The Quarterly Review of Economics and Finance 71 (2019) 107–113

Contents lists available at ScienceDirect

The Quarterly Review of Economics and Finance

journal homepage: www.elsevier.com/locate/qref

Holding Bitcoin longer: The dynamic hedging abilities of Bitcoin

Wing Hong Chan, Minh Le, Yan Wendy Wu *

Department of Economics, School of Business and Economics, Wilfrid Laurier University, Waterloo, ON, N2L 3C5, Canada

ARTICLE INFO

Article history:
Received 19 December 2017
Received in revised form 28 May 2018
Accepted 3 July 2018
Available online 5 July 2018

JEL classification:
G1
G11
G15

Keywords:
Bitcoin
Hedge
Risk management
Cryptocurrency
Frequency
Dependent
Frequency decomposition

ABSTRACT

This paper investigates whether Bitcoin can hedge and diversify risk against the Euro STOXX, Nikkei, Shanghai A-Share, S&P 500, and the TSX Index, and examines the dynamics of these abilities over different data frequencies. Pairwise GARCH models and constant conditional correlation models are used for daily, weekly, and monthly returns from October 2010 to October 2017. We find that Bitcoin is an effective strong hedge for all these indices under monthly data frequency. However, daily and weekly returns do not demonstrate strong hedge properties. Further frequency dependence model tests reveal that Bitcoin returns are strong hedgings against S&P and Euro indices over medium data frequency, and also against the Shanghai A-Share over low data frequency.

1. Introduction

Bitcoin has grown in both price and popularity since its introduction in 2009. On any given day, changes in Bitcoin can headline both finance and technology news. From its inception to the end of 2016, Bitcoin prices have remained under $1500.00 USD. However, the buying frenzy of 2017 lead Bitcoin prices to rise to over $18,000.00 USD, exhibiting major volatility on its way up (Fig. 1). One week after the Chicago Board of Exchange launched its Bitcoin future contract, CME, the world's largest futures exchange, launched its own Bitcoin futures contract. William Dudley, the President and CEO of Federal Reserve Bank of New York is exploring the idea of creating the bank's own digital currency. While the actual reasons for this price boom is up for debate, one common explanation known as the "Satoshi Cycle" suggests that there is a high correlation between Google searches for "Bitcoin" and the actual prices of Bitcoin (Fig. 2). Fig. 3 provides insight on the positive relationship between Bitcoin's price and the number of trades occurring that signifies growing market interest.

With Bitcoin's increasing popularity, understanding how its prices are correlated with other financial assets is of interest to investors, regulators and policy makers. Is Bitcoin a valuable asset to add to the portfolio? We investigate how Bitcoin can be used in risk management against certain equity markets. Specifically, we follow Baur and Lucey's (2010) research that defines an asset as exhibiting strong hedging features when it is negatively correlated to another asset, and exhibiting diversifying features when it is positively correlated with another asset. Dyhrberg (2016b) shows that Bitcoin can be used as a hedge against stocks in the Financial Times Stock Exchange (FTSE) Index, as well as the dollar-euro and dollar-sterling exchange rates. Bouri, Molnár, Azzi, Roubaud, and Hagfors (2017) report that Bitcoin's daily returns are negatively correlated to the Japanese and Asia Pacific stocks indices, but the correlations fade for weekly data. Both of these studies are based on 2010–2015 data. No study, to our knowledge, has investigated how the dramatic price increases in 2017 impact the hedging abilities of Bitcoin. We fill this gap by providing an up-to-date analysis of Bitcoin's hedging ability against several major equity markets. In additional to analyses using various GARCH models, we also provide the first empirical evidence on the dynamic hedging abilities of Bitcoin by decomposing the movements of the daily returns into high, medium, and low frequency movements using the frequency

* Corresponding author.
E-mail addresses: wchan@wlu.ca (W.H. Chan), lexx0847@mylaurier.ca (M. Le), wwu@wlu.ca (Y.W. Wu).

https://doi.org/10.1016/j.qref.2018.07.004
1062-9769/

Fig. 1. Daily Bitcoin price series from 2010 to 2017.

