TMA08 MST210 PSS85734 C.TOWNHILL

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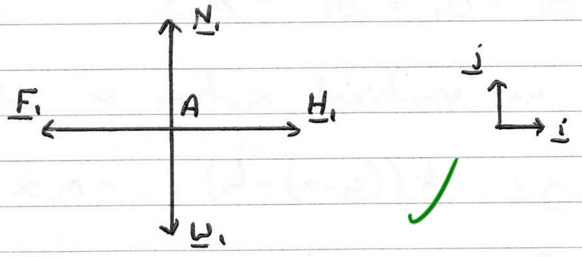
Question 1

a) The position of the centre of mass of the two particles is

$$\underline{r}_G = \frac{m_1 x + m_2 y}{m_1 + m_2}\,\underline{i}$$ ✓ 2/2

$\underline{r}_G$ is a vector

b) The force diagram for A is



✓

The forces in vector form are

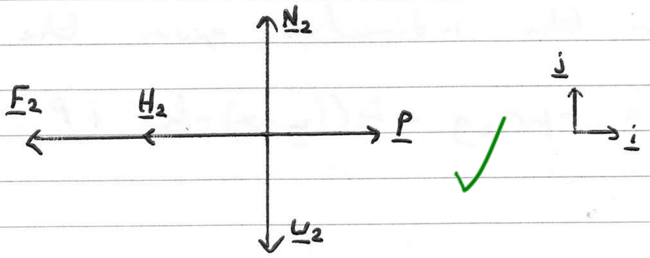
$$\underline{W}_1 = -m_1 g\,\underline{j} \quad , \quad \underline{N}_1 = -\underline{W}_1 = m_1 g\,\underline{j}$$ ✓ ✓

$$\underline{F}_1 = -\mu|\underline{N}_1|\,\underline{i} = -\mu m_1 g\,\underline{i}$$ ✓ ✓

& $$\underline{H}_1 = k((y-x)-l_0)\,\underline{i}$$ ✓

The spring force, $\underline{H}_1$, is internal, the other three, $\underline{W}_1$, $\underline{N}_1$ and $\underline{F}_1$ are external. ✓

The force diagram for B is



✓

Forces in vector form ✓

$$\underline{W}_2 = -m_2 g\,\underline{i} \quad , \quad \underline{N}_2 = -\underline{W}_2 = m_2 g\,\underline{j}$$ ✓ ✓

$\underline{F}_2 = -\mu |\underline{N}_2| \underline{i} = -\mu m_2 g \underline{i}$ ✓

$\underline{H}_2 = -k((y-x) - l_0)\underline{i}$, & $\underline{P} = P\underline{i}$ ✓ 5/5

The spring force, $\underline{H}_2$, is internal ✓ the other 4, $\underline{W}_2$, $\underline{N}_2$, $\underline{F}_2$ and $\underline{P}$ are external. ✓

c) From Newton's 2nd Law, for particle A then

$$\underline{F}_1 + \underline{W}_1 + \underline{N}_1 + \underline{H}_1 = m_1 \ddot{x}\underline{i}$$ vector equation

there is no vertical motion so $\underline{W}_1 + \underline{N}_1 = 0$

$$\Rightarrow -\mu m_1 g \underline{i} + k((y-x) - l_0)\underline{i} = m_1 \ddot{x}\underline{i}$$ ✓

Resolving for the i-direction gives the equation of motion for particle A

$$m_1 \ddot{x} = k((y-x) - l_0) - \mu m_1 g$$ ✓

Similarly for particle B 6/6

$$\underline{F}_2 + \underline{W}_2 + \underline{N}_2 + \underline{H}_2 + \underline{P} = m_2 \ddot{y}$$

As before, there is no movement vertically so

$$\underline{W}_2 + \underline{N}_2 = 0$$ ✓

$$\Rightarrow -\mu m_2 g \underline{i} - k((y-x) - l_0)\underline{i} + P\underline{i} = m_2 \ddot{y}\underline{i}$$

