MST210 TMA07 Q2 Marking Scheme Name: ***Christopher Townhill***

1. Specify	(i) define problem	3		***3***	
	(ii) features to investigate	2		***2***	
	A good clear introduction		5		***5***
2. Create	(i) outline approach	5		***5***	
	(ii) assumptions	7		***7***	
	(iii) variables/parameters	5		***5***	
	(iv) formulate mathematics	13		***13***	
	A good start. Your outline to your model was well reasoned. A comprehensive list of assumptions but are some a necessary condition of the model? A good basis for the model.		30		***30***
3. Do the Maths	(i) solve	4		***4***	
	(ii) draw graphs	2		***2***	
	(iii) derive a first model	2		***2***	
	(iv) use dimensional analysis	2		***2***	
	Your team found a good first model. Did you try simplifying the formula you obtained?		10		***10***
4. Interpret	(i)Collect relevant data and produce numerical results	4		***1***	
	(ii)Describe solution in words	1		***1***	
	(iii) carry out sensitivity analysis	3		***3***	
	(ii)Describe results to compare	2		***0***	
	I had to take marks off here as you did not give any actual numerical predicted values for your model, either for actual buildings or experimental conditions. The working and result in the appendix should be included here together with a whole range of distances for different heights, roof angles etc.		10		***5***
5. Evaluate	(i) Collect a second set of data	2		***2***	
	(ii) test by comparing	5		***3***	
	(iii) criticise model	3		***3***	
	(iv) review the assumptions	5		***3***	
	A good experiment but only one direct comparison between the predicted and actual value was given. Without a series of comparisons it is difficult to justify your comments in your review of the assumptions.		15		***11***
6. Revise	(i) justify revision	2		***1***	
	(i) describe intended revision	2		***2***	
	(ii) revise the model based on 5 (ii)	6		***4***	
	A logical idea for a revision but does this actually change the maths? All it will do is to add another symbol to the expression you already have.		10		***7***
7. Conc.	(i)clear statement of outcome	5		***5***	
	Good		5		***5***
	Total		**85**		*73*

Question 1

The four members in our group mainly communicated by email, and via a comments page we created within our group wiki. We also had a one off chat session in the OULive room, but not everyone could attend. We used the marking scheme in the TMA as a template to complete our wiki.

My main contribution to the group was the 'doing the maths' section, creating the model diagram in Visio, and all the graphs on a Mac application called Grapher. I also had input to the parameter / variable table, and I rewrote our assumptions which were later merged with another students.

We successfully completed our first model, and there was a relatively good division of labour because the contributing members had different qualities. □

Question 2

1 Specify the purpose of the model

Definition of the problem

We aim to find the maximum horizontal distance an object can fall from a building, having slid down the roof, and dropped off its edge. This will enable safety barriers to be placed beyond that distance, measured from the base of the building vertically below the roof edge, to protect people from falling debris.

Aspects of the problem to be investigated

The problem splits into two parts. Firstly, the investigation of how the initial roof position and velocity of the object determine its take-off velocity at the roof edge. Assumptions about a smooth roof surface and knowledge of its gradient will be required. Secondly, an investigation of the object's projectile motion in the air, under the influence of gravity, and its dependence on the take-off height. Assumptions ignoring air resistance and demanding an empty, flat landing area will be needed.

The final solution should be an expression for the maximum horizontal distance travelled by the object in terms of: the angle and height of the roof surface, initial roof position and velocity of the object, and possibly also the object's mass.

2 Create the model

Outline of the approach in the first model

To simplify the situation, we will treat the object as a particle, hence we can ignore air resistance.

Now, a simple model cannot be expected to cope with roof surface anomalies, and differing weather conditions. So we will only consider straight-line sliding motion, down a friction-free roof surface, in a direction perpendicular to the roof's edge. This will require one-dimensional linear motion equations with constant acceleration provided by a component of acceleration due to gravity.

Motion of an object at the edge of a roof can, in real life, be complicated by guttering, and also changes in slope profile near the edge. The simplified mathematical approach will be to treat the roof edge as a clean, sharp edge maintaining the previous roof gradient, and free of obstruction.

Once the object leaves the edge of the roof, until it lands, we need to treat it as projectile motion so we assume a clear object flight-path, free of obstruction. Since we have chosen a particle object, having no physical size, we can ignore air resistance. Therefore mathematically we have two-dimensional motion with constant horizontal velocity, and uniform acceleration vertically due to the force of gravity.

