# Report on Max-Cut

May 13, 2025

## 1 Introduction

### 1.1 The Max-Cut Problem

Given an undirected graph, $G = (V, U)$, where $V$ is the set of vertices and $U$ is the set of edges, and weights $w_{uv}$ associated with each edge $(u, v) \in U$, the maximum cut (MAX-CUT) problem consists in finding a nonempty proper subset of vertices $S \subset V$ ($S \neq \emptyset$), such that the weight of the cut $(S, S')$, given by

$$w(S, S') = \sum_{u \in S, v \in S'} w_{uv}$$

is maximized.

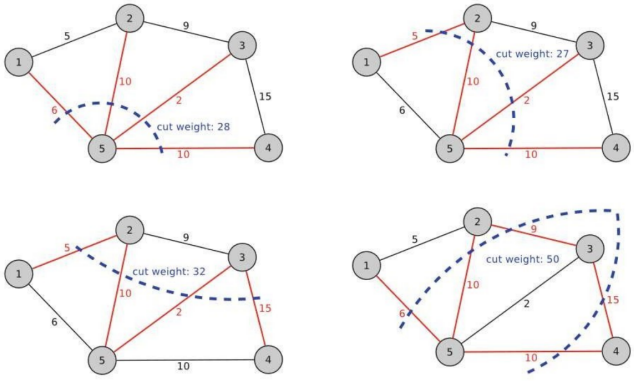


Figure 1: Example of the maximum cut problem on a graph with five vertices and seven edges. The maximum cut is $(S, S') = (\{1, 2, 4\}, \{3, 5\})$ and has a weight $w(S, S') = 50$.

### 1.2 Tasks Analysis

In this assignment, we have implemented GRASP algorithm (Greedy Randomized Adaptive Search Procedure) to find the best cut which maximizes cut weight.This algorithm has two phase construction phase and local search for optimization.We have implemented three constructive algorithm Greedy,Randomized and Semi greedy.

### 1.3 Randomized Heuristic

The randomized construction phase involves creating initial solutions using a randomized approach.

Initially, both partitions $X$ and $Y$ are empty. For each vertex $v \in V$, place $v$ in partition $X$ or $Y$ with uniform randomness (i.e., with probability $\frac{1}{2}$). The procedure terminates when all vertices are placed either in $X$ or $Y$.

As the construction phase is random, it may sometimes outperform greedy and semi-greedy approaches in terms of local optima (max-cut cost), though it might require more iterations to converge.

### 1.4 Greedy Heuristic

The greedy construction phase relies on a deterministic heuristic that makes locally optimal decisions at each step. It was run for only one iteration (construction followed by local search).

This algorithm converges faster due to its effective heuristic choices. However, the greedy approach may get stuck in local optima, making the final cut weight lower than more exploratory algorithms like GRASP.

### 1.5 Semi-greedy Heuristic

The semi-greedy construction phase combines aspects of both randomization and greedy heuristics.

This balances exploration and exploitation. The average number of iterations and local optima falls between the randomized and greedy approaches, reflecting a trade-off between quality and convergence speed.

## 2 Summary

| **Construction** | **Initial Solution Quality** | **Efficiency** | **Limitations** |
|---|---|---|---|
| Simple Randomized | Random | Bad | Inconsistent Performance |
| Simple Greedy | Good | Good | Traps in Local Optima |
| Semi-Greedy | Moderate | Moderate | Both to some extent |

