ICCS413 - Lecture 20

Association Rules & Frequent Itemsets

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Ref Credit: Association Rules & Sequential Patterns by Bing Liu, UIC

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Today

- Association Rules Mining
- Frequent Itemsets
- Apriori Algorithm

Association rule mining

- Proposed by Agrawal et al in 1993.
- It is an important data mining model studied extensively by the database and data mining community.
- Assume all data are categorical.
- No good algorithm for numeric data.
- Initially used for Market Basket Analysis to find how items purchased by customers are related.

Bread $\rightarrow$ Milk [sup = 5%, conf = 100%]

The model: data

- $I = \{i_1, i_2, \ldots, i_m\}$: a set of *items*.
- Transaction t :
  - t a set of items, and $t \subseteq I$.
- Transaction Database T: a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$.

Transaction data: supermarket data

- Market basket transactions:
  - t1: {bread, cheese, milk}
  - t2: {apple, eggs, salt, yogurt}
  - … …
  - tn: {biscuit, eggs, milk}
- Concepts:
  - An *item*: an item/article in a basket
  - *I*: the set of all items sold in the store
  - A *transaction*: items purchased in a basket; it may have TID (transaction ID)
  - A *transactional dataset*: A set of transactions

Transaction data: a set of documents

- **A text document data set. Each document is treated as a “bag” of keywords**

doc1:	Student, Teach, School
doc2:	Student, School
doc3:	Teach, School, City, Game
doc4:	Baseball, Basketball
doc5:	Basketball, Player, Spectator
doc6:	Baseball, Coach, Game, Team
doc7:	Basketball, Team, City, Game

The model: rules

- A transaction *t* contains *X*, a set of items (itemset) in *I*, if $X \subseteq t$.
- An association rule is an implication of the form:

$$X \rightarrow Y, \text{ where } X, Y \subset I, \text{ and } X \cap Y = \emptyset$$

- An itemset is a set of items.
  - E.g., X = {milk, bread, cereal} is an itemset.
- A *k*-itemset is an itemset with *k* items.
  - E.g., {milk, bread, cereal} is a 3-itemset

Rule strength measures

- Support: The rule holds with support *sup* in T (the transaction data set) if sup% of transactions contain $X \cup Y$.
  - $sup = \Pr(X \cup Y)$.
- Confidence: The rule holds in T with confidence *conf* if *conf*% of transactions that contain X also contain Y.
  - $conf = \Pr(Y \mid X)$
- An association rule is a pattern that states when X occurs, Y occurs with certain probability.

Support and Confidence

- Support count: The support count of an itemset *X*, denoted by *X.count*, in a data set *T* is the number of transactions in *T* that contain *X*. Assume *T* has *n* transactions.
- Then,

$$support = \frac{(X \cup Y).count}{n}$$

$$confidence = \frac{(X \cup Y).count}{X.count}$$

Goal and key features

- **Goal:** Find all rules that satisfy the user-specified *minimum support* (minsup) and *minimum confidence* (minconf).
- **Key Features**
  - Completeness: find all rules.
  - No target item(s) on the right-hand-side
  - Mining with data on hard disk (not in memory)

An example

t1:	Beef, Chicken, Milk
t2:	Beef, Cheese
t3:	Cheese, Boots
t4:	Beef, Chicken, Cheese
t5:	Beef, Chicken, Clothes, Cheese, Milk
t6:	Chicken, Clothes, Milk
t7:	Chicken, Milk, Clothes

- Transaction data
- Assume:

  minsup = 30%
  minconf = 80%
- An example frequent *itemset*:

{Chicken, Clothes, Milk} [sup = 3/7]
- Association rules from the itemset:

Clothes → Milk, Chicken [sup = 3/7, conf = 3/3]

… …

Clothes, Chicken → Milk, [sup = 3/7, conf = 3/3]

Transaction data representation

- A simplistic view of shopping baskets,
- Some important information not considered. E.g,
  - the quantity of each item purchased and
  - the price paid.

