Problem-solving Seminar

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Why such seminars?

- These seminars are inspired by the fact that Professor Donald E Knuth offered such seminars at Stanford University for a long time
- Sharpen analytical skill
- Inspire logical reasoning
- Improve problem solving skill
- Enable our students face interviews with better skill

Similar seminars

- Instructor(s) Profs. Hartley Rogers, Kiran Kedlaya, Richard Stanley MIT Course Number
- 18.S34 as Taught In Fall 2007 Level UG
- Math 491 - Mathematics Problem Solving Seminar Rutgers U 1 credit course
- A Programming and Problem-Solving-Seminar at Stanford U, 1976, 1977, 1985, 2012

Elements of Discrete Mathematics

l Logic, logical reasoning, analytical skill
l Aristotle (384bc-322bc) some 2400 years back and George Boole (1815-1864) The mathematical Analysis of Logic(1848) and The Laws of Thought (1854)
l Logic is the foundation of computer science

Elements of Discrete Mathematics

Proposition

Logic is the foundation of computer science

1. I am a liar.

2. Set consists of elements that do not belong to A.

3. There is a barber in a village who used to shave those who did not shave themselves. Who shaved the barber?

Logic Puzzles

- 4. Two roads emanate from the gate of a jail- one to town and the other back to the jail. Two guards standing on two sides of the road- one liar and the other truthful, and they know about each other. What is the single question to be asked to find the road to the town?
- 5. If they do not know about other's behaviour (truthful or liar) then what should be the question to ask?

Logic Puzzles

6. You arrive at a place where the road got divided into two branches one leading to Post Office and the other to a university, which not known. A man standing on the fork speaks the truth and lies alternately. What is the single question for finding the right branch?

7. If the man speaks the truth or lies consistently what could be the question?

Logic Puzzles

- 8. Three Greek philosophers arguing on a philosophical question fell asleep under a tree. A joker was passing by with a bucket of dye with which he painted their face and left. When they woke up they started laughing at each other. After a while one of them understood that his face is also painted. How could he understand that?

- 8a. Sabuj went 10 miles south then 10 miles east and 10 miles north to reach his home position. Where is the home?

Einstein Logic Puzzles

9. There are 5 ships in a port:

1. The Greek ship leaves at six and carries coffee.
2. The Ship in the middle has a black chimney.
3. The English ship leaves at nine.
4. The French ship with blue chimney is to the left of a ship that carries coffee.
5. To the right of the ship carrying cocoa is a ship going to Marseille.
6. The Brazilian ship is heading for Manila.
7. Next to the ship carrying rice is a ship with a green chimney.
8. A ship going to Genoa leaves at five.
9. The Spanish ship leaves at seven and is to the right of the ship going to Marseille.
10. The ship with a red chimney goes to Hamburg.
11. Next to the ship leaving at seven is a ship with a white chimney.
12. The ship on the border carries corn.
13. The ship with a black chimney leaves at eight.
14. The ship carrying corn is anchored next to the ship carrying rice.
15. The ship to Hamburg leaves at six.

Which ship goes to Port Said? Which ship carries tea?

Logic Puzzles

9a. 50 coins with 10 heads up should blindfoldedly be divided into two piles by you not necessarily having same number of coins and turn some coins upside down as you wish so that each pile contains same number of coins heads up. How can it be done?

9b. There are three light switches up in the attic of an old house... They control three light bulbs down in the basement. The problem is that you don't know which switch is connected to which bulb. You can make one trip down to the basement to figure this out. How are you going to do it?

10. How can you make 3 equilateral triangles with 6 match sticks where each side equals the length of the match stick?

Age Puzzles

11. Two friends meet each other after many years and came to learn that both are mathematicians and married. The first asked the second about the number of children to which 3 was the answer. When asked about their age the second replied with the number of windows in the neighbouring building. The first then said that he needs a little more information to find individual ages to which the reply was the youngest son has blue eyes. This was enough for the first mathematician to figure out ages. How?

Age Puzzles

12. Tommy: "How old are you, mamma?"

Mamma: "Let me think, Tommy. Well, our three ages add up to exactly seventy years."

Tommy: "That's a lot, isn't it? And how old are you, papa?"

Papa: "Just six times as old as you, my son."

