Lecture 3

Camera Models 2 & Camera Calibration

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Lecture 3

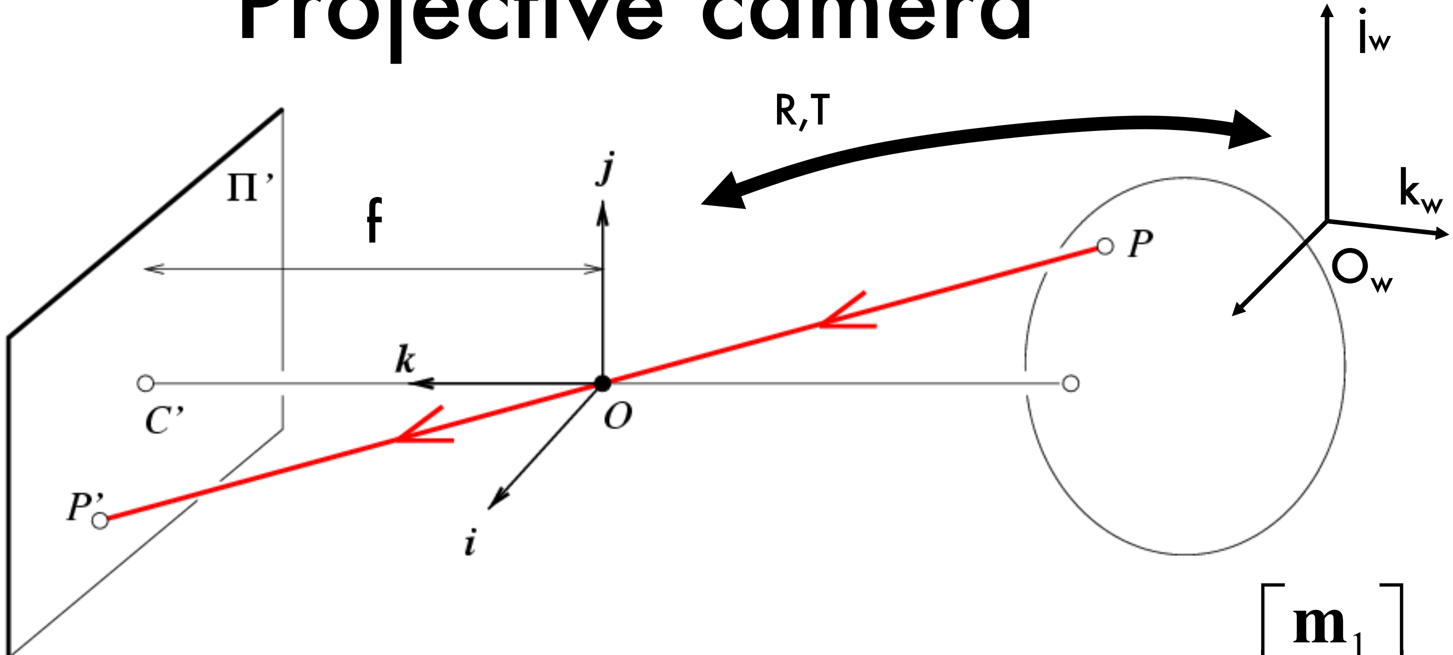
Camera Models 2 & Camera Calibration

- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 1 "Geometric Camera Calibration"
[HZ] Chapter 7 "Computation of Camera Matrix P"

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Projective camera



$$P'_{3\times1} = M\;P_w \quad = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} P_{w\,4\times1} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

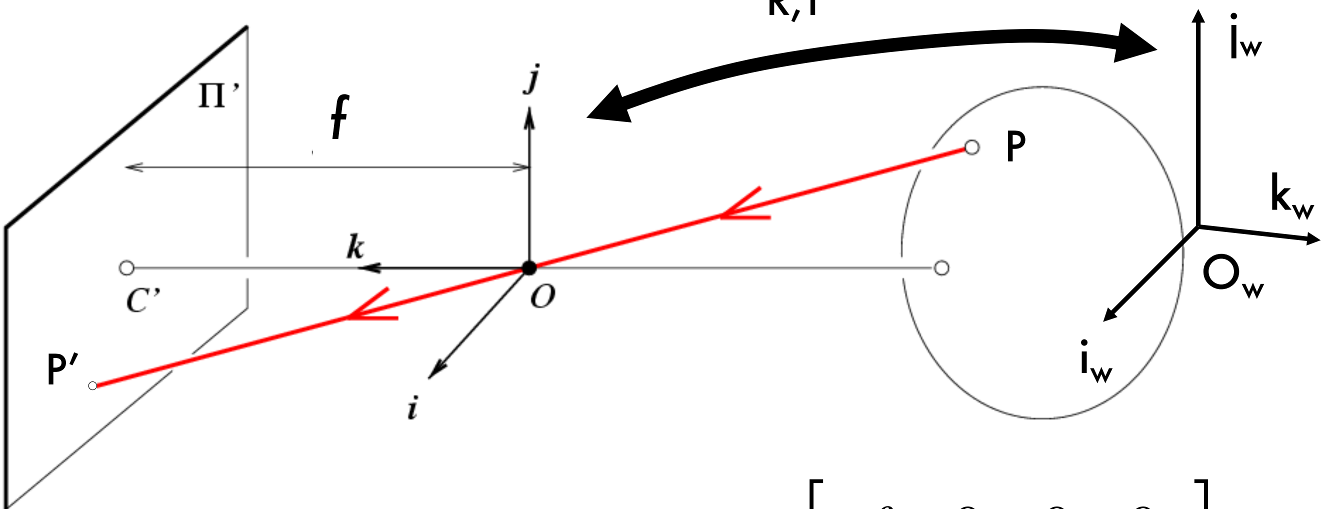
$$= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1 P_W \\ \mathbf{m}_2 P_W \\ \mathbf{m}_3 P_W \end{bmatrix}$$

euclidean

$$\xrightarrow{E} P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right)$$

Exercise!

what happens if both coordinate systems overlap?



$$M = K\begin{bmatrix} R & T \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow \; P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right) = \left(f\frac{x_w}{z_w}, f\frac{y_w}{z_w}\right)$$

$$P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Canonical Projective Transformation

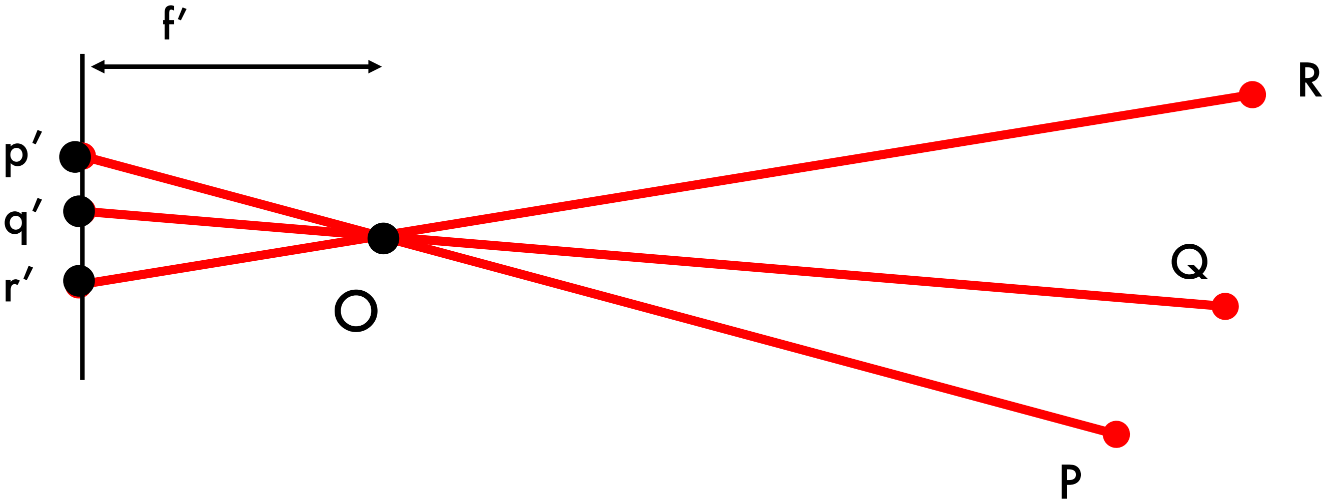
$$P' = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = M\,P$$

$$\Re^4 \xrightarrow{\mathrm{H}} \Re^3$$

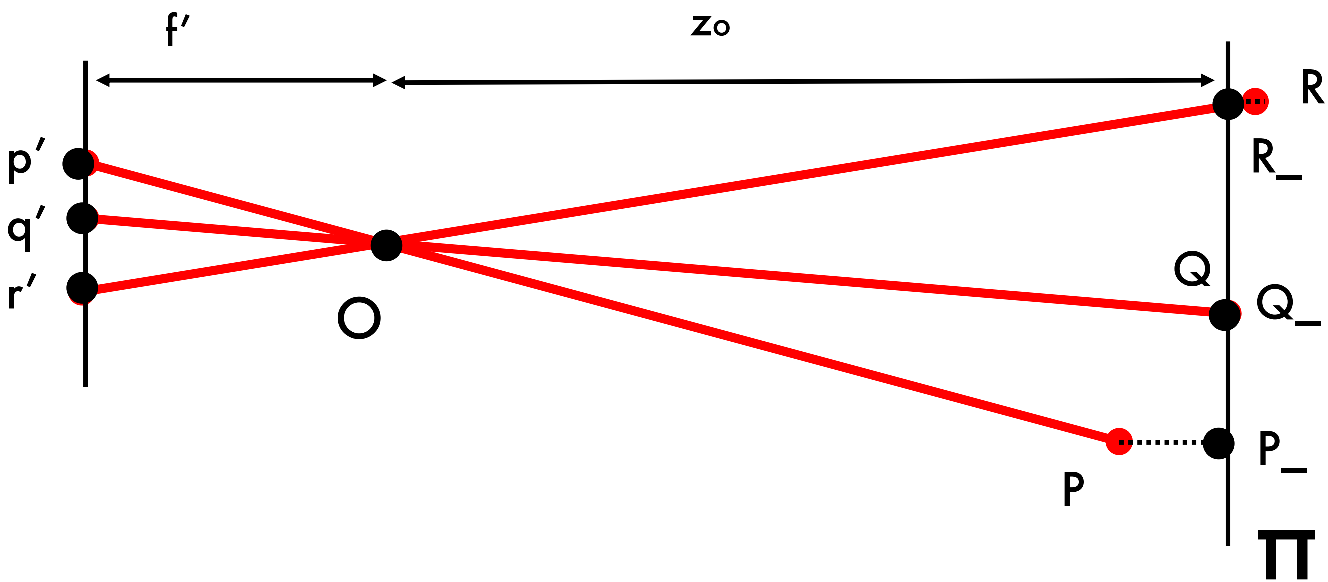
$$P_i' = \begin{bmatrix} \dfrac{x}{z} \\ \dfrac{y}{z} \end{bmatrix}$$