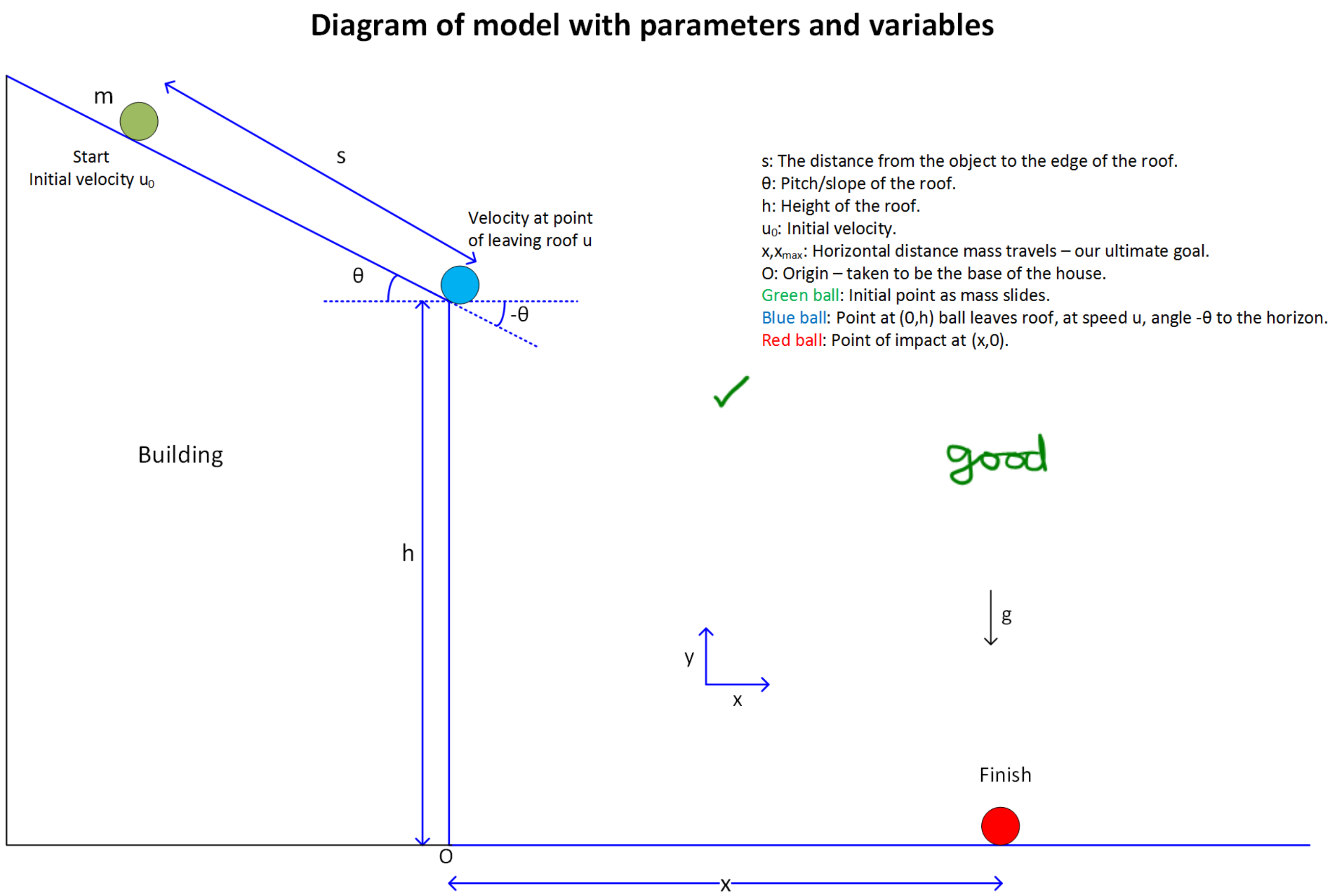
We want to treat the object's landing point as the required distance for the positioning of safety barriers. This distance is measured perpendicularly outwards from the wall below the roof edge. That means assuming the object does not bounce. Clearly an extra safety margin would probably be added to the measurement, but this is beyond the scope of our basic model. ✓

Assumptions not really necessary

1. The object on the roof is treated as a particle (which neither deforms nor breaks up during motion.)
2. The object starts from a position of rest on the roof (it is not dropped onto the roof).
3. The roof is strong enough to support the mass of the object without deforming or collapsing.
4. ✓ The roof surface can be treated as perfectly smooth so there is no friction between the roof and the moving object.
5. The roof has a constant slope and makes an acute angle with the horizontal.
6. The roof has a clean edge (there is no guttering which may affect the motion of the object).
7. ✓ The object slides with constant linear acceleration down the surface of the roof under the influence of gravity only.
8. Air resistance is ignored.
9. Any seasonal changes which might affect the motion of the object (e.g wind, rainfall) can be ignored.
10. ✓ There are no obstructions in the way of the object that would affect motion, and the object does not collide with anything before landing.
11. The projectile motion of the object in flight has constant velocity horizontally and constant acceleration vertically.
12. The landing area is horizontal and the landing point position is measured perpendicular to the wall of the house from an origin vertically below the take-off point. ✓
13. The object does not bounce upon impact with the landing area.

Definition of variables and parameters

Symbol	Definition	Unit	Variable/Parameter
m	Mass of the object dropped from the roof	Kg	P
θ	angle of the slope of the roof with the horizontal	rad	P
g	acceleration due to gravity	ms^{-2}	P
u	take-off velocity with which the object leaves the roof	ms^{-1}	V
u_0	initial velocity of the object at its starting point on the roof	ms^{-1}	P
h	vertical distance from the ground origin, at the base of the building, to the edge of the roof	m	P
s	the distance along the roof surface from the object to the edge of the roof, perpendicular to the roof edge	m	P
t	time of flight of the object falling through the air measured from t=0 at take-off from the roof edge	s	V
x	horizontal distance from the building outwards to the position of the object in flight	m	V
x_{max}	maximum horizontal distance from the base of the building origin to the landing place of the object	m	V
y	vertical distance from the ground upwards to the position of the object in flight	m	V



good

Figure 1: Diagram of model with parameters and variables

Formulation of mathematical relationships

The motion of the object is modeled in two parts:

1. The linear motion down the roof slope (from the starting position to $(0, h)$ in figure 1)
2. The projectile motion from the roof edge to the landing position (from $(0, h)$ to $(x_{max}, 0)$ in figure 1)

Part 1 evaluates the take-off velocity, u, of the object at the edge of the roof, at position $(0, h)$. This value is used in part 2 to evaluate the projectile motion of the object and eventual landing position at $(x_{max}, 0)$.

Part 1

By assumptions 3, 5, 7, 8, 9 and 10 there are no external conditions other than gravity which affect the motion of the object so the path of movement down the roof is linear. Newton's Second Law (handbook, page 40) is applied to the particle (assumption 1) taking into account the forces acting upon it (assumptions 4 and 7) to give an expression for the component of acceleration due to gravity acting down the slope of the roof as $g \sin(\theta)$.

