CS 3333 Mathematical Foundations Spring '11

Recitation 8 Practiced on: 3/2 5:30 - 6:20 pm

Permutations and Combinations

Note: These problems are designed for practice during a 50 minute recitation.

a) **Easy** problems: expected to be solved in *5 min*.
b) **Medium** problems: expected to be solved in *30 min*.
c) **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) List all the permutations of {a, b, c}. (Textbook [KR] Page 360: 1)
   Answer: Permutations are ordered arrangements. Thus we need to list all the ordered arrangements of all 3 of these letters. There are 6 such: abc, acb, bac, bca, cab, and cba. Note that we have listed them in alphabetical order.

2. (Easy, 3 min) How many permutations of {a, b, c, d, e, f, g} end with a? (Textbook [KR] Page 360: 3)
   Answer: If we want the permutation to end with a, then we may as well forget about the a, and just count the number of permutations of {b, c, d, e, f, g}. Each permutation of these 6 letters followed by a, will be a permutation of the desired type, and conversely. Therefore the answer is P(6, 6) = 6! = 720.

3. (Medium, 10 min) How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible? (Textbook [KR] Page 361: 9)
   Answer: We need to pick 3 horses from the 12 horses in the race, and we need to arrange them in order (first, second, and third), in order to specify the win, place and show. Thus there are P(12, 3) = 12*11*10 = 1320 possibilities.

4. (Medium, 10min) A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate? (Textbook [KR] Page 361: 13)
   Answer: We assume that the row has a distinguished head. Consider the order in which the men appear relative to each other. There are n men, and all of the P(n, n) = n! arrangements is allowed. Similarly, there are n! arrangements in which the women can appear. Now the men and women must alternate, and there are the same number of men and women; therefore there are exactly two

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possibilities: either the row starts with a man and ends with a woman or in the opposite way. We have three tasks to perform, then: arrange the men among themselves, arrange the women among themselves, and decide which sex starts the row. By the product rule there are $n! * n! * 2 = 2(n!)^2$ ways in which this can be done.

5. (Medium, 10min) In how many ways can a set of five letters be selected from the English alphabet? (Textbook [KR] Page 361: 15)
   Answer: We assume that a combination is called for, not a permutation, since we are told to select a set, not form an arrangement. We need to choose 5 thins from 26, so there are C(26, 5) = 26*25*24*23*22/5! = 65,780 ways to do so.

6. (Hard, 15 min) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.] (Textbook [KR] Page 361: 23)
   Answer: First position the men relative to each other. Since there are eight men, there are P(8, 8) ways to do this. This creates nine slots where a woman (but not more than one woman) may stand: in from of the first man, between the first and second men, …, between the seventh and eight men, and behind the eighth man. We need to choose five of these positions, in order, for the first through fifth woman to occupy (order matters, because the women are distinct people). This can be done is P(9, 5) ways. Therefore the answer if P(8, 8)*P(9, 5) = 8! * 9!/ 4! = 609,638,400.

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