Projective camera



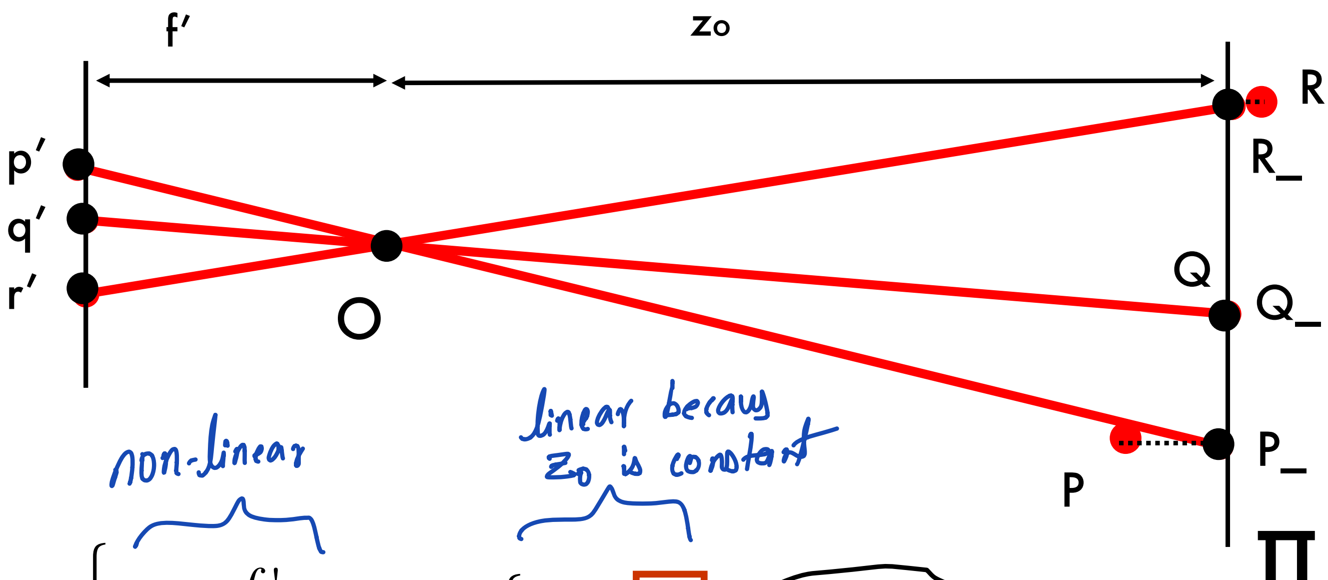
Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



Points projected to a reference plane π

Weak perspective projection

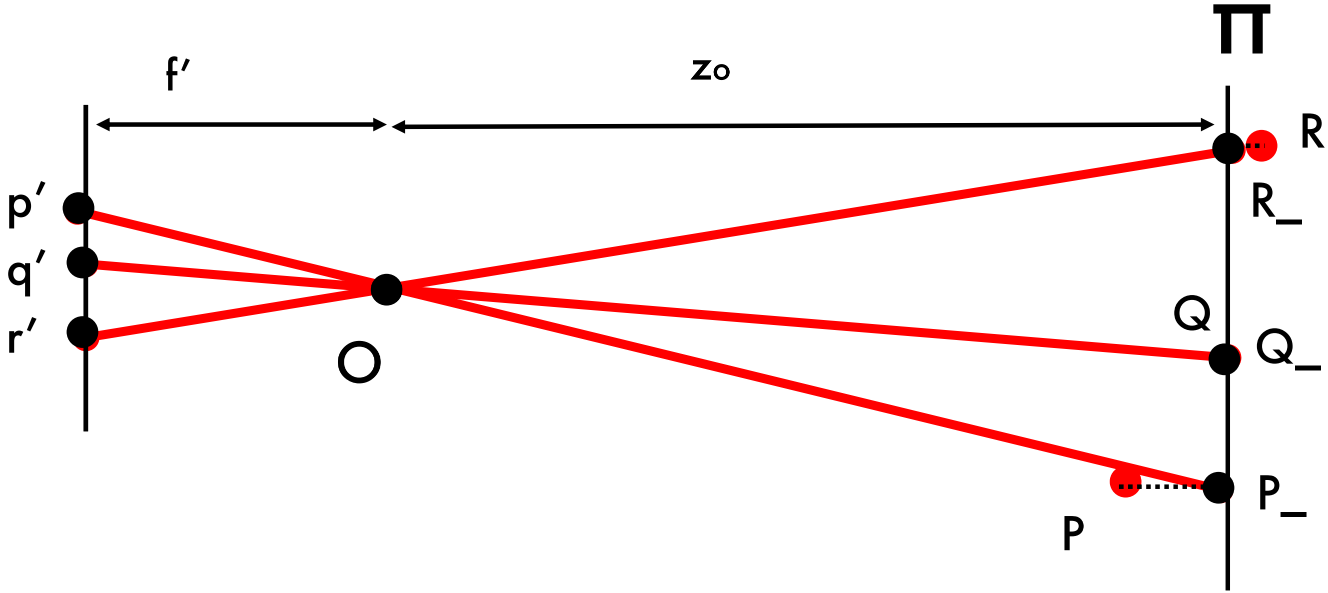


$$\underbrace{\begin{cases} x' = \dfrac{f'}{z}x \\ y' = \dfrac{f'}{z}y \end{cases}}_{\text{non-linear}} \rightarrow \underbrace{\begin{cases} x' = \boxed{\dfrac{f'}{z_0}}x \\ y' = \dfrac{f'}{z_0}y \end{cases}}_{\text{linear because } z_0 \text{ is constant}}$$

Magnification m

instead of projecting the actual points P, Q, R, we are projecting the points P_, Q_, R_ on the image plane.

Weak perspective projection



Projective (perspective) | Weak perspective

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & 1 \end{bmatrix} \rightarrow \quad M = \begin{bmatrix} \underset{3\times 3}{\mathbf{A}} & \underset{3\times 1}{\mathbf{b}} \\ \underset{1\times 3}{\mathbf{0}} & \underset{1\times 1}{\mathbf{1}} \end{bmatrix}$$

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1 P_W \\ \mathbf{m}_2 P_W \\ \mathbf{m}_3 P_W \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\xrightarrow{\mathbb{E}} \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right) \longrightarrow \text{non-linear}$$

Perspective

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \underset{4\times 1}{P_W} = \begin{bmatrix} \mathbf{m}_1 P_W \\ \mathbf{m}_2 P_W \\ 1 \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} & \mathbf{m}_1 & & \\ & \mathbf{m}_2 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\mathbb{E}} (\mathbf{m}_1 P_w, \mathbf{m}_2 P_w) \longrightarrow \text{linear}$$

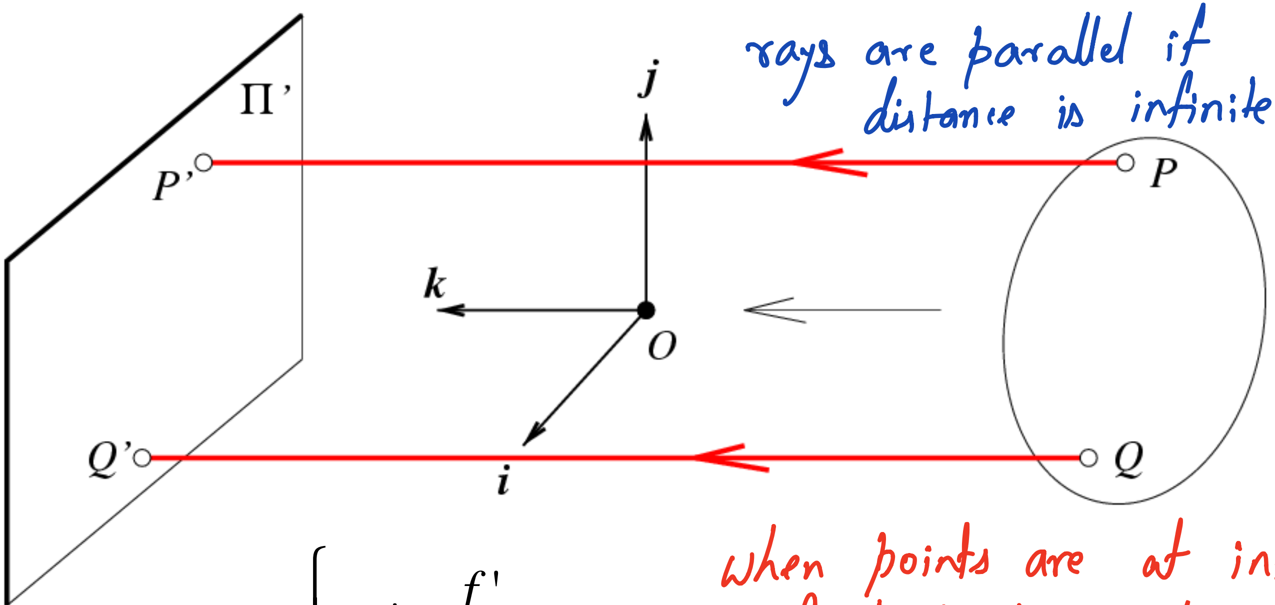
magnification

Weak perspective

Orthographic (affine) projection

Distance from center of projection to image plane is infinite

rays are parallel if distance is infinite



when points are at infinity, focal point is also at infinity

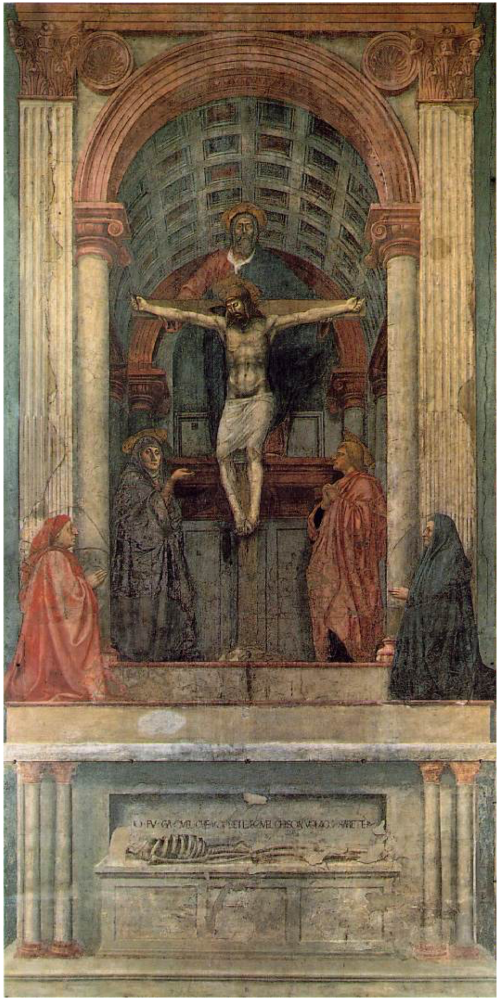
$$
\begin{cases} x' = \dfrac{f'}{z}x \\[2em] y' = \dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = x \\ y' = y \end{cases}
$$

