MST210/J

MST2101506F1

Module Examination 2015
Mathematical methods, models and modelling

Monday 8 June 2015 **10.00 am–1.00 pm**

Time allowed: 3 hours

You are **not** allowed to use a calculator in this examination.

There are THREE parts to this paper, each taking approximately one hour.

In each part of the paper the questions are arranged, as far as possible, in the order in which they appear in the module.

Part 1 consists of 19 computer-marked questions, each worth 2 marks. Mark your answers on the form provided using an HB pencil. Detailed instructions on filling in the computer-marked form are given overleaf.

Part 2 consists of 6 questions, each worth 5 marks.

Part 3 consists of 5 questions, each worth 16 marks. The marks from your best two answers will be added together to give a maximum of 32 marks for this part.

In Parts 2 and 3: Write your answers in **pen** in the answer book(s) provided. The marks allocated to each part of each question are given in square brackets in the margin. Unless you are directed otherwise in the question, you may use any formula or other information from the Handbook in your answers. Do **not** cross out any answers unless you have a better alternative answer – everything not crossed out may receive credit.

At the end of the examination: Check that you have written your personal identifier and examination number on each answer book used, and that you have written your personal identifier and name on the CME form. **Failure to do so may mean that your work cannot be identified.** Attach your signed desk record to the front of your answer book(s) using the round paper fastener, then attach the CME form and your question paper to the back of the answer book(s) using the flat paperclip.

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Instructions for filling in the computer-marked examination (CME) form

- You will find one CME form provided with this paper. The invigilator has a supply of spare forms if you should need any.
- You should use an **HB pencil** to make entries on the CME form. If you make any smudges or other spurious marks on the form that you cannot cancel out clearly, you should *ask the invigilator for a new form*, and transfer your entries to it.
- If you do not wish to answer a question, pencil across the 'don't know' cell ('?').
- If you think that a question is unsound in any way, pencil across the 'unsound' cell ('U'), *in addition* to pencilling across either an answer cell or the 'don't know' cell.
- For each question, you **must** pencil across **either** the required number of answer cells **or** the 'don't know' cell.
- We suggest that, in the first instance, you answer by pencilling across the relevant cells in the facsimile CME rows reproduced at back of this question paper. Check your answers before transferring them to your CME form.
- You should note that **no additional time** will be allowed at the end of the three-hour period for transferring your marks to the CME form.
- On Part 1 of the CME form, you must *write* in your personal identifier (NOT the examination number) and the 'assignment number' for this examination (**MST210 81**). You should also *pencil* across the cells in the two blocks in Part 1 of the form corresponding to your personal identifier and the assignment number given above. We suggest you check that Part 1 on the CME form has been completed correctly before moving on to Part 2 of the examination paper.
- Failure to follow the above instructions may mean that we will not be able to award you a mark for this part of the examination.
- Use the answer book(s) provided for any rough work.
- Incorrect answers will not be penalised, i.e. you will not lose marks for incorrect answers.

PART 1

You should attempt **all** questions in this part of the paper. Each question is worth 2 marks. You should answer in **pencil** on the CME form provided.

Question 1

Select the option that gives an expression for the integrating factor of the differential equation

$$\frac{dy}{dx} + 2\frac{y}{x} = \sin x$$

Options

A $e^{\frac{2}{x}}$ **B** $e^{\frac{1}{4}x^2}$ **C** x^2 **D** $2\ln x$

Question 2

Select the option that gives the angle between the two vectors $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $3\mathbf{j} + 4\mathbf{k}$.

Options

A $\arccos\left(-\frac{2}{45}\right)$ **B** $\arccos\left(\frac{2}{3}\right)$ **C** $\arccos\left(\frac{2}{45}\right)$ **D** $\arccos\left(-\frac{2}{3}\right)$

Question 3

A particle starts at the origin at time $t = 0$ and moves with its velocity at time t given by $\left(t^2 - \frac{1}{2}\sin(2t)\right)\mathbf{i}$. Select the option that gives its position at time t.

