Langage et algorithmique
- Arbre -

A. Malek TOUMI

toumiab@ensta-bretagne.fr

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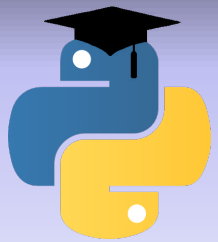
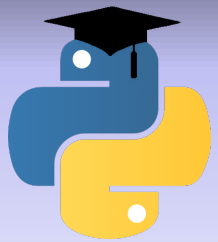
ENSTA Bretagne



Sommaire



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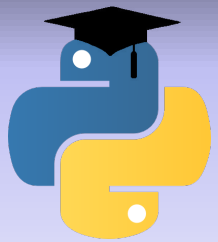
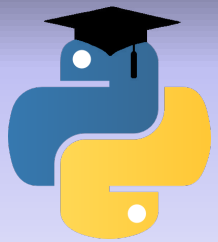


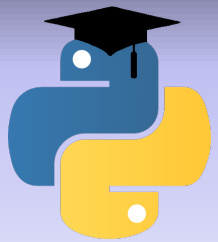
Pourquoi des arbres ?

- Structure dynamique : liste



Pourquoi des arbres ?

- Structure dynamique : liste
- Constat : coût élevé des opérations de base



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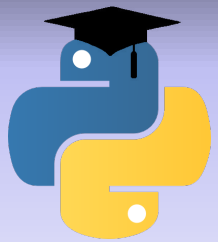
- Structure dynamique : liste
- Constat : coût élevé des opérations de base
- Objectif : trouver une structure de données plus efficace



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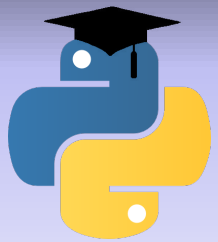

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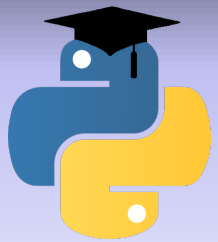
Arbre :

- Structure de données dynamique



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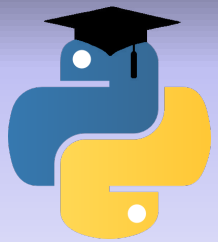
Arbre :

- Structure de données dynamique
- Bonne complexité pour les opérations de base

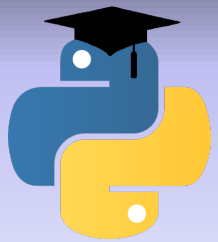


Pourquoi des arbres ?

- Structure dynamique : liste
- Constat : coût élevé des opérations de base
- Objectif : trouver une structure de données plus efficace

Arbre :

- Structure de données dynamique
- Bonne complexité pour les opérations de base
- Recherche, insertion, suppression : $\Theta(\log n)$

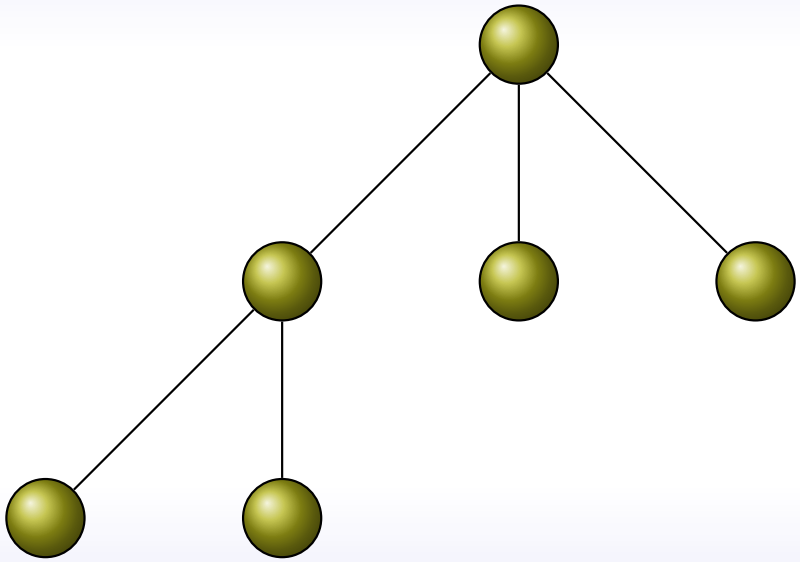


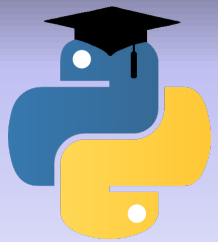
Définitions

Définition (Arbre)

Ensemble non vide de noeuds et d'arêtes tel que :

Définitions





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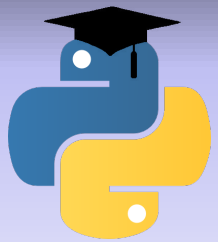

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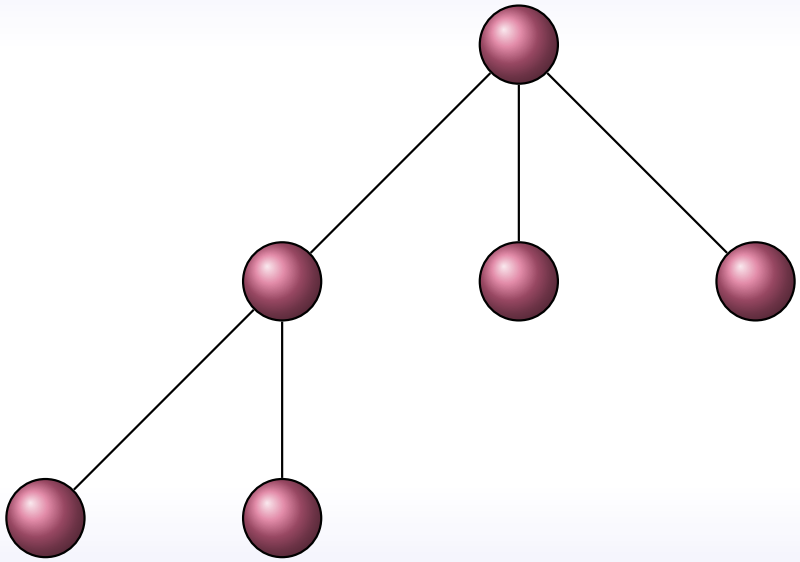

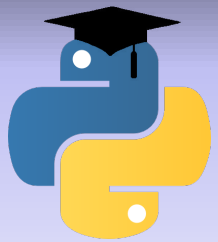

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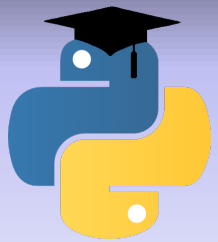
Définition (Arbre)

Ensemble non vide de noeuds et d'arêtes tel que :

- noeud : objet simple
- arête : relie deux noeuds (orienté)



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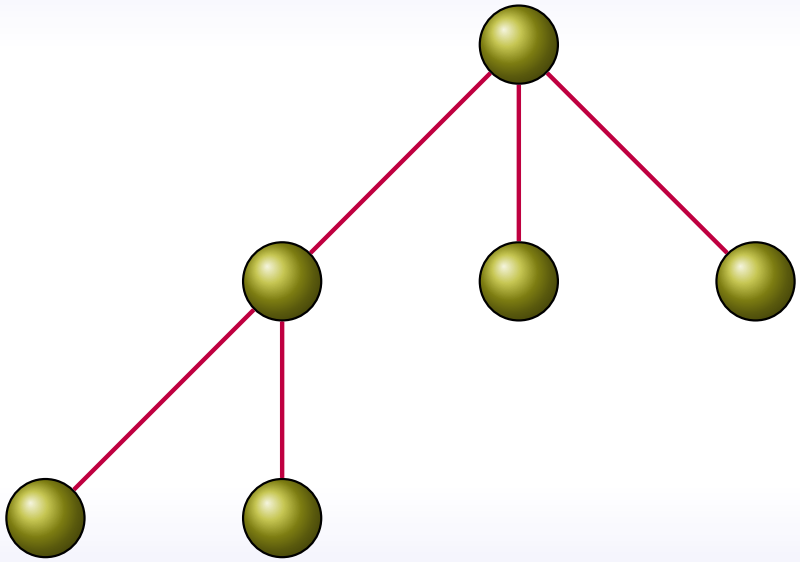
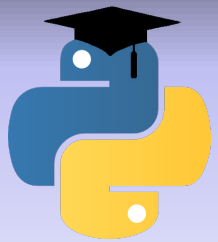


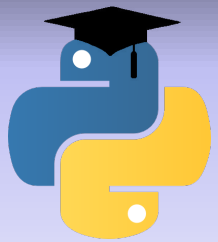


Définitions

Définition (Arbre)

Ensemble non vide de noeuds et d'arêtes tel que :

- noeud : objet simple
- arête : relie deux noeuds (orienté)
- branche : suite de noeuds reliés par des arêtes



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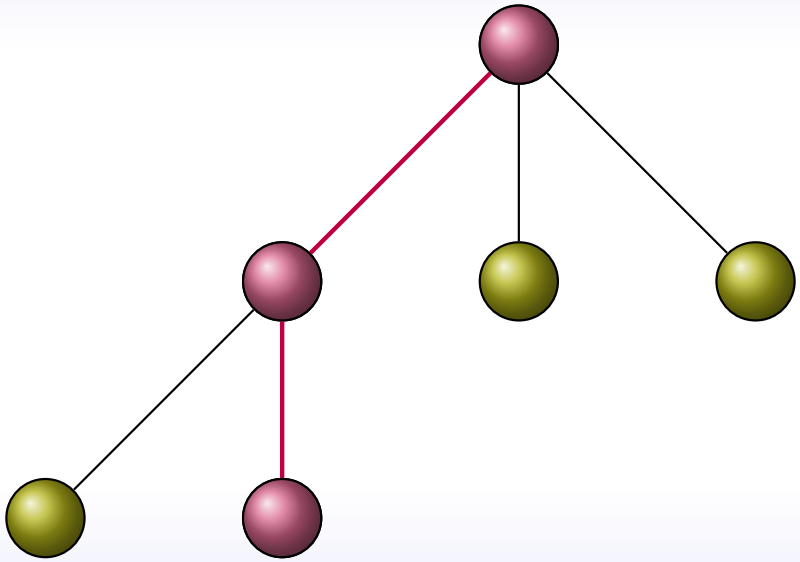

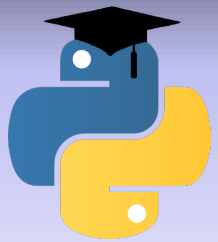

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Ensemble non vide de noeuds et d'arêtes tel que :

- noeud : objet simple
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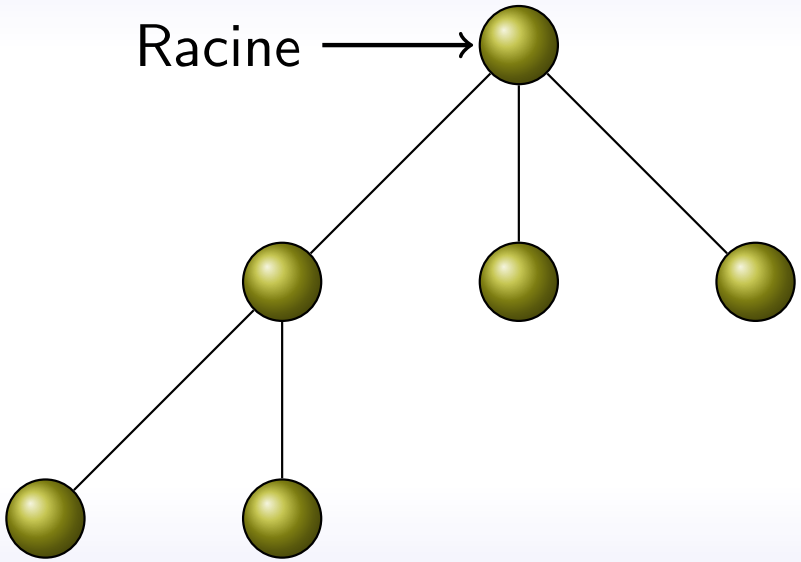
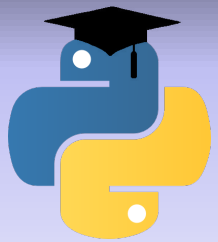
- branche : suite de noeuds reliés par des arêtes
- racine : noeud sans prédécesseur

Définitions





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- noeud : objet simple
- arête : relie deux noeuds (orienté)
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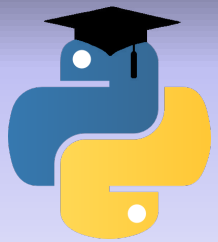
- racine : noeud sans prédécesseur
- tous les noeuds sont accessibles depuis la racine par une seule branche



Définitions

- Représentation : racine au sommet de l'arbre
- Relation père-fils entre les noeuds
- Bas de l'arbre : feuilles

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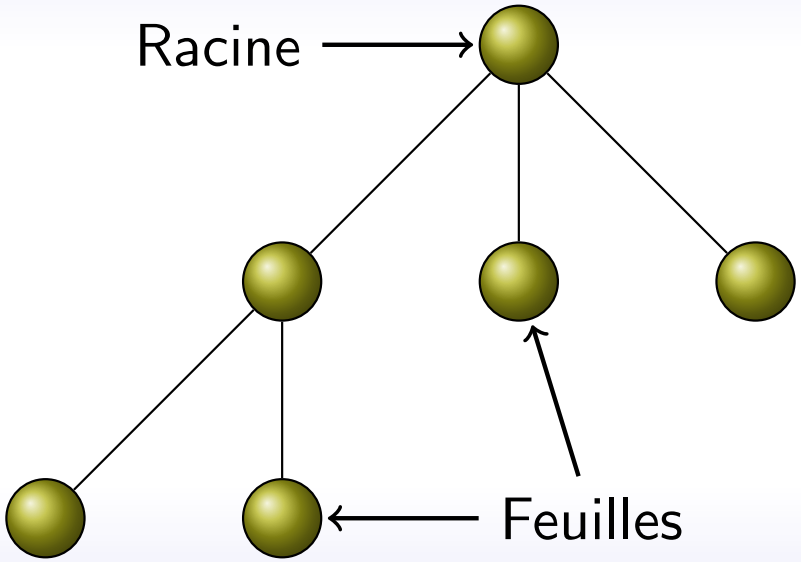