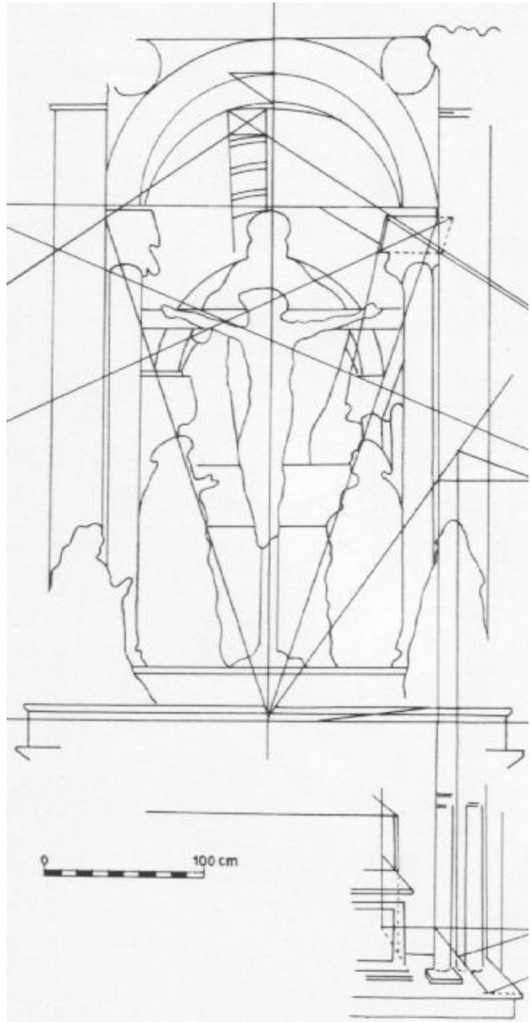
Pros and Cons of These Models

- Weak perspective results in much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.

- Pinhole perspective is much more accurate for modeling the 3D-to-2D mapping.
  - Used in structure from motion or SLAM.

One-point perspective





- Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

Credit slide S. Lazebnik

Weak perspective projection

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Weak perspective projection

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Lecture 3
Camera Calibration

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Projective camera

$$P' = M\,P_w \quad = \boxed{K}\left[\boxed{R \quad T}\right] P_w$$

Internal parameters

External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

11 degrees-of-freedom

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

5 dof 3 dof 3 dof

Goal of calibration

$$P' = M\,P_w \quad = \boxed{K}\begin{bmatrix} \boxed{R \quad T} \end{bmatrix} P_w$$