Options

A $\left(\frac{1}{3}t^3 + \frac{1}{4}\cos(2t)\right)\mathbf{i}$ **B** $\left(\frac{1}{3}t^3 + \frac{1}{4}\cos(2t) - \frac{1}{4}\right)\mathbf{i}$

C $\left(\frac{1}{3}t^3 + \frac{1}{4}\sin(2t)\right)\mathbf{i}$ **D** $\left(\frac{1}{3}t^3 + \frac{1}{4}\cos(2t) + \frac{1}{4}\right)\mathbf{i}$

Question 4

Consider using the method of Gaussian elimination to solve the following system of linear equations:

$$\begin{pmatrix} 3 & 18 & 0 \\ -7 & 14 & 7 \\ 14 & 19 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Select the option that gives the first step of the Gaussian elimination procedure.

Options

A $\mathbf{R}_{2a} = \mathbf{R}_2 - 7\mathbf{R}_1$ **B** $\mathbf{R}_{2a} = \mathbf{R}_2 - 3\mathbf{R}_1$

C $\mathbf{R}_{2a} = \mathbf{R}_2 - \frac{7}{3}\mathbf{R}_1$ **D** $\mathbf{R}_{2a} = \mathbf{R}_2 + \frac{7}{3}\mathbf{R}_1$

Question 5

Select the option that gives the eigenvalues of $\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$.

Options

A $2+i$ and $2-i$ **B** -5 and 1

C -1 and 5 **D** $-2-i$ and $-2+i$

Question 6

Consider the system of differential equations

$$\begin{cases} \dot{x} = 6x - 2y, \\ \dot{y} = -2x + 9y. \end{cases}$$

The coefficient matrix $\begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix}$ has eigenvectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, and the corresponding eigenvalues are 5 and 10.

Select the option that gives the general solution of this system.

Options

A $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{5t} + \beta \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{10t}$

B $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cos(5t) + \beta \begin{pmatrix} -1 \\ 2 \end{pmatrix} \sin(10t)$

C $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + \beta \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{10t}$

D $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} \sin(5t) + \beta \begin{pmatrix} -1 \\ 2 \end{pmatrix} \cos(10t)$

Question 7

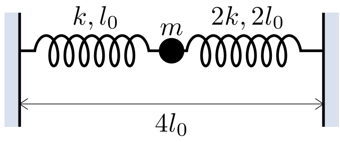
Select the option that gives $\partial f/\partial y$ for the function $f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$.

Options

A $\dfrac{-\frac{1}{2}xy}{\sqrt{x^2+y^2}}$ **B** $\dfrac{2xy}{(x^2+y^2)^{3/2}}$ **C** $\dfrac{-xy}{(x^2+y^2)^{3/2}}$ **D** $\dfrac{xy}{\sqrt{x^2+y^2}}$

Question 8

A particle is attached on its left to a spring of natural length l_0 and stiffness k and on its right to a spring of stiffness $2k$ and natural length $2l_0$. It lies on a smooth horizontal table, and the two spring ends are a fixed distance $4l_0$ apart, as shown below.



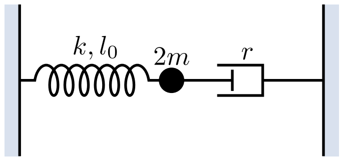
The particle is displaced to the midpoint position. Select the option that gives the force exerted on the particle by the right-hand spring.

Options

A $\mathbf{0}$ **B** $-2kl_0\,\mathbf{i}$ **C** $2kl_0\,\mathbf{i}$ **D** $-4kl_0\,\mathbf{i}$

Question 9

A particle of mass $2m$ is attached to a model spring of natural length l_0 and stiffness k, and to a model damper with damping constant r. The other ends of the spring and damper are fixed.



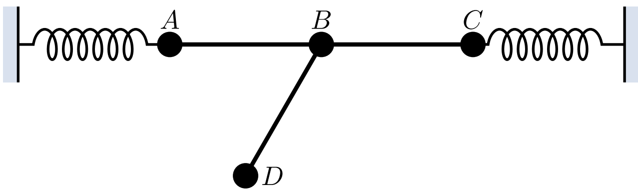
Select the option that gives the condition for strong damping in this system.