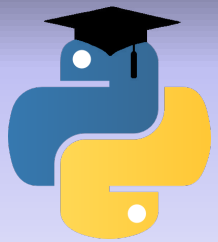

Définitions

- Représentation : racine au sommet de l'arbre
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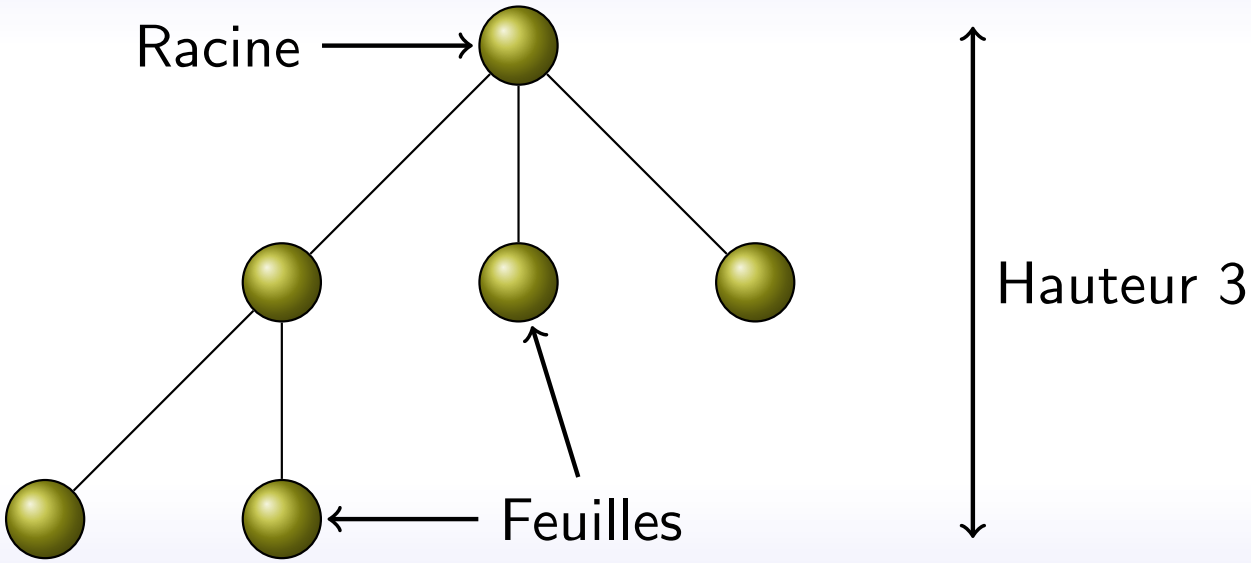
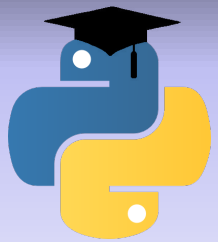
Définition (Hauteur)

Nombre de noeuds du plus long chemin entre la racine et les feuilles.

Définitions





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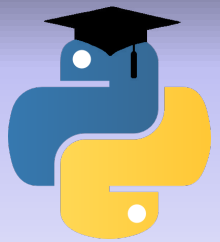


Définition

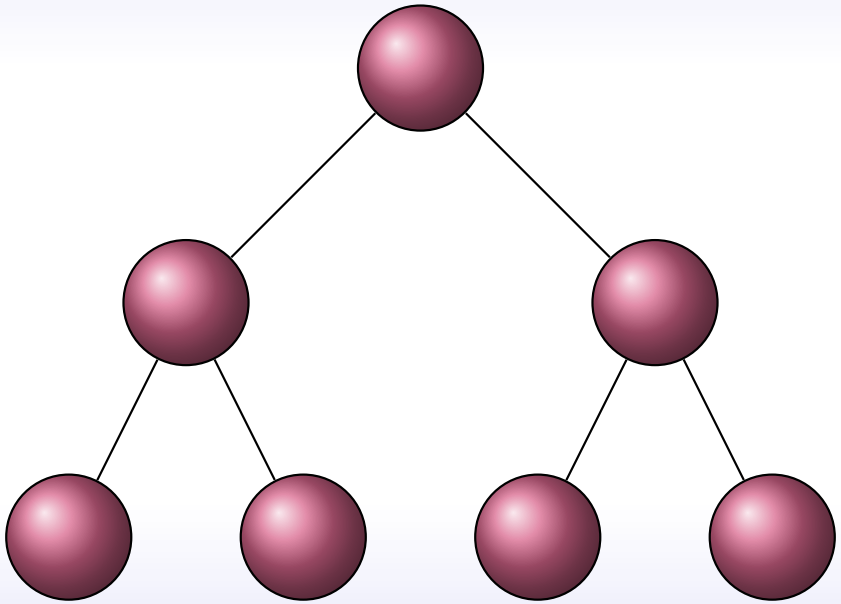
Définition (Arbre binaire)

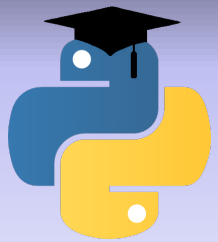
Un arbre binaire est un arbre dont chaque noeud possède au plus deux fils.

Définition (Arbre binaire complet)

Un arbre est complet si chaque noeud qui n'est pas une feuille possède exactement deux fils et où toutes les feuilles sont au même niveau.

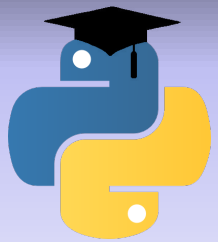
Définition





Propriétés

- Chaque noeud possède un fils gauche et un fils droit



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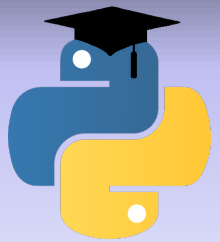
- Arbre de hauteur h



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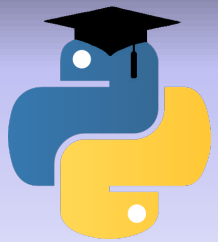
$\Rightarrow$ $2^h - 1$ noeuds (si complet)



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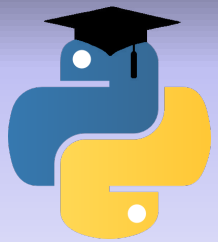

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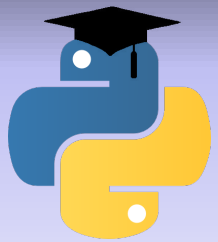
$\Rightarrow$ hauteur $\log n$



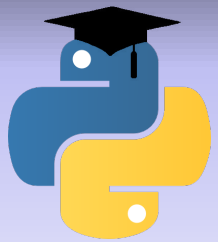
en Python

Code pour définir un noeud

```
class Node(object):

    ...

    ...
```



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Code pour définir un noeud

```
class Node(object):
    def __init__(self, valeur = 0):


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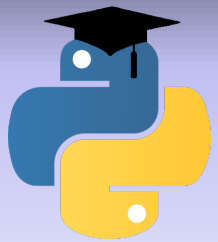

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class Node(object):
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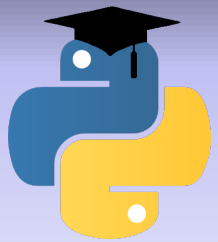

en Python

Code pour définir un noeud

```
class Node(object):
    def __init__(self, valeur = 0):
        self.val = valeur
        self.right = None
        self.left = None
        # self.height = 0
    ...



    ...
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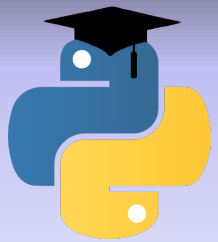

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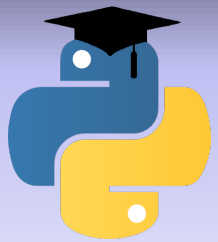
    ...

class Arbre(object):
    def __init__(self):

    ...
```



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class Arbre(object):
    def __init__(self):
        self.__root = None
    ...
```

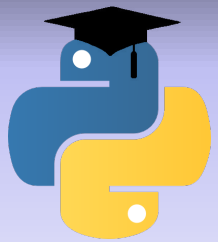

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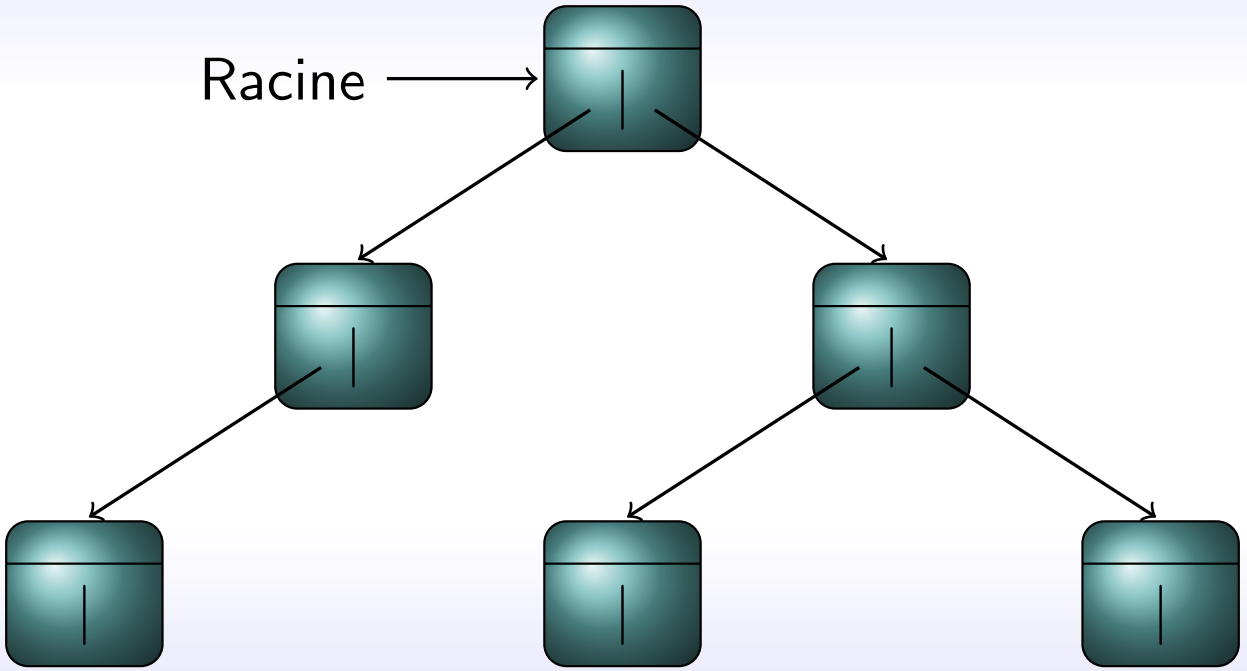
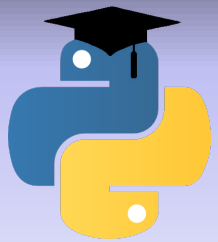
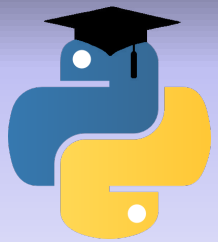




Généralisation

Arbres **n-aires** :

- chaque noeud possède au plus n fils.



Généralisation

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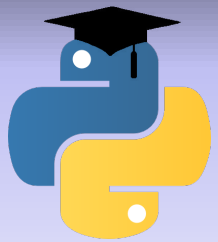

Généralisation

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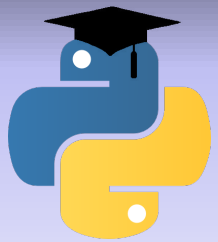
- Moins utile que les arbres binaires



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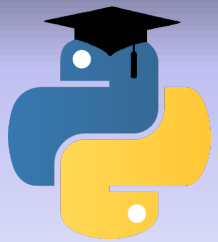
- Moins utile que les arbres binaires
- Exemple d'utilisation : **quadtree**



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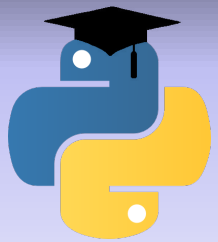


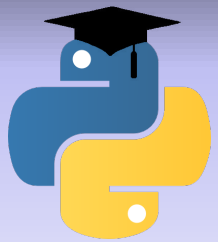
Définition

Définition (Arbre général)

Arbre pour lequel chaque noeud possède un nombre quelconque de fils (non borné **a priori**).

