M208 12J GTB2
examples

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In this question $\mathbb{C}^*$ is the group of non-zero complex numbers under multiplication and $\mathbb{C}$ is the group of all complex numbers under addition.

For each of the following functions, determine whether it is a homomorphism, justifying your answer.

(i) $\phi_1 : \mathbb{C} \to \mathbb{C}$
$z \mapsto |z|$

(ii) $\phi_2 : \mathbb{C}^* \to \mathbb{C}^*$
$z \mapsto \mathrm{Re}(z)$

(iii) $\phi_3 : \mathbb{C} \to \mathbb{C}$
$z \mapsto \mathrm{Re}(z)$

2 (a)

For each of the following functions we decide whether or not it is a group homomorphism, justifying the answer.

(i) $\phi_1 : (\mathbb{R},+) \to (\mathbb{Z},+)$
$r \mapsto [r]$, ie the greatest integer less than or equal to r

(ii) $\phi_2 : (\mathbb{R}^2,+) \to (\mathbb{R},+)$
$(x,y) \mapsto 2x + y$

(iii) $\phi_3 : V \to (\mathbb{R}^*,\times)$
$\begin{pmatrix} a & b \\ b & a \end{pmatrix} \mapsto a + b$

where $V = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} : a,b \in \mathbb{R}, a^2 \neq b^2 \right\}$ and the operation is matrix multiplication.

(b) For each homomorphism ϕ in (a) determine $\mathrm{Ker}(\phi)$ and $\mathrm{Im}(\phi)$.

(c) For each homomorphism ϕ in (a) identify the quotient group $G/\mathrm{Ker}(\phi)$ up to isomorphism (where G is the domain group of the homomorphism).