Resolving for the i-direction gives the equation of motion ✓

$$m_2 \ddot{y} = -\mu m_2 g - k((y-x) - l_0) + P$$

Question 1 (cont)

d) Using equation (10) of unit 19, the equation of motion of the centre of mass is

$$\underline{F}^{ext} = m\ddot{\underline{r}}_G$$

From part (a), the centre of mass is

$$\underline{r}_G = \frac{m_1 x + m_2 y}{m_1 + m_2} \underline{i}$$

differentiating twice gives us

$$\ddot{\underline{r}}_G = \frac{m_1 \ddot{x} + m_2 \ddot{y}}{m_1 + m_2} \underline{i}$$

From part (c) we know that

$$m_1 \ddot{x} = -\mu m_1 g + k((y-x) - l_0) \checkmark$$

and $$m_2 \ddot{y} = -\mu m_2 g - k((y-x) - l_0) + P$$

so $$m_1\ddot{x} + m_2\ddot{y} = -\mu(m_1+m_2)g + P$$

hence the equation of motion of the centre of mass is ✓

$$m\ddot{\underline{r}}_G = (-\mu(m_1+m_2)g + P)\underline{i}$$

To verify that this is equivalent as treating the system as a particle of mass m_1+m_2 subject to only external forces. Then the external forces are

$$\underline{F}_1, \underline{F}_2 \;\&\; \underline{P} \checkmark$$

$$\Rightarrow F^{ext} = -\mu m_1 g + \mu m_2 g + P$$

$$= -\mu(m_1+m_2)g + P \checkmark$$

3/3

hence they are equivalent.

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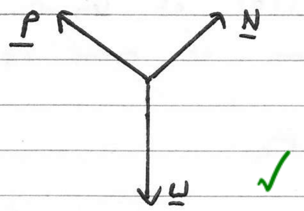
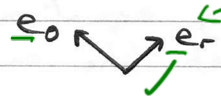
Question 2

a) The force diagram is





← vectors

What are $\underline{P}$ $\underline{N}$ $\underline{W}$? worth saying.

b) In terms of the unit vectors they are

$$\underline{P} = P e_\theta \quad , \quad \underline{N} = |\underline{N}| e_r$$

3/3

& $$\underline{W} = -mg\cos\theta\, e_\theta - mg\sin\theta\, e_r$$

c) Using Newton's 2nd Law then

vectors

$$m\ddot{\underline{r}} = \underline{P} + \underline{N} + \underline{W}$$

From equation (21) of unit 20

$$\ddot{\underline{r}} = -R\dot{\theta}^2 e_r + R\ddot{\theta} e_\theta$$

$$\Rightarrow \quad -mR\dot{\theta}^2 e_r + mR\ddot{\theta} e_\theta = -mg\cos\theta\, e_\theta - mg\sin\theta\, e_r + |\underline{N}| e_r + P e_\theta$$

Resolving radially, i.e e_r gives

$$-mR\dot{\theta}^2 = |\underline{N}| - mg\sin\theta$$

Resolving tangentially gives

$$mR\ddot{\theta} = P - mg\cos\theta$$

5/5

hence as required

$$\ddot{\theta} = \frac{P - mg\cos\theta}{mR}$$

Question 2 (cont)

d) From part (c)

$$\ddot{\theta} = \frac{P - mg\cos\theta}{mR} = f(\theta)$$

Now
$$\frac{d}{dt}\left(\dot{\theta}^2\right) = \frac{d(\dot{\theta}^2)}{d\theta}\frac{d\theta}{dt} = 2\dot{\theta}\ddot{\theta} = 2\theta f(\theta)$$