Exemple (Exemples d'utilisation)

- Représentation d'une arborescence de répertoires
- Représentation des possibilités pour un jeu

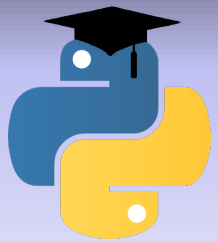


Définition

Code pour définir un noeud

```
class Node(object):

    def __init__(self, valeur = 0):
```

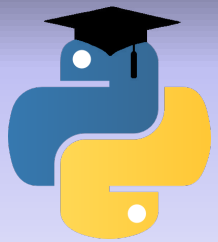


Définition

Code pour définir un noeud

```
class Node(object):

    def __init__(self, valeur = 0):
        self.val = valeur
```

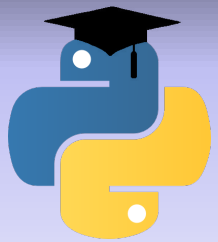


Définition

Code pour définir un noeud

```
class Node(object):

    def __init__(self, valeur = 0):
        self.val = valeur
        self.child = None
```



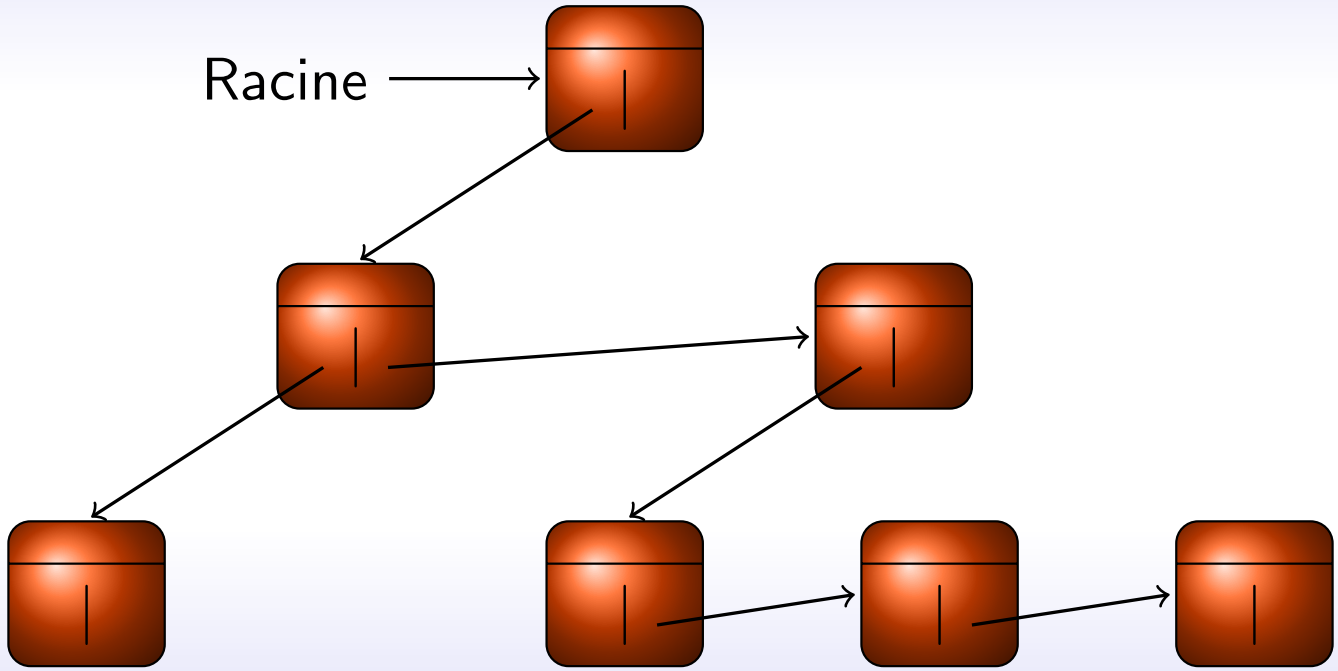
Définition

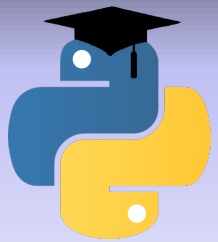
Code pour définir un noeud

```
class Node(object):

    def __init__(self, valeur = 0):
        self.val = valeur
        self.child = None
        self.brothers = []
```

Définition





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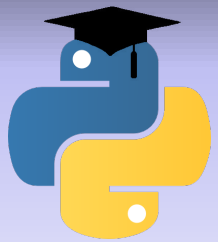

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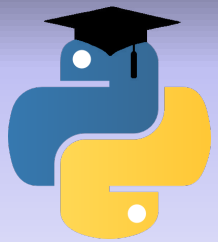
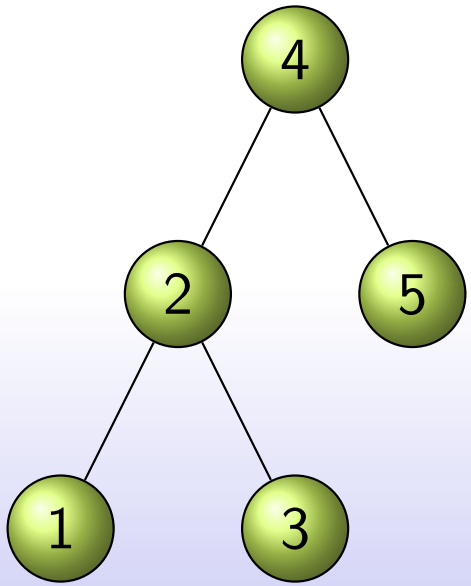
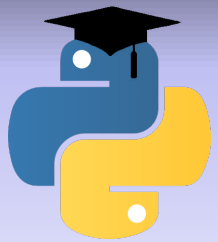


Définition

Définition (Arbre binaire de recherche)

Arbre binaire vérifiant la propriété : pour tout noeud n de l'arbre, tous les noeuds de son sous-arbre gauche sont inférieurs à n, tous les noeuds de son sous-arbre droit sont supérieurs à n.





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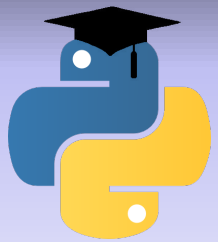
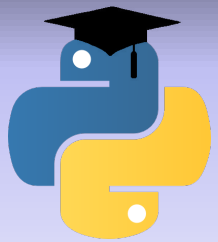


Recherche

Rechercher un élément :

- partir de la racine



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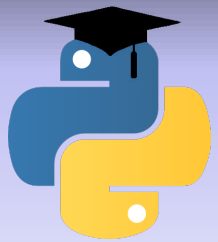
Rechercher un élément :

- partir de la racine
- comparer la valeur recherchée à l'élément courant



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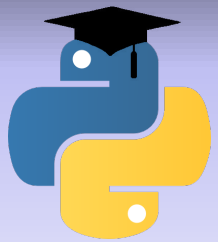
    - si elle est égale : fin



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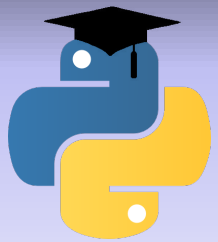
  - si elle est inférieure : chercher dans le sous-arbre gauche



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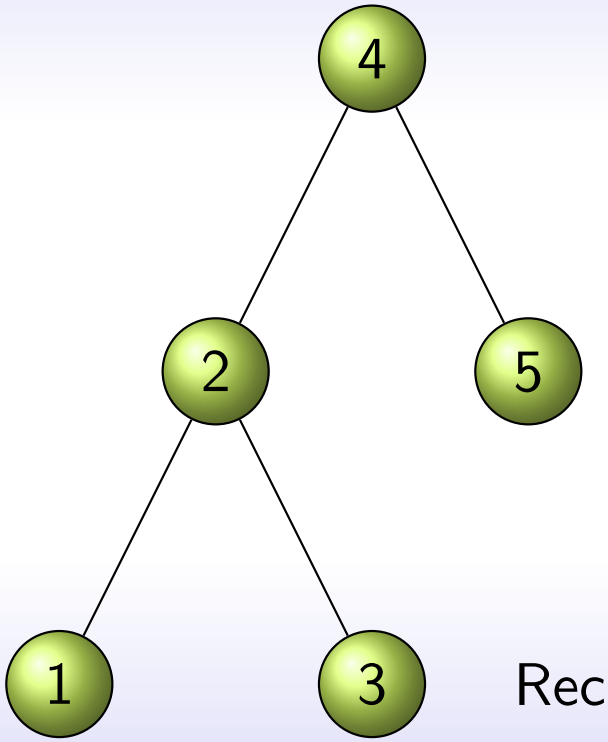
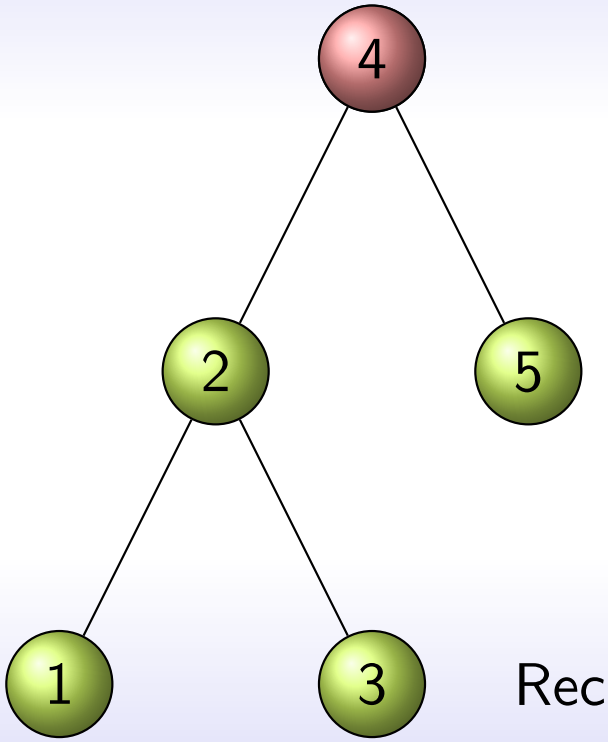
  - si elle est inférieure : chercher dans le sous-arbre gauche
  - sinon : chercher dans le sous-arbre droit

Exemple



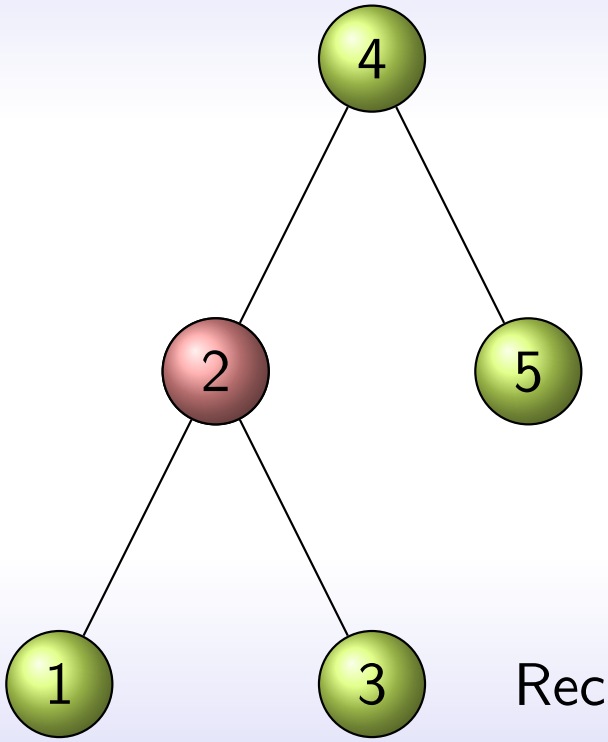
Recherche : 3

Exemple



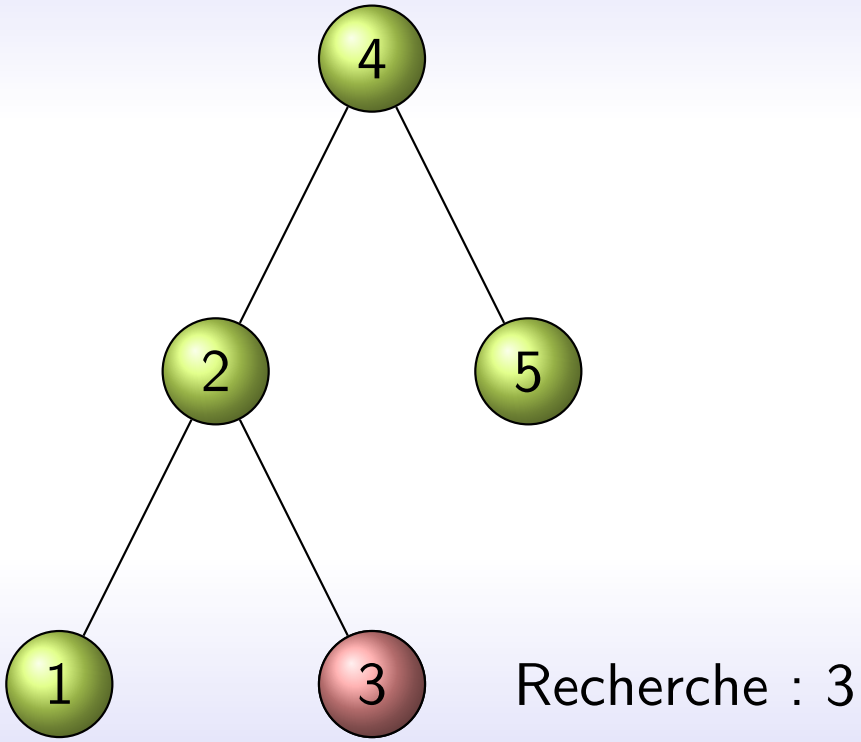
Recherche : 3

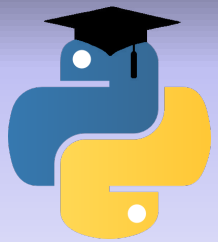
Exemple



Recherche : 3

Exemple

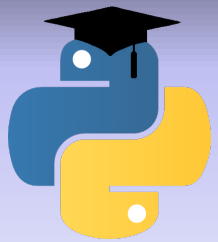




Complexité

Étude de l'algorithme :

- pire des cas : aller jusqu'à une feuille



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Étude de l'algorithme :

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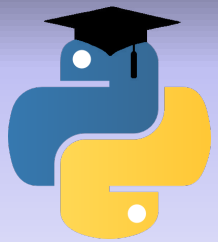
- complexité hauteur de l'arbre



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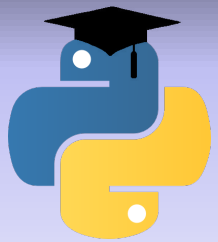
$\Rightarrow$ arbre complet : $\Theta(\log n)$