Options

A $r^2 > mk$ **B** $r^2 > 8mk$ **C** $r^2 > 2mk$ **D** $r^2 > 4mk$

Question 10

Consider the mechanical system consisting of four particles labelled A, B, C and D, as shown in the figure below. The particles are connected by either springs or rigid rods (which are shown in the figure by thick lines). Particles A, B and C are constrained to move along a straight line between the connections to the rigid walls. Particle D is freely hinged below particle B to move in a vertical plane.



Select the option that gives the number of degrees of freedom of the mechanical system.

Options

A 1 **B** 3 **C** 4 **D** 2

Question 11

Consider the system of non-linear differential equations

$$\frac{dx}{dt} = xy - 4x,$$
$$\frac{dy}{dt} = xy + x + y^2 - 16.$$

Select the option that gives a complete list of equilibrium points for this system.

Options

A $(4, 0)$ **B** $(0, -4),\ (0, 4)$

C $(1, -3),\ (1, 5)$ **D** $(-4, 0),\ (4, 0)$

Question 12

Consider the function $f(t) = -2t^2 + 1$ defined on the interval $-1 \leq t \leq 1$.

Select the option that best describes this function.

Options

A The function is neither even nor odd. **B** The function is odd.

C The function is even. **D** The function is both even and odd.

Question 13

Consider the function of two variables

$$u(x, t) = 2e^{-t}\sin(2x)$$

Select the option that gives a partial differential equation for which this function of two variables is a solution.

Options

A $\dfrac{d^2u}{dx^2} = -4\dfrac{du}{dt}$ **B** $\dfrac{d^2u}{dx^2} = 4\dfrac{du}{dt}$ **C** $\dfrac{d^2u}{dx^2} = -\dfrac{1}{4}\dfrac{d^2u}{dt^2}$ **D** $\dfrac{d^2u}{dx^2} = 4\dfrac{d^2u}{dt^2}$

Question 14

A scalar field is defined in cylindrical coordinates by

$$V(r, \phi, z) = rz\cos\phi.$$

Select the option that gives the $\mathbf{e}_\phi$-component of $\boldsymbol{\nabla} V$.

Options

A $z\cos\phi$ **B** $r\cos\phi$ **C** $-z\sin\phi$ **D** $-rz\sin\phi$

Question 15

A conservative vector field is given by

$$\mathbf{F} = \cos y\,\mathbf{i} - x\sin y\,\mathbf{j} - e^z\mathbf{k}.$$

Select the option that gives a potential function for $\mathbf{F}$.

Options

A $U(x,y,z) = x\cos y - e^z$

B $U(x,y,z) = -2x\cos y + e^z$

C $U(x,y,z) = x\sin y + e^z - \cos y$

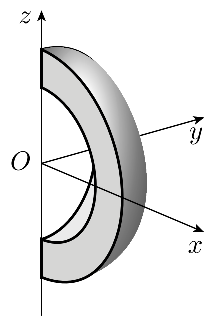
D $U(x,y,z) = -x\cos y + e^z$

Question 16

Select the option that includes the correct limits of integration for an integral in spherical polar coordinates for the volume of a segment of a thick spherical shell given in Cartesian coordinates defined by the inequalities

$$1 \le x^2+y^2+z^2 \le 4, \quad x \ge 0, \quad y \ge 0,$$

and depicted below.



Options

A $\displaystyle\int_{r=1}^{r=2}\left(\int_{\theta=0}^{\theta=\pi}\left(\int_{\phi=0}^{\phi=\pi} r^2\sin\theta\,d\phi\right)d\theta\right)dr$

B $\displaystyle\int_{r=1}^{r=4}\left(\int_{\theta=0}^{\theta=\pi}\left(\int_{\phi=0}^{\phi=\pi/2} r^2\sin\theta\,d\phi\right)d\theta\right)dr$

C $\displaystyle\int_{r=1}^{r=2}\left(\int_{\theta=0}^{\theta=\pi}\left(\int_{\phi=0}^{\phi=\pi/2} r^2\sin\theta\,d\phi\right)d\theta\right)dr$

D $\displaystyle\int_{r=1}^{r=4}\left(\int_{\theta=0}^{\theta=\pi}\left(\int_{\phi=0}^{\phi=\pi} r^2\sin\theta\,d\phi\right)d\theta\right)dr$

Question 17

A particle of mass $2m$ and velocity $u\mathbf{i}+v\mathbf{j}$ collides with a particle of mass m and velocity $U\mathbf{i}+V\mathbf{j}$. The two particles coalesce into a single particle. Select the option that gives the $\mathbf{i}$-component of the momentum of the single particle after the collision.