Since the acceleration down the slope of the roof is constant (assumption 7) the relationship between the velocity of the object at the edge of the roof, u, and the distance travelled down the roof, s, is given by

$$u = \sqrt{u_0^2 + 2g \cdot s \sin\theta} \quad \text{(adapted from handbook, page 39)}$$

By assumption 2, $u_0 = 0$ so this expression simplifies to

$$u = \sqrt{2g \cdot s \sin\theta} \tag{1}$$

Part 1 could alternatively been approached using conservation of energy. The final expression for u is unchanged but that approach emphasizes that u is independent of the particle's mass which cancels in the equations.

Part 2

By assumption 6 the object leaves the roof at position $(0, h)$ and by assumptions 8, 9, 10 and 11, the movement to the landing position (position $(x_{max}, 0)$) is unaffected by any external influences other than gravity. The object leaves the roof at an angle equal to that of the roof with the horizontal (assumption 5) but since the object leaves the roof below the horizontal the angle of launch becomes $-\theta$.

Since the landing position is perpendicular to the position of launch (assumption 12), and by assumption 13 the object rests where it lands, the flight of the object is modelled using projectile motion where the horizontal and vertical components (assumption 11) of this flight are given by

$$x = ut\cos\theta$$

and

$$y = h - ut\sin\theta - \frac{1}{2}gt^2$$

If we let $t = 0$ at the time of launch then an expression for the trajectory of the projectile, with launch speed u (from part 1), is given by

$-\theta$?

$$y = h + x\tan(-\theta) - x^2\frac{g}{2u^2}(1 + \tan^2\theta) \quad \text{(handbook, page 42)}$$

In the next section this equation is developed to give a final expression for the landing position x_{max}, in terms of the angle of launch, $-\theta$, height of building, h, and the distance travelled down the roof, s.

3 Do the mathematics

Solve of the equations

The mathematics will be approached in two parts, matching the previous section:

Part 1

The mathematics of part 1, involving the object starting from rest and undergoing constant acceleration down the roof slope is covered above, leading to a calculated take-off speed given by $u = \sqrt{2g \cdot s \sin\theta}$.

Part 2

As the mass leaves the roof it acts like a projectile, and has three properties, a speed, $u = \sqrt{2g \cdot s \sin\theta}$, a direction of angle $-\theta$, and an initial height h

Taking the origin as the base of the house, then at this point the mass is at $(0, h)$ i.e. the blue ball in the diagram.

From page 42 of the handbook the equation gives us the y-component for a projectile launched at time $t = 0$ from $(0, h)$, with launch speed u, and launch angle $-\theta$ above the horizon, which is,

$$y = h + x\tan(-\theta) - x^2 \frac{g}{2u^2}(1 + \tan^2\theta) \quad (2)$$

$$\Longrightarrow y = h - x\tan\theta - x^2 \frac{g}{2u^2}(1 + \tan^2\theta) \quad (3)$$

On impact $y = 0$, so $x = x_{max}$ and so: as $\tan -\theta = -\tan\theta$

$$x_{max}^2 \frac{g}{2u^2}(1 + \tan^2\theta) + x_{max}\tan(\theta) - h = 0 \quad (4)$$

We can solve this for x_{max} using the quadratic formula

$$x_{max} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ reference?

where

$$a = \frac{g}{2u^2}(1 + \tan^2\theta)$$
$$b = \tan(\theta)$$

and

$$c = -h$$

giving

$$x_{max} = \frac{-\tan\theta \pm \sqrt{\tan^2\theta + \frac{2gh(1+\tan^2\theta)}{u^2}}}{\frac{g}{u^2}\left(1 + \tan^2\theta\right)}$$ ✓

Now we need to substitute $u^2 = 2g \cdot s \cdot \sin(\theta)$ from part 1, at the same time noting that we are only interested in positive values of x_{max} to get:

$$x_{max} = \frac{-\tan\theta + \sqrt{\tan^2\theta + \frac{2gh(1+\tan^2\theta)}{2g \cdot s \cdot \sin\theta}}}{\frac{g}{2g \cdot s \cdot \sin\theta}\left(1 + \tan^2\theta\right)}$$

$$\implies x_{max} = \frac{-\tan\theta + \sqrt{\tan^2\theta + \dfrac{h(1+\tan^2\theta)}{s\cdot\sin\theta}}}{\dfrac{1}{2s\cdot\sin\theta}\left(1+\tan^2\theta\right)}$$

✓

Did you tru to simplify this?

$$\implies x_{max} = \frac{2s\cdot\sin\theta}{(1+\tan^2\theta)}\left(-\tan\theta + \sqrt{\tan^2\theta + \frac{h(1+\tan^2\theta)}{s\cdot\sin\theta}}\right) \tag{5}$$

To simplify, as they do in Unit 3 (page 226), let $z = \tan(\theta)$, and $L = \dfrac{u^2}{g} = 2s\sin\theta$. We also are only interested in the positive values of x, hence the final result.

Note really necessary as we want the formaula in tems of the basic parameters

$$x = \frac{L}{1+z^2}\left[-z + \sqrt{z^2 + \frac{2h}{L}(1+z^2)}\right] \tag{6}$$

Graphs

We produced four graphs detailed in the table below. The graphs were created using a Mac application called Grapher, and each graph is annotated with the parameter values used.

