CS 3333 Mathematical Foundations   Spring '11

Recitation 12   Practiced on: 4/4   5:30 - 6:20 pm

6.2 Probability Theory

Note: These problems are designed for practice during a 50 minute recitation.

a) **Easy** problems: expected to be solved in *5 min*.
b) **Medium** problems: expected to be solved in *30 min*.
c) **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don't annoy others. An electronic copy of these problems and solutions will be posted on the following URL: http://cs.utsa.edu/~btang/pages/teaching.html.

Solutions:

1. (Easy, 2 min) What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails? (Textbook [KR] Page 414: 1)
   Answer: we are told that p(H) = 3p(T), and p(H) + p(T) = 1. So p(T) = ¼ and p(H) = ¾ .

2. (Easy, 3 min) Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4. (Textbook [KR] Page 414: 3)
   Answer: The given information tells us that t = 3(1-t), since 1-t is the probability that some other number appear, so t = ¾. Each of 2 and 4 must have probability 3/8. Each of the other numbers must have probability (1-t)/4 = 1/16.

3. (Medium, 10min) Show that if E and F are events, then $p(E \cap F) \geq p(E) + p(F) - 1$. This is known as Bonferroni's Inequality. (Textbook [KR] Page 415: 13)
   Answer: The formula for the probability of the union of two events given in this section:
   $$p(E \cup F) = p(E) + p(F) - p(E \cap F)$$
   We know that $p(E \cup F) \leq 1$, since no event can have probability exceeding 1. Thus we have
   $$1 \geq p(E \cup F) = p(E) + p(F) - p(E \cap F).$$
   A little algebraic manipulation easily transforms this to the desired inequality.

4. (Medium, 10 min) If *E* and *F* are independent events, prove or disprove that $\bar{E}$ and *F* are necessarily independent events. (Textbook [KR] Page 415: 17)
   Answer: Since $E \cup \bar{E}$ is the entire sample space S, we can break the event F up into two disjoint events, $F = S \cap F = (E \cup \bar{E}) \cap F = (E \cap F) \cup (\bar{E} \cap F)$, using the distributive law. Therefore

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5. (Medium, 10min) What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive Os, given that the first bit is a I? (Assume the probabilities of a 0 and a 1 are the same.) (Textbook [KR] Page 415: 25)
   Answer:

6. (Hard, 15 min) Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if
   a) a 0 bit and a 1 bit are equally likely.
   b) the probability that a bit is a 1 is 0.6.
   c) the probability that the i th bit is a 1 is 1/2; for i = 1,2,3,...,10. (Textbook [KR] Page 416: 30)
   Answer:

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