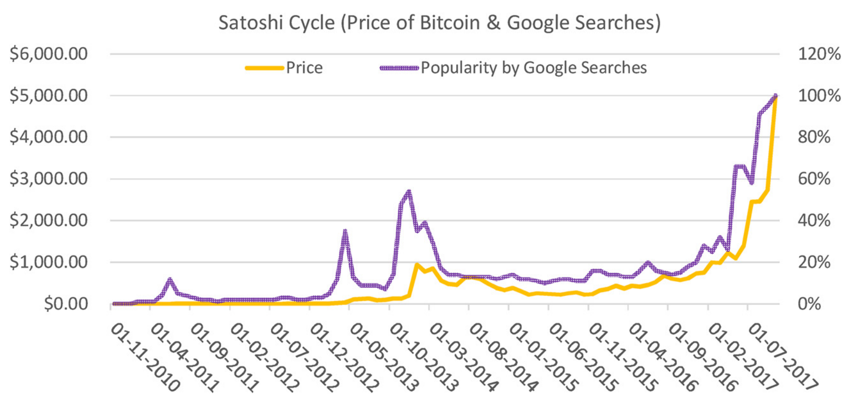

Fig. 2. Bitcoin price and Google searches from 2010 to 2017.

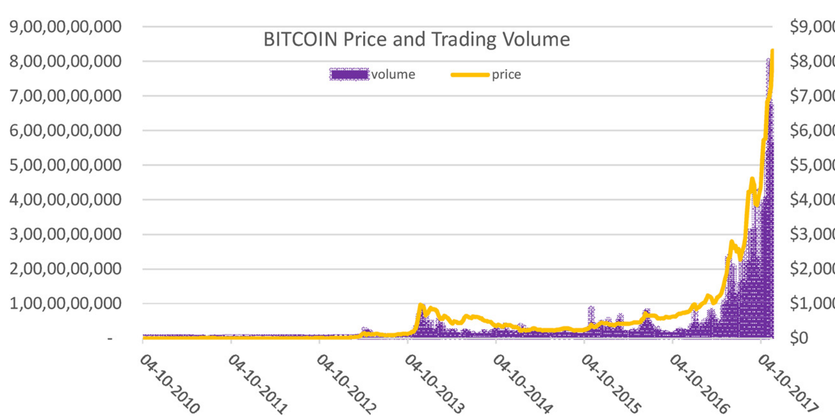

Fig. 3. Bitcoin price and volume from 2010 to 2017.

dependent model proposed by Ashley and Verbrugge (2009) and Tan and Ashley (1999).

Bitcoin daily price data from October 2010 (the earliest data available) to October 2017 was used to investigate how Bitcoin can hedge or diversify risk against the Euro STOXX, Nikkei, Shanghai A-Share, S&P 500, and TSX Indices conditional on daily, weekly, and monthly data frequencies. We use various models including GARCH and Constant Conditional Correlation (CCC) models. Firstly, we find that the correlations between Bitcoin and the index returns are insignificant over the daily and weekly horizons. This lack of correlation creates a possibility for investors to hedge some of the market risk, though it is only a weak hedge. Secondly, we find the hedging abilities of Bitcoin improve significantly for monthly frequencies. It is a strong hedge against the Euro STOXX, S&P 500, Shanghai A-Share, Nikkei, and TSX Indices for monthly returns through significant negative relationships. These findings suggest that Bitcoin is more effective at hedging against equity markets for monthly frequency than for daily and weekly frequencies.

To further investigate the correlation variations across different data frequencies, we conduct additional analyses by decomposing movements of various index daily returns into high, medium and low frequency components following Ashley and Verbrugge

(2009). We find that Bitcoin demonstrates different hedging abilities under these frequency dependence models; the low frequency hedging ability, longer than a month, becomes weaker. Bitcoin is only a strong hedge against Shanghai A-Share, and a weak hedge against other indices.

We contribute to the extant literature by showing that Bitcoin provides effective risk management functions under monthly data frequencies. This is the first study analyzing Bitcoin's hedging ability using monthly data and the first analysis on Bitcoin's hedging ability against TSX and Euro indices. Secondly, our sample covers 2010–2017, which enables us to find that Bitcoin's risk management abilities are more significant than the previous literature indicates. Thirdly, we provide the first empirical evidence using a frequency dependence approach and find that Bitcoin's hedging abilities are sensitive to model specifications.

The rest of the paper is organized as follows: Section 2 reviews the literature and develops hypotheses; Section 3 describes data; Section 4 explains the methodology; Section 5 presents results; and Section 6 concludes.