Options

A $mu + mU$

B $\frac{2}{3}mu + \frac{1}{3}mU$

C $2mu + mU$

D $mu + 2mU$

Question 18

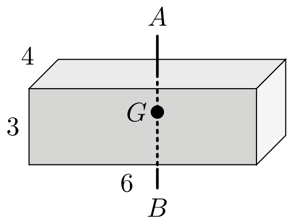
A particle of mass m is moving with a constant speed on a smooth horizontal circular wire of radius R. The unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ take their usual meanings. Which option gives the direction of the resultant force acting on the bead?

Options

A $-\mathbf{e}_r$ **B** $\mathbf{e}_\theta$ **C** $-\mathbf{e}_\theta$ **D** $\mathbf{e}_r$

Question 19

Consider a solid rectangular cuboid of mass M has width 6, height 3 and depth 4, as shown in the diagram below.



Select the option that gives the moment of inertia of the cuboid about the axis AB through its centre of mass G,

Options

A $\dfrac{45}{12}M$ **B** $\dfrac{52}{12}M$ **C** $\dfrac{61}{12}M$ **D** $\dfrac{25}{12}M$

PART 2

You should attempt **all** questions in this part of the paper. Each question is worth 5 marks. You should answer in **pen** in the answer book(s) provided.

Question 20

Consider the vectors

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}, \quad \mathbf{b} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(a) Calculate the scalar product of **a** and **b** and hence calculate the cosine of the angle between **a** and **b**. *[3]*

(b) Calculate the vector product of **a** and **b**. *[2]*

Question 21

Consider the matrix

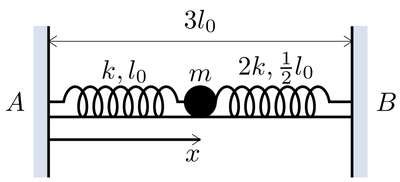
$$\mathbf{A} = \begin{pmatrix} 1 & -4 & 3 \\ -3 & 2 & -3 \\ 2 & 4 & 0 \end{pmatrix}.$$

(a) Given that the matrix **A** has an eigenvector $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, calculate the corresponding eigenvalue. *[2]*

(b) Given that one of the eigenvalues of the matrix **A** is 2, calculate a corresponding eigenvector. *[3]*

Question 22

A particle of mass m rests on an air track. It is connected by two springs to fixed points at A and B, which are a distance $3l_0$ apart.



The left-hand spring has natural length l_0 and stiffness k, whilst the right-hand spring has natural length $l_0/2$ and stiffness $2k$. Let x be the distance of the particle from A. Derive the equation of motion that x satisfies. *[5]*

Question 23

The system of non-linear differential equations

$$\dot{x} = \frac{xy}{\pi}$$
$$\dot{y} = \sin x + \cos y,$$

has an equilibrium point at $(0, \frac{1}{2}\pi)$.

(a) Calculate the Jacobian matrix of this system of equations and evaluate this matrix at the given equilibrium point. [*3*]

(b) Use your answer to part (a) to classify this equilibrium point. [*2*]

Question 24

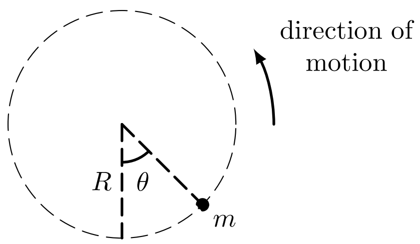
A vector field $\mathbf{v}$ is expressed in spherical coordinates as

$$\mathbf{v}(r, \theta, \phi) = 2r\cos\theta\,\mathbf{e}_r - r\sin\theta\,\mathbf{e}_\theta.$$

Calculate $\boldsymbol{\nabla} \times \mathbf{v}$ and hence show that $\mathbf{v}$ is everywhere conservative. [*5*]

Question 25

A stunt motorcyclist wishes to loop the loop, free-wheeling (i.e. no power from the engine to the wheels) on a circular track of radius R which lies in a vertical plane, as shown below.



