CS 3333 Mathematical Foundations   Spring '11

Recitation 10   Practiced on: 3/9   5:30 - 6:20 pm

5.5 Generalized Permutations and Combinations

Note: These problems are designed for practice during a 50 minute recitation.
   a) **Easy** problems: expected to be solved in *5 min*.
   b) **Medium** problems: expected to be solved in *30 min*.
   c) **Hard** problems: expected to be solved in *15 min*.
   During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed? (Textbook [KR] Page 379: 1)
    Answer: Since order is important here, and since repetition is allowed, this is a simple application of the product rule. There are 3 ways in which the first element can be selected, 3 ways in which the second element can be selected, and so on, with finally 3 ways in which the fifth element can be selected, so there are 3^5 = 243 ways in which the 5 elements can be selected. The general formula is that there are n^k ways to select k elements from a set of n elements, in order, with unlimited repetition allowed.

2.   (Easy, 3 min) How many ways are there to assign three jobs to five employees if each employee can be given more than one job? (Textbook [KR] Page 379: 5)
    Answer: We assume that the jobs and the employees are distinguishable. For each job, we have to decide which employee gets that job. Thus there are 5 ways in which the first job can be assigned, 5 ways in which the second job can be assigned and 5 ways in which the third job can be assigned. Therefore, by the multiplication principle there are 5^3 = 125 ways in which the assignments can be made. (Now that we do not require that every employee get at least one job.)

3. (Medium, 10min) How many ways are there to select three unordered elements from a set with five elements when repetition is allowed? (Textbook [KR] Page 379: 7)
    Answer: Since the selection is to be an unordered one, Theorem 2 applies. We want to choose r = 3 items from a set with n = 5 elements. Theorem 2 tells us that there are C(5+3-1, 3) = C(7, 3) = 35 ways to do so. (Equivalently, this problem is asking us to count the number of nonnegative integer solutions to x1 + x2 + x3 + x4 + x5 = 3, where xi represents the number of times that the ith element

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of the 5-element set gets selected.)

4.   (Medium, 10 min) How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels? (Textbook [KR] Page 380: 11)
   Answer: This can be solved by common sense. Since the pennies are all identical and the nickels are all identical, all that matters is the number of each type of coin selected. We can select anywhere from 0 to 8 pennies (and the rest nickels); since there are nine numbers in this range, the answer is 9. (The number of pennies and nickels is irrelevant, as long as each is at least eight.) If we want to use a high-powered theorem for this problem, we could observe that Theorem 2 applies, with n = 2 (there are two types of coins) and r = 8. The formula gives C(2+8-1, 8)=C(9, 8) = 9.

5.   (Medium, 10min) How many strings of 10 ternary digits (0, 1, or 2) are there that contain exactly two 0s, three 1s, and five 2s? (Textbook [KR] Page 380: 17)
   Answer: Theorem 3 applies here, with n = 10 and k = 3. The answer if therefore
$$\frac{10!}{2!\,3!\,5!} = 2520.$$

6.   (Hard, 15 min) How many different strings can be made from the letters in ABRACADABRA, using all the letters? (Textbook [KR] Page 380: 31)
   Answer: This is a direct application of Theorem 3, with n = 11, n1 = 5, n2 = 2, n3 = n4 = 1, and n5 = 2 (where n1 represents the number of A's, etc.). Thus the answer is 11!/(5!2!1!1!2!) = 83,160.

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