2. Literature review and hypotheses

Bitcoin, the first cryptocurrency, was created by an anonymous internet group operating under the pseudonym Satoshi Nakamoto and was initially introduced as an alternative to conventional currencies. It held an 89% share of all virtual currency market capitalization as of December 2016 (Bariviera, Basgall, Hasperué, & Naiouf, 2017) and is considered the most important cyptocurrency. Bitcoin prices over time are substantially more volatile than conventional currency. Blau (2017) finds that the volatility of Bitcoin prices doubles the average volatility of 51 regular currencies from July 2010 to June 2014. There are mixed findings on what drives the Bitcoin prices: Blau (2017) concludes that Bitcoin returns were unrelated to speculative trading, while Cheah and Fry (2015) show that Bitcoin's price exhibits speculative bubbles with the fundamental value being zero.

Bitcoin shares some common features with traditional currencies. Yermack (2013) evaluates the validity of Bitcoin as a currency against the three required functions of a currency. He states that although Bitcoin satisfies the function as a medium of exchange, it cannot be a store of value or a unit of account, which are two of the three attributes required for being considered a currency. Dyhrberg (2016a) investigates whether Bitcoin more resembles a commodity or a currency, and concludes that Bitcoin returns have a significant positive reaction to the US Federal Funds rate, similar to the US dollar. Bitcoin is also found to provide risk-management capabilities against dollar-pound and dollar-euro exchange rates, similar to those qualities Tully and Lucey (2007) found in gold. Therefore, Dyhrberg concludes that Bitcoin can be classified as something in between the US dollar and gold, and can be a useful tool for portfolio management. Luther and Salter (2017) analyze the increase in Bitcoin app downloads after the Cyprus bailout announcement. While Bitcoin app downloads increased in both the US and Cyprus after the bailout announcement, increases were greater in the US, suggesting that Bitcoin is not replacing the currencies of countries with troubled banks.

The other line of research analyzes Bitcoin's hedging ability as an asset. Dyhrberg (2016b) uses daily data to test Bitcoin's hedging ability against some UK-related assets including dollar-euro and dollar-pound exchange rates, as well as the FTSE index. The paper shows that Bitcoin's return is uncorrelated to the FTSE Index in both lagged and contemporaneous returns, while the exchange rates positively lead the return on Bitcoin. These findings indicate that Bitcoin could be a weak hedge against UK assets. Using daily and weekly price index data, Bouri et al. (2017) show that Bitcoin has the ability to hedge against the Nikkei, the MSCI Pacific and the commodity index. However, the hedging ability is not present in the weekly data. Bouri et al. (2017) caution that the diversification ability of Bitcoin is not constant over time and future studies on the time-varying nature of these risk-management abilities are necessary.

Based on the findings from Bouri et al. (2017) and Dyhrberg (2016b), we formulate our hypotheses as follows:

H1. Bitcoin can hedge and diversify against certain assets among S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro Index.

H2. The hedging and diversification abilities of Bitcoin differ under different data frequencies.

3. Data

Bitcoin (BTC) daily price data from October 2010 to October 2017 was retrieved from the Coindesk Price Index (2017). Five indices including S&P 500 (GSPC), Nikkei (N225), Shanghai A-Share (SSE Composite), TSX (GSPTSE), and Euro Index (STOXX) are chosen to represent different regions whose currencies top the share of Bitcoin trade. Bitcoin trading against the Chinese Yuan accounted for most of Bitcoin's trading volume until China started to clamp down on digital currency exchanges in early 2017, eventually banning the trading of Bitcoin in September of 2017. Japan's Yen then took over as the largest trading volume with the Japanese regulators adopted digital currency-friendly rules. The US Dollar and Euro are also among the top five most active Bitcoin trading currencies (Russo & Migliozzi, 2017). Daily, weekly, and monthly prices are sourced from Yahoo. Our sample consists of 1828 observations for daily frequency, 366 observations for weekly frequency, and 85 observations for monthly frequency for all assets under study. Augmented-Dickey Fuller tests suggest that the daily, weekly, and monthly prices for all assets under study exhibit a unit-root. Logarithmic difference in the prices is used to bring all assets' price series into the return series. Fig. 4 plots the monthly return of Bitcoin and chosen indices.