Internal parameters

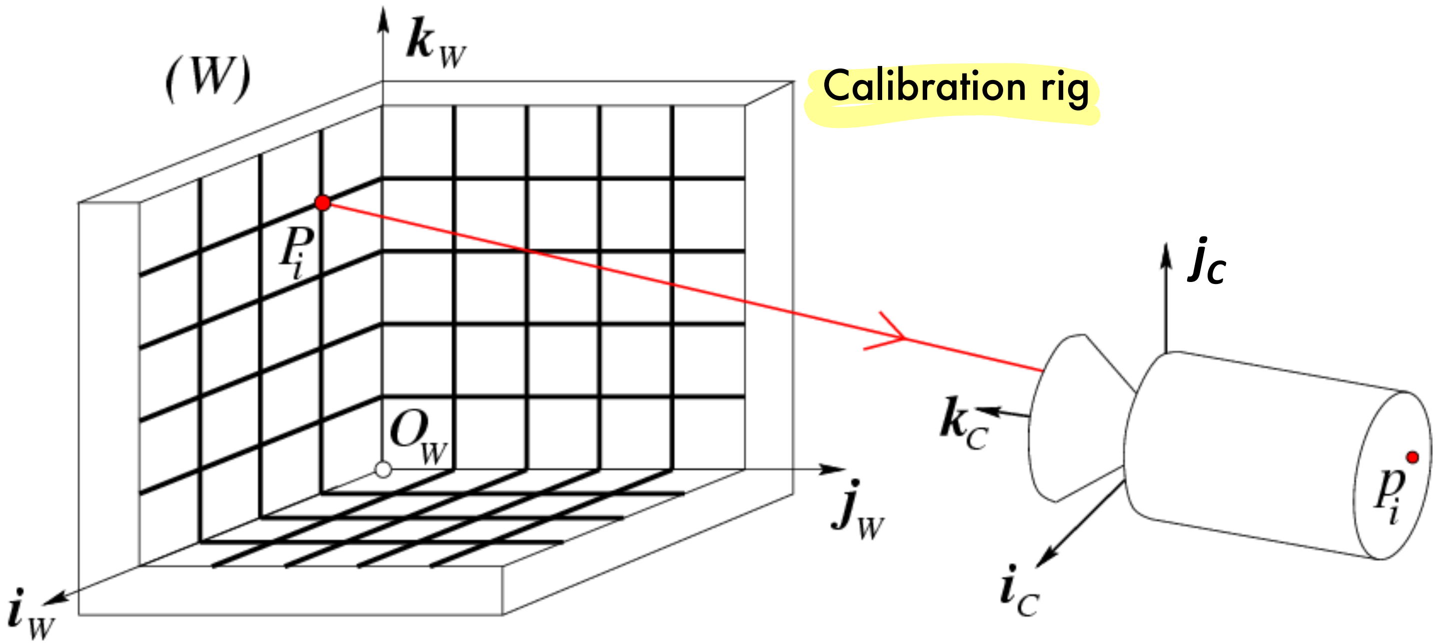
External parameters

Estimate intrinsic and extrinsic parameters from 1 or multiple images

Change notation:
$P = P_w$

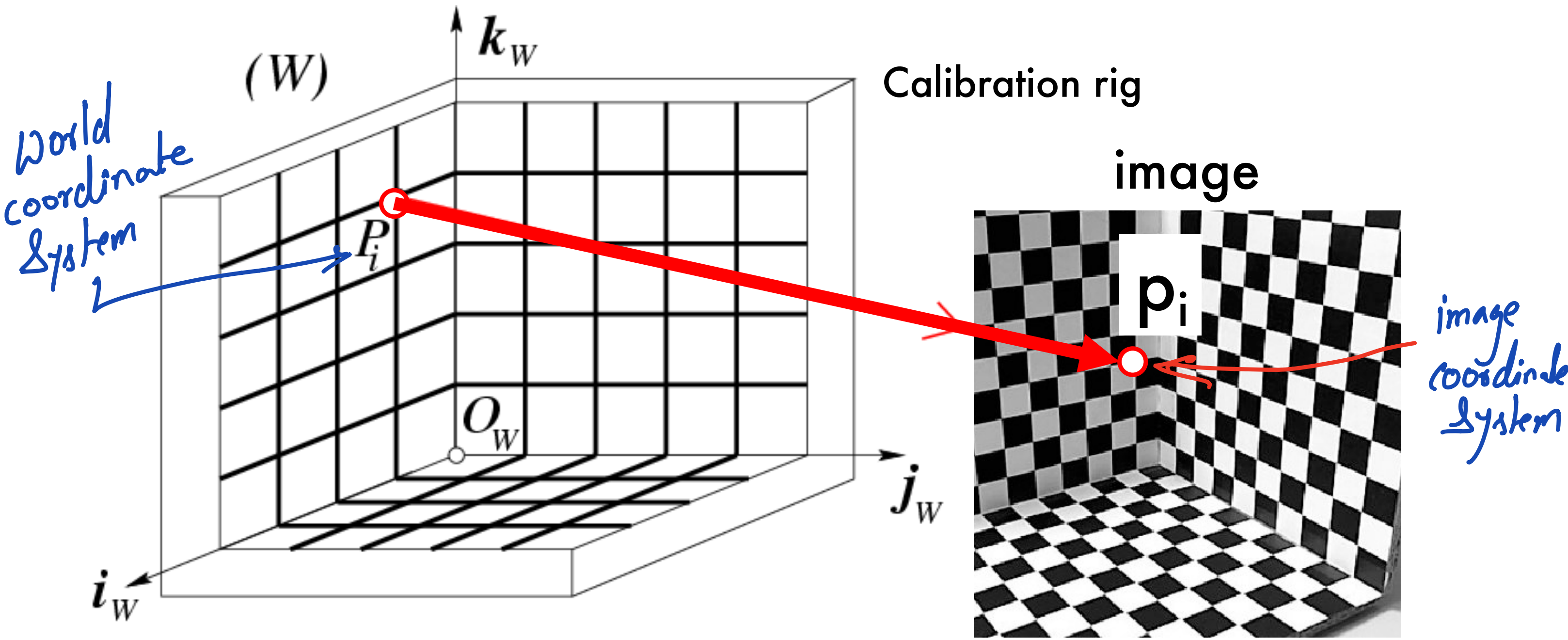
$p = P'$

Calibration Problem



- P_1… P_n with known positions in $[O_w, i_w, j_w, k_w]$

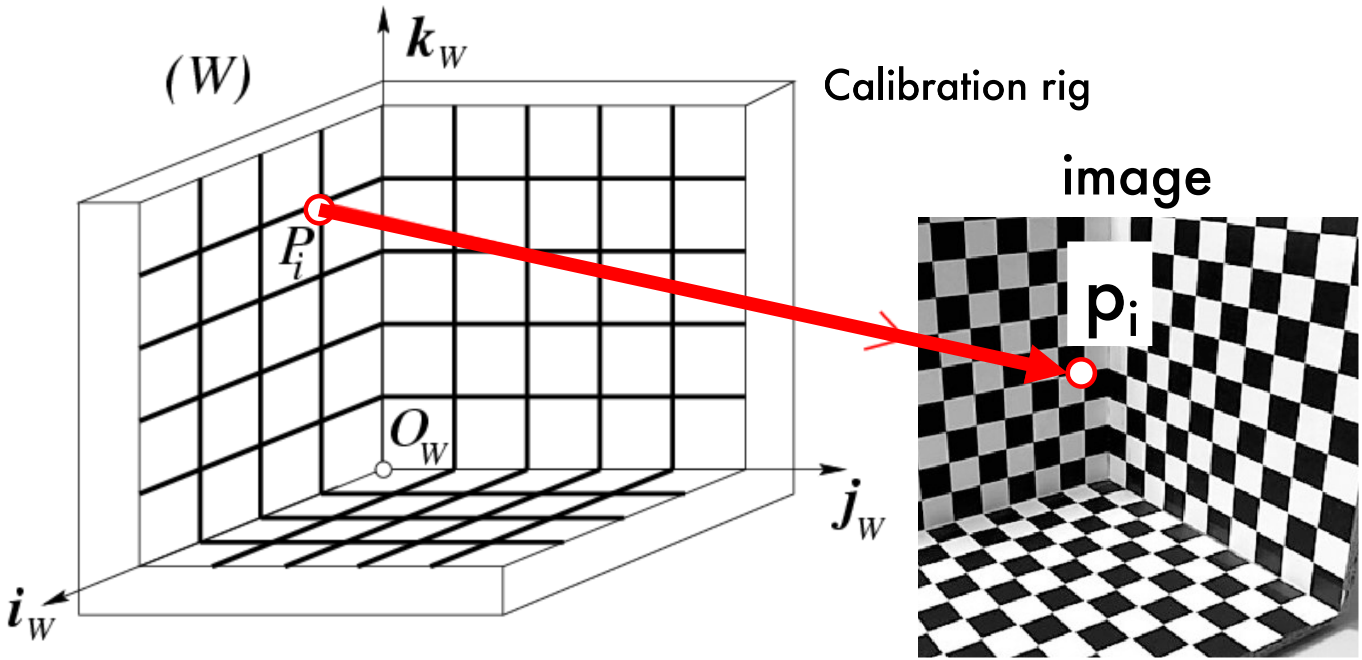
Calibration Problem



- $P_1 \ldots P_n$ with **known** positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ **known** positions in the image

Goal: compute intrinsic and extrinsic parameters

Calibration Problem

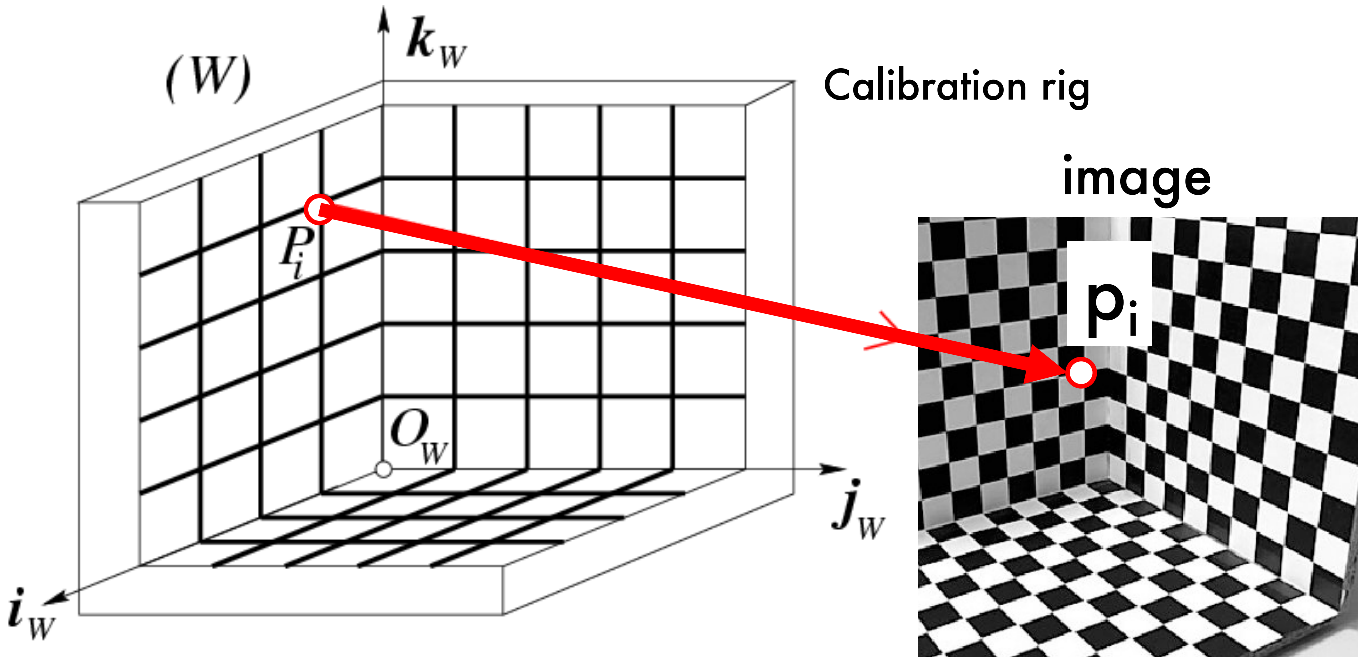


from each correspondence we get 2 equations and constraints.
(x,y)

How many correspondences do we need?

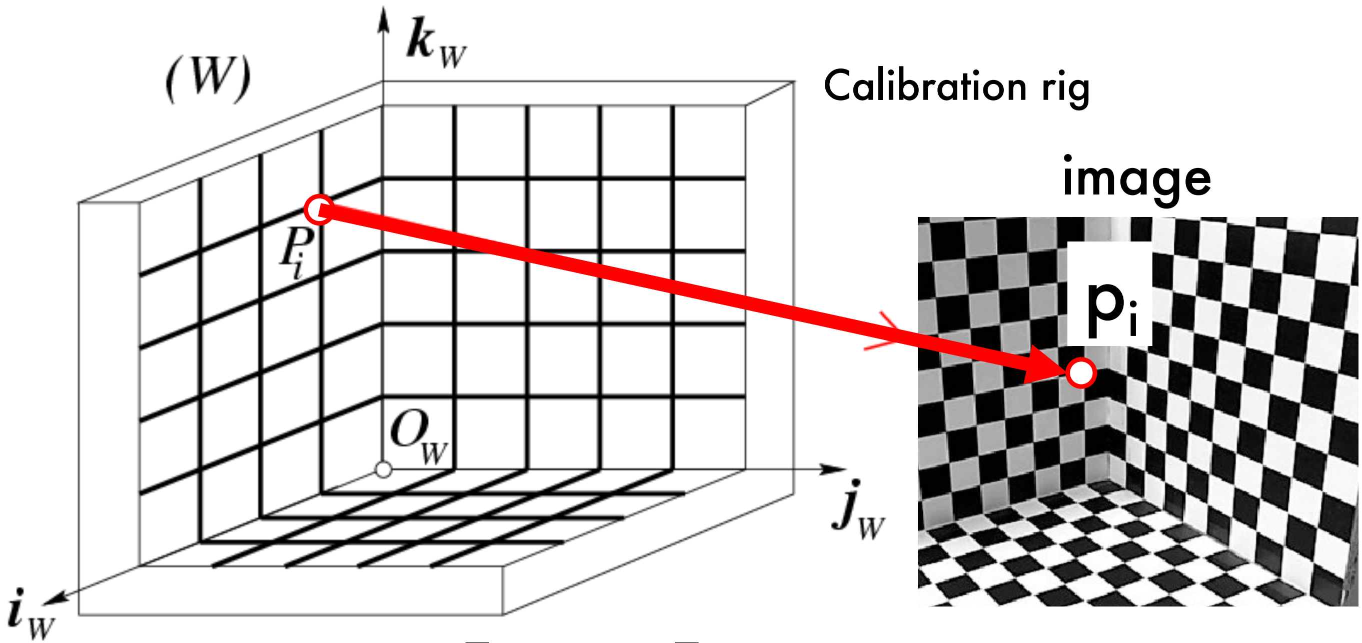
- M has 11 unknowns
- We need 11 equations
- 6 correspondences would do it

Calibration Problem



In practice, using more than 6 correspondences enables more robust results

Calibration Problem



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 \, \mathrm{P_i}}{\mathbf{m}_3 \, \mathrm{P_i}} \\ \dfrac{\mathbf{m}_2 \, \mathrm{P_i}}{\mathbf{m}_3 \, \mathrm{P_i}} \end{bmatrix} = M \; P_i \qquad \mathrm{M} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

[Eq. 1]

Calibration Problem

[Eq. 1]

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow u_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_1 \, P_i \rightarrow u_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_1 \, P_i = 0$$

$$v_i = \frac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow v_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_2 \, P_i \rightarrow v_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_2 \, P_i = 0$$

[Eqs. 2]

Calibration Problem

$$
\begin{cases}
u_1(\mathbf{m}_3\,P_1) - \mathbf{m}_1\,P_1 = 0 \\
v_1(\mathbf{m}_3\,P_1) - \mathbf{m}_2\,P_1 = 0 \\
\vdots \\
u_i(\mathbf{m}_3\,P_i) - \mathbf{m}_1\,P_i = 0 \\
v_i(\mathbf{m}_3\,P_i) - \mathbf{m}_2\,P_i = 0 \\
\vdots \\
u_n(\mathbf{m}_3\,P_n) - \mathbf{m}_1\,P_n = 0 \\
v_n(\mathbf{m}_3\,P_n) - \mathbf{m}_2\,P_n = 0
\end{cases}
$$

if we have n correspondences, we get 2∗n equations

[Eqs. 3]

Block Matrix Multiplication

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} \\ \mathrm{B}_{21} & \mathrm{B}_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$
\begin{cases}
-u_1(\mathbf{m}_3\, P_1) + \mathbf{m}_1\, P_1 = 0 \\
-v_1(\mathbf{m}_3\, P_1) + \mathbf{m}_2\, P_1 = 0 \\
\vdots \\
-u_n(\mathbf{m}_3\, P_n) + \mathbf{m}_1\, P_n = 0 \\
-v_n(\mathbf{m}_3\, P_n) + \mathbf{m}_2\, P_n = 0
\end{cases}
\longrightarrow
\boxed{\overset{\text{known}}{\mathbf{P}}\,\overset{\text{unknown}}{\mathbf{m}} = 0}
$$

[Eq. 4]