Label	Description
Figure 2	Trajectories of mass for a 'typical' result for a house and bungalow.
Figure 3	Plot of x_{max} against θ
Figure 4	Plot of x_{max} against h
Figure 5	Plot of x_{max} against s

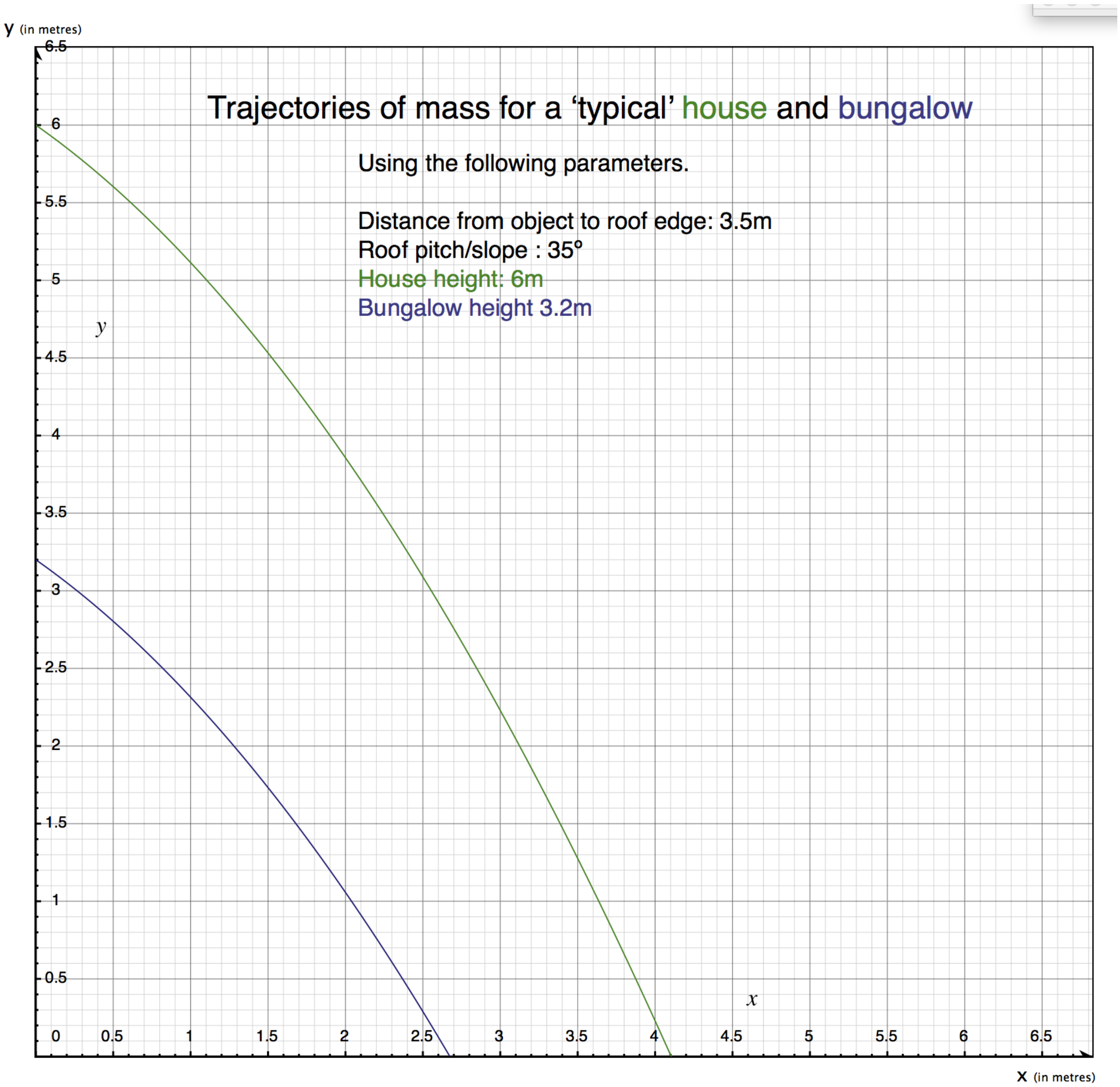


Figure 2: Trajectories of mass for a 'typical' result for a house and bungalow.

As we have not yet to put any values into the model. These graphs should not have any scales on the axes.

