**CS 3333 Mathematical Foundations Fall 2011**

**Homework 2 Solutions 24 points**

**Integers and Algorithms, Pages 229--231: 2 a & b, 4 c & d, 6, 8, 18, 20, 24 e & f, 26, 49 (use 11102 and 10102 as the numbers to be added), 50, 53 (use the algorithm given in the answers section, page S-22, of the textbook to multiply or compare 11102 and 10102).**

2. (2 points) Convert these integers from decimal notation to binary notation.

(a) 321

Answer: To convert from decimal to binary, we successively divided by 2. We write down the remainders so obtained from right to left; that is the binary representation of the given number.

Since 321/2 is 160 with a remainder of 1, the rightmost digit is 1. Then since 160/2 is 80 with a remainder 0, the second digit from the right is 0. We continue in this manner, obtaining successive quotients of 40, 20, 10, 5, 2, 1, and 0, and remainder of 0, 0, 0, 0, 1, 0, and 1. Putting all these remainders in order from right to left we obtain (1 0100 0001)2 as the binary representation.

(b) 1023

Answer:

Since 1023/2 is 511 with a remainder of 1, the rightmost digit is 1. Then since 511/2 is 255 with a remainder of 1, the second digit from the right is 1. We continue in this manner, obtaining successive quotients of 127, 63, 31, 15, 7, 3, 1, and 0, and remainder of 1, 1, 1, 1, 1, 1, 1 and 1. Putting all these remainders in order from right to left we obtain (11 1111 1111)2 as the binary representation.

Alternatively, we might notice that 1023=1024-1=210-1. Therefore the binary representation is 1 less than (100 0000 0000 )2, which is clearly (11 1111 1111)2.

4. (2 points) Convert these integers from binary notation to decimal notation.

(c) 11 1011 1110

Answer: 2+4+8+16+32+128+256+512=958

(d) 111 1100 0001 1111

Answer: 1+2+4+8+16+1024+2048+4096+8192+16384=31775

6. (1 point) Convert (BADFACED) 16 from its hexadecimal expansion to its binary expansion.

Answer: Following Example 6, we simply write the binary equivalents of each digit. Since (A)16=(1010)2, (B)16=(1011)2, (C)16=(1100)2, (D)16=(1101)2, (E)16=(1110)2, (F)16=(1111)2, we have (BADFACED)16=(10111010110111111010110011101101)2. Following the convention shown in Exercise 3 of grouping binary digits by fours, we can write this in a more readable form as 1011 1010 1101 1111 1010 1100 1110 1101.

8. (3 points) Convert each of these integers from binary notation to hexadecimal notation.

(a) 1111 0111

Answer: F7

(b) 1010 1010 1010

Answer: AAA

(c) 111 0111 0111 0111

Answer: 7777

18. (3 points) Convert (12345670)8 to its hexadecimal expansion and (ABB093BABBA)16 to its octal expansion.

Answer: We look through binary in each case. (12345670)8=(001 010 011 100 101 110 111 000)2=(0010 1001 1100 1011 1011 1000)2=(29CBB8)16 and (ABB093BABBA)16=(1010 1011 1011 0000 1001 0011 1011 1010 1011 1011 1010)2=(010 101 011 101 100 001 001 001 110 111 010 101 110 111 010)2=(253541116725672)8

20. (3 points) Use Algorithm 5 to find 11644 mod 645.

Answer: Since 644 = (10 1000 0100)2, we need to multiply together 114 mod 645, 11128 mod 645, 11512 mod 645, reducing modulo 645 at each step. We compute by repeatedly squaring: 112 mod 645 = 121, **114 mod 645 = 1212 mod 645 = 451**, 118 mod 645 = 4512 mod 645 = 226, 1116 mod 645 = 2262 mod 645 = 121. At this point we notice that 121 appeared earlier in our calculation, so we have 1132 mod 645 = 451, 1164 mod 645 = 226, **11128 mod 645 = 121**, 11256 mod 645 = 451, **11512 mod 645 = 226**. Thus our final answer will be the product of 451(equals to 114 mod 645), 121(equals to 11128 mod 645) and 226(equals to 11512 mod 645), reduced modulo 645. We compute these one at a time: 451 ∙ 121 mod 645 = 391, and 391 ∙ 436 mod 645 = 1. So 11644 mod 645 = 1.

24. (2 points) Use the Euclidean algorithm to find

(e) gcd(1529,14038).

Answer: To apply the Euclidean algorithm, we divide the larger number by the smaller, replace the larger by the smaller and the smaller by the remainder of this division, and repeat this process until the remainder is 0. At that point, the smaller number is the greatest common divisor.

gcd(1529,14038)=gcd(1529,277)= gcd(277,144)=gcd(144,133)= gcd(133,11) = gcd(11,1)= gcd(1,0)= 1.

(f) gcd(11111,111111)

Answer: gcd(11111,111111)=gcd(11111,1) =gcd(1,0)= 1.

26. (2 points) How many divisors are required to find gcd(34, 55) using the Euclidean algorithm?

Answer: We need to divide successively by 55, 34, 21, 13, 8, 5, 3, 2, 1, so 9 divisions are required.

49. (2 points) Add 11102 and 10102by working through each step of the algorithm for addition given in the text.

Answer: Note that n = 5. Initially the carry is c = 0, and we start the for loop with j = 0. Since a0 = 0 and b0 = 0, we set d to be ⎣ (0+0+0)/2⎦ = 0; then s0 = 0+0+0-2∙0 = 0, and finally c = 0. At the end of the first pass, then, the right-most digit of the answer has been determined to be a 0, and there is a carry of 0 into the next column.

Now j = 1, and we compute d to be ⎣ (1+1+0)/2⎦ = 1; whereupon s1 becomes s0 = 1+1+0-2∙1 = 0, and c is set to 1. Thus far we have determined that the last two bits of the answer are 00, and there is a carry of 1 into the next column.

The next 2 passes through the loop are similar. As a result of the pass:

j=2, d=1, s2=0, c=1;

j=3, d=1, s3=1, c=1;

At this point the loop is terminated, and when we execute the final step, s4=1. Thus the answer is (1 1000)2 .

50. (2 points) Multiply (1110)2 and (1010)2 by working through each step of the algorithm for multiplication given in the text.

Answer: The partial products are 11100 and 1110000, namely 1110 shifted one place and three places to the left. We add these two numbers, obtaining 10001100.

53. (2 points) Devise an algorithm that, given the binary expansions of the integers a and b, determines whether a > b, a = b, or a < b.(use the algorithm given in the answers section, page S-22, of the textbook to multiply or compare 11102 and 10102.)

Answer:

1) Note that n=3, we start executing the algorithm with k=3. Here are the values of variables at the end of each loop:

k=3, a3=1, b3=1;

k=2, a2=1, b2=0;

The loop ends when k=2 and a2>b2, so print “a is greater than b”.