1 Use the definition of a null sequence to prove that the sequence $\{a_n\}$ given by

$$a_n = \frac{(-1)^n}{3n^3 - 25}, \quad n = 1, 2, \ldots, \text{ is null.}$$

2 Determine the limits of the following sequences, $\{a_n\}$. You should state clearly any results or rules used. In every case, $n = 1, 2, 3, \ldots$

(a) $a_n = \dfrac{2n^2 + 5n - 3}{2 + 3n - n^2}$ (b) $a_n = \dfrac{n^2 + 5(2^n)}{n^3 - 3(2^n)}$

(c) $a_n = \dfrac{1 + 2n - 3n^2}{2n^2 + n + 1}$ (d) $a_n = \dfrac{n + 2(3^n) + 3(2^n)}{n^2 - 5(2^n) + 4(3^n)}$

(e) $a_n = \dfrac{5n^3 - 3n + 6}{2n^3 + 4n - 1}$ (f) $a_n = \dfrac{3^n + 3(n!)}{2^n + n^3 - 2(n!)}$

(g) $a_n = \dfrac{n^2 - 2n! + 5}{n! - 2^n - 4n^3}$ (h) $a_n = \dfrac{2n^2 + n - 3}{8n^2 + 2n + 3}$

(i) $a_n = \dfrac{2n^3 + 5n - 4}{6n^3 + 2n^2 - 3}$ (j) $a_n = \dfrac{2n^2 + 5n - 3(n!)}{3^n - n! - 3n^3}$

3 Prove that, as $n \to \infty$,

(a) $a_n = n^2 - \dfrac{4}{n} + 2^n$ tends to ∞;

(b) $a_n = 2 - 3(n!) + 4n^2$ tends to $-\infty$.

4 Prove that the sequences below are divergent.

(a) $a_n = \dfrac{1-(-1)^n}{1-2^{-n}}$ (b) $a_n = \dfrac{3n^2+(-1)^{n+1}n!}{2n+4(n!)}$

5 Determine whether each of the following sequences $\{a_n\}$ is convergent, stating the limit of the sequence (if a limit exists). You should state clearly any result or test that you may use.

(a) $a_n = \dfrac{n!+2^n}{n^2+3(n!)+1}, \quad n=1,2,\ldots$ (b) $a_n = \dfrac{n^2+4^n-4}{n^3+3^n-5}, \quad n=1,2,\ldots$

(c) $a_n = \dfrac{(-1)^n n^3}{4n^3+n+1}, \quad n=1,2,\ldots$ (d) $a_n = \dfrac{3^n+5n^2-3}{4^n+3n+1}, \quad n=1,2,\ldots$

(e) $a_n = \dfrac{n(-1)^n+2}{3n+3}, \quad n=1,2,\ldots$ (f) $a_n = \dfrac{n!+1}{1+n}, \quad n=1,2,\ldots$