Homogenous linear system

1x4

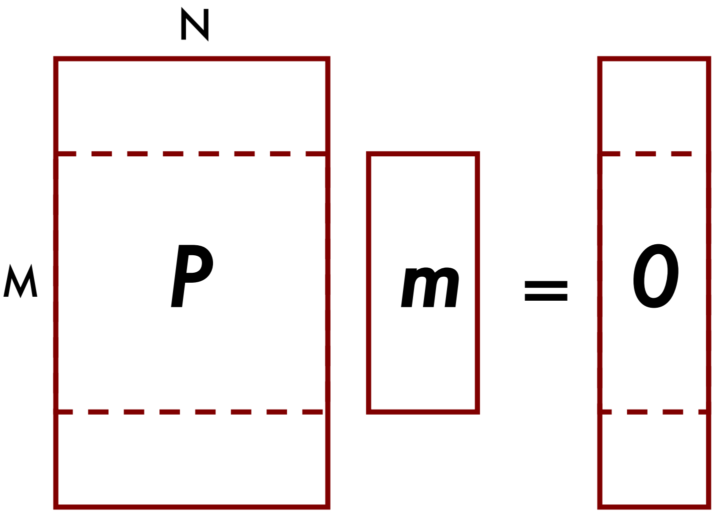
$$
\mathbf{P} \overset{\text{def}}{=} \begin{pmatrix}
\mathrm{P}_1^T & \mathbf{0}^T & -u_1 \mathrm{P}_1^T \\
\mathbf{0}^T & \mathrm{P}_1^T & -v_1 \mathrm{P}_1^T \\
 & \vdots & \\
\mathrm{P}_\mathrm{n}^T & \mathbf{0}^T & -u_n \mathrm{P}_\mathrm{n}^T \\
\mathbf{0}^T & \mathrm{P}_\mathrm{n}^T & -v_n \mathrm{P}_\mathrm{n}^T
\end{pmatrix}_{2n \times 12}
$$

4x1

$$
\boldsymbol{m} \overset{\text{def}}{=} \begin{pmatrix}
\mathbf{m}_1^{\mathrm{T}} \\
\mathbf{m}_2^{\mathrm{T}} \\
\mathbf{m}_3^{\mathrm{T}}
\end{pmatrix}_{12 \times 1}
$$

Homogeneous M x N Linear Systems

M=number of equations = 2n
N=number of unknown = 11



Rectangular system (M>N)

- 0 is always a solution
- To find non-zero solution

Minimize $|\mathbf{P}\,\mathbf{m}|^2$

under the constraint $|\mathbf{m}|^2 = 1$

Constrain the L2 norm of m to a unit value for normalization. Also, to avoid finding a zero-solution.

Calibration Problem

$$\mathbf{P}\ \mathbf{m} = 0$$

- How do we solve this homogenous linear system?
- Via SVD decomposition!

Calibration Problem

$$\boxed{\mathbf{P}}\,\mathbf{m} = 0$$

SVD decomposition of P

$$\boxed{\mathrm{U}_{2n\times12}\ \mathrm{D}_{12\times12}\ \mathrm{V}^{\mathrm{T}}{}_{12\times12}}$$

Last column of V gives $\boldsymbol{m}$

Why? See pag 592 of HZ

$$\boldsymbol{m} \overset{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^{\mathrm{T}} \\ \mathbf{m}_2^{\mathrm{T}} \\ \mathbf{m}_3^{\mathrm{T}} \end{pmatrix}$$

$$\boldsymbol{m} \longrightarrow M$$

Extracting camera parameters

$$M = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} \rho$$

Scale

M has 11 degrees-of-freedom but 12 (3×4) elements. So one element is always unknown (Scaling factor). So we disentangle that by taking it apart.

Extracting camera parameters

for derivation, see:

See [FP], Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = K[R \quad T]$$

$$\underbrace{\hphantom{xxxxxx}}_{A} \qquad \underbrace{\hphantom{xxxxxx}}_{\mathbf{b}}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \qquad u_o = \rho^2(\mathbf{a}_1 \cdot \mathbf{a}_3)$$

$$v_o = \rho^2(\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3)\cdot(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Extracting camera parameters

See [FP], Sec. 1.3.1

$$\frac{M}{\rho} = \left(\underbrace{\begin{matrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T \\ \boldsymbol{r}_3^T \end{matrix}}_{A} \quad \underbrace{\begin{matrix} \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ t_z \end{matrix}}_{\mathbf{b}} \right) = K[R \quad T]$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\theta$$

Extracting camera parameters

See [FP], Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

$$\underbrace{}_{A} \qquad \underbrace{}_{\mathbf{b}}$$

$$\mathrm{K} = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \frac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

Estimated values

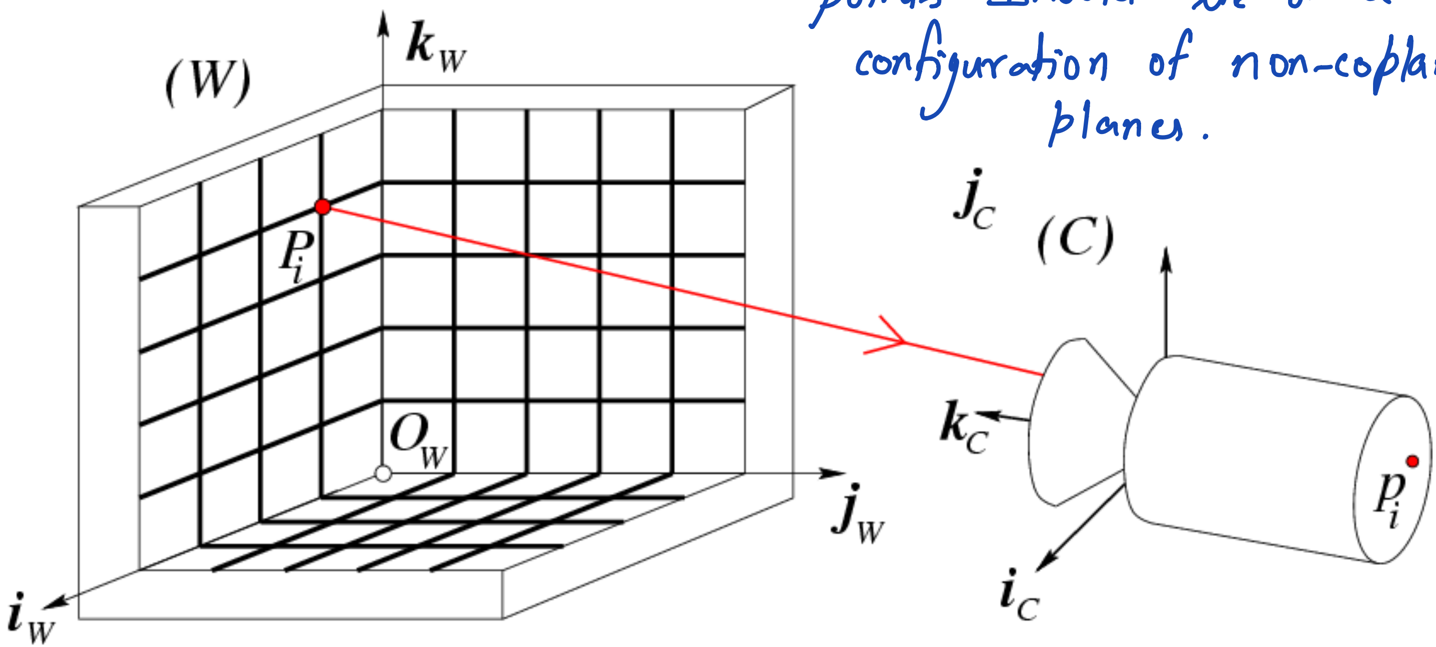
Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad \mathrm{T} = \rho\, \mathrm{K}^{-1} \mathbf{b}$$

Degenerate cases

points should lie on a configuration of non-coplanar planes.



- P_i's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces [FP] section 1.3

Lecture 3
Camera Calibration

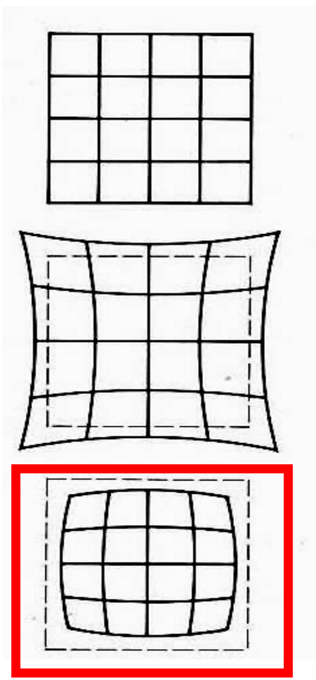
- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 1 "Geometric Camera Calibration"
[HZ] Chapter 7 "Computation of Camera Matrix P"

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Radial Distortion

- Image magnification (in)decreases with distance from the optical axis
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

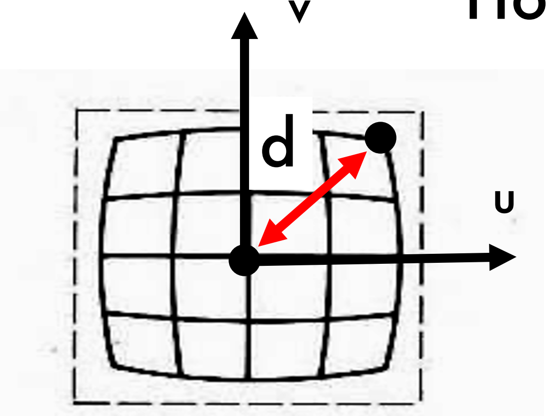
Barrel

(fish-eye)

Radial Distortion

Image magnification decreases with distance from the optical center

How do we model that?



$$\underbrace{\overbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{S_\lambda} M\, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{\text{no-longer linear}} = p_i$$

Distortion coefficient

$$\lambda = 1 \pm \underbrace{\sum_{p=1}^{3} \kappa_p d^{2p}}_{\text{Polynomial function}} \qquad \text{[Eq. 5]}$$

$$d^2 = a\,u^2 + b\,v^2 + c\,u\,v \qquad \text{[Eq. 6]}$$

To model radial behavior

Radial Distortion

$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{M}}_{\mathrm{Q}} \mathrm{P}_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \mathrm{p}_i \qquad \mathrm{Q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

Is this a linear system of equations?

$$\mathrm{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \mathrm{P}_i}{\mathbf{q}_3 \mathrm{P}_i} \\[2ex] \dfrac{\mathbf{q}_2 \mathrm{P}_i}{\mathbf{q}_3 \mathrm{P}_i} \end{bmatrix} \rightarrow \begin{cases} u_i \mathbf{q}_3 P_i = \mathbf{q}_1 P_i \\ v_i \mathbf{q}_3 P_i = \mathbf{q}_2 P_i \end{cases}$$

