CS 3333 Mathematical Foundations Spring '11

Recitation 7 Practiced on: 2/21 5:30 - 6:20 pm

Counting

Note: These problems are designed for practice during a 50 minute recitation.

a) **Easy** problems: expected to be solved in *5 min*.
b) **Medium** problems: expected to be solved in *30 min*.
c) **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) 1. There are 18 mathematics majors and 325 computer science majors at a college.
   a) How many ways are there to pick two representatives so that one is a mathematics major and the other is a computer science major?
      Answer: 18 * 325 = 5850.
   b) How many ways are there to pick one representative who is either a mathematics major or a computer science major? (Textbook [KR] Page 344: 1)
      Answer: 18 + 325 = 343.

2. (Easy, 3 min) Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco? How many of these pairs involve more than one airline? (Textbook [KR] Page 344: 5)
   Answer: 6 * 7 = 42.

3. (Medium, 15 min) How many positive integers between 50 and 100
   a) are divisible by 7? Which integers are these?
      Answer: $\lfloor 100/7 \rfloor = 14$ integers less than 100 that are divisible by 7, and $\lfloor 50/7 \rfloor = 7$ of them are less than 50 as well. 14 – 7 = 7 numbers between 50 and 100 that are divisible by 7. They are 56, 63, 70, 77, 84, 91, and 98.
   b) are divisible by 11? Which integers are these?
      Answer: $\lfloor 100/11 \rfloor - \lfloor 50/11 \rfloor = 9 - 4 = 5$. They are 55, 66, 77, 88, 99.
   c) are divisible by both 7 and 11? Which integers are these? (Textbook [KR] Page 345: 19)
      Answer: $\lfloor 100/77 \rfloor - \lfloor 50/77 \rfloor = 1 - 0 = 1$. It is 77.

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4. (Medium, 15 min) How many strings of three decimal digits
   a) do not contain the same digit three times?
      Answer: there are 10 strings that consist of the same digit three times (000, 111, …, 999). Therefore there are 990 strings that do not.
   b) begin with an odd digit?
      Answer: 5 * 10 * 10 = 500.
   c) have exactly two digits that are 4s? (Textbook [KR] Page 345: 23)
      Answer: Here we need to choose the position of the digit that is not a 4 (3 ways) and choose that digit (9 ways). Therefore there are 3 * 9 = 27 such strings.

5. (Hard, 15 min) How many bit strings of length 10 either begin with three 0s or end with two 0s? (Textbook [KR] Page 346: 43)
   Answer: $2^7 + 2^8 - 2^5 = 352$ such strings. The idea behind this principle here is that the strings that both begin with three 0s and end with two 0s were counted twice when we add 2^7 and2^8, so we need to subtract for the over-counting.

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