CS 3333 Mathematical Foundations Spring '11

Recitation 7b Practiced on: 2/23 5:30 - 6:20 pm

Counting

Note: These problems are designed for practice during a 50 minute recitation.

a) **Easy** problems: expected to be solved in *5 min*.
b) **Medium** problems: expected to be solved in *30 min*.
c) **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) 1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends. (Textbook [KR] Page 353: 1)
   Answer: Because there are six classes, but only five weekdays, the pigeonhole principle shows that at least two classes must be held on the same day.
2. (Easy, 3 min) Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4. (Textbook [KR] Page 353: 5)
   Answer: There are four possible remainders when a integer is divided by 4 (these are pigeonholes here): 0, 1, 2, or 3. Therefore, by the pigeonhole principle at least two of the five given remainders (these are pigeons) must the same.
3. (Medium, 15 min) Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.
   a) Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.
      Answer: if this statement were not true, then there would be at most 8 from each class standing, for a total of at most 24 students. This contradicts the fact that there are 25 students in the class.
   b) Show that there are at least three freshmen, at least 19 sophomores, or at least five juniors in the class. (Textbook [KR] Page 353: 19)
      Answer: if this statement were not true, then there would be at most 2 freshmen, at most 18 sophomores, and at most 4 juniors, for a total of at most 24 students. This contradicts the fact that there are 25 students in the class.

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4. (Medium, 15 min) There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed? (Textbook [KR] Page 354: 31)
   Answer: The 38 time periods are the pigeon holes, and the 677 classes are the pigeons. By the generalized pigeonhole principle there is some time period in which at least $\lceil 677/38 \rceil = 18$ classes are meeting. Since each class must meet in a different room, we need 18 rooms.

5. (Hard, 15 min) Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. (Here, the assumptions about cables and computers are the same as in Textbook [KR] Page 350: Example 9.) Justify your answer. (Textbook [KR] Page 354: 35)
   Answer: Label the computers C_1 through C_{100}, and label the printers P_1 through P_{20}. If we connect C_k to P_k for k = 1, 2, …, 20 and connect each of the computers C_{21} through C_{100} to all the printers, then we have used a total of 20 + 80 * 20 = 1620 cables. Clearly this is sufficient, because if computers C_1 through C_{20} need printers, then they can use the printers with the same subscripts, and if any computers with higher subscripts need a printer instead of one or more of these, then they can use the printers that are not being used, since they are connected to all the printers.
   Now we must show that 1619 cables are not enough. Since there are 1619 cables and 20 printers, the average number of computers per printer is 1619/20, which is less than 81. Therefore some printers must be connected to fewer than 81 computers (the average of a set of numbers cannot be bigger than each of the numbers in the set). That means it is connected to 80 or fewer computers, so there are at least 20 computers that are not connected to it. If those 20 computers all needed a printer simultaneously, then they would be out of luck, since they are connected to at most the 19 other printers.

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