Model the motorcycle plus rider as a particle of mass m, whose position on the track is described by the angle θ. Neglect friction and air resistance.

(a) Draw a force diagram indicating all the forces acting on the particle. [*1*]

(b) Express the forces in component form using plane polar unit vectors. [*2*]

(c) Show that the equation of motion of the particle resolved in the radial direction is given by

$$mR\dot{\theta}^2 = |\mathbf{N}| - mg\cos\theta,$$

where $|\mathbf{N}|$ is the magnitude of the normal reaction force of the track on the motorcycle. Find the corresponding transverse equation of motion of the particle. [*2*]

PART 3

You should attempt **two** questions in this part of the paper. Each question is worth 16 marks. All of your answers will be marked, and the marks from your best two answers will be added together, giving a maximum of 32 marks from this part. You should answer in **pen** in the answer book(s) provided.

Question 26

Consider the function

$$f(x, y) = 2x^2 + xy^2 - 6xy + 5x.$$

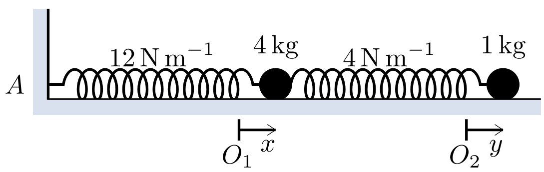
(a) Calculate the partial derivatives $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$. [5]

(b) Find the stationary points of f. [5]

(c) Classify each of the stationary points. [6]

Question 27

Consider the longitudinal vibrations of two particles of mass 4 kg and 1 kg respectively. The left-hand particle is attached by a model spring of stiffness $12\,\mathrm{N\,m^{-1}}$ to a fixed point A. The two particles are connected by a model spring of stiffness $4\,\mathrm{N\,m^{-1}}$. Choose to measure the displacements x and y of the two particles from their respective equilibrium positions O_1 and O_2, as shown in the diagram.



(a) Draw a force diagram for each particle, showing all of the forces acting on the particles. [2]

(b) Express the changes in the forces from equilibrium due to the springs in terms of the variables and parameters above. [3]

(c) Hence show that the equation of motion of the system can be written as

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \text{where } \mathbf{A} = \begin{pmatrix} -4 & 1 \\ 4 & -4 \end{pmatrix}.$$ [3]

(d) Find the eigenvalues of the matrix $\mathbf{A}$ and hence write down the normal mode angular frequencies of the system. [4]

(e) Find the eigenvectors of the matrix $\mathbf{A}$ and hence write down whether each normal mode is in-phase, or phase-opposed. [4]

Question 28

A periodic function $f(t)$ is defined by

$$f(t) = \begin{cases} 0, & -\pi < t \leq 0, \\ t, & 0 < t \leq \pi, \end{cases}$$

and $f(t + 2\pi) = f(t)$.

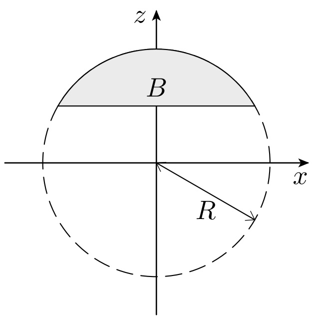
(a) Sketch the graph of f over three periods. State the fundamental period of f. State whether f is even, odd or neither of these. [4]

(b) State the values of $f(-17\pi)$ and $f(12\pi)$, explaining your reasoning. [3]

(c) Find the Fourier series, $F(t)$, for the function $f(t)$. [9]

Question 29

A region B is defined to be the part of the sphere of radius R that lies above a horizontal plane $z = R/2$, which is halfway between the upper pole and the equator. The region B is shown as the shaded region in the side view below, where x, y and z are Cartesian coordinates.