No! why? q contains λ [Eqs.7]

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \end{bmatrix} \longrightarrow \underbrace{X}_{\text{measurements}} = f(\underbrace{Q}_{\text{parameters}}) \quad \text{[Eq .8]}$$

i=1…n

f( ) is the nonlinear mapping

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution (because of local minir
- Newton requires the computation of J, H Jacobian & Hessian
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, P_i}{\mathbf{q}_3 \, P_i} \\ \dfrac{\mathbf{q}_2 \, P_i}{\mathbf{q}_3 \, P_i} \end{bmatrix} \longrightarrow X = f(Q) \quad \text{[Eq .8]}$$

i=1…n

measurements

parameters

f( ) is the nonlinear mapping

A possible algorithm

assuming no distortion

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \end{bmatrix} \longrightarrow X = f(Q) \quad \text{[Eq .8]}$$

i=1…n

measurements

parameters

f( ) is the nonlinear mapping

Typical assumptions:

- zero-skew, square pixel
- u_o, v_o = known center of the image

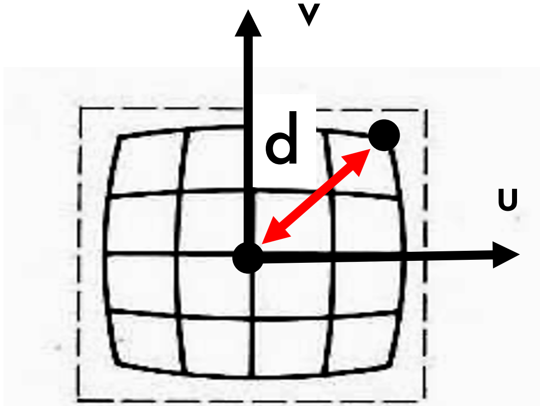
Radial Distortion

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, P_i}{\mathbf{q}_3 \, P_i} \\ \dfrac{\mathbf{q}_2 \, P_i}{\mathbf{q}_3 \, P_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

Can we estimate m_1 and m_2 and ignore the radial distortion?

radial distortion effects radially and hence, the slope is invariant to the distortion

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

Radial Distortion Tsai [87]

Estimating m_1 and m_2...

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix} \rightarrow \frac{u_i}{v_i} = \frac{\dfrac{(\mathbf{m}_1 \, P_i)}{(\mathbf{m}_3 \, P_i)}}{\dfrac{(\mathbf{m}_2 \, P_i)}{(\mathbf{m}_3 \, P_i)}} = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_2 \, P_i}$$

[Eq .9]

[Eq .10]

$$\begin{cases} v_1(\mathbf{m}_1 \, P_1) - u_1(\mathbf{m}_2 \, P_1) = 0 \\ v_i(\mathbf{m}_1 \, P_i) - u_i(\mathbf{m}_2 \, P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 \, P_n) - u_n(\mathbf{m}_2 \, P_n) = 0 \end{cases}$$

linear homogeneous system

[Eq .11]

$$L\,\mathbf{n} = 0$$

Get $\mathbf{m}_1$ and $\mathbf{m}_2$ by SVD

$$\mathbf{L} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \mathrm{P}_1^T & -u_1 \mathrm{P}_1^T \\ v_2 \mathrm{P}_2^T & -u_2 \mathrm{P}_2^T \\ \vdots & \vdots \\ v_n \mathrm{P}_n^T & -u_n \mathrm{P}_n^T \end{pmatrix}$$

$$\mathbf{n} = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \end{bmatrix}$$

Radial Distortion

Once that $\mathbf{m}_1$ and $\mathbf{m}_2$ are estimated...

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$\mathbf{m}_3$ is non linear function of $\mathbf{m}_1, \mathbf{m}_2, \lambda$

Using Newton or Levenberg-Marquadt method.

There are some degenerate configurations for which m_1 and m_2 cannot be computed

Lecture 3
Camera Calibration

- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

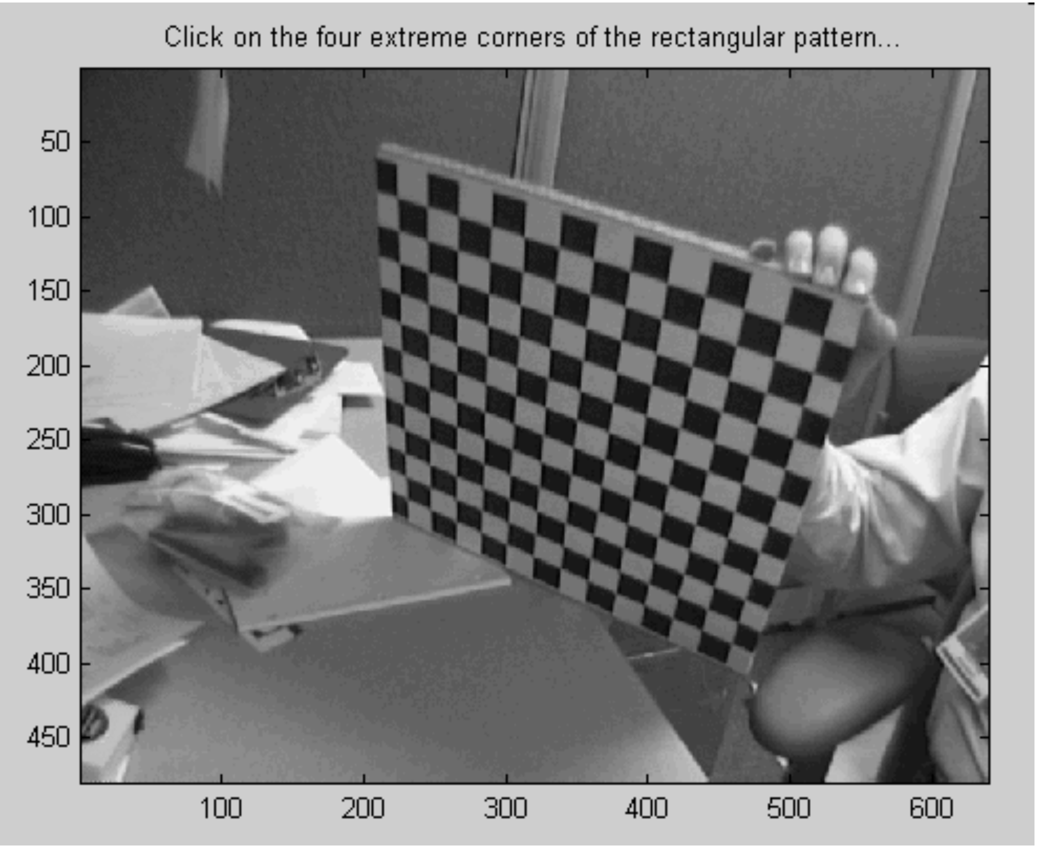
Reading: [FP] Chapter 1 “Geometric Camera Calibration”
[HZ] Chapter 7 “Computation of Camera Matrix P”

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Calibration Procedure

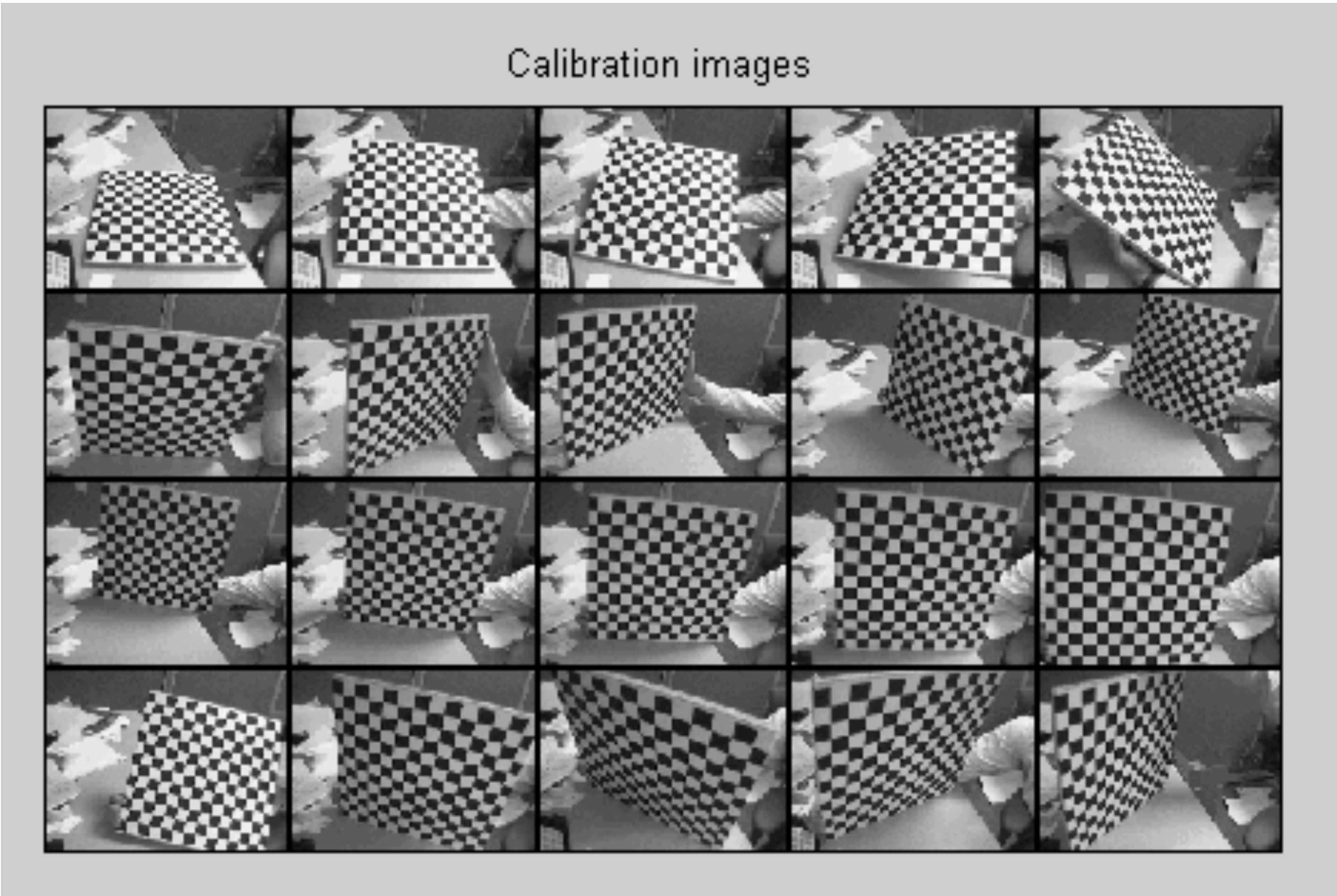
Camera Calibration Toolbox for Matlab
J. Bouguet – [1998-2000]