So
$$\dot{\theta}^2 = 2\int f(\theta)\,d\theta$$

$$\Rightarrow 2\int \frac{P - mg\cos\theta}{mR}\,d\theta$$

$$\Rightarrow \frac{2P\theta - 2mg\sin\theta}{mR}$$

Now from part (c) we know that

$$-mR\dot{\theta}^2 = |\underline{N}| - mg\sin\theta$$

Substituting gives

$$-mR\left[\frac{2P\theta - 2mg\sin\theta}{mR}\right] = |\underline{N}| - mg\sin\theta$$

$$\Rightarrow 2mg\sin\theta - 2P\theta = |\underline{N}| - mg\sin\theta$$

hence the magnitude of the normal reaction is

$$|\underline{N}| = 3mg\sin\theta - 2P\theta$$

4/4

e) Initially $\theta = 0$. If $P = \frac{1}{2}mg$ then $|\underline{W}| > P$, so the particle would fall vertically down, hence leaving the cylinder at $\theta = 0$ Why 2

2/3

(e) The particle leaves the surface of the cylinder when the normal reaction is zero. With the given force $\mathbf{P}$ we have $|\mathbf{N}| = mg(3\sin\theta - \theta)$, so this is zero when $3\sin\theta - \theta = 0$. To find which root of this equation is appropriate we look at the sign of the acceleration. Using the result from part (c) we have

$$\ddot{\theta} = \frac{\frac{1}{2}mg - mg\cos\theta}{mR} = \frac{g}{R}\left(\tfrac{1}{2} - \cos\theta\right).$$

As $\theta = 0$ when $t = 0$, the initial acceleration is negative and so the particle moves clockwise.

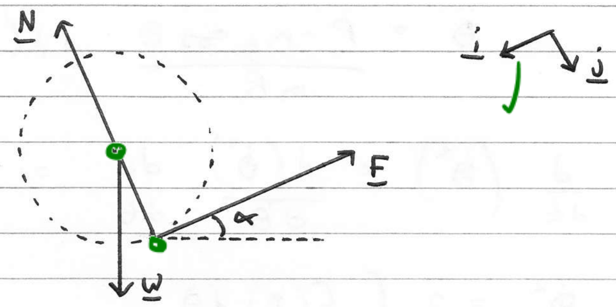
> The solution $\theta = 0$ of the equation $3\sin\theta - \theta = 0$ is the appropriate solution of the equation, and hence the particle leaves the surface when $\theta = 0°$, to the nearest degree.

Question 3

16/17

a) The force diagram for the sphere is

you were asked to show the point of action. ie for $\underline{F}$ and $\underline{N}$ the point of contact between sphere and surface for $\underline{W}$ centre.



2/3

b) Applying Newton's second law then

$$m\ddot{x}\,\underline{i} = \underline{W} + \underline{N} + \underline{F}$$

$$\Rightarrow mg(\sin\alpha\,\underline{i} + \cos\alpha\,\underline{j}) - |\underline{N}|\,\underline{j} - |\underline{F}|\,\underline{i}$$

resolving for the i-direction gives

$$m\ddot{x} = mg\sin\alpha - |\underline{F}|$$

2/2

So the linear acceleration is

$$\ddot{x} = g\sin\alpha - \frac{|\underline{F}|}{m}$$

c) From the force diagram in part (a) we can see that both the weight, $\underline{W}$ and the normal force, $\underline{N}$, cut through the centre of the sphere so the torque for these two forces is zero

However, the torque for the friction force is

$$\Gamma = (R\underline{j}) \times (-F\underline{i}) = R|\underline{F}|\,\underline{k}$$

