CS 3333 Mathematical Foundations Spring '11

Recitation 3 Practiced on: 1/26 5:30 - 6:20 pm

Division and Modular Arithmetic, Primes, GCDs and LCMs

Solution:

All books are identified by an **International Standard Book Number (ISBN)**, a 10-digit code $X_1X_2 \ldots X_{10}$, assigned by the publisher. (This system will change in 2007 when a new 13-digit code will be introduced.) These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, a 1-digit check digit that is either a digit or the letter X (used to represent 10). This check digit is selected so that $\sum_{i=1}^{10} ix_i \equiv 0 \ (mod\ 11)$ and is used to detect errors in individual digits and transposition of digits.

1. The ISBN of the fifth edition of Elementary Number Theory and Its Applications is 0-32-123Q072, where Q is a digit. Find the value of Q. (Textbook [KR] Page 210: 34)

   Answer: We know that $1 \cdot 0 + 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 1 + 5 \cdot 2 + 6 \cdot 3 + 7 \cdot Q + 8 \cdot 0 + 9 \cdot 7 + 10 \cdot 2 \equiv 0 \pmod{11}$. This simplifies to $127 + 7Q \equiv 0 \pmod{11}$. We subtract 127 from both sides and simplify to $7Q \equiv 5 \pmod{11}$, since $-127 = -12 \cdot 11 + 5$. It is now a simper matter to use trial and error (or the methods to be introduced in Section 3.7) to find that $Q = 7$ (since $49 \equiv 5 \pmod{11}$).

Bo Tang
Mail: CS Department, UT San Antonio, San Antonio TX 78249, USA
Phone: 210-458-5592 Email: btang[at]cs.utsa.edu Homepage: http://cs.utsa.edu/~btang/