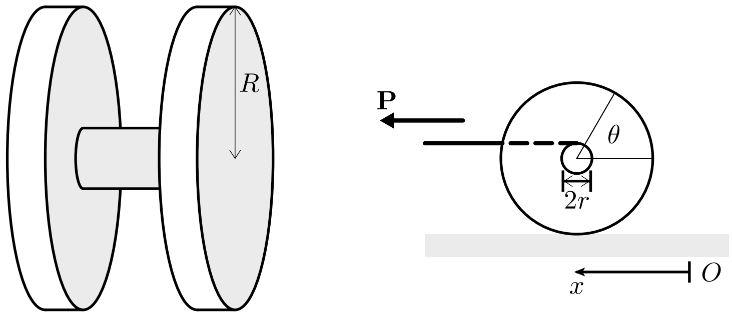
(a) Use cylindrical polar coordinates to show the volume of the region B is $\frac{5}{24}\pi R^3$. [9]

(b) Evaluate the integral $\displaystyle\int_B z\, dV$ in cylindrical polar coordinates. [4]

(c) Use your answers to parts (a) and (b) to find the centre of mass of an object of uniform density ρ_B occupying the region B (in terms of the radius R). [3]

Question 30

A wheel-axle assembly consists of a pair of heavy wheels, each of radius R and mass M, joined by a rigid axle of radius r and mass m. The wheels rest on a rough surface such that the wheels roll without slipping. The wheels start with the centre of the axle above a point O. After the wheel-axle assembly has moved a distance x the axle has turned through an angle θ, as shown in the following diagram.



A rope is wound tightly anticlockwise around the axle. The free end of the rope is subjected to a constant horizontal force of magnitude P. The rope is wound halfway between the wheels (i.e. at the middle of the axle) so that the tension force due to the rope is in the plane of the above diagram (which in turn means that the subsequent motion is in the direction x marked). Model the rope as a model string and assume that it unwinds without slipping from the top of the axle, as shown in the right-hand diagram.

(a) Draw a force diagram showing all of the forces acting on the wheel-axle assembly and mark the point of action of each force. [3]

(b) Show that the moment of inertia I of the wheel-axle assembly about the centre of the axle is given by

$$I = MR^2 + \tfrac{1}{2}mr^2.$$ [2]

(c) Apply Newton's second law to obtain an equation for the friction force at the point of contact F in terms of the acceleration $\ddot{x}$ and other parameters. [3]

(d) Write down the rolling condition for this motion and use it to obtain a relationship between $\ddot{x}$ and $\ddot{\theta}$. [2]

(e) Calculate the torques of each force about an axis running through the centre of the axle. [3]

(f) Show that the acceleration of the wheel-axle assembly is given by

$$\ddot{x} = \frac{PR(R+r)}{3MR^2 + mR^2 + \frac{1}{2}mr^2}.$$ [3]

[END OF QUESTION PAPER]

COMPUTER MARKED EXAMINATION FORM

08-10

PART 1

PERSONAL IDENTIFIER
(NOT Examination Number)

ASSIGNMENT NUMBER
(as given on the question paper)

NAME and INITIALS

PERSONAL IDENTIFIER

A	0	0	0	0	0	0	0
B	1	1	1	1	1	1	1
C	2	2	2	2	2	2	2
D	3	3	3	3	3	3	3
E	4	4	4	4	4	4	4
F	5	5	5	5	5	5	5
G	6	6	6	6	6	6	6
H	7	7	7	7	7	7	7
K	8	8	8	8	8	8	8
L	9	9	9	9	9	9	9
M							X
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W							
X							
Y							

MODULE AND ASSIGNMENT NUMBER

A	A	A	A	0	0	0		0	–
B	B	B	B	1	1	1		1	–
C	C	C	C	2	2	2	2	2	–
D	D	D	D	3	3	3	3	3	
E	E	E	E	4	4	4	4	4	
F	F	F	F	5	5	5	5	5	
G	G	G	G	6	6	6	6	6	
H	H	H	H	7	7	7	7	7	
K	K	K	K	8	8	8	8	8	
L	L	L	L	9	9	9	9	9	
M	M	M	M						
N	N	N	N						
P	P	P	P						
R	R	R	R						
S	S	S	S						
T	T	T	T						
U	U	U	U						
W	W	W	W						
X	X	X	X						
Y	Y	Y	Y						
Z	Z	Z	Z						

OFFICE USE ONLY

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

IMPORTANT NOTES

1. Please check that all of this form has been completed including the coloured boxes in Part 1 with your Personal Identifier, Module and Assignment Number.
2. Only use an HB pencil to complete this form and press firmly.
3. Shown opposite are examples of the correct marks to use. Do not use any other types of mark.