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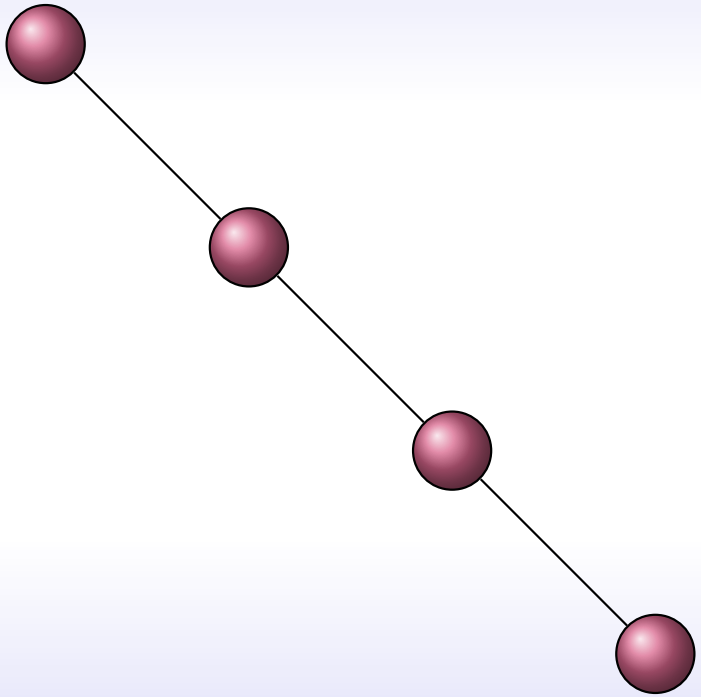
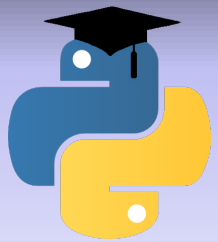
$\Rightarrow$ arbre complet : $\Theta(\log n)$

- arbre le pire : peigne

Complexité





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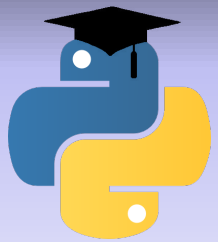
- arbre le pire : peigne
- complexité : $\Theta(n)$



Algorithme itératif

Algorithme 1: Recherche itérative dans un ABR

Entrées : entier val

$n \leftarrow racine$;

tant que $n \neq None$ **et** $n.val \neq val$ **faire**

    si $val < n.val$ **alors**

        $n \leftarrow n.filsGauche$;

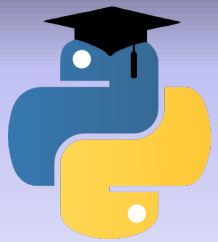
    sinon

        $n \leftarrow n.filsDroit$;

    fin

fin

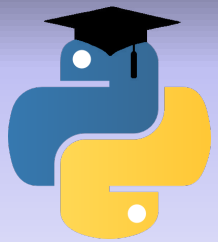
retourner n `// n a la valeur recherchée ou n vaut None`



Algorithme récursif

Fonction rechRec(entier val, Noeud cour)

si *cour* = *None* **ou** *cour.val* = *val* **alors**
    retourner cour;
sinon si *val* < *cour.val* **alors**
    retourner rechRec(val, n.filsGauche);
sinon
    retourner rechRec(val, n.filsDroit);
fin



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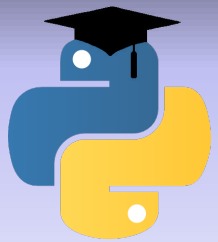
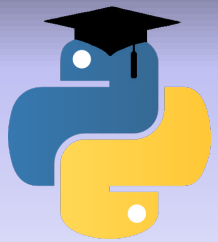


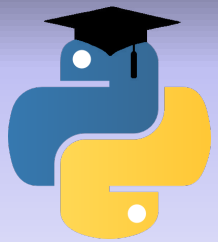
Principe

- parcourir un arbre : appliquer un traitement à tous les noeuds



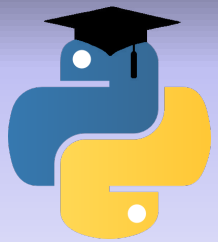
Principe

- parcourir un arbre : appliquer un traitement à tous les noeuds
- algorithme itératif : difficile à écrire



Principe

- parcourir un arbre : appliquer un traitement à tous les noeuds
- algorithme itératif : difficile à écrire
- algorithme récursif : très simple



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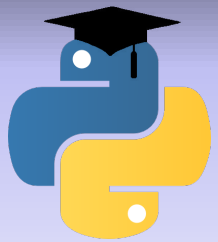
  - principe : 3 traitements $\Rightarrow$ 2 sous-arbres et noeud courant



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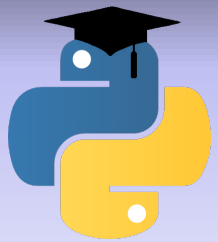
  - noeud courant avant le fils gauche $\Rightarrow$ préfixe



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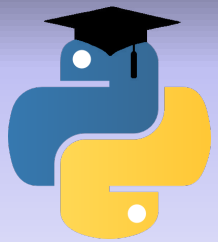
  - noeud courant avant le fils gauche $\Rightarrow$ préfixe
  - noeud courant entre les fils $\Rightarrow$ infixe



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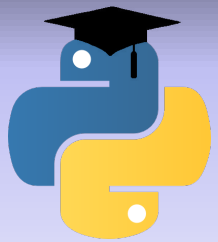
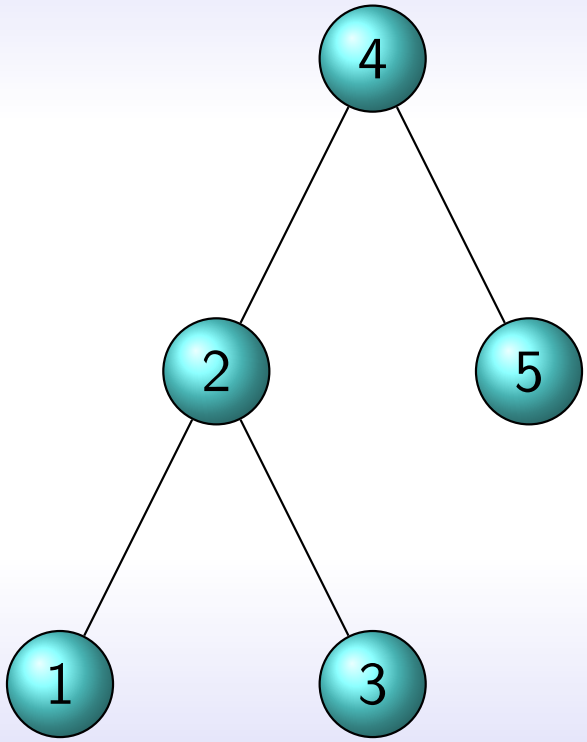
  - principe : 3 traitements $\Rightarrow$ 2 sous-arbres et noeud courant
  - noeud courant avant le fils gauche $\Rightarrow$ préfixe
  - noeud courant entre les fils $\Rightarrow$ infixe
  - noeud courant après le fils droit $\Rightarrow$ postfixe



Exemple : parcours infixe



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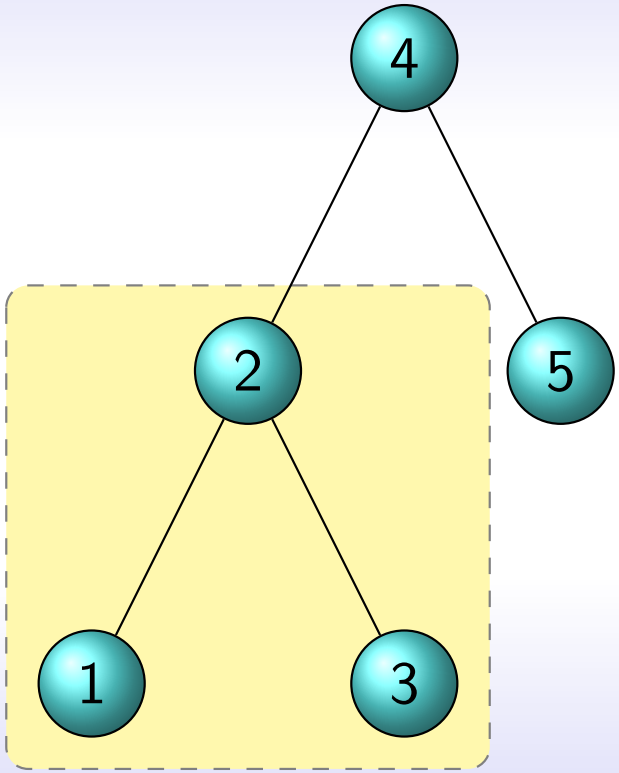

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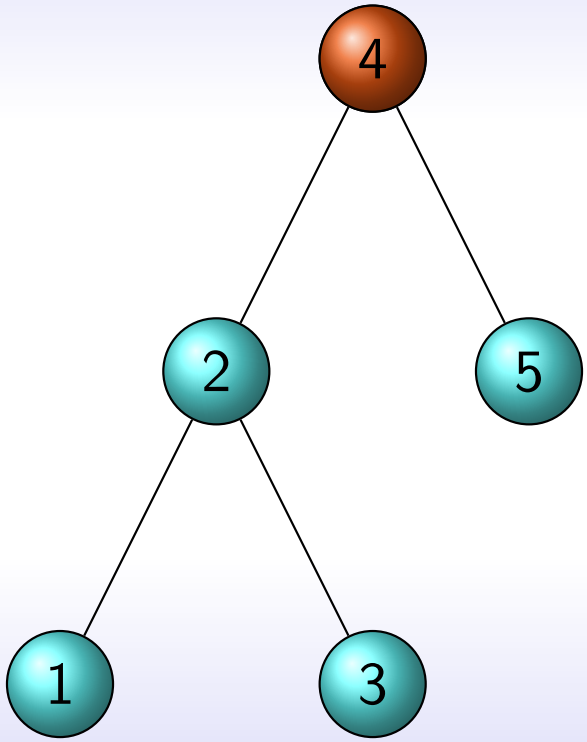

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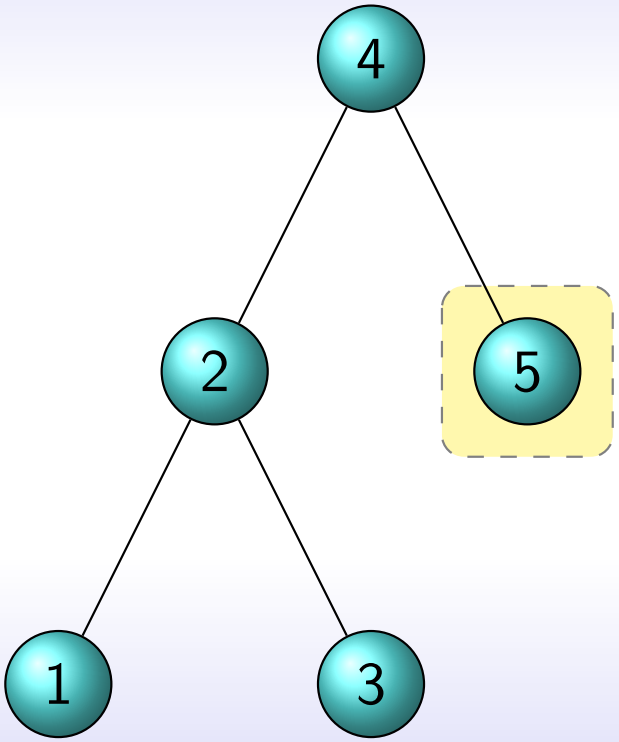

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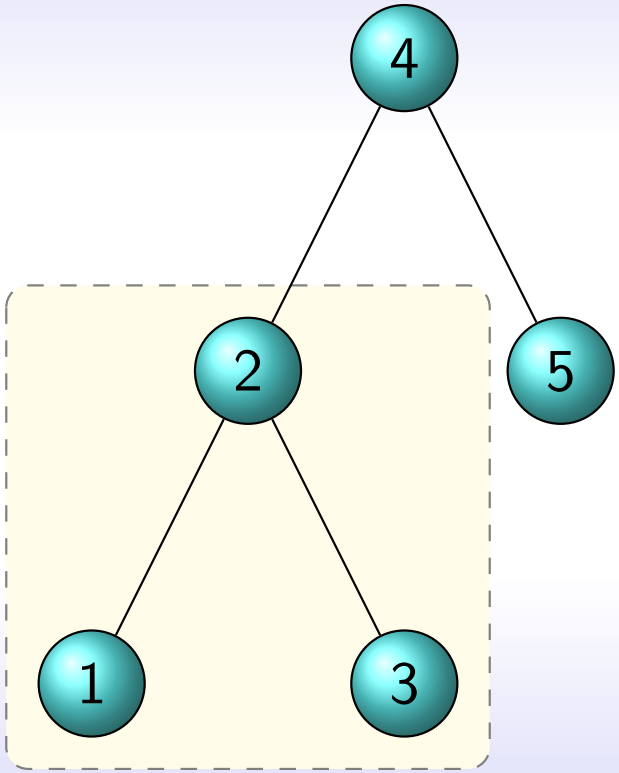

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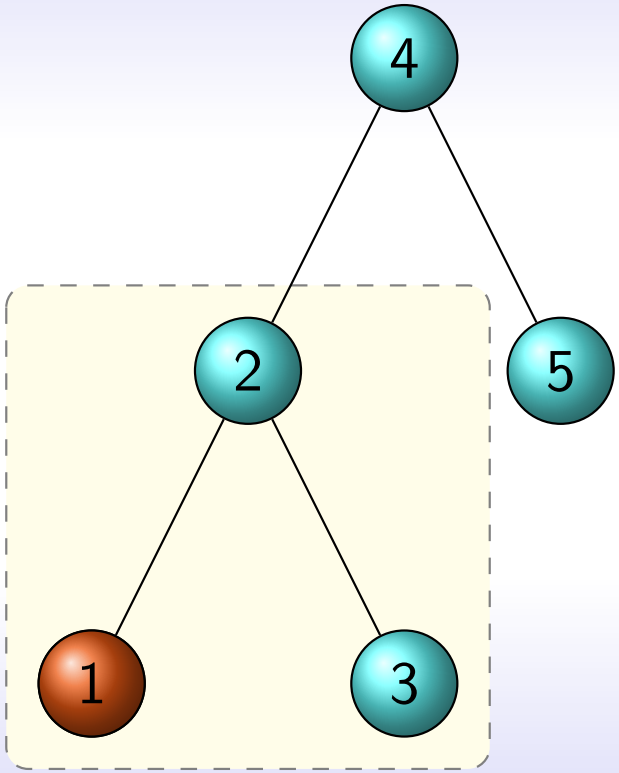
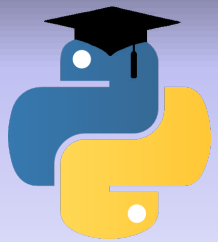
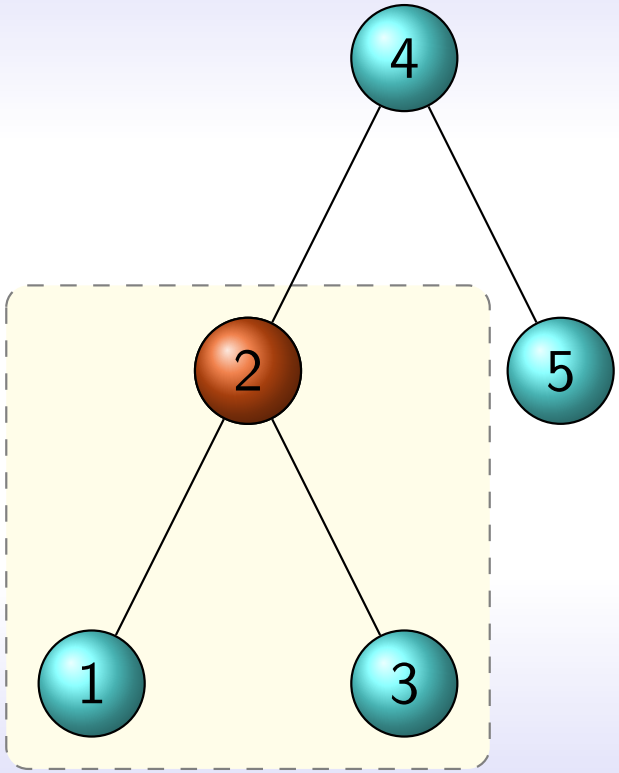