3/3

Question 3 (cont)

d) Note that as the sphere rotates anti-clockwise then the distance travelled, x is

$$x = R\theta$$

differentiating twice gives

$$\ddot{x} = R\ddot{\theta}$$

Now from table 1 in unit 21 the moment of inertia for a homogenous solid sphere is

$$I = \frac{2}{5}MR^2$$

and from part (c) we know that

$$\underline{\Gamma}^{rel} = R|\underline{F}|\underline{k}$$

so $$\Gamma^{rel}_{axis} = R|\underline{F}|$$

From the equation of relative rotational motion we know

$$\Gamma^{rel}_{axis} = I\ddot{\theta}$$

$$\Rightarrow \quad R|\underline{F}| = \frac{2}{5}MR^2\ddot{\theta}$$

but $$\ddot{\theta} = \frac{\ddot{x}}{R}$$

so, $$R|\underline{F}| = \frac{2}{5}MR\ddot{x}$$

therefore $$|\underline{F}| = \frac{2}{5}M\ddot{x}$$

Now from part (b) we know

$$M\ddot{x} = Mg\sin\alpha - |\underline{F}|$$

$$\Rightarrow \quad M\ddot{x} = Mg\sin\alpha - \frac{2}{5}M\ddot{x}$$

$$\Rightarrow \quad \frac{7}{5}\ddot{x} = g\sin\alpha$$

hence as required the acceleration of the sphere is

$$\ddot{x} = \frac{5}{7} g \sin\alpha$$

6/6

e) From unit 3 we have the equation

$$x = x_0 + V_0 t + \frac{1}{2} a t^2$$

where $x_0 = V_0 = 0$, and $x = L$, and $a = \ddot{x}$

Which rearranged

$$\Rightarrow \quad t^2 = \frac{2L}{\ddot{x}}$$

$$\Rightarrow \quad \frac{2L}{\frac{5}{7}\sin\alpha}$$

3/3

hence the time it would take the sphere roll the distance L is

$$t = \sqrt{\frac{14L}{5g\sin\alpha}}$$

5/5

Question 4

$$\frac{dy}{dx} = \frac{e^{-y}}{x^2}$$

a) The differential equation is of the form

$$\frac{dy}{dx} = g(x)\,h(y)$$

so we can use the method of separation of variables

Question 4 (cont)

$$\Rightarrow e^y\,dy = x^{-2}\,dx \quad ✓$$

$$\Rightarrow \int e^y\,dy = \int x^{-2}\,dx$$

$$\Rightarrow e^y = -x^{-1} + c \quad ✓$$

taking the log of both sides give the general solution explicitly

$$y = \ln\left(c - \frac{1}{x}\right) \quad ✓$$

$$= \ln\left(\frac{cx-1}{x}\right) \quad ✓$$

b) Given $y(1) = 0$, then substituting gives

$$0 = \ln(c-1)$$

$$\Rightarrow c = 2 \quad ✓$$

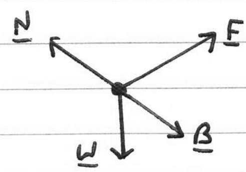
hence the particular solution is

$$y = \left(\frac{2x-1}{x}\right) \quad ✓$$

Question 5

3/5

a) The force diagram is



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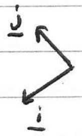


✓

$\underline{N} = |N|\,\underline{j}$ $\quad \underline{F} = -\mu|N|\,\underline{i}$ ✗

Now $\underline{N} = mg\underline{j}$ ✗, $\underline{F} = -\mu mg\underline{i}$ ✗, $\underline{B} = -B\underline{j}$ ✓

and $\underline{W} = mg(\sin\alpha\,\underline{i} - \cos\alpha\,\underline{j})$ ✓

b) From Newton's second law

$$m\ddot{x}\underline{i} = \underline{N} + \underline{W} + \underline{B} + \underline{F}$$ ✓

$$m\ddot{x}\underline{i} = |\underline{N}|\underline{j} + mg(\sin\alpha\,\underline{i} - \cos\alpha\,\underline{j}) - B\underline{j} - \mu|\underline{N}|\underline{i}$$

$$\Rightarrow \quad \cancel{mg\underline{j}} + mg(\sin\alpha\,\underline{i} - \cos\alpha\,\underline{j}) - \mu mg\underline{i} - B\underline{j} \times$$

resolving for the i-direction then

$$m\ddot{x} = mg\sin\alpha\,\underline{i} - \mu|\underline{N}| \quad (1)$$

$$m\ddot{x} = mg\sin\alpha - \mu mg \times$$

hence the acceleration is Resolve in $\underline{j}$

$$\ddot{x} = g\sin\alpha - \mu g \qquad 0 = -mg\cos\alpha + |\underline{N}| - B.$$

$$|\underline{N}| = mg\cos\alpha + B$$

Substitute this into (1) $\quad \ddot{x} = g\sin\alpha - \mu g\cos\alpha + \frac{B}{m}$