# 3 Plots or visualizations

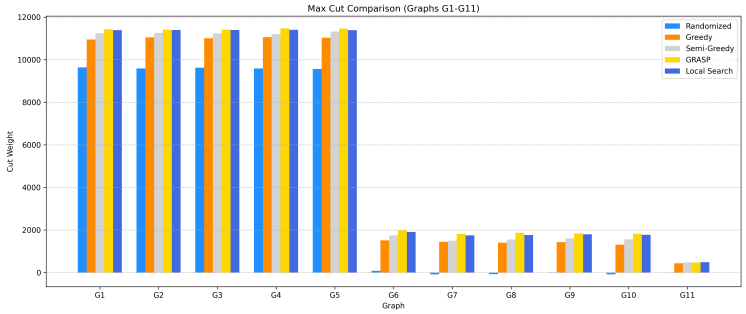


Figure 2: Comparison plot for graph 1-11

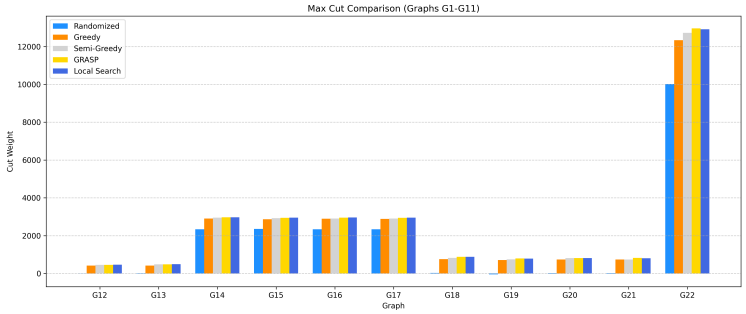


Figure 3: Comparison plot for graph 12-22

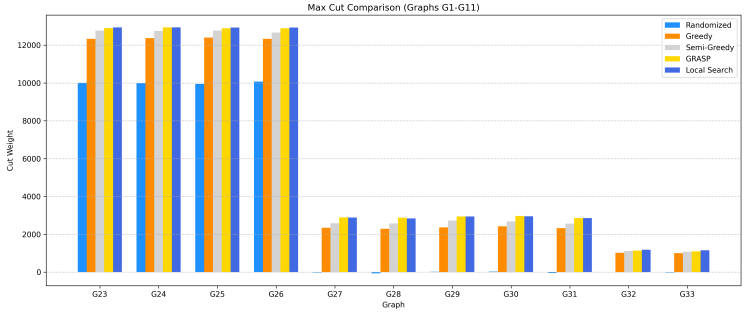


Figure 4: Comparison plot for graph 23-33

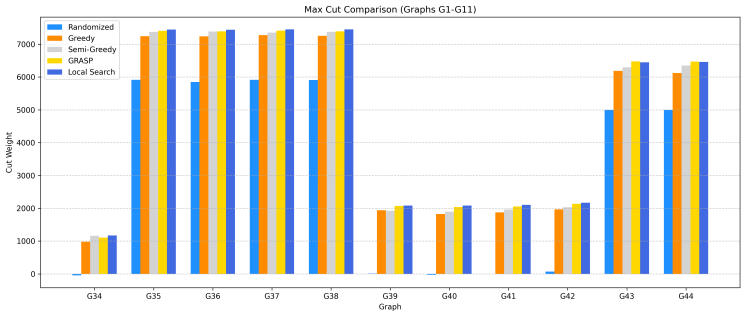


Figure 5: Comparison plot for graph 34-44

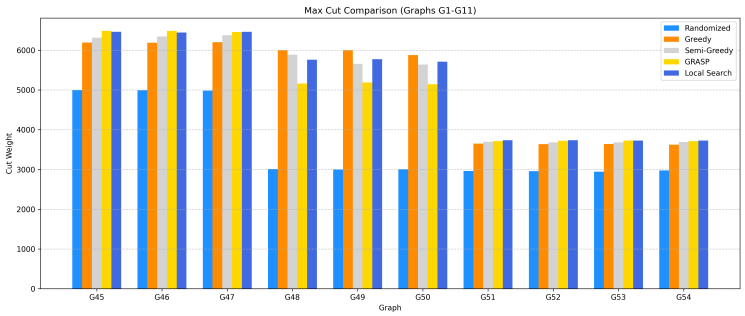


Figure 6: Comparison plot for graph 45-54