1 Since the top face has three symmetries and there are two places for it, the number of symmetries is 6. The elements are shown below in their conjugacy classes.

Direct		Indirect	
C_1	e	C_4	(12)
C_2	(345)	C_5	$(12)(345)$
C_3	(354)	C_6	$(12)(354)$

At a first sight it may appear that (345) should be in the same conjugacy class as (354) since each is a rotation of a third of a revolution. However the reflection does not change the direction of rotation and a check using each element of the symmetry group will show that they are not conjugate.

By the Theorem of Lagrange, the possible orders of non-trivial subgroups are 2 and 3. Since we are considering both normal and non-normal subgroups, it is helpful to notice that the group contains an element of order 6, namely (12)(345). This means that the group is cyclic. This in turn makes the group Abelian. Finally this implies that all its subgroups are normal and since each element is in a conjugacy class of its own, the earlier observation about (345) and (354) not being in the same conjugacy class is confirmed.

A normal subgroup of order 2 is $\{e,(12)\}$.

A normal subgroup of order 3 is $\{e,(345),(354)\}$.

The first of these is $C_1 \cup C_4$ and the second is $C_1 \cup C_2 \cup C_3$. *Of course the fact that this is an Abelian group is enough to ensure that all subgroups are normal.*

2 The square face at the front has 8 symmetries and there are two places for it. The total number of symmetries is 16.

	Direct		Indirect
C_1	e	C_6	(15)(26)(37)(48)
C_2	$\begin{cases}(1234)(5678)\\(1432)(5876)\end{cases}$	C_7	$\begin{cases}(14)(23)(58)(67)\\(12)(34)(56)(78)\end{cases}$
C_3	(13)(24)(57)(68)	C_8	$\begin{cases}(13)(57)\\(24)(68)\end{cases}$
C_4	$\begin{cases}(18)(27)(36)(45)\\(16)(25)(38)(47)\end{cases}$	C_9	(17)(28)(35)(46)
C_5	$\begin{cases}(15)(28)(37)(46)\\(17)(26)(35)(48)\end{cases}$	C_{10}	$\begin{cases}(1638)(2745)\\(1836)(2547)\end{cases}$

The symmetries are found in their conjugacy classes as follows:
C_1 *contains the identity.*
C_2 *contains the quarter turns about the axis through the midpoints of the squares.*
C_3 *contains the half turn about the axis through the midpoints of the squares.*
C_4 *contains the half turns about the axes passing through the midpoints of opposite rectangular faces.*
C_5 *contains the half turns about axes through the midpoints of opposite long edges.*
C_6 *contains the reflection in a vertical plane halfway between the square faces.*
C_7 *contains the reflections in planes passing through the midpoints of edges of the squares.*
C_8 *contains the reflections in planes passing through opposite long edges.*
C_9 *contains the reflection in the point at the centre of the cuboid. This can also be seen as the composition* $(13)(24)(57)(68)\circ(15)(26)(37)(48)$.
C_{10} *contains the elements usually known as the composites. They cannot be described by a single reflection or rotation. They are found by taking the composition of each element of* C_2 *with the reflection* (15)(26)(37)(48).

By the theorem of Lagrange the possible orders of non-trivial subgroups are 2, 4 and 8.
A normal subgroup of order 2 is $C_1 \cup C_6$. *There are in total three such subgroups, the other two being* $C_1 \cup C_3$ *and* $C_1 \cup C_9$
A non-normal subgroup of order 2 is $\{e, (18)(27)(36)(45)\}$. *There are another seven each containing the identity and an element of order 2 where the element is not in a conjugacy class on its own.*
A normal subgroup of order 4 is $C_1 \cup C_2 \cup C_3$. *I was reasonably sure that this was unique but have found* $C_1 \cup C_3 \cup C_8$. *Fortunately, in this course, we are not asked to find all the normal subgroups but it can be of interest to try to be exhaustive.*
A non-normal subgroup of order 4 is
$\{e, (14)(23)(58)(67), (15)(26)(37)(48), (18)(27)(36)(45)\}$. *This is the set of symmetries*

keeping the top face fixed. Another example would keep the front face fixed. There may be others.
A normal subgroup of order 8 is $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5$. *This contains all the direct symmetries.*
All subgroups of order 8 are automatically normal. This is because the index of such a subgroup is 2 and any subgroup of index 2 is normal.

(A common error when looking for normal subgroups is to look for unions of conjugacy classes with one of the classes containing e and where the order of the union is a divisor of the order of the group and to stop the process when this has been done. It is also necessery to check that the union has given a subgroup. The union is frequently not closed.)