Tommy: "Shall I ever be half as old as you, papa?"

Papa: "Yes, Tommy; and when that happens our three ages will add up to exactly twice as much as to-day."

Tommy: "And supposing I was born before you, papa; and supposing mamma had forgot all about it, and hadn't been at home when I came; and supposing——"

Mamma: "Supposing, Tommy, we talk about bed. Come along, darling. You'll have a headache."

Now, if Tommy had been some years older he might have calculated the exact ages of his parents from the information they had given him. Can you find out the exact age of mamma?

Age Puzzles

- 13. "My husband's age," remarked a lady the other day, "is represented by the figures of my own age reversed. He is my senior, and the difference between our ages is one-eleventh of their sum."
- Ezaz: "Do you know, when Towfiq married eighteen years ago Towfiq was three times as old as his wife, and to-day he is just twice as old as she?"
  Akbar: "Then how old was Mrs. Towfiq on the wedding day?"
  Can you answer Akbar's question?
- I have celebrated my birthday 10 times. How old can I be tomorrow?
- "I've always been 45 years older than your Dad", said Grandma to young Trickle. "But now the two digits in my age, both prime, are the reverse of the two in your Dad's age". How old is Grandma?

Age Puzzles

l 14. Together, Grandma and Grandpa are 140 years old. How old is Grandma if Grandpa was twice as old as Grandma was when Grandpa was as old as Grandma is now?

l 15. In the last 6 birthdays grandma's age was divisible by grandson's age. How old was grandma in the last of these birthdays?

16. There was a grand mother in a village who had a grand child.Upon asking her grand child's age she told that she is as older as many days old as her daughters age in weeks and as many days as her own age in years.The sum of the three is

Age Puzzles

17. "Mother, I wish you would give me a bicycle," said a girl of twelve the other day. Mariam: "Do you know, dear, that in seven years' time our combined ages will be sixty-three years?"

Masha: "Is that really so? And yet it is a fact that when you were my present age you were twice as old as I was then. I worked it out last night." Now, what are the ages of Masha and Mariam?"Now, then, Topon, how old is Rajesh?" Mildred's young man asked her brother. "Well, five years ago," was the youngster's reply, "sister was four times older than the dog, but now she is only three times as old." Can you tell Rajesh's age?

17a. A old lady had three grandchildren the difference between two children was 3 years. Her eldest grandchild was 3 times elder than the   youngest one and the elder one 2 years more than the sum of the other two. Then what is the age of the eldest child. ?

Age Puzzles

17b. 4 years before Paul's age is 3 times the Alice age and the present age of Paul is 6 times the Alice. What is the presents Paul's age ?

17c. One year ago, I was in my prime. Twelve years ago, I was a power of a prime. Now, my digits are powers of primes.

17d. Sumit was as old as last two digits of his birthday in 2014. But his father was half as old. When was his father born?

Age Puzzles

17e. Teacher and a student had the following conversation. Teacher said: "Mrs Chowdhury was visited by three women today. Can you work out how old they are, if you know that the product of their ages is 2450, and that together they are as old as you are?" After a deep consideration, the student said: "No, I can't." Then the teacher said: "Of course you can't, but if I tell you that the oldest woman is older than Mrs Chowdhury, you should be able to work it out."

Logic Puzzles

18. A couple invited 5 couples to a dinner party in which those who did not know each other shook hands. While leaving the land lord asked everybody else how many people he/she shook hands with to which everybody answered with a different number. How many people did the land lord shook hands with? Only assumption that could be made is that husband and wife know each other.

Logic Puzzles

19. Three mathematicians are applying for a job. There are five hats, three white, two black. They're lined up, and a hat is placed on each. The first person in line cannot see any hat; the second in line sees only the hat of the person in front of him; the third person sees only the hats of the two people in front of her. The first person to correctly figure out what color hat he has gets the job; you guess wrong and you are killed. Assume these are INTELLIGENT mathematicians, and that they will do the logically correct thing at each stage — if something can be deduced, they will figure it out. After a long pause, the first person, who cannot see any hats, says he knows the color of his hat. What is the color, and how does he know?