5/5

<u>Question 6</u>

$$\underline{A} = \begin{pmatrix} -8 & 7 & -3 \\ 4 & -5 & -3 \\ 4 & -7 & -1 \end{pmatrix}$$

a) Given $\underline{v}^* = (-1, 1, 1)^T$ we need to find λ where

$$A\underline{v} = \lambda\underline{v}$$

So

$$\begin{pmatrix} -8 & 7 & -3 \\ 4 & -5 & -3 \\ 4 & -7 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ -12 \\ -12 \end{pmatrix}$$ ✓

$\Rightarrow \lambda = -12$, hence the corresponding eigenvalue is $\lambda = -12$ ✓

b) Given that one eigenvalues is -4 then to find the corresponding eigenvector is need to solve

$$\det(A - \lambda\underline{I}) = 0$$

Question 6 (cont)

$\lambda = -4$

$$\Rightarrow A - \lambda I = \begin{pmatrix} -4 & 7 & -3 \\ 4 & -1 & -3 \\ 4 & -7 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

$$\Rightarrow \begin{aligned} -4a + 7b - 3c &= 0 \quad -① \\ 4a - b - 3c &= 0 \quad -② \\ 4a - 7b + 3c &= 0 \quad -③ \end{aligned}$$

② + ③ $\Rightarrow$ $8a - 8b = 0 \rightarrow a = b$

① $\Rightarrow$ $b = c$

hence a corresponding eigenvector is $(1, 1, 1)^T$

5/5

Question 7

$$f(x,y) = 2x^2 + xy^2 - 6xy + 5x + 2$$

a) For $(1, 3)$ to be a stationary point we need to verify

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$$

Now $\dfrac{\partial f}{\partial x} = 4x + y^2 - 6y + 5$

and $\dfrac{\partial f}{\partial y} = 2xy - 6x$

substituting gives

$$\frac{\partial f(1,3)}{\partial x} = 4 + 9 - (6 \times 3) + 5 = 0$$

and $\dfrac{\partial f(1,3)}{\partial y} = 2(1 \times 3) - 6 = 0$

hence $(1,3)$ is a stationary point of f.

b) To classify the stationary point then let

$$A = f_{xx} = 4, \quad B = f_{xy} = 2y - 6$$

& $C = f_{yy} = 2$

then as $AC - B^2 = 4 \times 2 - (2 \times 3 - 6)^2 > 0$

so $(1,3)$ is a local minimum

5/5

Question 8

a) From equation (25) of unit 9, the total mechanical energy is

$$E = \tfrac{1}{2}mv^2 + U(x)$$

where $U(x)$ is the potential energy of the sum of spring energy and the gravitational potential energy.

These are $U(x) = -mgx + \tfrac{1}{2}k(x-1)^2$

which are 9 and 10 in the handbook page 67

hence the total mechanical energy for the system is

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}k(x-1)^2 - mgx$$

b) If at $x = 2\,m$ then $v = 1\,ms^{-1}$ the total mechanical energy of the system is

$$E = \tfrac{1}{2} \times 2 \times 1^2 + \tfrac{1}{2} \times 4(2-1)^2 - 2 \times 10 \times 2$$

$$= 1 + 2 - 40$$

Question 8 (cont)

so $E = -37$

Using the law of conservation of mechanical energy then E is constant.