Many mining algorithms

- There are a large number of them!!
- They use different strategies and data structures.
- Their resulting sets of rules are all the same.
  - Given a transaction data set *T*, and a minimum support and a minimum confident, the set of association rules existing in *T* is uniquely determined.
- Any algorithm should find the same set of rules although their computational efficiencies and memory requirements may be different.
- We study only one: the Apriori Algorithm

The Apriori algorithm

- **The best known algorithm**
- **Two steps**:
  - Find all itemsets that have minimum support (*frequent itemsets*, also called large itemsets).
  - Use frequent itemsets to generate rules.

- E.g., a frequent itemset

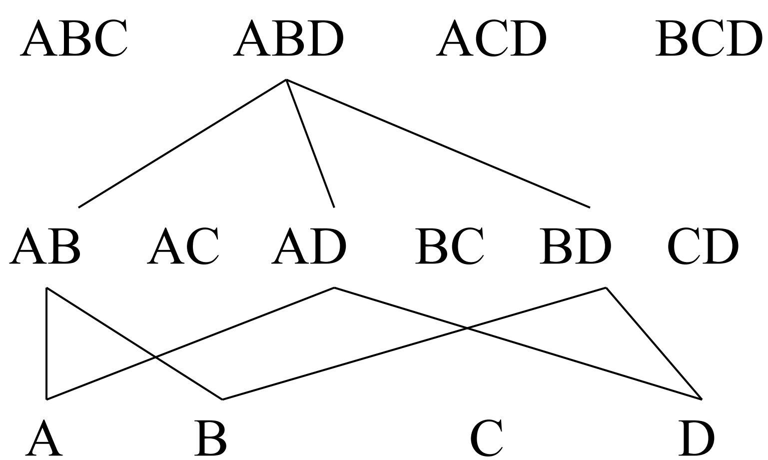
  {Chicken, Clothes, Milk} [sup = 3/7]

  and one rule from the frequent itemset

  Clothes → Milk, Chicken [sup = 3/7, conf = 3/3]

Step 1: Mining all frequent itemsets

- A frequent *itemset* is an itemset whose support is ≥ minsup.
- Key idea: The apriori property (downward closure property): any subsets of a frequent itemset are also frequent itemsets



The Algorithm

- Iterative algo. (also called level-wise search): Find all 1-item frequent itemsets; then all 2-item frequent itemsets, and so on.
  - In each iteration k, only consider itemsets that contain some k-1 frequent itemset.

- Find frequent itemsets of size 1: F_1
- From $k = 2$
  - C_k = candidates of size k: those itemsets of size k that could be frequent, given F_{k-1}
  - F_k = those itemsets that are actually frequent, $F_k \subseteq C_k$ (need to scan the database once).

Example - Finding frequent itemsets

Dataset T
minsup=0.5

TID	Items
T100	1, 3, 4
T200	2, 3, 5
T300	1, 2, 3, 5
T400	2, 5

itemset:count

1. scan T ➔ C_1: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3

➔ F_1: {1}:2, {2}:3, {3}:3, {5}:3

➔ C_2: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}

2. scan T ➔ C_2: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2

➔ F_2: **{1,3}**:2, **{2,3}**:2, **{2,5}**:3, **{3,5}**:2

➔ C_3: {2, 3,5}

3. scan T ➔ C_3: **{2, 3, 5}**:2 ➔ F_3: **{2, 3, 5}**

Details: ordering of items

- The items in *I* are sorted in lexicographic order (which is a total order).
- The order is used throughout the algorithm in each itemset.
- $\{w[1], w[2], \ldots, w[k]\}$ represents a k-itemset w consisting of items $w[1], w[2], \ldots, w[k]$, where $w[1] < w[2] < \ldots < w[k]$ according to the total order.

