M208 12J GTB3
examples

1 Consider the group

$G = \left\{ \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix} : a \in \mathbb{R}^* \right.$, under matrix multiplication.

It acts on $\mathbb{R}^2$ with the definition that for $g = \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix} \in G$ and $(x, y) \in \mathbb{R}^2$ then

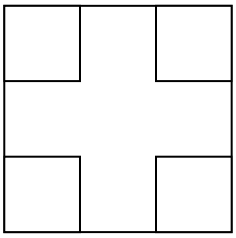
$g \wedge (x, y) = (ax + (1-a)y, y)$.

(a) Assume that G is a group but establish that $\wedge$ is a group action.

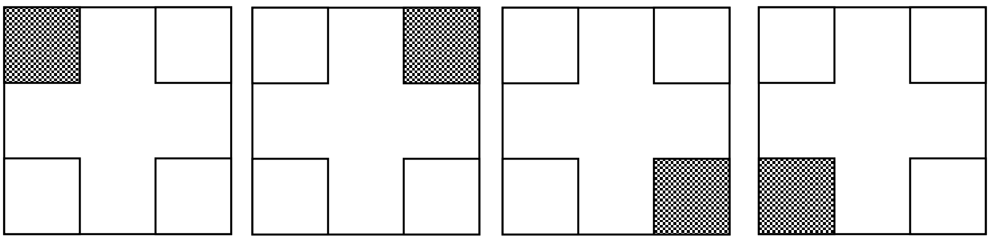
(b) Find the orbits of (1, 0), (0, 1) and (1, 1) and deduce all the orbits of the action and give a geometric description of each of them.

(c) Finally, find the stabilisers of (1, 0), (0, 1) and (1, 1).

2 Square napkins are to be made from five pieces of cloth, arranged as below, each piece coloured black or white. How many different napkins can be made? Two shadings will be considered to be the same pattern if a symmetry of the square will move one to the other.



eg



are all examples of just one pattern.

You may use two methods to answer this question.

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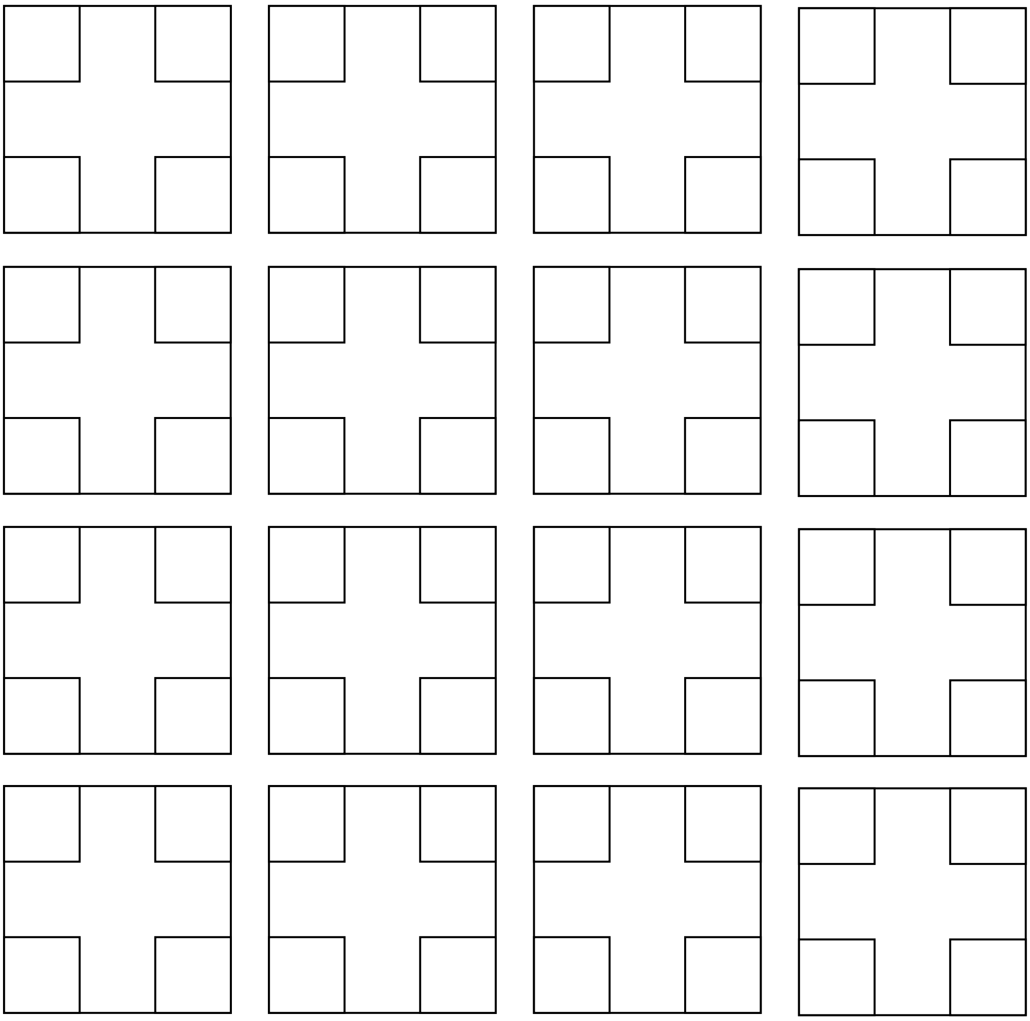
(a) Use some, but not all, of the blanks below to draw one example of each pattern.



(b) Use the Counting Theorem.