So at the springs natural length, $x = 1$

$\Rightarrow \quad -37 = v^2 + 0 - 20$

$\Rightarrow \quad v^2 = -17$

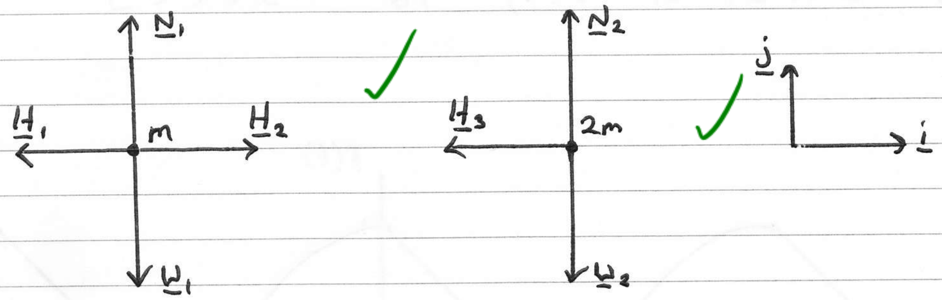
hence the value of v is not real, implying the system never reaches $x = 1$.

Therefore the spring is never compressed during the motion.

Question 9

5/5

a) The force diagrams are



remember ΔH when we move from equilibrium position

b) Applying Newton's second law of motion then

$$\left.\begin{aligned} m\ddot{\underline{r}}_1 &= \Delta\underline{H}_1 + \Delta\underline{H}_2 + \underline{N}_1 + \underline{W}_1 \\ \text{and} \quad 2m\ddot{\underline{r}}_2 &= \Delta\underline{H}_3 + \underline{N}_2 + \underline{W}_2 \end{aligned}\right\} \quad (1)$$

where $r_1 = x_1 \underline{i}$ and $r_2 = y_1 \underline{i}$

Using Hooke's law then

$$\underline{H}_1 = 2k((l_0 + x_1) - l_0)\,\hat{s} = -2kx_1\,\underline{i}$$

& $$\underline{H}_2 = k((l_0 + y_1 - x_1) - l_0)\,\hat{s} = k(y_1 - x_1)\,\underline{i}$$