Logic Puzzles

20. 10 prisoners were given a chance to freedom. The next day in the morning they will be placed in a line so that anyone can see heads of all standing in the front. Then they will be given caps on their heads from a collection of many white and black caps. The person at the back of the line will be asked first about the colour and a correct answer will give him freedom. In this way the last person asked will be the person standing in the front of the line. How could at least 9 of them get freedom?

Logic Puzzles

- 21. What happens if there are n prisoners?
- 22. What happens if there are hats of m colours?
- 23. Two players play the following game in a round table with equal sized round coins. The first player places a coin. Then alternately coins are placed by the players never overlapped. The player placing the coin last is the winner. How should the game be played?

Logic Puzzles

24. Abir and Bornil just became friends with Sonda, and they want to know when her birthday is. Sonda gives them a list of 10 possible dates. May 15, May 16, May 19 June 17, June 18 July 14, July 16 August 14, August 15, August 17. Sonda then tells Abir and Bornil separately the month and the day of her birthday respectively. Abir: I don't know when Sonda's birthday is, but I know that Bornil does not know too. Bornil: At first I don't know when Sonda's birthday is, but I know now. Abir: Then I also know when Sonda's birthday is. When is Sonda's birthday?

Logic Puzzles

25. There are four men who want to cross a bridge. They all begin on the same side. You have 17 minutes to get all of them across to the other side. It is night. There is one flashlight. A maximum of two people can cross at one time. Any party who crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth, it cannot be thrown, etc. Each man walks at a different speed. A pair must walk together at the rate of the slower man

Man 1: 1 minute to cross, Man 2: 2 minutes to cross
Man 3: 5 minutes to cross, Man 4: 10 minutes to cross
For example, if Man 1 and Man 4 walk across first, 10 minutes have elapsed when they get to the other side of the bridge. If Man 4 returns with the flashlight, a total of 20 minutes have passed, and you have failed the mission.

Clock Puzzles

26. The long hand of a clock points exactly at a full minute, while the short hand is exactly two minutes away. What time is it?

27. In how many distinct places do hour and minute hands exactly coincide?

28. In how many distinct places do hour and minute hands point exactly to opposite directions?

29. In how many distinct positions swapping hour and minute hands give valid time?

30. How many times do hour and minute hands make equal angle with the vertical line?

Clock Puzzles

- 31. A clock goes at the right speed, but backwards. How many times each week does it tell the right time?
- 32. A circular field is to be divided among three brothers as equally as possible. But the divisions will be represented by the three hands of the clock. Find the time representing the best distribution.
- 33. How many minutes is it until six o'clock if fifty minutes ago it was four times as many minutes past three o'clock?

Clock Puzzles

34. A friend pulled out his watch and said, "This watch of mine does not keep perfect time; I must have it seen to. I have noticed that the minute hand and the hour hand are exactly together every sixty-five minutes." Does that watch gain or lose, and how much per hour?

34a. A clock was officially put into use when it showed 6 o'clock. Soon it was noticed that the hour hand and minute hand had been interchanged and attached to the wrong axes. The result was that the hour hand moved with a speed twelve times higher than the minute hand. When the clock maker arrived, a remarkable thing happened: on the moment he inspected the clock, it showed exactly the right time again.

The Question: If the clock started at 6 o'clock in the correct position, then what was the first moment that it showed the correct time again?

Computer Puzzles

l 35. You have two variables A and B containing two real values. You want to swap contents without using dummy variable. How?

l 36. If type of data are not known how can you swap without dummy variables?

l 37. You have an array of data and a permutation array saying where to place each data item as follows

l 1 2 3 4 5 6 7 8

l A B C D E F G H Data array

l 3 2 4 5 1 7 8 6 Permutation array

l E 2 D C A H F G   Output array

Puzzles with math Analysis

38. A robot standing in front of an infinite wall with only one gate somewhere, which robot can identify only when it steps just in front of it. Find a strategy in which it can identify the gate in minimum number of steps in the worst case assuming it to be n (unknown) steps either to the left or to the right.

39. Which one is greater e^pi or pi^e?

Puzzles with math Analysis

- 40. 4 dogs standing on the corners of a square with side length 1 km. Each dog started running towards the one at the right directly and blindfold half the distance and opened their eyes to change the direction and do the same until they cannot run anymore. What is the ultimate destination for them? How far can they run?
- 41. Whatabout n dogs standing in the corners of an equilateral n-gon?