Details: the algorithm

```
Algorithm Apriori(T)
    C_1 ← init-pass(T);
    F_1 ← {f | f ∈ C_1, f.count/n ≥ minsup};    // n: no. of transactions in T
    for (k = 2; F_{k-1} ≠ ∅; k++) do
        C_k ← candidate-gen(F_{k-1});
        for each transaction t ∈ T do
            for each candidate c ∈ C_k do
                if c is contained in t then
                    c.count++;
            end
        end
        F_k ← {c ∈ C_k | c.count/n ≥ minsup}
    end
return F ← U_k F_k;
```

Apriori candidate generation

- The candidate-gen function takes F_{k-1} and returns a superset (called the candidates) of the set of all frequent *k*-itemsets. It has two steps
  - *join* step: Generate all possible candidate itemsets C_k of length *k*
  - *prune* step: Remove those candidates in C_k that cannot be frequent.

Candidate-gen function

Function candidate-gen($F_{k\text{-}1}$)

$C_k \leftarrow \emptyset$;

forall $f_1, f_2 \in F_{k\text{-}1}$

with $f_1 = \{i_1, \ldots, i_{k\text{-}2}, i_{k\text{-}1}\}$

and $f_2 = \{i_1, \ldots, i_{k\text{-}2}, i'_{k\text{-}1}\}$

and $i_{k\text{-}1} < i'_{k\text{-}1}$ **do**

$c \leftarrow \{i_1, \ldots, i_{k\text{-}1}, i'_{k\text{-}1}\}$; // join f_1 and f_2

$C_k \leftarrow C_k \cup \{c\}$;

for each (k-1)-subset s of c **do**

if ($s \notin F_{k\text{-}1}$) **then**

delete c from C_k; // prune

end

end

return C_k;

An example

- F_3 = {{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {1, 3, 5}, {2, 3, 4}}
- After join
  - C_4 = {{1, 2, 3, 4}, {1, 3, 4, 5}}
- After pruning:
  - C_4 = {{1, 2, 3, 4}}

    because {1, 4, 5} is not in F_3 ({1, 3, 4, 5} is removed)

Step 2: Generating rules from frequent itemsets

- Frequent itemsets ≠ association rules
- One more step is needed to generate association rules
- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,
  - Let $B = X - A$
  - $A \rightarrow B$ is an association rule if
    - Confidence$(A \rightarrow B) \geq$ minconf,

      $\text{support}(A \rightarrow B) = \text{support}(A \cup B) = \text{support}(X)$

      $\text{confidence}(A \rightarrow B) = \text{support}(A \cup B) / \text{support}(A)$

Generating rules: an example

- Suppose {2,3,4} is frequent, with sup=50%
  - Proper nonempty subsets: {2,3}, {2,4}, {3,4}, {2}, {3}, {4}, with sup=50%, 50%, 75%, 75%, 75%, 75% respectively
  - These generate these association rules:
    - 2,3 → 4, confidence=100%
    - 2,4 → 3, confidence=100%
    - 3,4 → 2, confidence=67%
    - 2 → 3,4, confidence=67%
    - 3 → 2,4, confidence=67%
    - 4 → 2,3, confidence=67%
    - All rules have support = 50%

Generating rules: summary

- To recap, in order to obtain $A \rightarrow B$, we need to have support($A \cup B$) and support(A)
- All the required information for confidence computation has already been recorded in itemset generation. No need to see the data *T* any more.
- This step is not as time-consuming as frequent itemsets generation.

On Apriori Algorithm

Seems to be very expensive

- Level-wise search
- K = the size of the largest itemset
- It makes at most K passes over data
- In practice, K is bounded (10).

More on association rule mining

- Clearly the space of all association rules is exponential, $O(2^m)$, where m is the number of items in *I*.
- The mining exploits sparseness of data, and high minimum support and high minimum confidence values.
- Still, it always produces a huge number of rules, thousands, tens of thousands, millions, ...

Improving Apriori Algo.

- Further reduce the size of the candidate sets C_i for i >= 2
- Simultaneously find F_1, F_2, F_3, .. in one or two passes, rather than a pass per level.