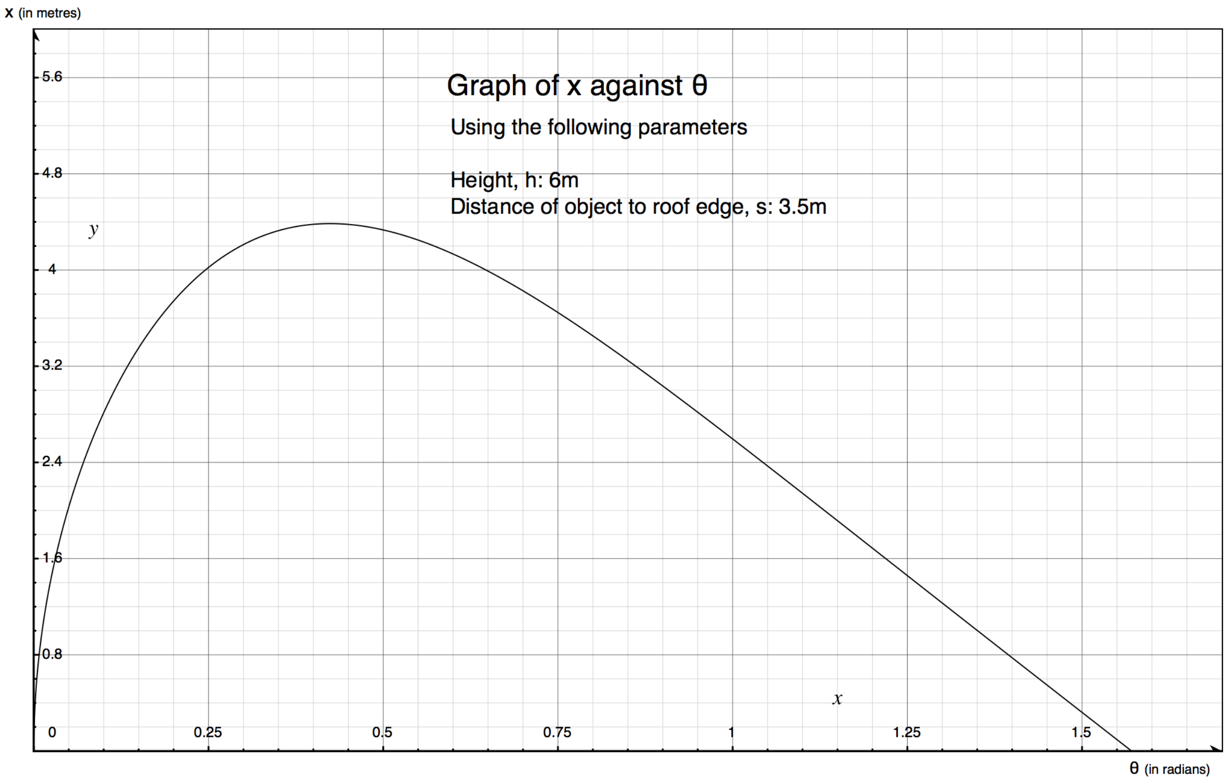


Figure 3: Plot of x_{max} against θ

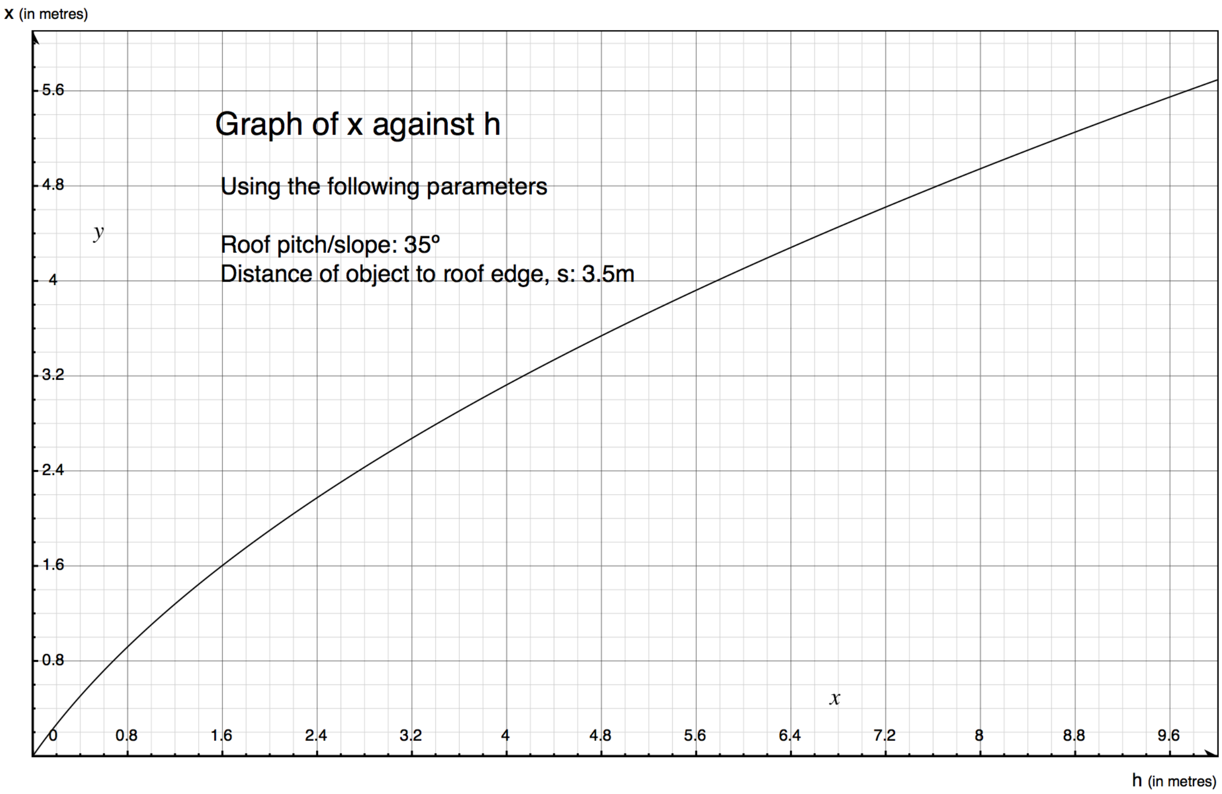


Figure 4: Plot of x_{max} against h

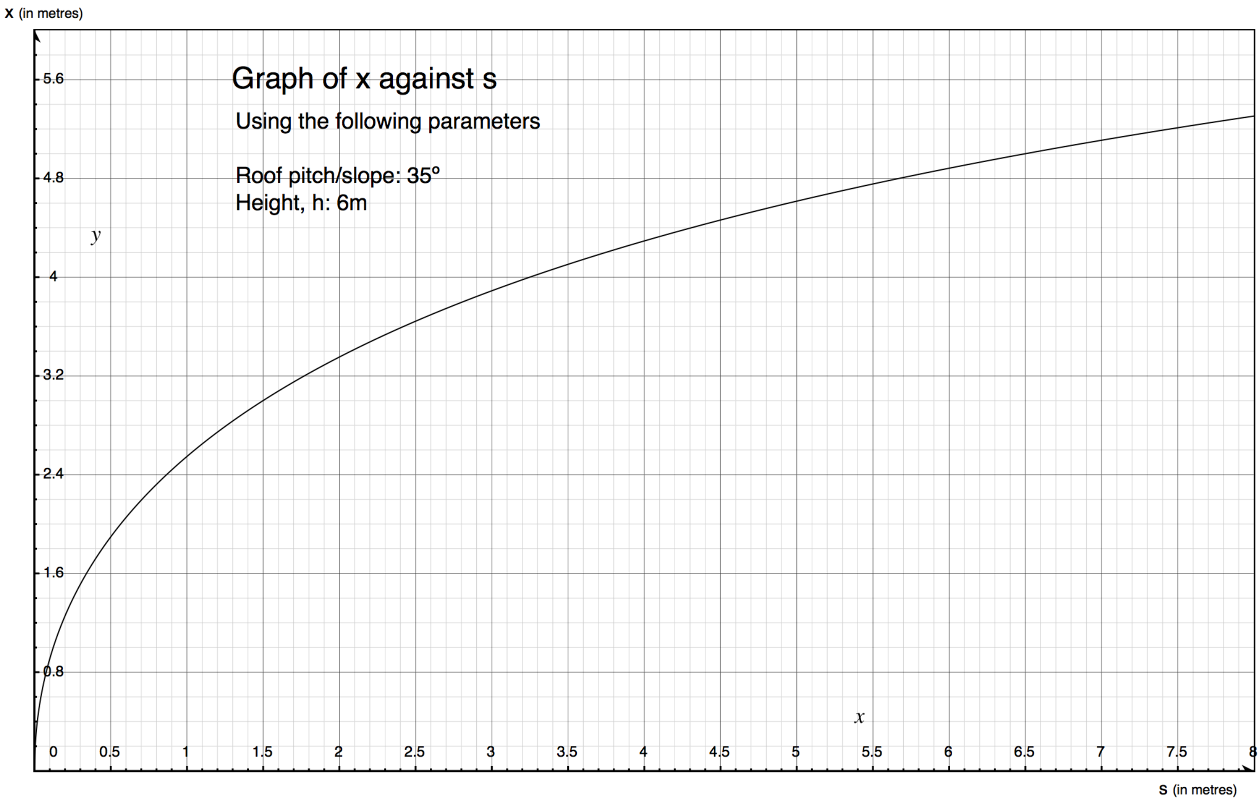


Figure 5: Plot of x_{max} against s

Figure 2 : Trajectories of mass for a 'typical' result for a house and bungalow.

This graph shows two plots, simulating the trajectory for a 'typical' house, and bungalow. Besides the height difference the other parameters are the same in both plots, hence the initial velocity of the object, u, is the same.

The plots show that the objects would continue with a horizontal direction whilst beginning to fall under the influence of gravity, before hitting the ground at $y = 0$.

Figure 3 : Plot of x_{max} against θ

This graph shows the values of x given for θ over the range of $0 \leq \theta \leq \pi/2$, and for a house of height 6m and a slope length of 3.5m. As we can see at the boundaries, $\theta = 0$ and $\theta = \pi/2$ that $x = 0$. We can interpret $\theta = 0$ as the house having a flat roof, and the second case is when the roof is vertical.

From the graph we can see that x increases with θ until it reaches a local maximum. x then descreases back to zero when $\theta = \pi/2$

Figure 4 : Plot of x_{max} against h

This graph shows how x varies against h. Inspecting the graph for values $0 < h < 1$ it appears linear. However, as h increases the increase in x seems to slow. On inspection of equation (5), we can see that, with all other parameters constant that $x \propto \sqrt{h}$.

Figure 5 : Plot of x_{max} against s

This graph shows how x varies against s. Inspecting the graph x seems to increase rapidly, and then as s increases further then x still increases, but at a slowe rate.

Dimensional analysis

Using the equation (6) which is repeated here:

$$x = -\frac{L}{1+z^2}\left[z + \sqrt{z^2 + \frac{2h}{L}(1+z^2)}\right]$$

We must check that the left and right hand sides are dimensionally equal.

The left hand side is x measured in metres, hence, $[x] = \mathrm{L}$

On the right hand side we have, h, $L = 2s\sin\theta$, and $z = \tan\theta$, so the dimensions are: $[h]$ =L, $[L]$ =L, and $[z] = 1$. Substituting gives