Puzzles with math Analysis

- 42. If length of two sides of a right angled triangle differ by 17, and sides are of whole number length what is the smallest length of its hypotenuse?
- 43. Area of top of a box is 240 m^2, front is 300 m^2 and end is 180 m^2 what is smallest linear dimensions?
- 44. A man's age at death was 1/32nd of his birth year. What would have been his age in year 2000?

Puzzles with math Analysis

45. A woman with four children bought a sack of peanuts. To the oldest child, a boy, she gave one peanut and 1/4 of what remained; and to each of the other children she did the same. (gave 1 peanut + 1/4 of the rest) The second child was a girl, the third a boy, and the last a girl. It was found that the boys had received 100 more peanuts than the girls. What was the initial number of peanuts?

Puzzles with math Analysis

46. Mr Rahim has 2 sons, 3 daughters and 7 grandchildren. If he divides his money in whole numbers among his sons then Tk 1 remains, while doing the same thing amongst his daughters results in Tk 2 as remainder, and dividing among his grandchildren leaves him with Tk 4. What is the minimum amount of money in his pockets?

Puzzles with math Analysis

- 47. There are some pigeons in the roof of Hall A and Hall B. If two pigeon come from Hall B to A then number of pigeons in A becomes half that remains in B. Whereas if 3 pigeons from A go to B then number of pigeons in B becomes thrice that remains in A. What is the initial number of pigeons in Hall A?
- 48. A man bought 3 items and paid the price by multiplying individual prices. When he bought the fourth item he again multiplied all 4 prices and in both the cases got the right amount. What are the prices?

Puzzles with math Analysis

- 49. A five digit number is represented by ABCDE where each letter represents a digit. If we add the digit 1 in front of ABCDE, we get 1ABCDE. The product of 1ABCDE and 3 is the six digit number ABCDE1. Find the value of the original number ABCDE.

- 50. Two brothers sold some coconuts each at the price of number of coconuts sold. Then starting with elder brother they took 20 taka notes alternately. When the elder brother took the last 20 taka note he had to give his knife to younger brother so that the division was equal. What is

Puzzles with math Analysis

- 51. Mr and Mrs A are 120 km apart. A bee is on Mr A's nose. The couple cycle towards each other, Mr A at 25km/h and Mrs A and 15km/h. The bee dashes from Mr A's nose to Mrs A's nose and back again and so on at 60km/h. How far does the bee travel before the cyclists crash?
- 52. A cube is painted white and cut into 27 small cubes. How many of these little cubes are painted on 1 side, 2 sides and 3 sides?

Puzzles with math Analysis

53. If a pipe can fill the tank within 6 hrs but due to leak it took 30 min more now if the tank is full how much time will it take to get emptied through the leak?

53a. We want to find the smallest, right-angled triangle for which holds:
The lengths of the sides are whole numbers.
The circumference is the square of a whole number. The area is a whole number to the power of three. To help you a bit: the length of the hypotenuse is 240.
The Question: What are the dimensions of this

Puzzles with math Analysis

- 54. What is the area of the largest triangle that can be inscribed in a triangle?
- 55. What is the area of the largest equilateral triangle that can be inscribed in a unit square?
- 56. What is the length of the largest   stick that can be crawled along a corridor of a square building whose outer side length is 40 m whereas inner side length is 35 m?

Puzzles with math Analysis

- 57. A pencil, an eraser, and a notebok together cost one dollar. A notebook costs more than two pencils. Three pencils cost more than four erasers. Three erasers cost more than a notebook. What are their prices?
- 58. Given a pile of coins two players play the following game. The first player does not take all but at least 1. In subsequent moves alternatively next player must remove at least one ball and not more than double the opponent took in the preceding move. How can this game be optimally played?

Puzzles with math Analysis

- 59. How can I get the answer 24 by only using the numbers 8,8,3,3. You can use add, subtract, multiply, divide, and parentheses. Bonus rules: also allowed are logarithms, factorials and roots
- 60. Given a pile of coins two players play the following game. The first player does not take all but at least 1. In subsequent moves alternatively next player must remove at least one ball and not more than double the opponent took in the preceding move. How can this game be optimally played?