Table 1 outlines the summary statistics for the daily, weekly and monthly data for all assets. Bitcoin's mean returns are 0.805% for daily data, 4.049% for weekly returns, and 23.6% for monthly returns. The difference between minimum and maximum returns for Bitcoin is 100.7% for daily frequency, 183.1% for weekly, and 415.7% for monthly. As expected, Bitcoin returns exhibit much higher volatility than the stock indices. The Ljung Box Q statistics for Bitcoin returns are all significant, showing that there is autocorrelation in Bitcoin's return for all three frequencies. The Ljung Box Q^2 statistics testing autocorrelation on squared return series are significant for daily and weekly Bitcoin returns, but insignificant for Bitcoin monthly returns. These imply that GARCH effects exist in daily and weekly returns, but not in the monthly returns.

To further verify whether GARCH effects are needed in the analysis, we performed ARCH tests on the 1st and 3rd lag of the residuals of the corresponding OLS regressions and report the test results in Table 2. The p-values of ARCH (1) and ARCH (3) are significant for daily and weekly index returns, but insignificant for monthly returns. These are consistent with the Ljung Box statistics of Table 1 indicating that the GARCH model is needed for daily and weekly returns, but not for monthly returns.

Table 3 reports the correlation matrix for Bitcoin and equity indices returns. For daily data, the only negative correlation is between the Bitcoin return and the Nikkei index return. For weekly data, Bitcoin returns are negatively correlated with the returns of Shanghai A-Share, TSX and Euro indices. For monthly data, the Bitcoin returns are negatively correlated with all the equity indices,

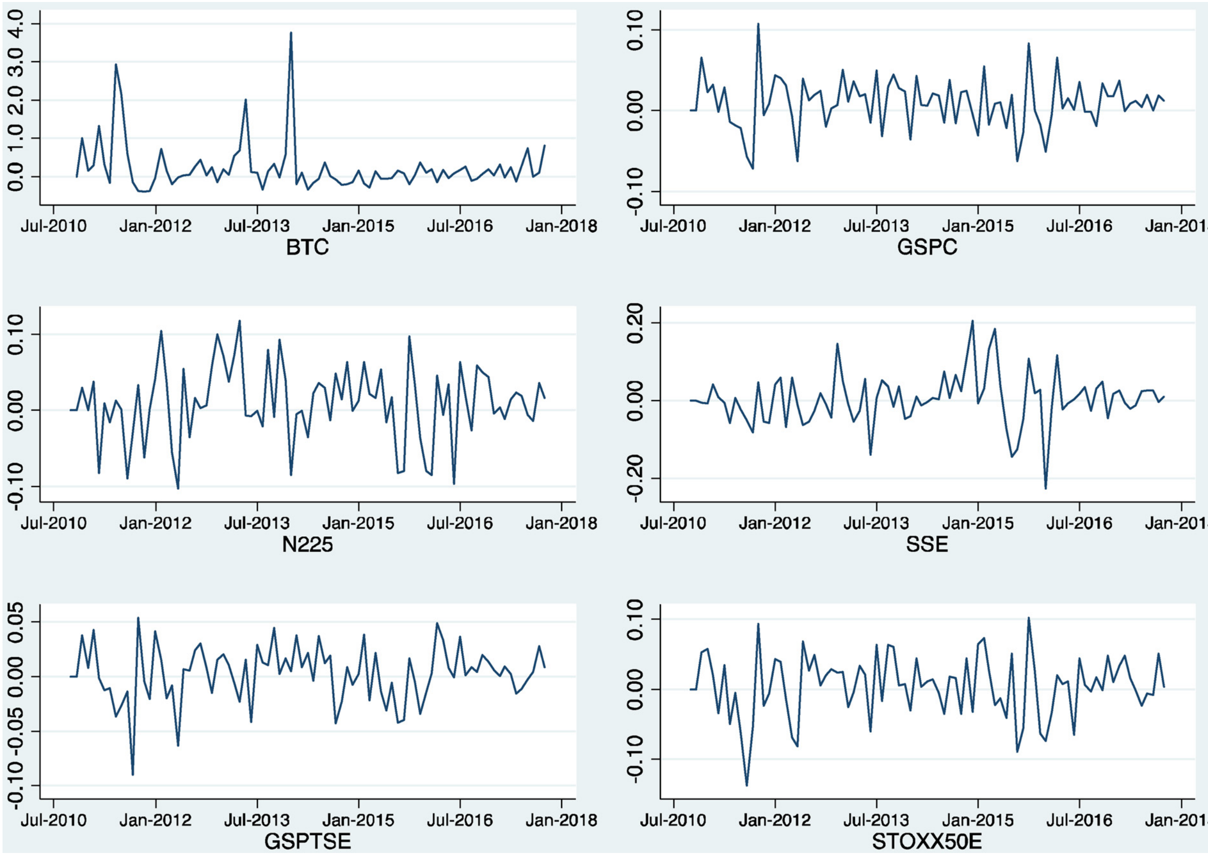

Fig. 4. Monthly return for all assets under study from October 2010 to October 2017.