$$\mathrm{L} = \frac{\mathrm{L}}{1+1^2}\left(1 + \sqrt{1^2 + \frac{\mathrm{L}}{\mathrm{L}}(1+1^2)}\right)$$
$$\implies = \frac{\mathrm{L}}{1}\left(1+\sqrt{1}\right)$$
$$\implies = \mathrm{L}$$

Hence both sides are dimensionally equal to L, which is measured in metres.

4 Interpret the results

Data

From research on the internet and various other places including a conversation with a Building Engineer, it is clear that typical parameter values are:

Angle of roof (θ): The angle for a pitched roof is defined as being greater than 10° and less than 75° (BSI, p5). A typical roof may vary between 30° and 42° and may be higher for living spaces (e.g. loft conversions). To accommodate plain tiles, such as those used on most modern houses, the BSI advises that rafter pitches should not be less than 35° (BSI, p43). A typical range for the angle of the roof in this model is hence $35° \leq \theta \leq 42°$ and a typical value is 35°.

Height of wall (h): The height of the house is dependent on the type of house, age, and number of storeys. A 2010 survey on homes in England (DCLC, p19) reported the average ceiling height for a living room as being 2.5m. Allowing extra height for floor, roof and cavity spaces it is therefore reasonable to assign a typical external floor-to-roof value of 3.2m for a single-storey house and of 6m for a 2-storey house.

Length of roof (s): This is related to the base dimensions of the house and the angle of the roof, So using trigonometry we could write $s = \frac{1}{2} \times (\frac{\text{width of house}}{\cos\theta})$, where width of house might be the distance from front wall to back wall.

Mass of object (m): The mass of the object is dependent on the type of object used but is likely to be a common item used in roofing projects and one regularly placed or left on the surface of a roof. Taking a claw hammer as an appropriate object a quick survey of different brands available from a DIY shop (www.diy.com) indicates a typical range of $0.45 \leq \text{m} \leq 0.57$ kg for the mass of the object in the model and a typical value of 0.51 kg.