Puzzles with math Analysis

61. A father, in his will, left his money to his children in the following manner:
$1000 to the first born and 1/10 of what then remains, then $2000 to the second born and 1/10 of what then remains, then $3000 to the third born and 1/10 of what then remains, and so on. When this was done each child had the same amount. How many children were there?

Puzzles with math Analysis

- 62. Mosharaf's house is on a circular road as his university is. So he avails of the first bus that comes to the stoppage. Buses ply every 20 minutes and runs deterministically. Out of 20 times he caught the anticlockwise bus 18 times? Can you reason why?
- 63. Arrange the digits 1-9 into a 9-digit number such that for each N from 1 to 9, the first N digit number is divisible by N.

Puzzles with math Analysis

64. Suppose you have a grassy field, and cows eat grass at a constant rate. Keep in mind, the grass keeps growing continuously. 48 cows can clear all the grass off the field in 90 days. 120 cows can clear all the grass off the field in 30 days.

How many cows would be needed to clear all of the grass in 16 days?

Puzzles with Geometry

- 65. Given a line segment, a line parallel to it, and a straightedge, divide the segment into N equal segments. (With a straightedge, you are allowed only to draw straight lines. You are not allowed to mark off distances on the straightedge.)
- 66. Find the midpoint of two pints A and B using only pencil compass and no straight edge drawing apparatus.

Puzzles with Geometry

l 67. You are in a convex dense forest. You have equipment to make any geometric trajectory. How are you coming out of the forest in minimum time in the worst case?

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Puzzles with Geometry

68. A man in the death bed told his idle son that he has kept gold hidden in a forest where there are two identical trees and a stone. If he walks from the stone to one of the trees and equal distance perpendicularly to get a point A, and do the same with the other tree to get point B. Stone is hidden in the mid point of the straight line joining A and B. The son could find the trees and not the stone. How to get the gold?

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Puzzles with math Analysis

- 69. You are in the intersection of n roads one of which will lead to freedom after traversing an unknown distance. You need to reach that point in minimum time in the worst case. How are you going to do it?

Puzzles with math Analysis

70. You have a jeep to reach a destination 20 kms inside a forest with no gas station. Your jeep runs 1km/gallon. You are allowed to make depots. The jeep can carry only 10 gallons. How can it reach the destination using minimum fuel?

Puzzles with math Analysis

70a. Farmer Bob owns a piece of grassland and three animals: a cow, a goat, and a goose. Bob discovered the following:

When the cow and the goat graze on the field together, there is no more grass after 45 days.

When the cow and the goose graze on the field together, there is no more grass after 60 days.

When the cow grazes on the field alone, there is no more grass after 90 days.

When the goat and the goose graze on the field

Puzzles with measures

- 71. From 12 metal balls with one defective in weight you should find the defective ball and the defect in 3 measures given measuring pans.
- 72. 11 baskets each contains not less than 10 apples. Each apple of 10 baskets weighs 100 gms each whereas each of the remaining basket weighs 90 gms each. Measuring pans and weights are available. Find the basket with 90 gm apples in one measure.

Puzzles with measures

73. There are 8 marbles that weigh 1 ounce each, and 1 marble that weighs 1.1 ounces. The marbles are all uniform in size, appearance, and shape. You have a balance that contains 2 trays. You are only able to use the scale 2 times. How do you determine which marble is the heaviest using only the scale and marbles in 2 weighings?

74. One of 9 otherwise identical balls is overweight. How can it be identified after 2 weighings with an old balance?

Puzzles with measures

- 75. One of 27 otherwise identical balls is overweight. How can it be identified after 3 weighings with an old balance?
- 76. n>2 intertwined electrical wires are hanging from a skyscraper. You need to identify both ends of each wire by making connections among wires. What is the minimum number of times you will require to move up and down to do the job?

Some Physics Puzzles

- 77. Motion of a train is expected to be translatory how can it them move along a circular track?
- 78. High and low tides occur due to position of moon. How many times a day is the high tide?
- 79. When you get down from a moving bus you jump forward. What should you do if you have a sack of salt?
- 80. Is it possible that a spectator standing by the side of a rail track can see an object in the train moving at a speed of 200 kms an hour when speed of the train is only 100?