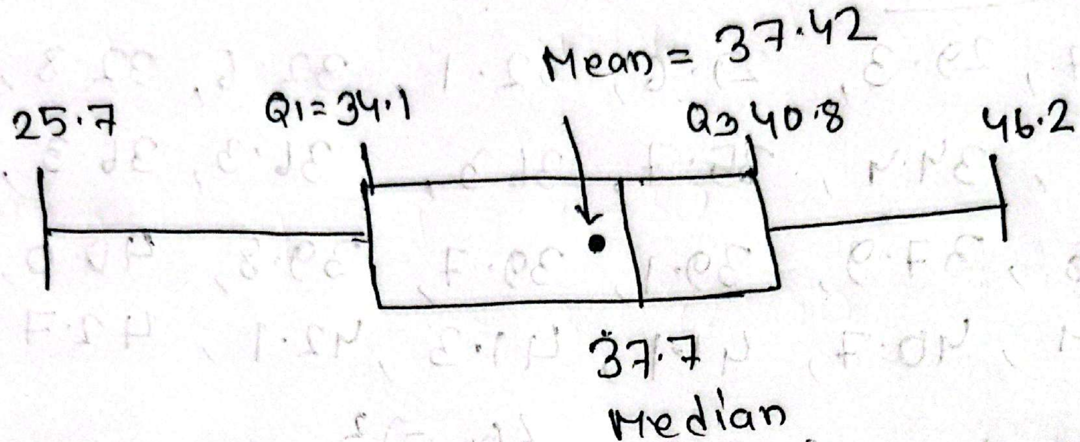
$CV = \frac{s}{\bar{x}} \times 100$

$= \left(\frac{4.695}{37.42}\right) 100\%$

$= 12.55\%$

Ans

(d).



The ~~digi~~ distribution is negatively skewed as the gap between minimum and median (12) is longer than maximum and median (8.5) and mean < median.

(e). Range = 46.2 − 25.7 = 20.5

Num of class = 5.

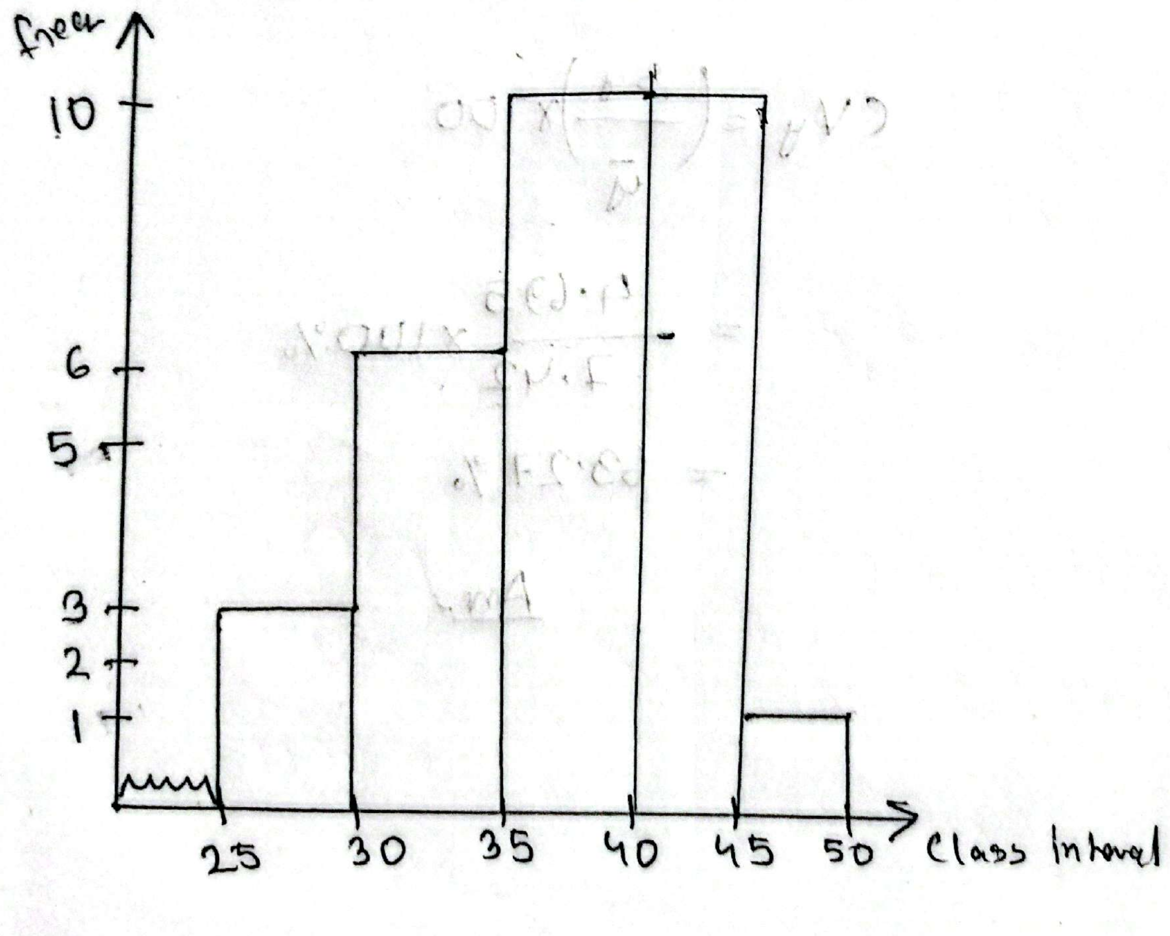
So, class interval $= \frac{20.5}{5} = 4.1 \approx 5$

Frequency distribution table :

Class Interval	Frequency
25-30	3
30-35	6
35-40	10
40-45	10
45-50	1

Total = 30

The histogram shows a peak in the 35-45 range. The distribution is Negatively Skewed (Left Skewed).



(f) Statistics for $y = (x - 30)$.

Mean of y, $\bar{y} = \bar{x} - 30$

$= 37.42 - 30 = 7.42$

Median of y, $M_y = 37.7 - 30 = 7.7$

Variance of y, $s_y^2 = s_x^2 \cancel{30}$

$= 22.041$

Coefficient of Variation of y,

$$CV_y = \left(\frac{s_y}{\bar{y}}\right) \times 100$$

$$= \frac{4.695}{7.42} \times 100\%$$

$$= 63.27\%$$

Ans.