http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples

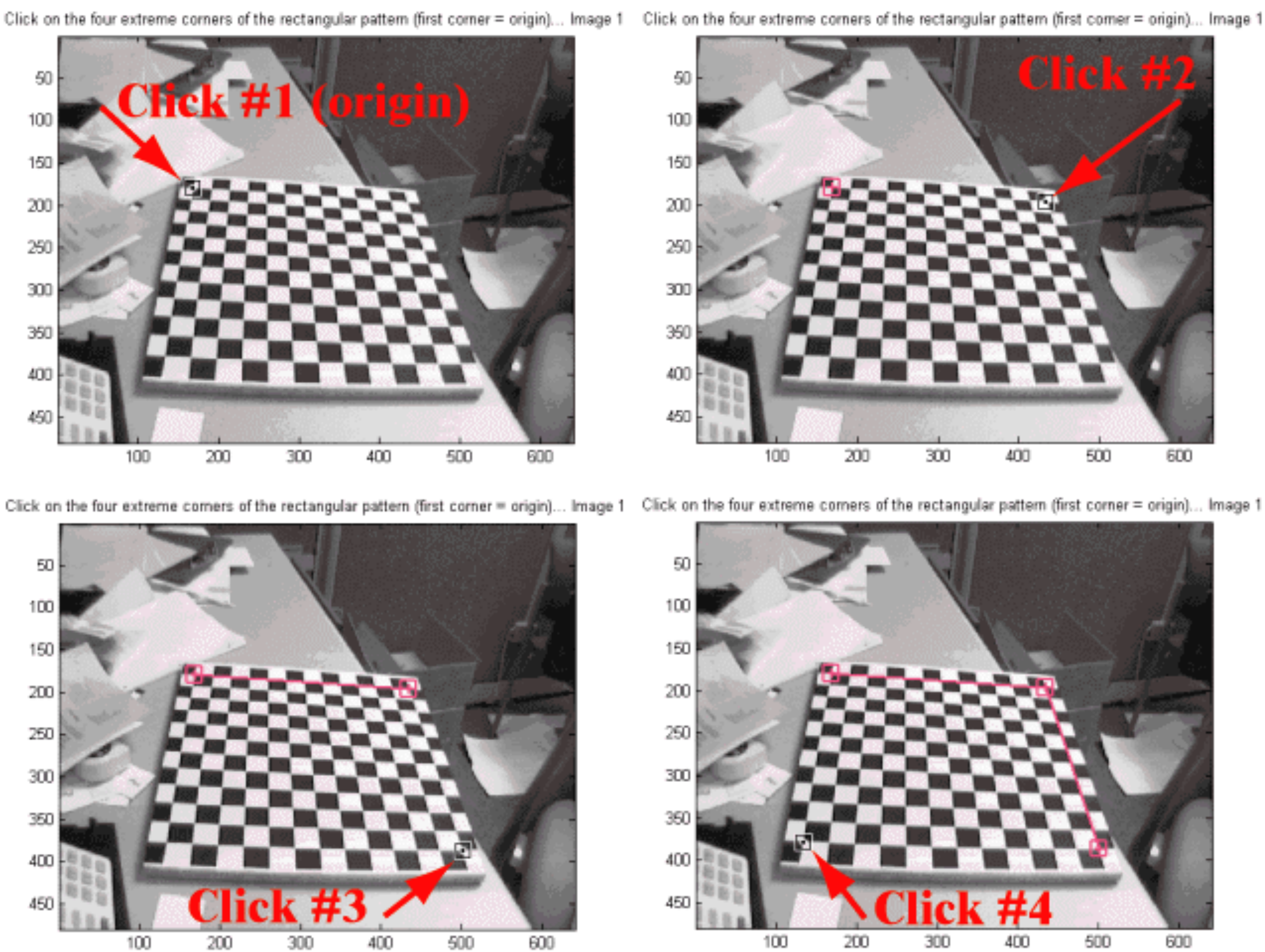


Single plan is not enough.

