CS 3333 Mathematical Foundations Spring '11

Recitation 13 Practiced on: 4/11 & 4/13 5:30 - 6:20 pm

Discrete Random Variables

Note: These problems are designed for practice during a 50 minute recitation.

- a) Easy problems: expected to be solved in 5 min.
- b) Medium problems: expected to be solved in 30 min.
- c) Hard problems: expected to be solved in 15 min.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 5 min) Let X be a random variable giving the number of aces in a random draw of 4 cards from an ordinary deck of 52 cards. Construct a table showing the probability distribution(Textbook [S3] Page 66: 2.42)

Answer:
$$p(x_0 = 0) = \frac{C(48,4)}{C(52,4)}$$
, $p(x_1 = 1) = \frac{C(48,3)C(4,1)}{C(52,4)}$, $p(x_2 = 2) = \frac{C(48,2)C(4,2)}{C(52,4)}$, $p(x_3 = 3) = \frac{C(48,1)C(4,3)}{C(52,4)}$, $p(x_4 = 2) = \frac{C(48,0)C(4,4)}{C(52,4)}$.

2. (easy, 5 min) Obtain the distribution function for Problem 1. (Textbook [S3] Page 66: 2.45)

Answer:
$$f(x) = \begin{cases} \frac{C(48,4)}{C(52,4)} & x = 0\\ \frac{C(48,3)C(4,1)}{C(52,4)} & x = 1\\ \frac{C(48,2)C(4,2)}{C(52,4)} & x = 2\\ \frac{C(48,1)C(4,3)}{C(52,4)} & x = 3\\ \frac{C(48,0)C(4,4)}{C(52,4)} & x = 4 \end{cases}$$

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3. (Medium, 10min) Obtain the cumulative distribution function (CDF) for Problem 1.

Answer:
$$F(x) = \begin{cases} \frac{\frac{C(48,4)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} & 0 \le x < 1 \\ \frac{\frac{C(48,3)C(4,1)}{C(52,4)} + \frac{C(48,4)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} & 1 \le x < 2 \end{cases}$$

$$\frac{\frac{C(48,2)C(4,2)}{C(52,4)} + \frac{\frac{C(48,3)C(4,1)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} + \frac{\frac{C(48,4)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} & 2 \le x < 3 \end{cases}$$

$$\frac{\frac{C(48,1)C(4,3)}{C(52,4)} + \frac{\frac{C(48,2)C(4,2)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} + \frac{\frac{C(48,3)C(4,1)}{C(52,4)}}{\frac{C(52,4)}{C(52,4)}} & 3 \le x < 4 \end{cases}$$

- 4. (Medium-Hard, 30min) Table below shows the distribution function of a random variable X. Determine:
 - a) the probability function,
 - b) $P(1 \le X \le 3)$,
 - c) $P(X \ge 2)$,
 - d) P(X < 3),
 - e) P(X > 1.4).

| x | 1 | 2 | 3 | 4 |
|------|-----|-----|-----|---|
| F(x) | 1/8 | 3/8 | 3/4 | 1 |

a.
$$f(x) = \begin{cases} 1/8 & x = 1\\ 1/4 & x = 2\\ 3/8 & x = 3\\ 1/4 & x = 4 \end{cases}$$

- b. 3/4
- c. 7/8
- d. 3/8
- e. 7/8