Exemple : parcours infixe



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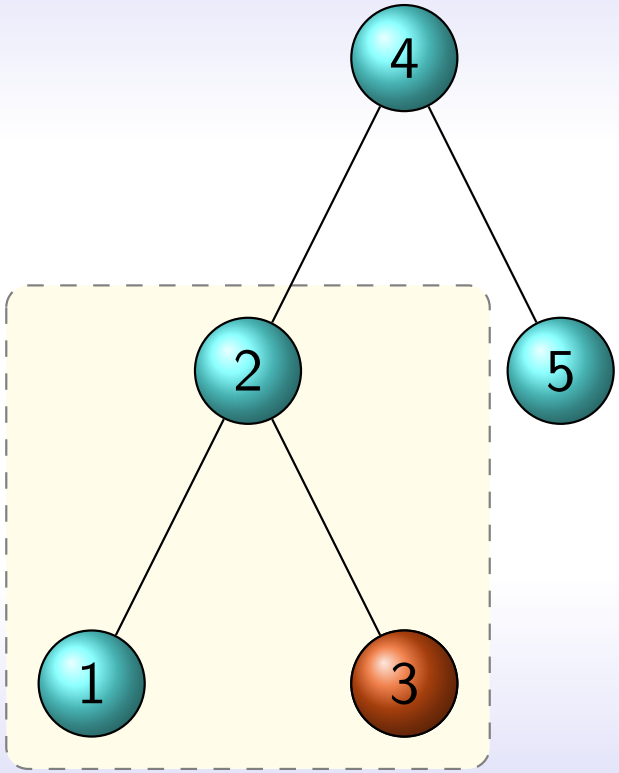
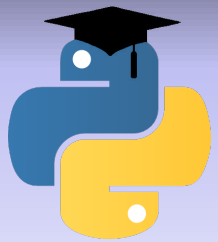
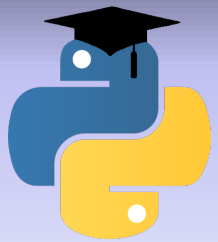




Algorithmes

Procédure infixe(Noeud cour)

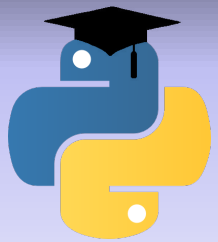
si *cour* $\neq$ *None* **alors**
    infixe(cour.filsGauche);
    traiter **cour** ;
    infixe(cour.filsDroit);
fin



Algorithmes

Procédure prefixe(Noeud cour)

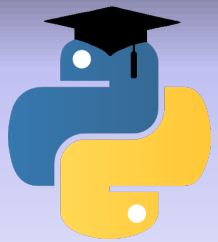
si *cour* $\neq$ *None* **alors**
    traiter **cour** ;
    prefixe(cour.filsGauche);
    prefixe(cour.filsDroit);
fin



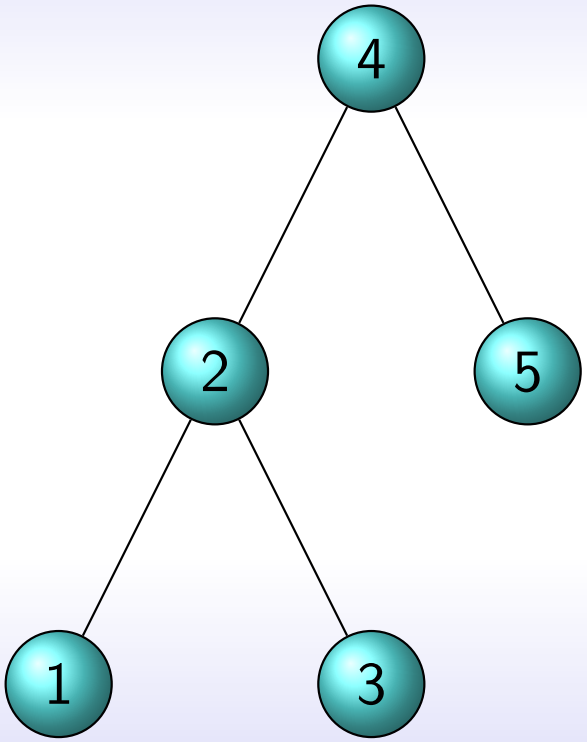
Algorithmes

Procédure postfixe(Noeud cour)

si *cour* $\neq$ *None* **alors**
    postfixe(cour.filsGauche);
    postfixe(cour.filsDroit);
    traiter **cour** ;
fin



Exemple : parcours infixe



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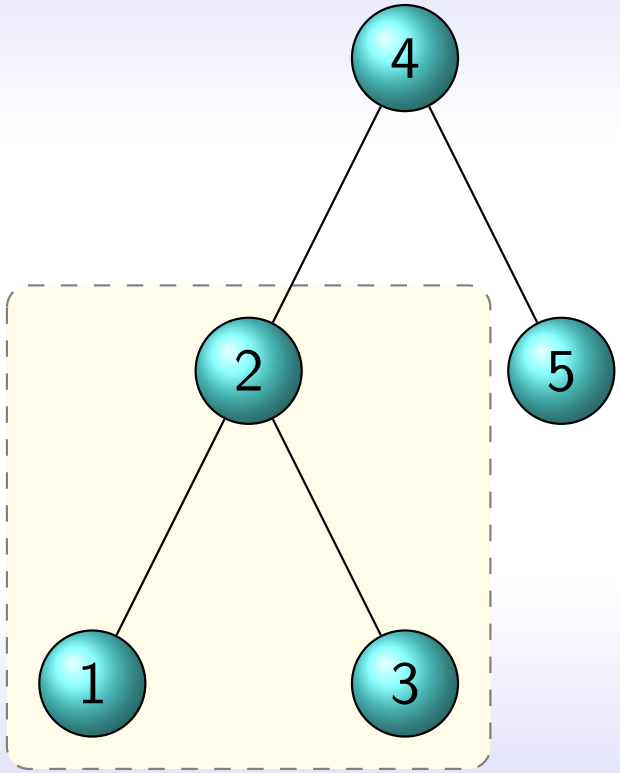
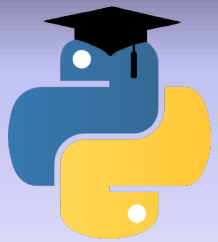
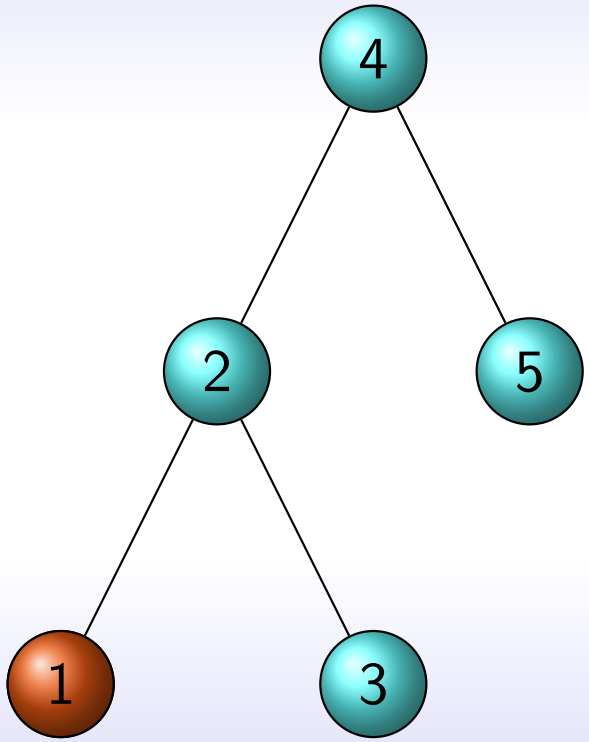