Table 1
Summary statistics for daily, weekly, and monthly data.

	Mean	S.D.	Minimum	Maximum	Count	Q Stat	p-value	Q^2 Stat	p-value
Daily Returns									
BTC	0.805	6.573	−35.840	64.817	1828	6.690	0.010	48.441	0.000
GSPC	0.047	0.885	−6.663	4.741	1828	7.376	0.007	135.054	0.000
N225	0.054	1.325	−10.554	7.709	1828	5.204	0.023	92.758	0.000
SSE	0.021	1.361	−8.491	5.764	1828	0.003	0.954	71.921	0.000
GSPTSE	0.014	0.778	−4.039	4.020	1828	5.081	0.024	40.186	0.000
STOXX	0.021	1.258	−8.617	6.075	1828	1.628	0.202	17.397	0.000
Weekly Returns									
BTC	4.049	16.173	−42.830	140.269	366	44.932	0.000	14.229	0.000
GSPC	0.228	1.829	−7.189	7.389	366	3.917	0.048	11.603	0.001
N225	0.252	2.745	−11.100	9.207	366	0.510	0.475	10.786	0.001
SSE	0.098	2.954	−14.667	9.537	366	4.726	0.030	24.381	0.000
GSPTSE	0.076	1.719	−6.530	5.369	366	3.789	0.052	13.371	0.000
STOXX	0.101	2.671	−11.055	10.953	366	2.174	0.140	6.895	0.009
Monthly Returns									
BTC	23.579	65.508	−38.733	377.014	85	4.215	0.040	0.910	0.340
GSPC	0.958	3.114	−7.176	10.772	85	1.263	0.261	1.002	0.317
N225	0.983	4.847	−10.274	11.800	85	0.874	0.350	0.992	0.319
SSE	0.426	6.526	−22.649	20.566	85	3.659	0.056	0.501	0.479
GSPTSE	0.264	2.531	−8.966	5.405	85	0.362	0.547	0.336	0.562
STOXX	0.460	4.390	−13.792	10.242	85	0.530	0.467	3.349	0.067

BTC, GSPC, N225, SSE, GSPTSE and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX indices respectively, calculated as the logarithmic differences of price. Q and Q^2 stats refer to Ljung Box tests on return and squared return series for autocorrelation.

and the size of the correlation is much bigger than that of the daily and weekly returns.

4. Methodology

We first employ the following GARCH process to investigate the pairwise relationship between Bitcoin returns and the return of Euro STOXX, S&P 500, TSX Index, Shanghai A-Share and Nikkei, one stock index at a time.

$$BTCReturn_t = \alpha_o + \beta_1 BTCReturn_{t-1} + \beta_2 IndexReturn_t + \beta_3 IndexReturn_{t-1} + \varepsilon_t \quad (1)$$

Table 2
Arch test statistics for daily, weekly, and monthly data.

	ARCH(1)	p-value	ARCH(3)	p-value	Q stat	p-value	Q^2 stat	p-value
Daily Returns								
GSPC	80.095	0.000	68.050	0.000	0.579	0.447	49.047	0.000
N225	78.815	0.000	68.070	0.000	0.573	0.449	47.956	0.000
SSE	78.919	0.000	67.789	0.000	0.576	0.448	48.053	0.000
GSPTSE	78.877	0.000	68.270	0.000	0.631	0.427	48.173	0.000
STOXX	78.920	0.000	67.223	0.000	0.679	0.410	48.648	0.000
Weekly Returns								
GSPC	42.019	0.000	45.919	0.000	0.597	0.440	42.471	0.000
N225	38.707	0.000	42.322	0.000	0.566	0.452	39.123	0.000
SSE	41.673	0.000	45.786	0.000	0.633	0.426	42.121	0.000
GSPTSE	41.218	0.000	45.388	0.000	0.644	0.422	41.661	0.000
STOXX	41.976	0.000	46.044	0.000	0.629	0.428	42.428	0.000
Monthly Returns								
GSPC	0.769	0.380	1.142	0.767	0.035	0.852	0.806	0.369
N225	0.628	0.428	0.927	0.819	0.014	0.907	0.659	0.417
SSE	1.029	0.310	1.404	0.705	0.038	0.845	1.079	0.299
GSPTSE	0.883	0.347	1.190	0.755	0.034	0.853	0.925	0.336
STOXX	0.742	0.389	1.037	0.792	0.037	0.847	0.778	0.378

