Note: These problems are designed for practice during a 50 minute recitation.

1. **Easy** problems: expected to be solved in *5 min*.
2. **Medium** problems: expected to be solved in *30 min*.
3. **Hard** problems: expected to be solved in *15 min*.

During the recitation, you may discuss the problems with your peers and the TA. Please control your volume and don’t annoy others. An electronic copy of these problems and solutions will be posted on the following URL: <http://cs.utsa.edu/~btang/pages/teaching.html>.

**Solutions**:

1. (3 min) Convert these integers from decimal notation to binary notation. (Textbook [KR] Page 229: 1 a & b)
   1. 231.  
      ***Answer***: we can compute that 20 + 21 + 22 + 25 + 26 + 27 = 231. So (231)10 = (1110 0111)2.
   2. 4532.  
      ***Answer***: we can compute that 22 + 24 + 25 + 27 + 28 + 212 = 4532. So (4532)10 = (1 0001 1011 0100)2.
2. (2 min) Convert these integers from hexadecimal notation to binary notation. (Textbook [KR] Page 229: 5 a & b)
   1. 80E.  
      ***Answer***: (80E)16 = (1000 0000 1110)2.
   2. 135AB.  
      ***Answer***: (135AB)16 = (0001 0011 0101 1010 1011)2.
3. (10 min)Use *Algorithm 5* to find 7644 mod 645. (Textbook [KR] Page 230: 19, hint: Page 226-227)  
   ***Answer:*** Since 644 = (10 1000 0100)2, we need to multiply together 74 mod 645, 7128 mod 645, 7512 mod 645, reducing modulo 645 at each step. We compute by repeatedly squaring: 72 mod 645 = 49, **74 mod 645 = 492 mod 645 = 466**, 78 mod 645 = 4662 mod 645 = 436, 716 mod 645 = 4362 mod 645 = 466. At this point we see a pattern with period 2, so we have 732 mod 645 = 436, 764 mod 645 = 466, **7128 mod 645 = 436**, 7256 mod 645 = 466, **7512 mod 645 = 436**. Thus our final answer will be the product of 466(equals to 74 mod 645), 436(equals to 7128 mod 645) and 436(equals to 7512 mod 645), reduced modulo 645. We compute these one at a time: 466 ∙ 436 mod 645 = 1, and 1 ∙ 436 mod 645 = 436. So 7644 mod 645 = 436.
4. (10 min)Use the Euclidean algorithm to find: (Textbook [KR] Page 230: 23 e & f)
   1. gcd(1000,5040).  
      ***Answer***: gcd(5040, 1000) = gcd(1000, 40) = gcd(40, 0) = 40.
   2. gcd(9888,6060).  
      ***Answer***: gcd(9888, 6060) = gcd(6060, 3828) = gcd(3828, 2232) = gcd(2232, 1596) = gcd(1596, 636) = gcd(636, 324) = gcd(324, 312) = gcd(312, 12) = gcd(12, 0) = 12.
5. (10 min)Multiply (1110)2 and (1010)2 by working through each step of the algorithm for multiplication given in the text. (Textbook [KR] Page 231: 50, Hint: Page 224-225)  
   ***Answer***: The partial products are 11100 and 1110000, namely 1110 shifted one place and three places to the left. We add these two numbers, obtaining 10001100.
6. (15 min)How many bit operations does the comparison algorithm from Exercise 53 use when the larger of a and b has n bits in its binary expansion? (Textbook [KR] Page 231: 54 , hint: use the algorithm given in the answers section, page S-22, of the textbook)  
   ***Answer***: In the worst case, each bit of a has to be compared to each bit of b, so O(n) comparisons are needed. An exact analysis of the procedure given in the solution to Exercise 53 shows that n+1 comparisons of bits are needed in the worst case, assuming that the logical “and” condition in the **while** loop is evaluated efficiently from left to right (so that a0 is not compared to b0 there).  
     
   ***Reference***: solution pseudo-code for Exercise 53:
7. **procedure** compare(a, b: nonnegative integers)
8. i := n – 1
9. **while** i > 0 and ai = bi
10. i := i – 1
11. **if** ai > bi **then** answer := "a > b”
12. **else** **if** ai < bi **then** answer := "a < b”
13. **else** answer := "a = b”
14. {the answer is recorded in answer}