Gravity, (g): Taken as the standard 9.81ms^{-2}

Now we will check our model using equation (5) and consider the three parameters in turn, to predict the distance the object would fall. The three parameters we will consider are the height of the roof, h, the length of the slope, s, and the pitch of the roof, θ.

First consider parameter h

From equation (5), if $h = 0$ then the model predicts $x = 0$, this is because

$$x = \frac{2s \cdot \sin(\theta)}{(1 + \tan^2\theta)} \left(-\tan\theta + \sqrt{\tan^2\theta + \frac{h(1 + \tan^2\theta)}{s \cdot \sin(\theta)}} \right)$$

inspecting the right hand bracket

$$\left(-\tan\theta + \sqrt{\tan^2\theta + \frac{h(1 + \tan^2\theta)}{s \cdot \sin(\theta)}} \right)$$

as $h = 0$ then this reduces to

$$\Longrightarrow = \left(-\tan\theta + \sqrt{\tan^2\theta} \right)$$
$$\Longrightarrow = 0$$

Hence, the model correctly predicts that when $h = 0$ then $x = 0$

Inspecting equation (5) then the model predicts that as h increases then $x \propto \sqrt{h}$

Second, consider parameter s

Again from equation (5) we can see that if $s = 0$ then the model correctly predicts that $x = 0$. If θ and h are sensible values and remain constant then as s increases we can see that $x \propto s + \sqrt{s}$

Finally, consider parameter θ

When $\theta = 0$, then $\tan\theta = 0$, substituting this into equation (5) then the model predicts that $x_{max} = 0$. This is correct as it corresponds to the object not moving at all, as the roof would be flat.

However, when $\theta = \pi/2$, then this simulates a vertical drop. Looking at equation (5) again, we can see that $x_{max} = 0$ because

$$\lim_{\theta \to \pi/2} \tan\theta = \infty$$

then

$$\implies \lim_{\theta \to \pi/2} x = 0$$

Further, looking at figure 3 there is a local maximum, this indicates that there is one value of θ which would give the largest value for x_{max}.

But where are your model predictions for a range of buildings? These are essential here.

Sensitivity Analysis

We will now check the sensitivity analysis for each of the three parameters in turn, for either absolute and relative sensitivity, depending on their dimensions.

First let us consider h, which is measured in metres, the same units as x, so we will only consider absolute sensitivity.

Using the empiricial method then when h=6m, s=3.5m, and θ=0.61 rad, then substituting into equation (5) gives $x_{max} = 4.10657$m

If we now make a small change in h, say δh=0.05, then $h + \delta h$=6.05m, s=3.5m, and θ=0.61 rad, then

$$\implies x_{max} + \delta x_{max} = 4.12902\text{m}$$

so,

$$\frac{\delta x}{\delta h} = \frac{4.10657 - 4.12902}{6.05 - 6} = 0.45\text{m}$$

As the empirical absolute sensitivity is less than 5 we can say that the distance, x, is relatively insensitive to small changes in h.

Now let us consider small changes in s. As with s, the dimensions are the same, so we'll only consider absolute sensitivity.

As above our initial values are $h = 6$m, $s = 3.5$m, $\theta = 0.61$ rad, and $x = 4.10657$m
If we now make a small change in s, say $\delta s = 0.05$, then $s + \delta s$=3.55m, h=6.0m, and θ=0.61 rad, then

$$\implies x_{max} + \delta x_{max} = 4.12656\text{m}$$

so,

$$\frac{\delta x}{\delta h} = \frac{4.12656 - 4.10657}{3.55 - 3.5} = 0.40\text{m}$$

Again, as the empirical absolute sensitivity is less then 5 we can say that the distance, x, is relatively insensitive to small changes in s.

Finally let us consider small changers in θ, which is dimensionless, so we'll consider relative sensitivity.

Again our initial values are $h = 6$m, $s = 3.5$m, $\theta = 0.61$ rad, and $x = 4.10657$m. Adding $\delta\theta = 0.01$ gives a new value of $x = 4.07905$

First our absolute value is

$$\frac{\delta\theta}{\delta x} = \frac{(4.10657 - 4.07905)}{(0.62 - 0.61)} = 2.752\text{m}$$

So, the relative sensitivity is

$$\frac{\theta}{x}\frac{\delta\theta}{\delta x} = \frac{0.61}{4.10657} \times \frac{(4.10657 - 4.07905)}{(0.62 - 0.61)} = 0.41\text{m}$$

As the relative sensitivity is less than 5 we can say that the distance, x, is relatively insensitive to small changes in θ

5 Evaluate model

Comparison with reality

From appendix 2 my experimental outcome was a distance of x_{max}=1.65m. However, the model predicts a higher value of x_{max}=1.80m. This discrepancy could be due to the fact that our first model ignores friction and air resistance, as both these forces would reduce the acceleration, and hence decrease the value of x_{max}.

Where is the maths for this. You need to compare over a range of values. Not just one.

Criticism of the model

I will now review our assumptions in turn.

1. Treating the object as a particle could lead to significant errors, for example a yard brush could fall horizontally covering a wide area.
2. The object need not start from rest, which could effect the objects x_{max}.
3. This assumption is essential, but doesn't impact on the model.
4. Ignoring any friction the roof has an impact on the model.
5. The roof having a constant slope is essential for the model to calculate the velocity of the object.
6. Most roofs have guttering, and an improved model would need to include an object clipping the guttering.
7. This is a reasonable assumption.
8. Air resistance would impact on the model and should not be ignored.
9. Seasonal changes can be ignored, a strong wind would have an adverse effect, but strong winds can occur in any season.
10. Assuming there are no obstructions in the way of the object is an important.
11. The assumption that the vertical acceleration is constant is no longer valid because air resistance should not be ignored.
12. The landing area not being horizontal does impact on the model, it's equivalent of changing the parameter h
13. If the object does roll or bounce the barrier would do its job and stop the object, so this has no impact of the model.