BTC, GSPC, N225, SSE, GSPTSE, and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX index respectively, calculated as the logarithmic differences of price. ARCH(1) and ARCH (3) refer to the ARCH tests on OLS residuals from the baseline model for 1st and 3rd lag. Q and Q^2 stats refer to Ljung Box tests on raw and squared residuals for autocorrelation.

Table 3
Correlation matrix.

	BTC	GSPC	N225	SSE	GSPTSE	STOXX
Daily						
BTC	1.00					
GSPC	0.04	1.00				
N225	−0.01	0.14	1.00			
SSE	0.02	0.13	0.23	1.00		
GSPTSE	0.03	0.75	0.17	0.16	1.00	
STOXX	0.03	0.63	0.23	0.13	0.55	1.00
Weekly						
BTC	1.00					
GSPC	0.02	1.00				
N225	0.00	0.53	1.00			
SSE	−0.01	0.22	0.21	1.00		
GSPTSE	−0.07	0.75	0.46	0.19	1.00	
STOXX	−0.01	0.77	0.58	0.18	0.67	1.00
Monthly						
BTC	1.00					
GSPC	−0.15	1.00				
N225	−0.14	0.59	1.00			
SSE	−0.09	0.33	0.38	1.00		
GSPTSE	−0.07	0.71	0.33	0.34	1.00	
STOXX	−0.14	0.74	0.65	0.26	0.60	1.00

BTC, GSPC, N225, SSE, GSPTSE and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX indices respectively, calculated as the logarithmic differences of price.

Where $\varepsilon_t \sim N(0, \delta_t^2)$

$$\delta_t^2 = \omega_0 + \beta_4 \varepsilon_{t-1}^2 + \beta_5 \delta_{t-1}^2$$

$BTCReturn_t$ is the Bitcoin return at time t and $IndexReturn_t$ is the Index return at time t. The error term is normally distributed with a mean of zero and a variance of δ_t^2. The conditional variance (δ_t^2) follows a GARCH (1, 1) process.

The sign and significance of coefficients β_2 disclose Bitcoin's hedging and diversification features. As discussed in Baur and Lucey (2010), a significant negative correlation implies a strong hedging relationship; a significant positive correlation can be interpreted as a diversifier, and insignificant correlation implies a weak hedge relationship. The strong hedging feature is most desired as it is the least complicated to apply in real world application to mitigate risk.

Table 4
GARCH (1, 1) results for Bitcoin daily, weekly and monthly returns.

VARIABLES	GSPC	N225	SSE	GSPTSE	STOXX
	Panel A: daily returns				
BTC_{t-1}	0.017	0.018	0.015	0.016	0.013
	(0.023)	(0.022)	(0.023)	(0.023)	(0.023)
$Index\_Return_t$	−0.052	−0.052	0.019	0.043	0.093
	(0.087)	(0.070)	(0.043)	(0.103)	(0.067)
$Index\_Return_{t-1}$	0.119	0.064	−0.100*	−0.043	0.082
	(0.124)	(0.076)	(0.057)	(0.125)	(0.075)
Constant	0.003***	0.003***	0.003***	0.000	0.000***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
log likelihood	2847	2847	2848	2847	2848
	Panel B: weekly returns				
BTC_{t-1}	0.228***	0.228***	0.232***	0.246***	0.235***
	(0.057)	(0.056)	(0.056)	(0.055)	(0.056)
$Index\_Return_t$	0.167	0.054	0.067	−0.318	−0.028
	(0.283)	(0.164)	(0.136)	(0.273)	(0.200)
$Index\_Return_{t-1}$	0.385	0.173	−0.069	0.103	0.133
	(0.258)	(0.170)	(0.120)	(0.288)	(0.175)
Constant	0.010**	0.001	0.011**	0.010**	0.011**
	(0.000)	(0.005)	(0.000)	(0.005)	(0.000)
log likelihood	282.4	281.8	281.3	282.1	281.3
	Panel C: Monthly returns				
BTC_{t-1}	0.573***	0.411**	0.343***	0.434***	0.348**
	(0.136)	(0.169)	(0.126)	(0.138)	(0.172)
$Index\_Return_t$	−2.024*	−0.890**	−0.547*	−2.278*	−1.427**
	(1.045)	(0.449)	(0.327)	(1.193)	(0.578)
$Index\_Return_{t-1}$	0.523	−0.142	−0.109	2.420***	1.427**
	(0.595)	(0.629)	(0.473)	(1.086)	(0.725)
Constant	0.003	−0.001	−0.003	0.055	−0.002
	(0.008)	(0.035)	(0.028)	(0.034)	(0.016)
log likelihood	−57.90	−59.31	−60.48	−58.28	−56.69