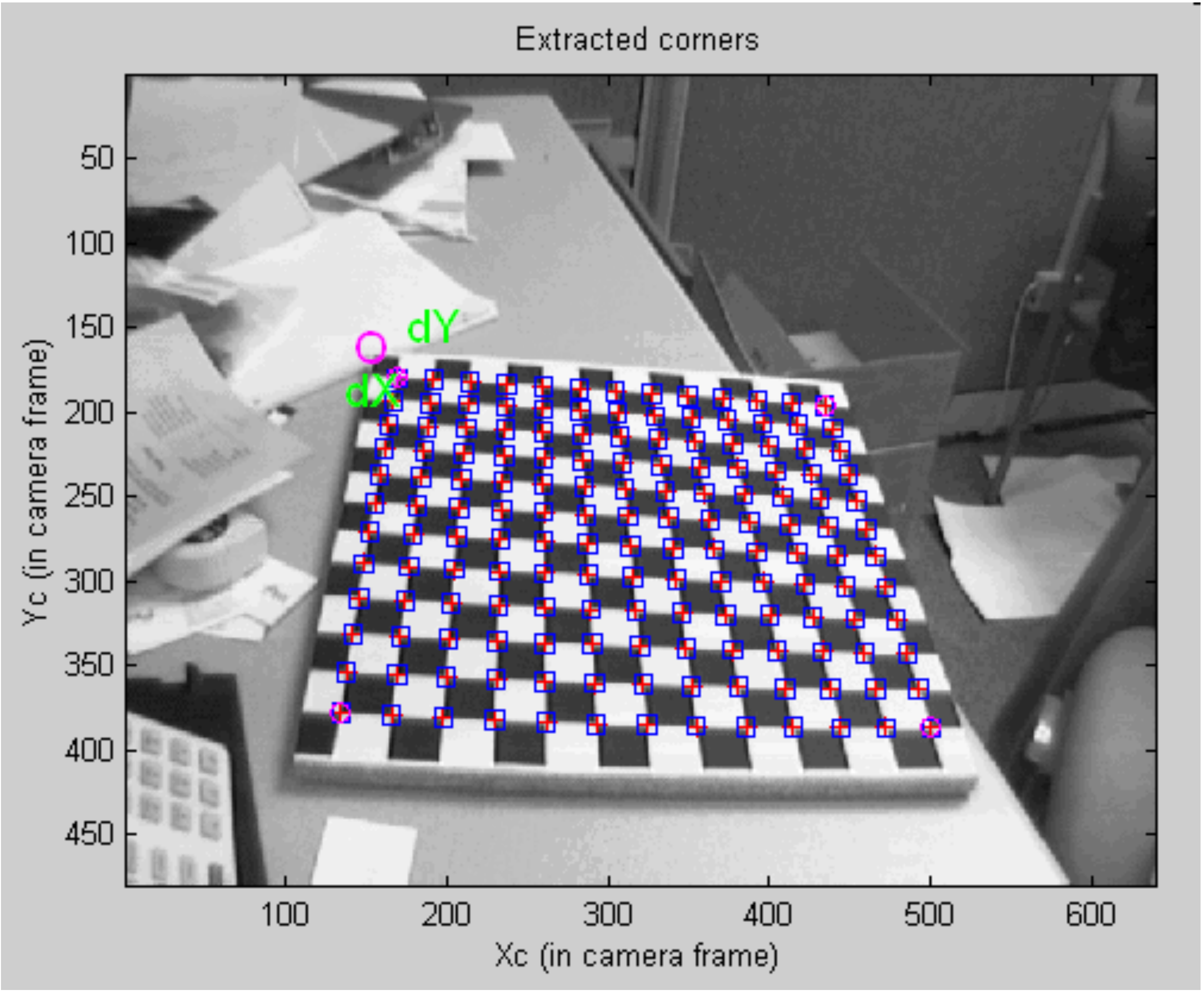
Calibration Procedure



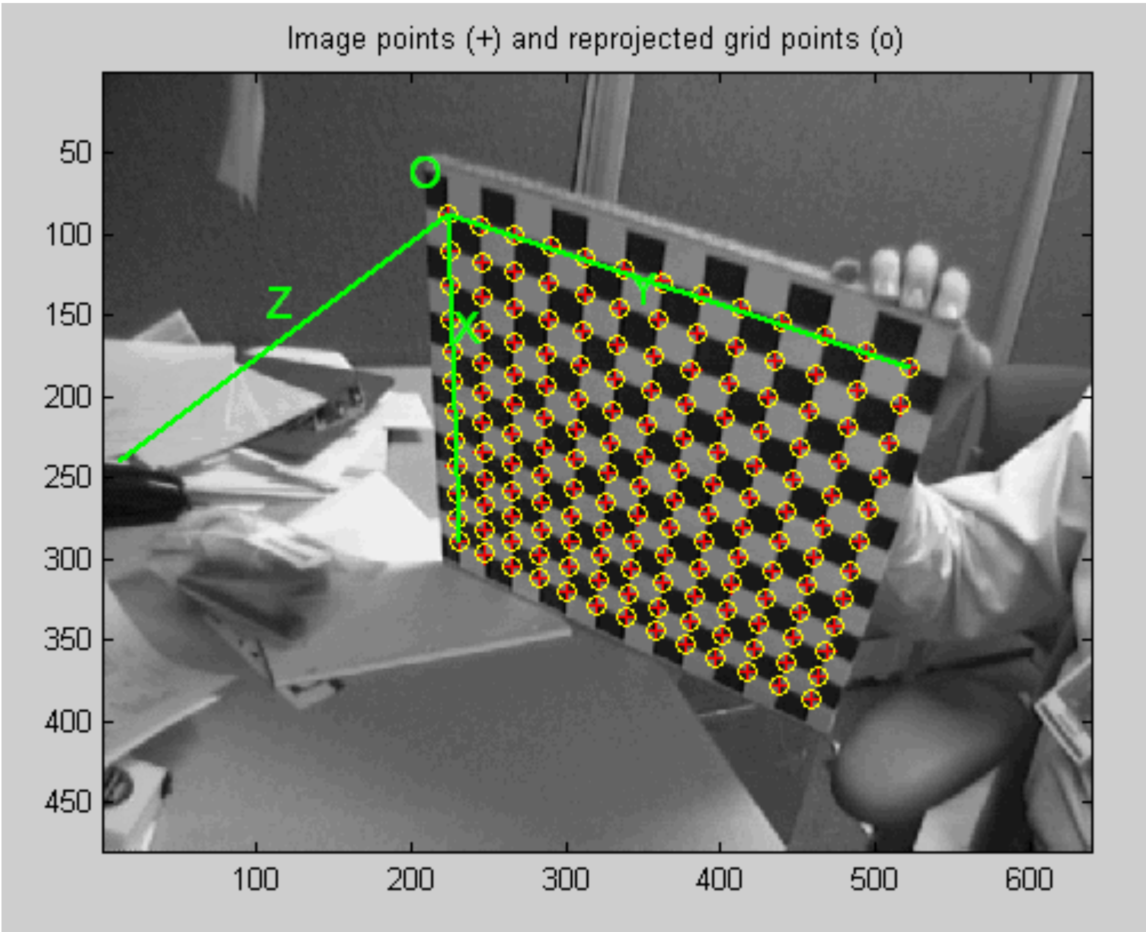
Calibration Procedure



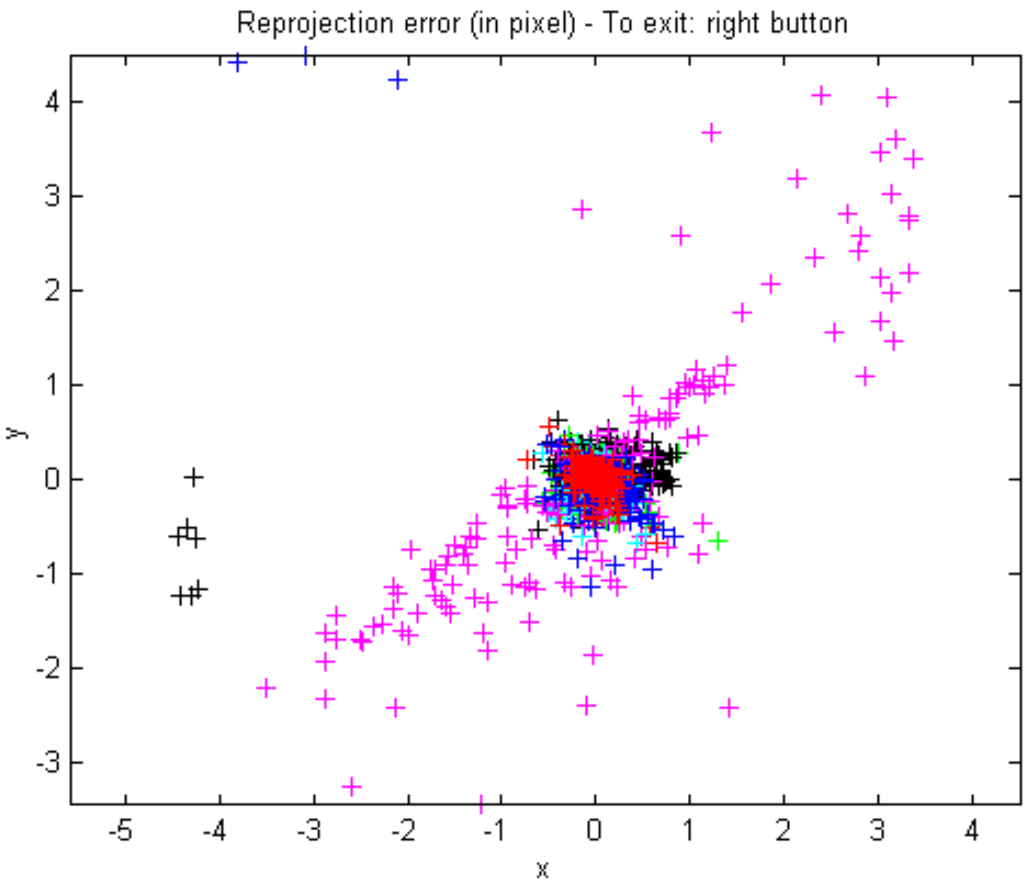
Calibration Procedure



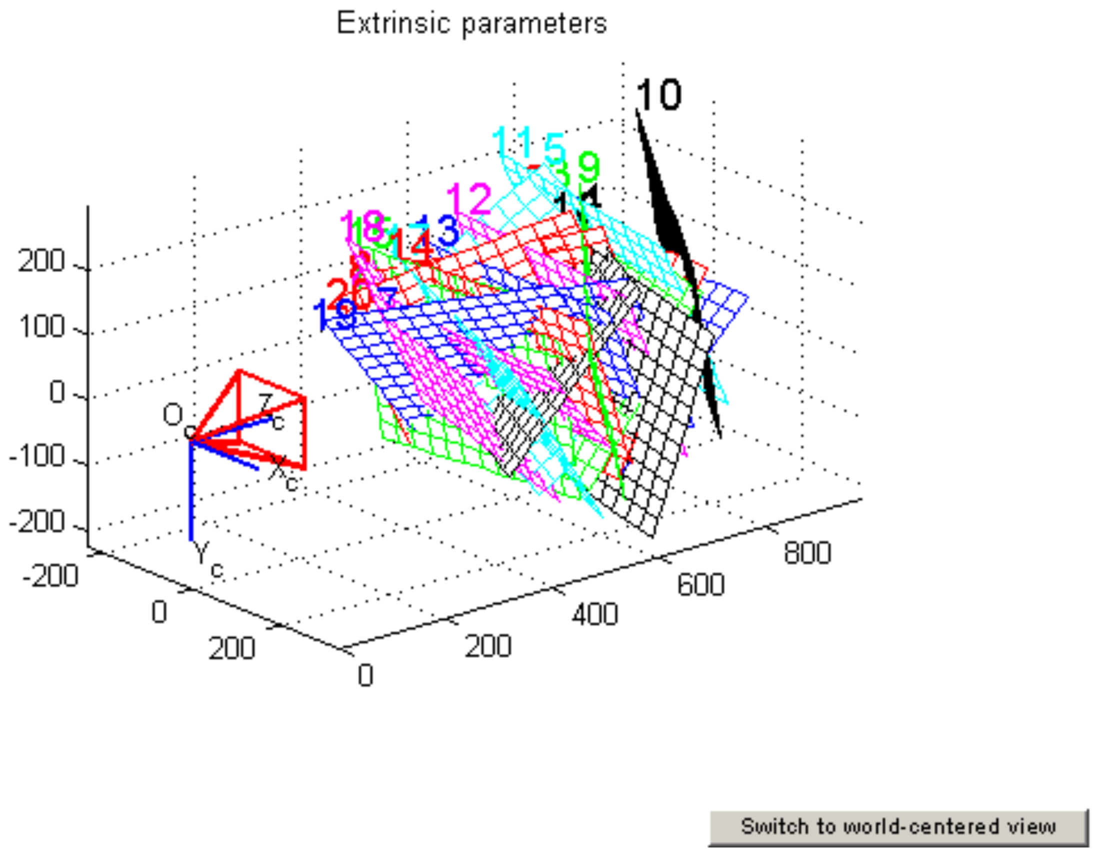
Calibration Procedure



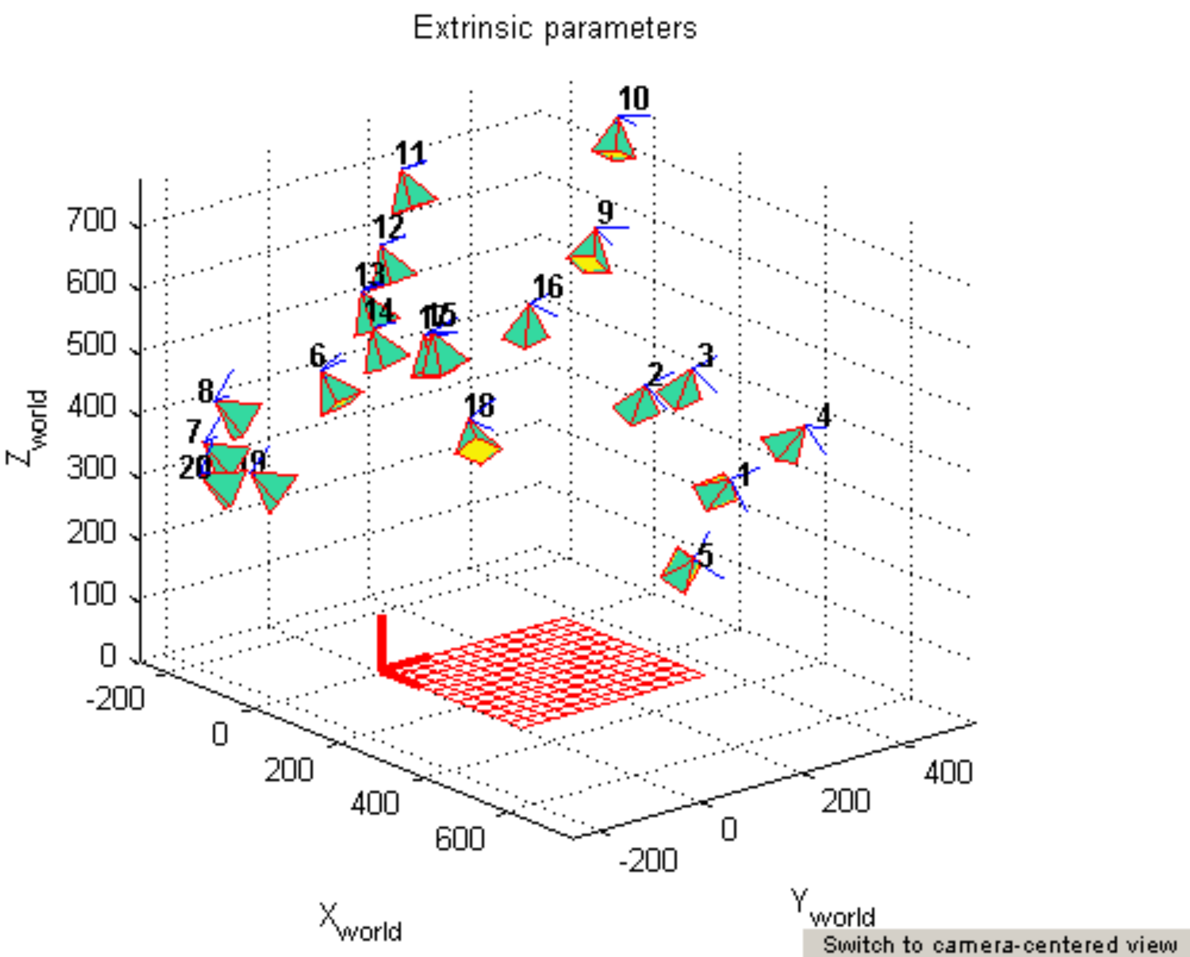
Calibration Procedure



Calibration Procedure



Calibration Procedure



Next lecture

- Single view reconstruction