These reviews must be linked to the comparison test that did. They need justifying in light of the tests.

Review the assumptions

We can see that assumptions 1, 2, 4, 8 and 11 all impact on the model to varying degrees.

6 Revise model

Description of the revision

From our revision of assumption 1, we are now treating the object as a smooth spherical object of diameter D. So we now need to add the radius of the object to x_{max} to ensure the safety distance is sufficient.

Ignoring assumption 2 leads the object to possibly having an initial velocity, u_0. Also, we are now ignoring assumption 8, so air resistance will need to be considered for a smooth spherical object.

Assumption 4 is now ignored, so we'll need to consider friction as the object slides down the roof. This gives rise to a new parameter, μ', which is the sliding coefficient of friction.

We could also consider adding guttering, which could potentially 'catch' the object and prevent it from falling, however, I will not be adding it to my model. ✓

The revised model

Incorporating our reviewed assumptions leads to the following revised equations.

First, we are now ignoring assumption 2, so we need to introduce an initial speed, u_0. This changes equation (1) to

$$u = \sqrt{u_0^2 + 2g \cdot s \sin\theta} \quad ✓ \tag{7}$$

$$\implies u^2 = u_0^2 + 2g \cdot s \sin\theta \tag{8}$$

As we are now incorporating friction in to our model, we need to introduce a new parameter, μ', which is the coefficient of sliding friction. So re-examining part 1, the component of acceleration due to gravity with friction taken into consideration is now.

$$\begin{aligned} a &= g\sin\theta - \mu' g\cos\theta \\ \implies &= g(\sin\theta - \mu'\cos\theta) \end{aligned}$$

So, equation (8) now becomes

$$u^2 = u_0^2 + 2(g(\sin\theta - \mu'\cos\theta)) \cdot s$$

Next, from our review of assumption 1, we now have another new parameter, the diameter of the object, D, measured in metres.

For air resistance we have two possibilities, linear and quadratic. In our model $D|v| > 10^{-5}$, so we'll only consider the quadratic model for air resistance. Also, we'll only consider air resistance for the trajectory part of our object.

From page 212 of Unit 3, the equation of motion of an object falling from rest under gravity with quadratic air resistance is

$$\begin{aligned} ma &= mg - c_2 D^2 u^2 \\ \implies a &= g - \frac{c_2 D^2 u^2}{m} \end{aligned}$$

So in part 2, rather than use just g in equation (3) we would need to use ✓

$$g - \frac{c_2 d^2 u^2}{m}$$

giving

$$\implies y = h - x\tan\theta - x^2 \left[\frac{g - \dfrac{c_2 D^2 u^2}{m}}{2u^2} \right] (1 + \tan^2\theta)$$

where

$$u^2 = u_0^2 + 2(g(\sin\theta - \mu'\cos\theta)) \cdot s$$

Finally, on impact $y = 0$, so $x = x_{max}$ and so giving an equation which is a quadratic in x:

$$x_{max}^2 \left[\frac{g - \dfrac{c_2 D^2 u^2}{m}}{2u^2} \right] (1 + \tan^2\theta) - x_{max}\tan\theta - h = 0 \tag{9}$$

Does this model actually change the maths? Surely all you have done is included another parameter.

7 Conclusions

Our model predicts that as the parameters, h, and s increase, so does x. This makes sense, as if either the height of the building, or the initial speed the object beings to fall, then you would expect x to increase. However, this is not the case for the angle of the roof, θ. As θ increases from zero, then x increases, until at such a point where it reaches a local maximum. ✓

The revised model introduced two new parameters, D, and μ, so this should give a more accurate prediction of where to place safety barriers. However, typical values for both these would need to be found.

My experimental results are very limited, so I would treat them with some suspicion. I would reccommend that more experimental data is collected, or use the model's predictions with caution. ✓

In conclusion, I think the first model we provided gives a reasonable approximation, but the revised model would provide a far more realistic answer. ✓

8 References

BSI-British Standards Institution (2014) BS 5534:2014. Slating and tiling for pitched roofs and vertical cladding - Code of practice. London: BSI.

DCLC-Department for Communities & Local Government (2012) English Housing Survey: HOMES. London: National Statistics. Available: https://www.gov.uk/government/statistics/english-housing-survey-homes-report-2010

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9 Appendix One: The experiment

I collected my data by performing my own experiment with the help of my wife. Here is a list of the equipment I used.

1. Computer mouse ball
2. Ladders
3. tape measure x 2 ✓
4. long hollow cardboard tube 2.09m long
5. pen and paper

Before the experiment I measured the fixed values, e.g. the height, h, the length of the tube, s. I laid one tape measure on the drive, so my wife could read the values of x as the mouse ball hit the drive.

Once on the roof, I placed one end of the cardboard tube on the edge of the garage roof, and the other end at a set point on my chest. I then dropped the mouse ball down the cardboard tube, and my wife took the measurement. I repeated this 4 times.

Unfortunately I could only repeat the experiment four times because we lost the mouse ball.

10 Appendix Two: Experimental data ✓

My results are tabled below.

h (in metres)	θ (in radians)	s (in metres)	x (in metres)
2.74	0.75	2.09	1.57
2.74	0.75	2.09	1.70
2.74	0.75	2.09	1.74
2.74	0.75	2.09	1.60

Taking an average from the four results then from my experiment $x = 1.65$m

Based on the values I used in my experiment x should be 1.80m

$$x_{max} = \frac{(2 \times 2.09) \cdot \sin(0.75)}{(1 + \tan(0.75)^2)} \left(- \tan 0.75 + \sqrt{\tan(0.75)^2 + \frac{2.74(1 + \tan(0.75)^2)}{2.09 \times \sin(0.75)}} \right)$$

$$\implies = 1.80$$

✓

□

Did you do any calculations for any other sets of parameters.?