CS 3333 Mathematical Foundations Spring '11

Recitation 11 Practiced on: 3/30 5:30 - 6:20 pm

6.1 An Introduction to Discrete Probability

Note: These problems are designed for practice during a 50 minute recitation.

a) **Easy** problems: expected to be solved in *5 min*.
b) **Medium** problems: expected to be solved in *30 min*.
c) **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) What is the probability that a card selected from a deck is an ace? (Textbook [KR] Page 398: 1)

   Answer: There are 52 equally likely cards to be selected, and 4 of them are aces. Therefore the probability is $4/52 = 1/13 \approx 7.7\%$.

2. (Easy, 3 min) What is the probability that a five-card poker hand does not contain the queen of hearts? (Textbook [KR] Page 398: 9)

   Answer: We saw in Example 11 of Section 5.3 that there are C(52, 5) possible poker hands, and we assume by symmetry that they are all equally likely. In order to solve this problem, we need to compute the number of poker hands that do not contain the queen of hearts. Such a hand is simply an unordered selection from a deck with 51 cards in it (all cards except the queen of hearts), so there are C(51, 5) such hands. Therefore the answer to the question is the ratio

$$\frac{C(51,5)}{C(52,5)} = \frac{47}{52} \approx 90.4\%.$$

3. (Medium, 10min) What is the probability that a die never comes up an even number when it is rolled six times?? (Textbook [KR] Page 399: 21)

   Answer: Looked at properly, this is the same as Exercise 7. There are 2 equally likely outcomes for the parity on the roll of a die – even and odd. Of the $2^6 = 64$ parity outcomes in the roll of a die 6 times, only one consists of 6 odd numbers. Therefore the probability is 1/64.

4. (Medium, 10 min) What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7? (Textbook [KR] Page 399: 23)

   Answer: We need to count the number of positive integers not exceeding 100 that are divisible by 5 or 7. Using an analysis similar to Exercise 21e in Section 5.1, we see that there are $\lfloor 100/5 \rfloor = 20$

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numbers in that range divisible by 5 and $\lfloor 100/7 \rfloor = 14$ divisible by 7. However, we have counted the numbers 35 and 70 twice, since they are divisible by 5 and 7. Therefore there are 20 + 14 – 2 = 32 such numbers. Now since there are 100 equally likely numbers in the set, the probability of choosing one of these 32 numbers is $32/100 = 0.32$.

5. (Medium, 10min) In a superlottery, players win a fortune if they choose the eight numbers selected by a computer from the positive integers not exceeding 100. What is the probability that a player wins this superlottery? (Textbook [KR] Page 399: 29)
   Answer: There is only one winning choice of numbers, namely the same 8 numbers the computer chooses. Therefore the probability of winning is $1/C(100, 8) \approx 1/(1.86 \times 10^{11})$.

6. (Hard, 15 min) Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of9 when three dice are rolled? (Textbook [KR] Page 399: 37)
   Answer: Reasoning as Example 2, we see that there are 4 ways to get a total of 9 when two dice are rolled: (6, 3), (5, 4), (4, 5), and (3, 6). There are $6^2 = 36$ equally likely possible outcomes of the roll of two dice, so the probability of getting a total of 9 when two dice are rolled is $4/36 \approx 0.111$. For three dice, there are $6^3 = 216$ equally likely possible outcomes, which we can represent as ordered triples (a, b, c). We need to enumerate the possibilities that give a total of 9. They are
   first die turn out to be 6, then (6, 2, 1) and (6, 1, 2);
   first die turn out to be 5, then (5, 2, 2), (5, 3, 1) and (5, 1, 3);
   first die turn out to be 4, then (4, 4, 1), (4, 3, 2), (4, 2, 3), and (4, 1, 4);
   first die turn out to be 3, then (3, 5, 1), (3, 4, 2), (3, 3, 3), (3, 2, 4), and (3, 1, 5);
   first die turn out to be 2, then (2, 6, 1), (2, 5, 2), (2, 4, 3), (2, 3, 4), (2, 2, 5), and (2, 1, 6);
   first die turn out to be 1, then (1, 6, 2), (1, 5, 3), (1, 4, 4), (1, 3, 5), and (1, 2, 6).
   Therefore, there are totally 25 possible outcomes giving a total of 9. This tells us that the probability of rolling a 9 when three dice are thrown is $25/216 \approx 0.116$, slightly larger than the corresponding value for two dice.

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