& $$\underline{H}_3 = -H_2 = -k(y_1 - x_1)\,\underline{i}$$

Substituting these into ① gives us the equations of motion for each particle

$$m\ddot{x} = -3kx_1 + k(\cancel{x_2}\, y_1 - x_1)$$

$$2m\ddot{y} = -k(y_1 - x_1)$$

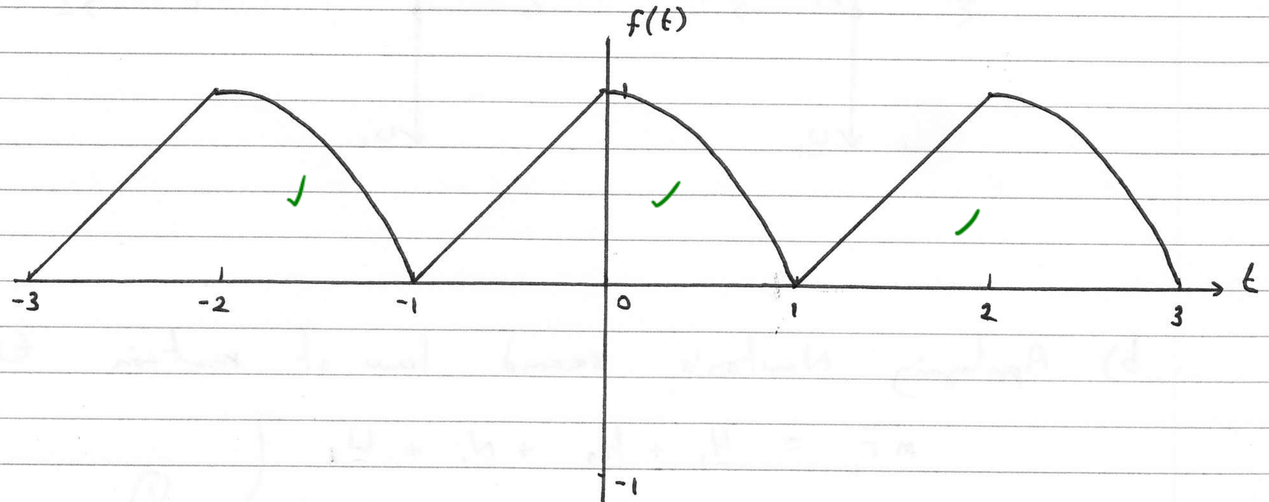
5/5

__Question 10__

$$f(t) = \begin{cases} t+1 & -1 < t \leq 0 \\ -t^2+1 & 0 < t \leq 1 \end{cases}$$

$$f(t+2) = f(t)$$

a) Sketch of $f(t)$ for $-3 \leq t \leq 3$



Question 10 (cont)

As $f(-t) \neq f(t)$ or $f(-t) \neq -f(t)$ then the function is neither odd or even.

b) From page 80 of the handbook

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

so,

$$A_0 = \frac{1}{2}\int_{-1}^{0} t+1\,dt + \frac{1}{2}\int_{0}^{1} -t^2+1\,dt$$

$$\Rightarrow \frac{1}{2}\left[\left[\frac{1}{2}t^2+t\right]_{-1}^{0} + \left[-\frac{t^3}{3}+t\right]_0^1\right]$$

$$= \frac{1}{2}\left[0-\left(\frac{1}{2}-1\right) + \left(-\frac{1}{3}+1\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}+\frac{2}{3}\right]$$

hence $A_0 = \frac{7}{12}$

$\frac{5}{5}$

Question 11

$$\underline{v}(r,\theta,\phi) = 3r^2\sin\theta\cos\phi\,\underline{e}_r + r^2\cos\phi\cos\theta\,\underline{e}_\theta - r^2\sin\phi\,\underline{e}_\phi$$

From page 87 of the handbook the curl of a vector field in spherical coordinates is

$$\nabla\times\underline{F} = \left(\frac{1}{r}\frac{\partial F_\phi}{\partial F_\theta} - \frac{1}{r\sin\theta}\frac{\partial F_\theta}{\partial\phi} + \frac{\cot\theta}{r}F_\phi\right)\underline{e}_r$$

$$+\left(-\frac{\partial F_\phi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{1}{r}F_\phi\right)\underline{e}_\theta$$

$$+\left(\frac{\partial F_\theta}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial\theta} + \frac{1}{r}F_\theta\right)\underline{e}_\phi \qquad -①$$

where $\quad \underline{v}_r = 3r^2 \sin\theta \cos\phi$

$\underline{v}_\theta = r^2 \cos\phi \cos\theta$

and $\quad \underline{v}_\phi = -r^2 \sin\phi$ ✓

Now $\quad \dfrac{\partial \underline{v}_\phi}{\partial \theta} = 0$, ✓ $\quad \dfrac{\partial \underline{v}_\theta}{\partial \phi} = -r^2 \sin\phi \cos\theta$

$\dfrac{\partial \underline{v}_\phi}{\partial r} = -2r\sin\phi$ ✓, $\quad \dfrac{\partial \underline{v}_r}{\partial \phi} = -3r^2 \sin\theta \sin\phi$ ✓

& $\quad \dfrac{\partial \underline{v}_\theta}{\partial r} = 2r\cos\phi\cos\theta$ ✓, $\quad \dfrac{\partial \underline{v}_r}{\partial \theta} = 3r^2 \cos\theta\cos\phi$

