

Question 1

(a) Determine whether the following limits exist:

(i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1},$

(ii) $\lim_{x \rightarrow 0} \frac{\sin x}{e^{\sin x} - 1}$

(b) Using the standard limits $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1$ and $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1$, find the value of the following limit:

$$\lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x(e^x - 1)} \right).$$

(c) Prove that the following function does not tend to a limit as x tends to 0.

$$f(x) = \frac{3x^3 - 4|x|}{2x}$$

(d) Use the ε - δ definition of continuity to prove that

$$f(x) = 4x^2 + 2x$$

is continuous at the point $c = 3$.

(e) Prove that the function

$$f(x) = \frac{x}{1 + 2x}$$

is uniformly continuous on the interval $[0, 2]$, stating any results from the course which you use.