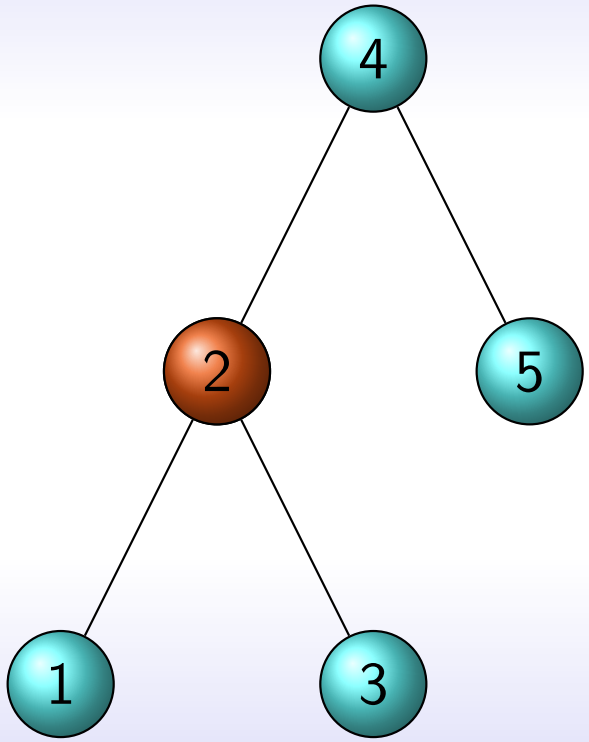


Exemple : parcours infixe



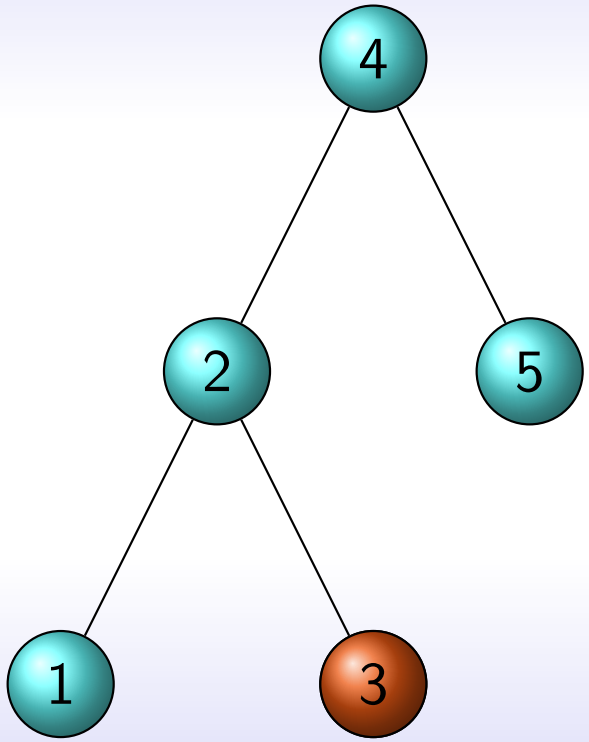
1

Exemple : parcours infixe



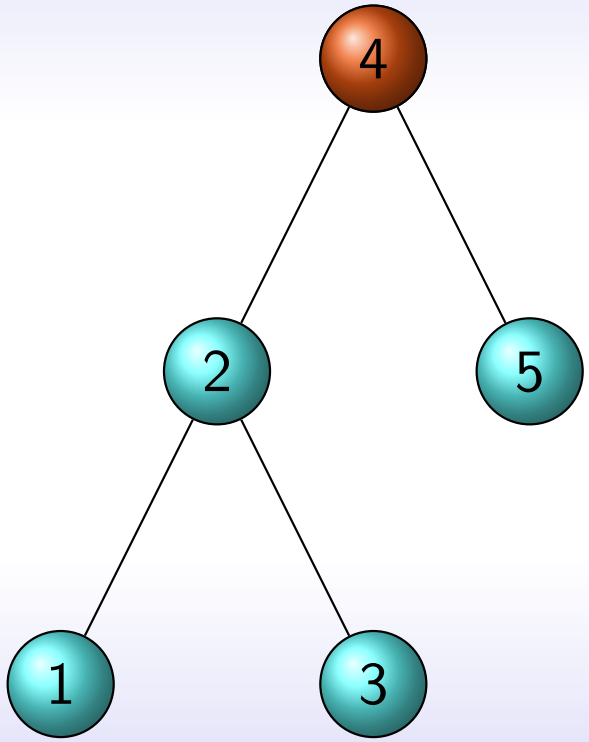
1 2

Exemple : parcours infixe



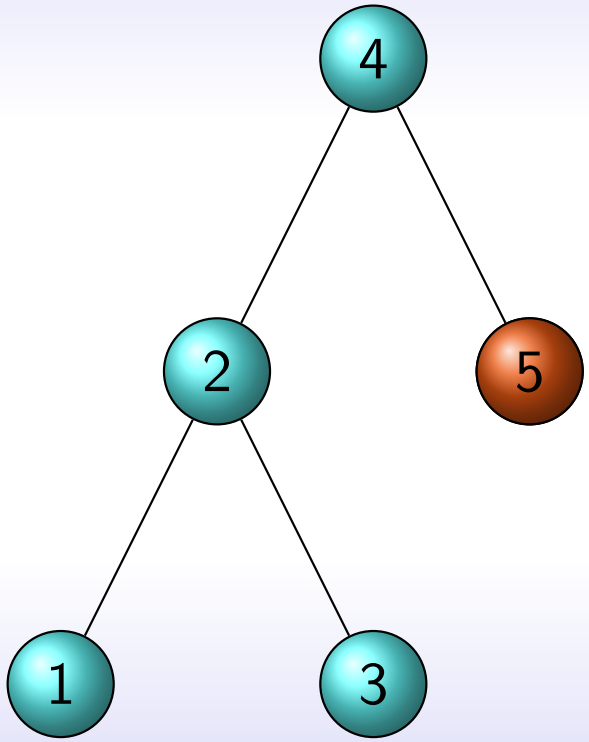
1 2 3

Exemple : parcours infixe



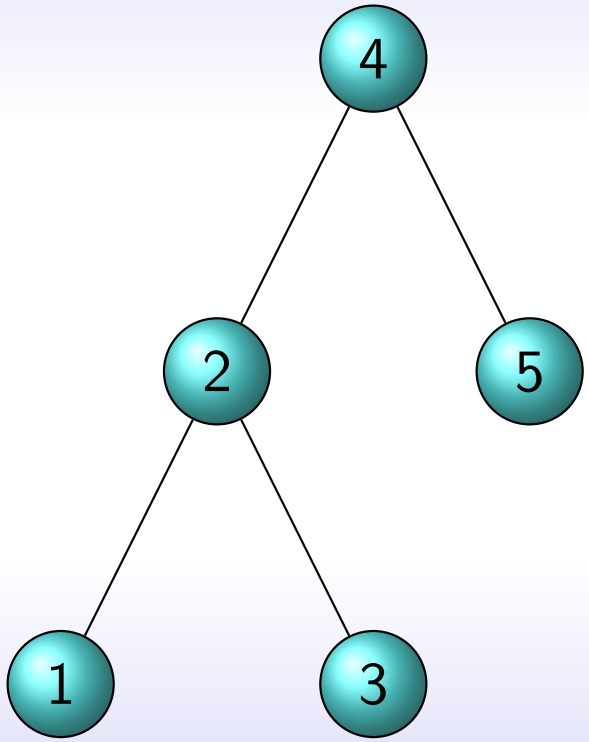
1 2 3 4

Exemple : parcours infixe

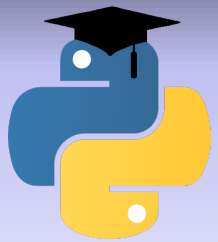


1 2 3 4 5

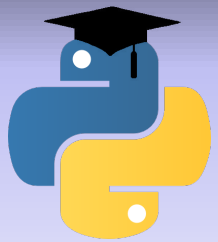
Exemple : parcours infixe



1 2 3 4 5



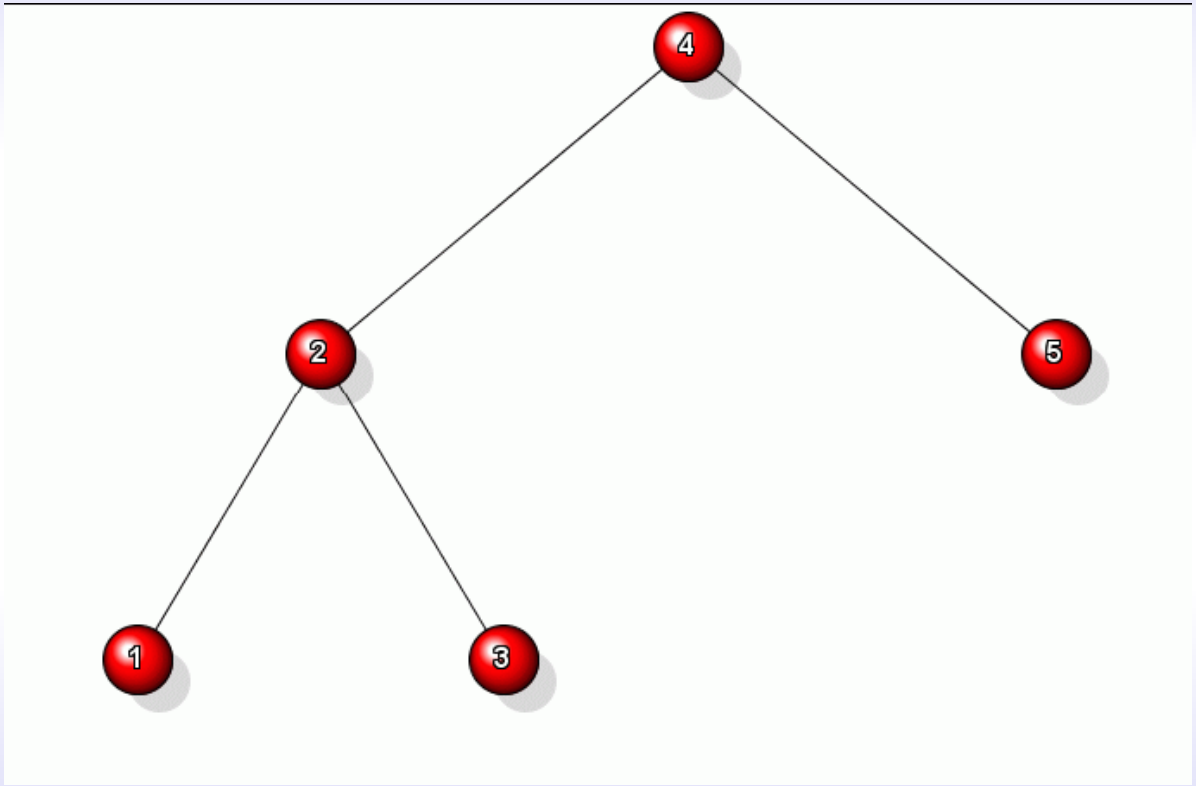
Sommaire

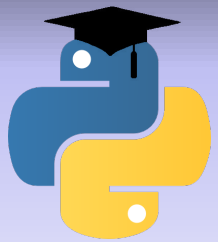


Principe

- Insertion uniquement sur les feuilles
- Principe :
  - se placer sur la bonne feuille (déplacement itératif ou récursif)
  - ajouter le fils
- principe similaire à l'insertion dans une liste

Demo

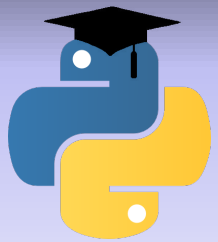




Algorithme

Algorithme 2: Insertion dans un arbre binaire de recherche

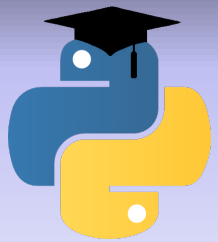
Entrées : noeud val
si *arbre* $= \emptyset$ **alors**
  racine $\leftarrow$ *val*;
sinon
  insert(val, racine) `// insère val sous la racine`
fin



Algorithme

Procédure insert(noeud val,noeud pere)

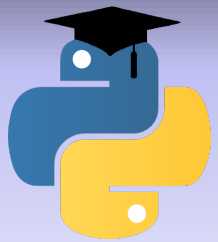
```
n ← pere;
tant que n ≠ None faire
    pere ← n;
    si val < n alors
        n ← n.filsGauche;
    sinon
        n ← n.filsDroit;
    fin
fin
// maintenant le fils de pere est None
```



Algorithme

Procédure insert(noeud val,noeud pere)

```
// ... maintenant le fils de pere est None
si val < pere alors
    pere.filsGauche ← val;
sinon
    pere.filsDroit ← val;
fin
```



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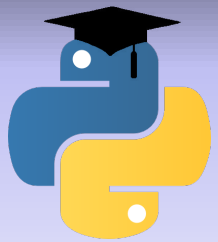
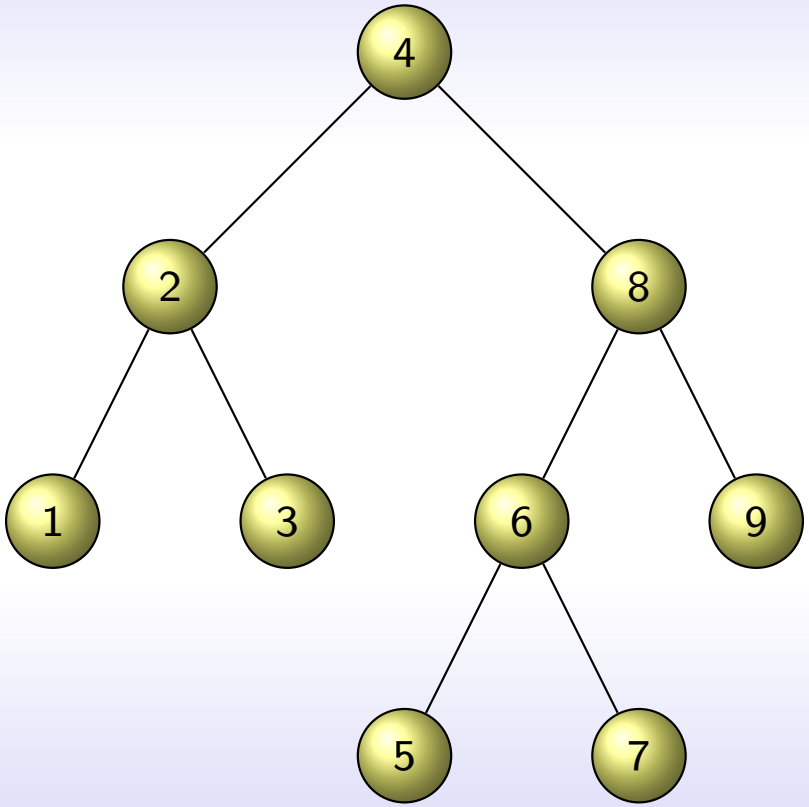
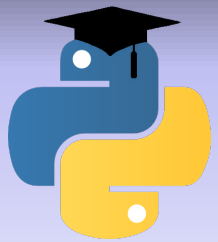


Principe

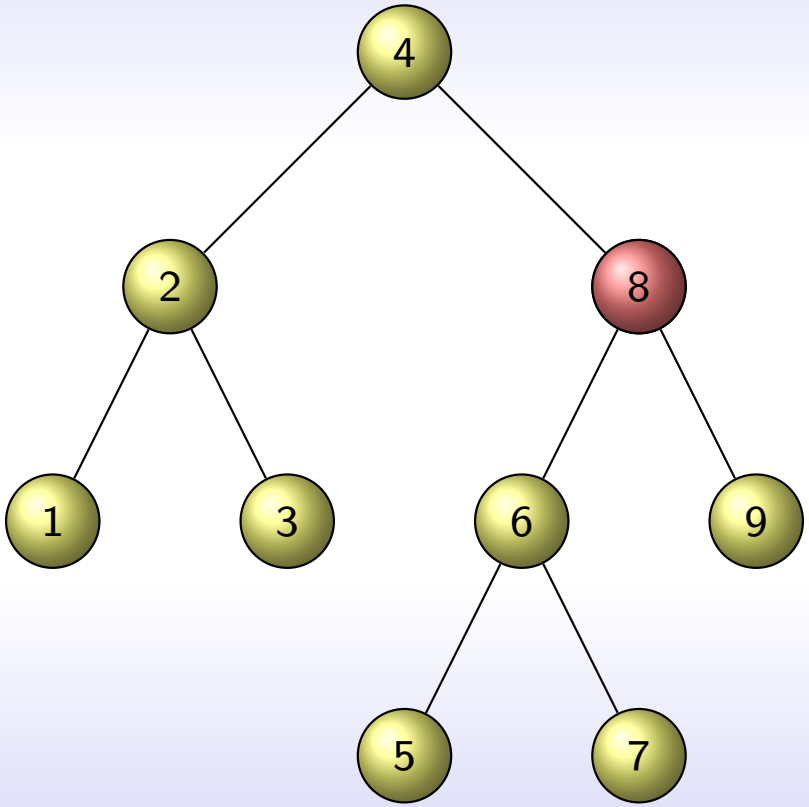
- se placer au dessus de l'élément à supprimer
- remplacer le noeud par le sous-arbre gauche
- réinsérer le sous-arbre droit