CORRECT MARK

Pencil across between dots A

CANCELLATION MARK

To cancel a mark pencil in the coloured part of the cell. Do not rub out

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PLEASE DO NOT PUNCH HOLES IN THIS FORM

Fold this form INWARDS along the dotted line

PART 2

	ANSWER											ANSWER											ANSWER									
1	A	B	C	D	E	F	G	H	?	U	21	A	B	C	D	E	F	G	H	?	U	41	A	B	C	D	E	F	G	H	?	U
2	A	B	C	D	E	F	G	H	?	U	22	A	B	C	D	E	F	G	H	?	U	42	A	B	C	D	E	F	G	H	?	U
3	A	B	C	D	E	F	G	H	?	U	23	A	B	C	D	E	F	G	H	?	U	43	A	B	C	D	E	F	G	H	?	U
4	A	B	C	D	E	F	G	H	?	U	24	A	B	C	D	E	F	G	H	?	U	44	A	B	C	D	E	F	G	H	?	U
5	A	B	C	D	E	F	G	H	?	U	25	A	B	C	D	E	F	G	H	?	U	45	A	B	C	D	E	F	G	H	?	U
6	A	B	C	D	E	F	G	H	?	U	26	A	B	C	D	E	F	G	H	?	U	46	A	B	C	D	E	F	G	H	?	U
7	A	B	C	D	E	F	G	H	?	U	27	A	B	C	D	E	F	G	H	?	U	47	A	B	C	D	E	F	G	H	?	U
8	A	B	C	D	E	F	G	H	?	U	28	A	B	C	D	E	F	G	H	?	U	48	A	B	C	D	E	F	G	H	?	U
9	A	B	C	D	E	F	G	H	?	U	29	A	B	C	D	E	F	G	H	?	U	49	A	B	C	D	E	F	G	H	?	U
10	A	B	C	D	E	F	G	H	?	U	30	A	B	C	D	E	F	G	H	?	U	50	A	B	C	D	E	F	G	H	?	U
11	A	B	C	D	E	F	G	H	?	U	31	A	B	C	D	E	F	G	H	?	U	51	A	B	C	D	E	F	G	H	?	U
12	A	B	C	D	E	F	G	H	?	U	32	A	B	C	D	E	F	G	H	?	U	52	A	B	C	D	E	F	G	H	?	U
13	A	B	C	D	E	F	G	H	?	U	33	A	B	C	D	E	F	G	H	?	U	53	A	B	C	D	E	F	G	H	?	U
14	A	B	C	D	E	F	G	H	?	U	34	A	B	C	D	E	F	G	H	?	U	54	A	B	C	D	E	F	G	H	?	U
15	A	B	C	D	E	F	G	H	?	U	35	A	B	C	D	E	F	G	H	?	U	55	A	B	C	D	E	F	G	H	?	U
16	A	B	C	D	E	F	G	H	?	U	36	A	B	C	D	E	F	G	H	?	U	56	A	B	C	D	E	F	G	H	?	U
17	A	B	C	D	E	F	G	H	?	U	37	A	B	C	D	E	F	G	H	?	U	57	A	B	C	D	E	F	G	H	?	U
18	A	B	C	D	E	F	G	H	?	U	38	A	B	C	D	E	F	G	H	?	U	58	A	B	C	D	E	F	G	H	?	U
19	A	B	C	D	E	F	G	H	?	U	39	A	B	C	D	E	F	G	H	?	U	59	A	B	C	D	E	F	G	H	?	U
20	A	B	C	D	E	F	G	H	?	U	40	A	B	C	D	E	F	G	H	?	U	60	A	B	C	D	E	F	G	H	?	U