Substituting into ① gives

$$\nabla \times \underline{v} = \left(\frac{1}{r} \times 0 - \frac{1}{r\sin\theta}\left(-r^2 \sin\phi\cos\theta\right) + \frac{\cot\theta}{r}\left(-r^2 \sin\phi\right)\right)\underline{e}_r$$ ✓ ✓

$$+ \left(-(-2r\sin\phi) + \frac{1}{r\sin\theta}\left(-3r^2 \sin\theta\sin\phi\right) - \frac{1}{r}\left(-r^2\sin\phi\right)\right)\underline{e}_\theta$$ ✓

$$+ \left(2r\cos\phi\cos\theta - \frac{1}{r}\left(3r^2\cos\theta\cos\phi\right) + \frac{1}{r}\left(r^2\cos\phi\cos\theta\right)\right)\underline{e}_\phi$$

$$\Rightarrow \left(0 + r\sin\phi\cot\theta - r\cot\theta\sin\phi\right)\underline{e}_r$$ ✓

$$+ \left(2r\sin\phi - 3r\sin\phi + r\sin\phi\right)\underline{e}_\theta$$ ✓

$$+ \left(2r\cos\phi\cos\theta - 3r\cos\phi\cos\theta + r\cos\phi\cos\theta\right)\underline{e}_\phi$$ ✓

$$= 0\underline{e}_r + 0\underline{e}_\theta + 0\underline{e}_\phi$$ ✓

$$= \underline{0} \text{ (vector)}$$

Hence by the curl test, as curl $\underline{v} = 0$ then $\underline{v}$ is everywhere conservative. ✓

Question 12

4/5

Let A be the particle of mass m, and B the particle of mass 3m

Let $\underline{r}_1 = \dot{x}_1 \underline{i}$ be the velocity of particle A before the collision, where $\dot{x}_1 = 5$ ✓

and let $\dot{\underline{R}}_1 = \dot{X}_1 \underline{i}$ and $\dot{\underline{R}}_2 = \dot{X}_2 \underline{i}$ be the velocities of particles A and B respectively after the collision.

By the principle of conservation of linear momentum we have

$$m\dot{x}_1 \underline{i} = m\dot{X}_1 \underline{i} + 3m\dot{X}_2 \underline{i} \checkmark$$

Resolving in the i-direction and dividing by m gives

$$\dot{x}_1 = \dot{X}_1 + 3\dot{X}_2 \quad \checkmark$$

$$\Rightarrow \quad \dot{X}_1 + 3\dot{X}_2 = 5 \quad -①$$

As the collision is elastic then the kinetic energy is conserved. So after impact we have

$$\frac{1}{2}m\dot{x}_1^2 = \frac{25}{2}m = \frac{1}{2}m\dot{X}_1^2 + \frac{1}{2}3m\dot{X}_2^2 \checkmark$$

$$\Rightarrow \quad \dot{X}_1^2 + 3\dot{X}_2^2 = 25 \quad -②$$

✓

$$① \rightarrow \quad \dot{X}_1 = 5 - 3\dot{X}_2$$

Substituting into eq ② gives

$$(5 - 3\dot{X}_2)^2 + 3\dot{X}_2^2 = 25$$

$$\Rightarrow \quad 9\dot{X}_2^2 - 30\dot{X}_2 + 25 + 3\dot{X}_2^2 = 25$$

$$\Rightarrow \quad 12\dot{X}_2^2 - 30\dot{X}_2 = 0 \quad \checkmark$$

$$\Rightarrow \quad 6\dot{X}_2(2\dot{X}_2 - 5) = 0$$

So either $\dot{X}_2 = 0$, or $\dot{X}_2 = \frac{5}{2}$ ✓

Now $\dot{X}_2 = 0$ is impossible, so $\dot{X}_2 = \frac{5}{2}$ ✓

① $\Rightarrow \dot{x}_1 = \frac{5}{2}$

Hence, after the collision both particles have velocity $\frac{5}{2}\underline{i}$ ✗

no one is $\frac{5}{2}\underline{i}$ the other $-\frac{5}{2}\underline{i}$

I leave you to check this!