**CS 3333 Mathematical Foundations Fall 2010**

**Homework 2 Solutions 14 points**

**Number Systems, P229-231**

2. (2 points) Convert these integers from decimal notation to binary notation.

(b) 1023

Answer: To convert from decimal to binary, we successively divided by 2. We write down the remainders so obtained from right to left; that is the binary representation of the given number.

Since 1023/2 is 511 with a remainder of 1, the rightmost digit is 1. Then since 511/2 is 255 with a remainder of 1, the second digit from the right is 1. We continue in this manner, obtaining successive quotients of 127, 63, 31, 15, 7, 3, 1, and 0, and remainder of 1, 1, 1, 1, 1, 1, 1 and 1. Putting all these remainders in order from right to left we obtain (11 1111 1111)2 as the binary representation.

Alternatively, we might notice that 1023=1024-1=210-1. Therefore the binary representation is 1 less than (100 0000 0000 )2, which is clearly (11 1111 1111)2.

(c) 100632

Answer: We could carry out the same process as in part (b). The result is (1 1000 1001 0001 1000)2.

4. (2 points) Convert these integers from binary notation to decimal notation.

(b) 10 1011 0101

Answer: 1+4+16+32+128+512=693

(d) 111 1100 0001 1111

Answer: 1+2+4+8+16+1024+2048+4096+8192+16384=31775

6. (1 point) Convert (BADFACED) 16 from its hexadecimal expansion to its binary expansion.

Answer: Following Example 6, we simply write the binary equivalents of each digit. Since (A)16=(1010)2, (B)16=(1011)2, (C)16=(1100)2, (D)16=(1101)2, (E)16=(1110)2, (F)16=(1111)2, we have (BADFACED)16=(10111010110111111010110011101101)2. Following the convention shown in Exercise 3 of grouping binary digits by fours, we can write this in a more readable form as 1011 1010 1101 1111 1010 1100 1110 1101.

10. (1 point) Convert (1 1000 0110 0011)2 from its binary expansion to its hexadecimal expansion.

Answer: Following Example 6, we simply write the hexadecimal equivalents of each group of four binary digits. Note that we group from the right, so the left-most group, which is just 1, becomes 0001. Thus we have (0001 1000 0110 0011)2 = (1863)10.

18. (3 points) Convert (12345670)8 to its hexadecimal expansion and (ABB093BABBA)16 to its octal expansion.

Answer: We look through binary in each case. (12345670)8=(001 010 011 100 101 110 111 000)2=(0010 1001 1100 1011 1011 1000)2=(29CBB8)16 and (ABB093BABBA)16=(1010 1011 1011 0000 1001 0011 1011 1010 1011 1011 1010)2=(010 101 011 101 100 001 001 001 110 111 010 101 110 111 010)2=(253541116725672)8

24. (3 points) Use the Euclidean algorithm to find

(d) gcd(1529,14039)

Answer: To apply the Euclidean algorithm, we divide the larger number by the smaller, replace the larger by the smaller and the smaller by the remainder of this division, and repeat this process until the remainder is 0. At that point, the smaller number is the greatest common divisor.

gcd(1529,14039)=gcd(1529,278)= gcd(278,139)=gcd(139,0) = 139

(f) gcd(11111,111111)

Answer: gcd(11111,111111)=gcd(11111,1) =gcd(1,0)= 1

26. (2 points) How many divisors are required to find gcd(34, 55) using the Euclidean algorithm?

Answer: We need to divide successively by 55, 34, 21, 13, 8, 5, 3, 2, 1, so 9 divisions are required.