Exemple

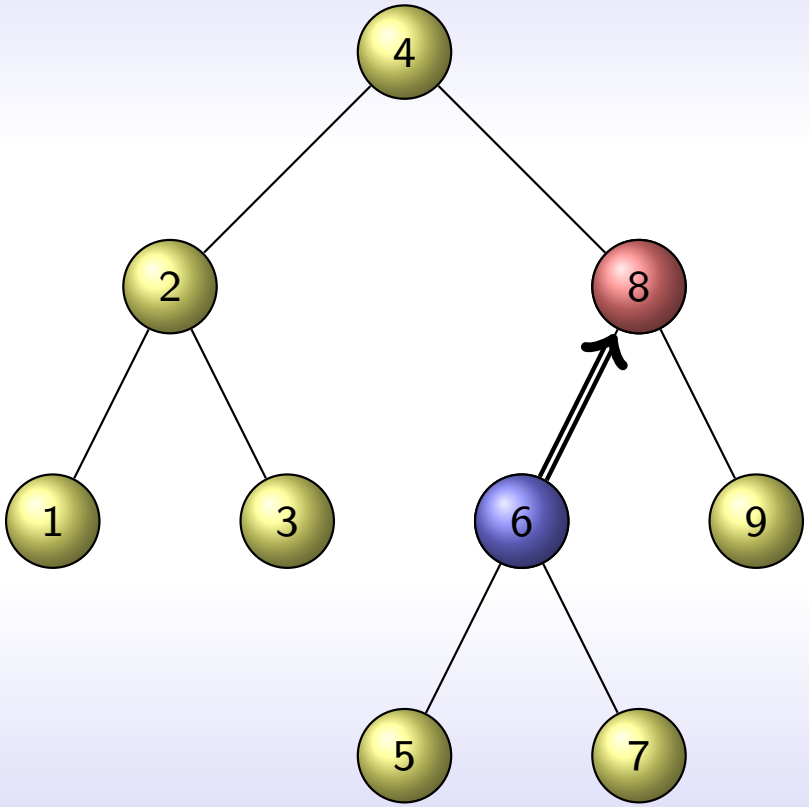


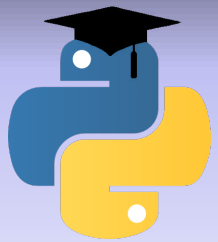


Exemple



Exemple





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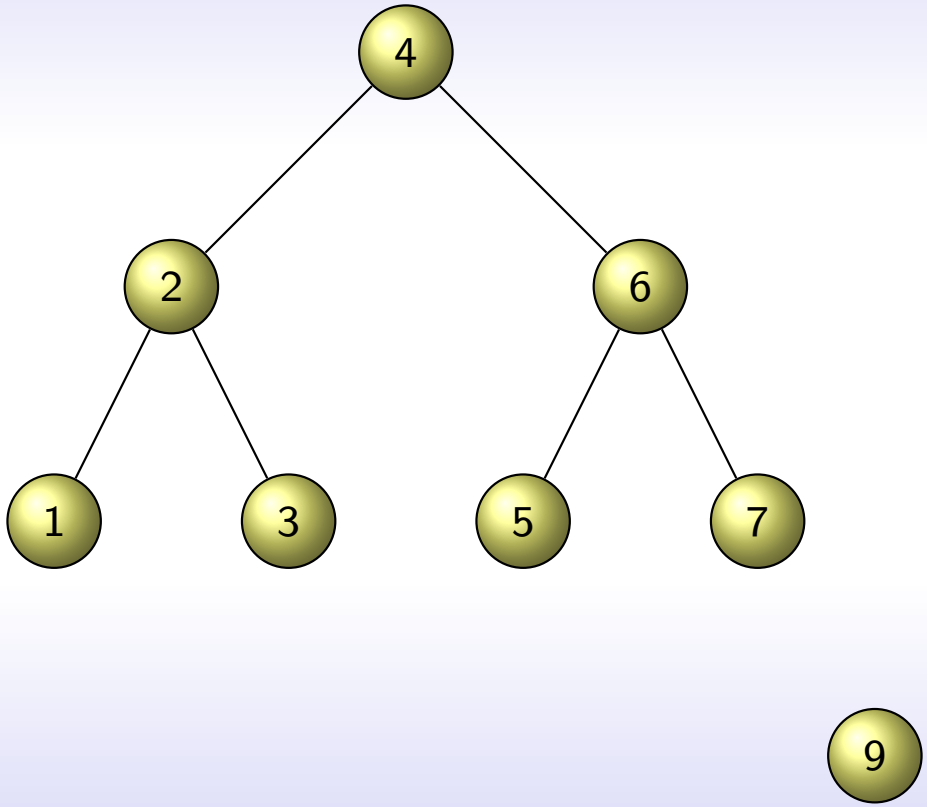
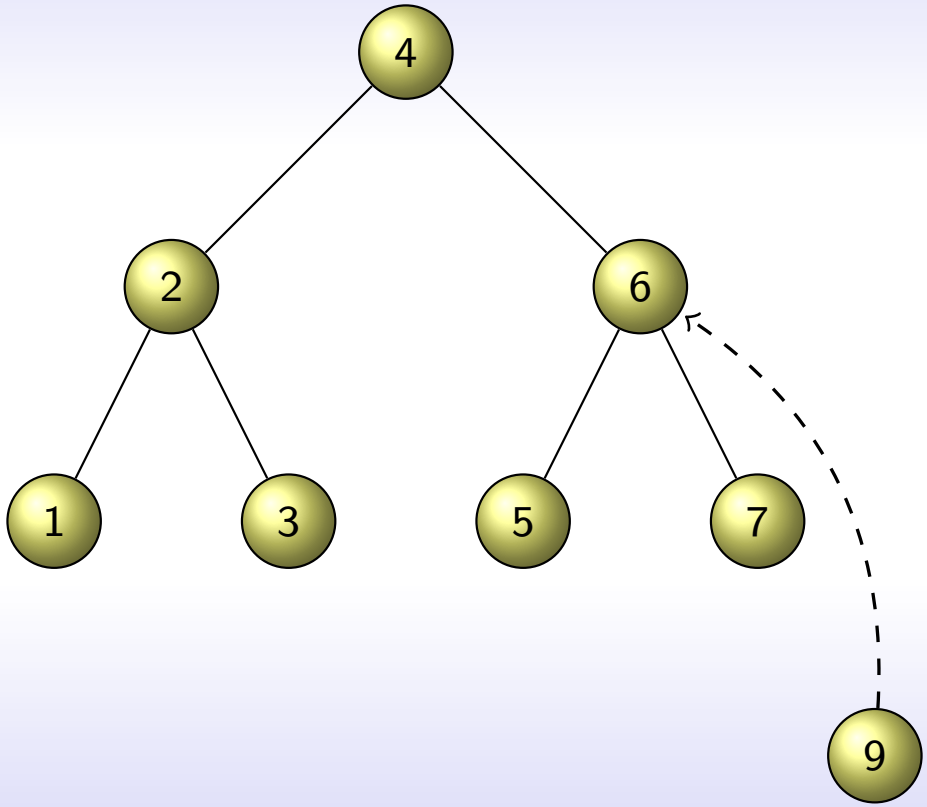
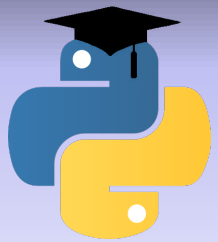
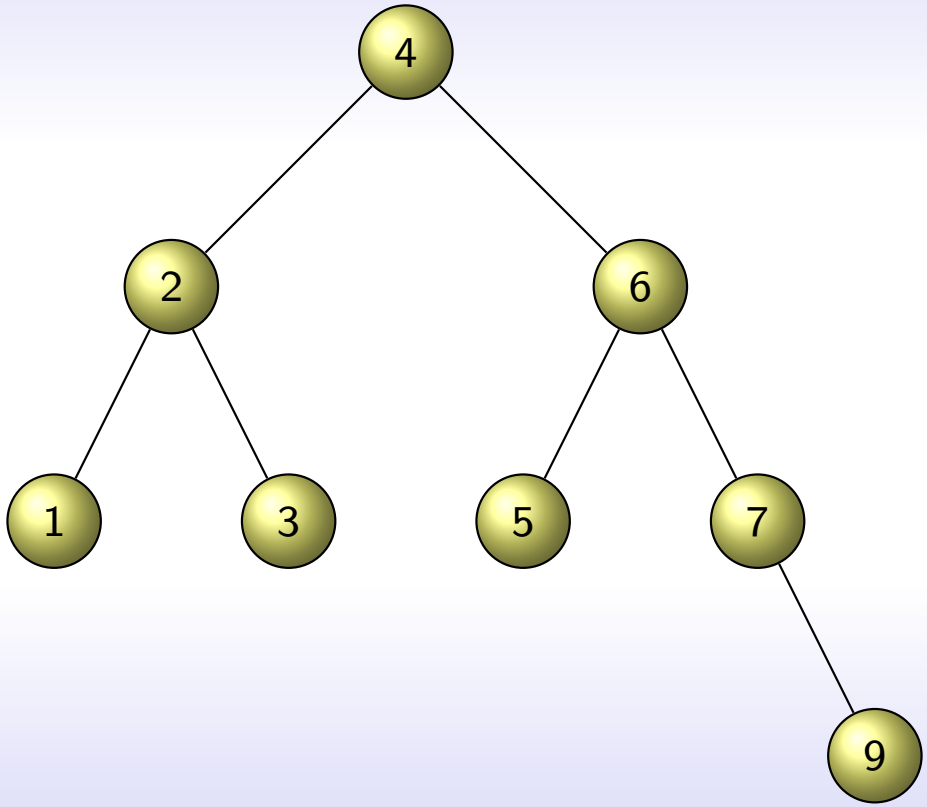


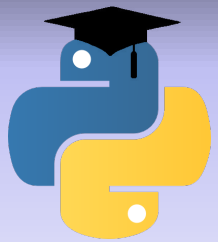
Exemple





Exemple





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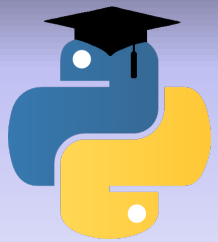

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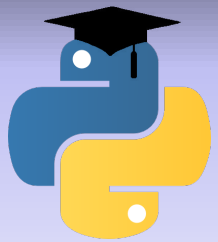


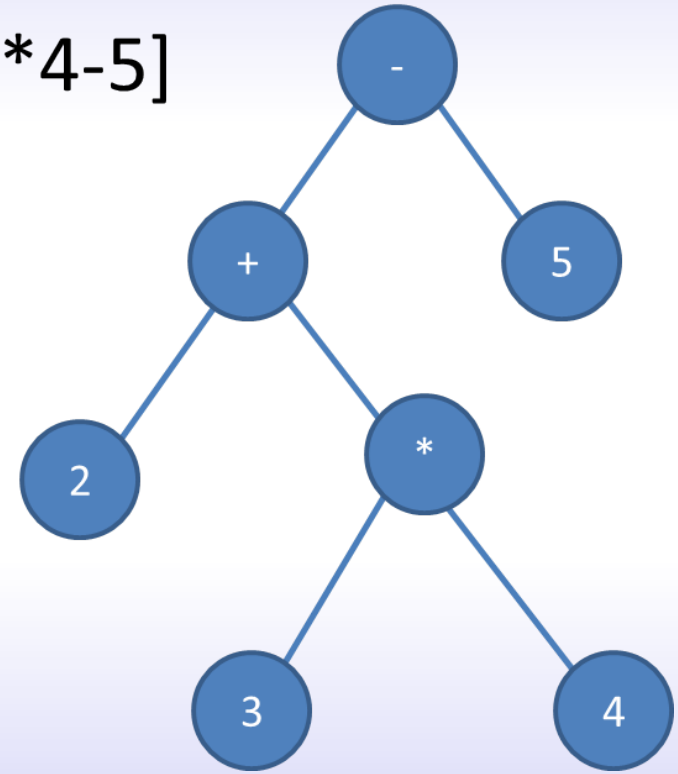
Arbre syntaxique

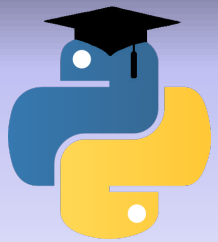
- Arbre représentant un phrase
- ou un programme ou une partie d'un programme
- exemple : expression arithmétique
- Sert à analyser la phrase/le programme

Exemple : $2 + 3 * 4 - 5$

Arbre syntaxique

[2+3*4-5]





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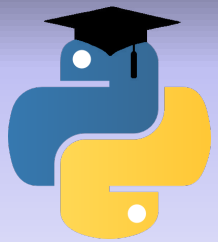
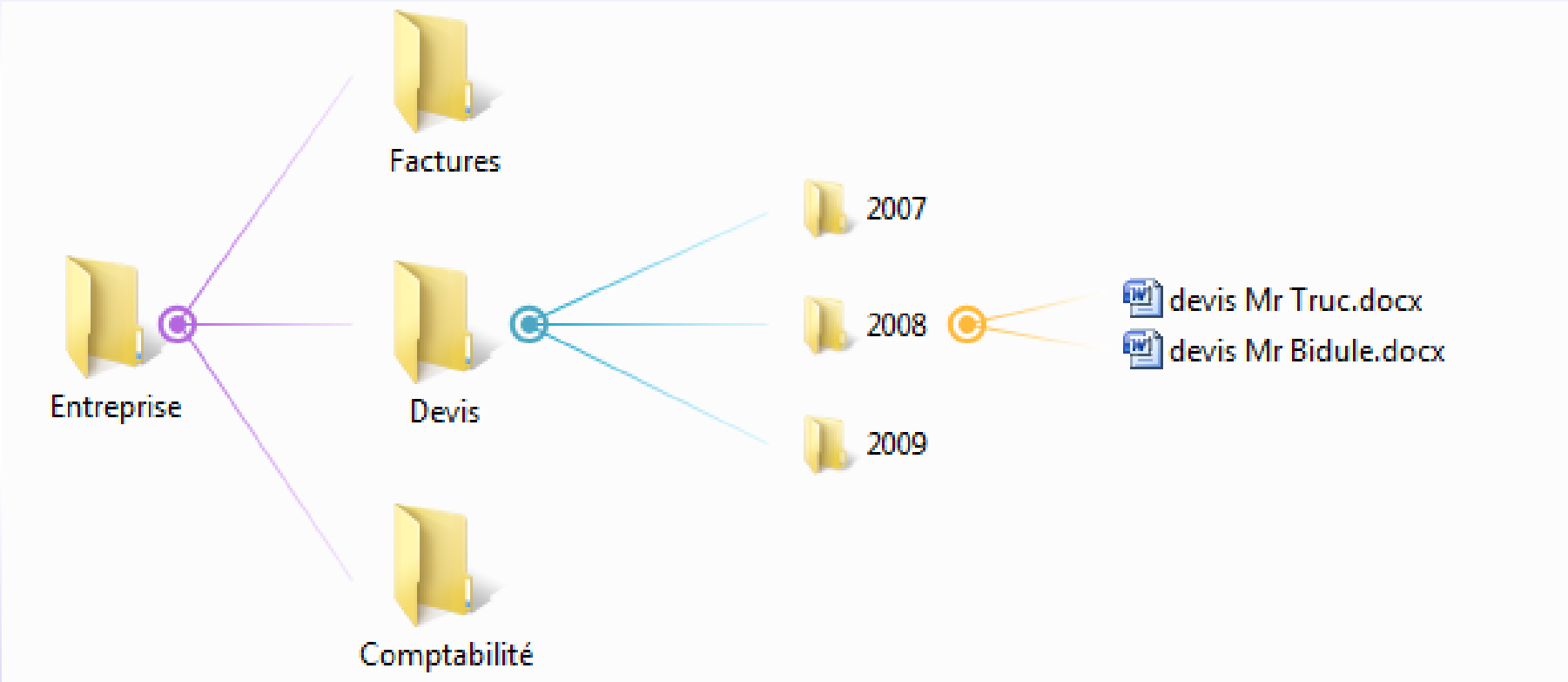


Fichiers

- Représentation sous forme d'arbre général
- Fichiers : feuilles
- Répertoires : noeuds intermédiaires

Fichiers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Entreprise
Devis
2007
2008
2009
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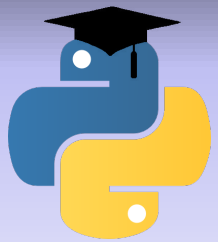
devis Mr Bidule.docx
Comptabilité



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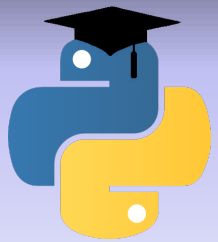
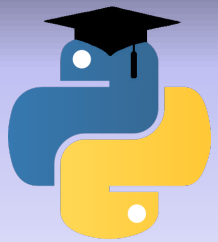


Intérêt

- Arbre binaire : efficace si complet
- Arbre binaire complet : pas toujours possible

$\Rightarrow$ notion d'arbre équilibré



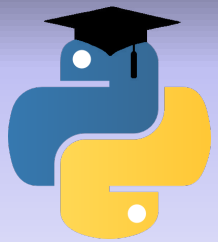
Intérêt

- Arbre binaire : efficace si complet
- Arbre binaire complet : pas toujours possible

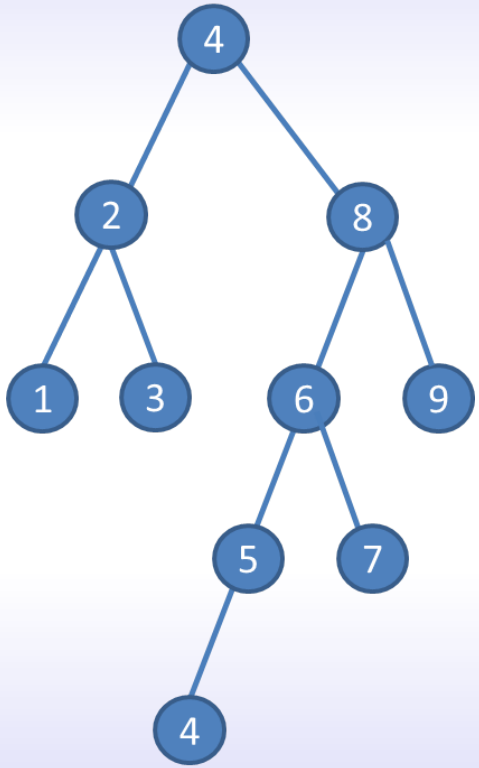
$\Rightarrow$ notion d'arbre équilibré

Définition (Arbre équilibré)

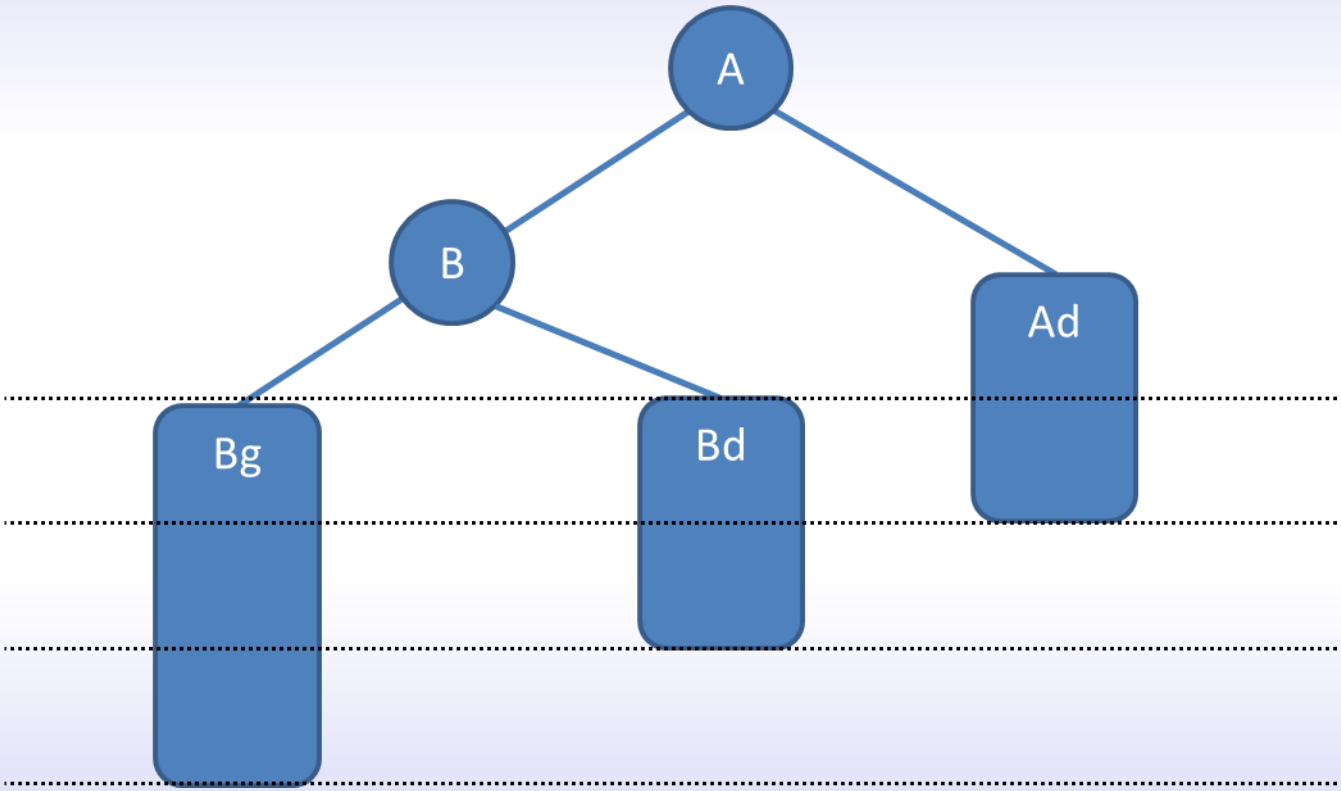
Arbre binaire est équilibré : la différence entre les hauteurs des fils gauche et droit de tout noeud ne peut excéder 1.



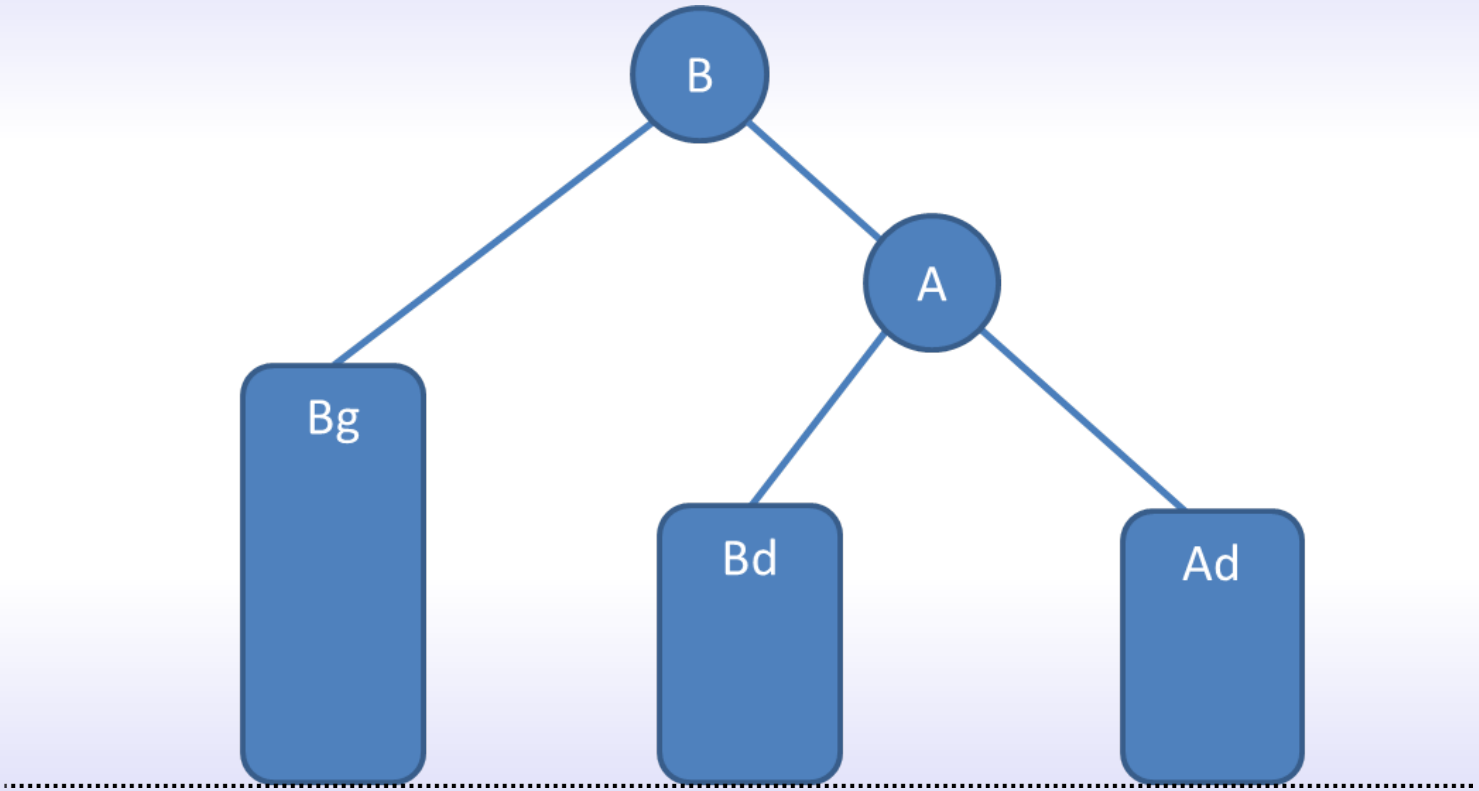
Exemple : arbre déséquilibré



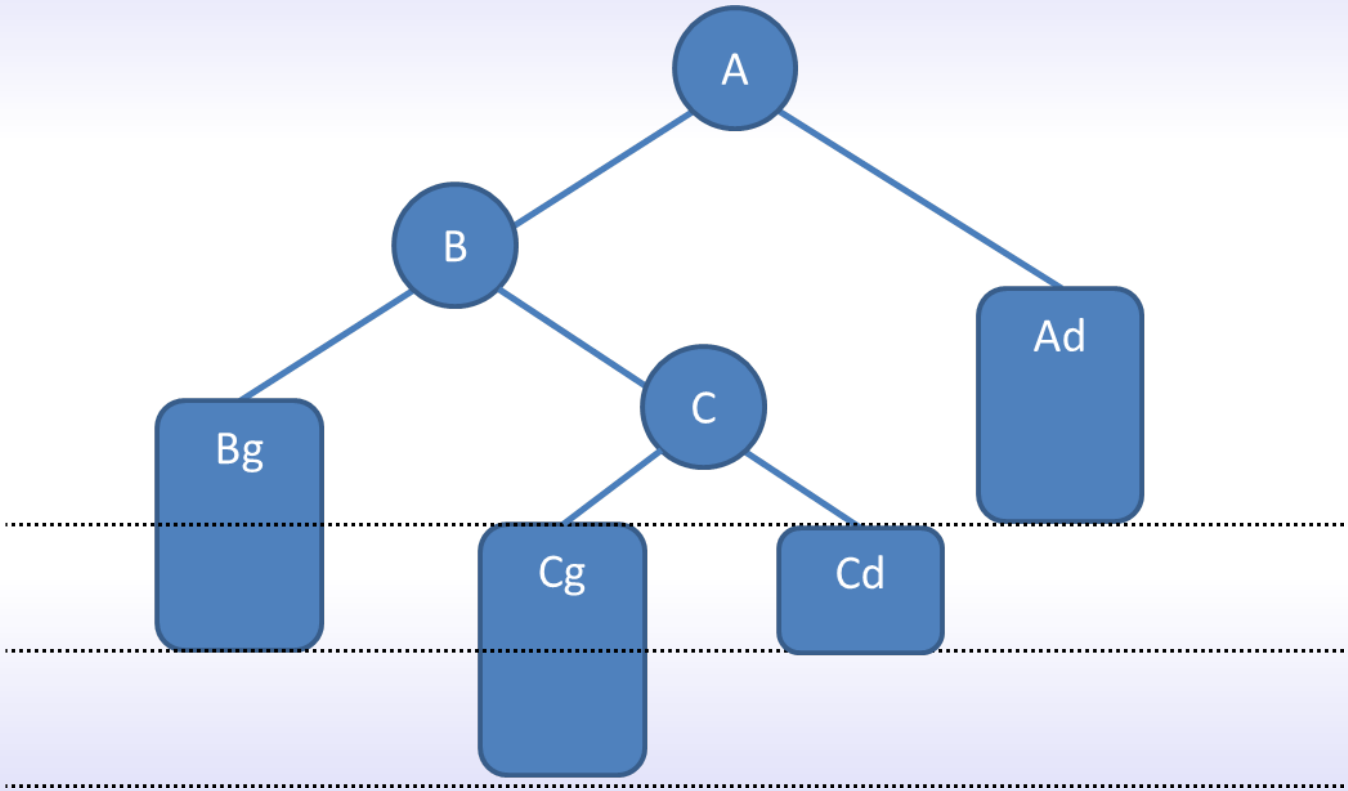
Équilibrage d'un arbre



Équilibrage d'un arbre



Équilibrage d'un arbre



Équilibrage d'un arbre