BTC, GSPC, N225, SSE, GSPTSE and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX indices respectively, calculated as the logarithmic differences of price. BTC_{t-1} is the lagged Bitcoin return. $Index\_Return_t$ and $Index\_Return_{t-1}$ are the contemporaneous and lagged index returns. Standard errors are reported in parentheses. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

5. Results

5.1. GARCH model

GARCH (1,1) regressions are run for daily, weekly, and monthly Bitcoin returns. Table 4 outlines these testing results. The tests yield statistically significant ARCH and GARCH terms for all assets and frequencies. We observe that β_2 for the daily and weekly regressions are all insignificant, indicating that the daily and weekly returns are uncorrelated with all these indices and Bitcoin could only be used as a weak hedge. These findings are similar to Dyhrberg (2016b) that shows Bitcoin return is uncorrelated to FTSE index.

The monthly results demonstrate multiple significant coefficients. The correlation between Bitcoin and chosen indices are all negatively significant. With a one percent monthly return movement in these indices, Bitcoin returns move in the opposite direction by −2.024, −0.89, −0.547, −2.278 and −1.427 percent respectively, suggesting that Bitcoin is a strong hedge against these indices over monthly horizon. Investors could hold Bitcoin to strategically offset negative return movements in these assets. However, only investors who hold Bitcoins for longer periods can benefit from these hedging benefits.

5.2. Constant conditional correlation (CCC) model

To allow for interdependence between the Bitcoin return and index return, we also consider the Constant Conditional Correlation Model of Bollerslev (1990) given by

adding a second equation to form a bivariate model as following,

$$Index_t = \alpha_0^* + \beta_2^* BTC_{t-1} + \beta_3^* Index_{t-1} + \varepsilon_t^* \tag{2}$$

where the error term ε_t^* is correlated with ε_t in Eq. (1) and the constant correlation between the error terms is denoted by ρ_{12}. The bivariate error structure $e_t = \begin{bmatrix} \varepsilon_t & \varepsilon_t^* \end{bmatrix}'$ has a standard form of

$$e_t = H_t^{1/2} \nu_t \tag{3}$$

$$H_t = D_t^{1/2} \, R \, D_t^{1/2} \tag{4}$$

$H_t^{1/2}$ refers to the Cholesky factor of the conditional covariance matrix H_t and ν_t is a 2×1 vector of normal, independent, and identically distributed innovations; D_t is a diagonal matrix of conditional GARCH variances; and R is a matrix of constant unconditional correlation of the two error terms.

We estimated both constant conditional correlation (CCC) and dynamic conditional correlation (DCC) models. The DCC model does not converge, especially for monthly frequency data, and the log likelihood does not improve over the CCC models. This non-convergence is likely caused by lacking data to estimate the dynamic correlation between residual terms. The Bitcoin return data might not have a regular structure to support the dynamics of correlation.

Therefore, we estimate the CCC (1,1) model for all three frequencies and report the results in Table 5. The ARCH and GARCH terms of these regressions are all significant, which are not reported for conserving space. The correlation coefficients (ρ_{12}) are significant for most of the CCC regressions, which suggests that it is necessary to model the interdependence between Bitcoin and market indices explicitly instead of estimating a single equation. The CCC coefficients will be more efficient than the single equation GARCH model.

Similar to the GARCH results, the correlations between Bitcoin daily (and weekly) returns and index returns all stay insignificant, indicating that Bitcoin is a weak hedge against all chosen indices. Consistent with the GARCH model results, monthly Bitcoin returns demonstrate significant negative relationships with all indices. With a one percent change in the monthly return of S&P 500, Nikkei, Shanghai A-Share, TSX Index, and Euro STOXX indices, Bitcoin returns change in the opposite direction by −2.350, −2.219, −0.573, −3.476, and −1.501 respectively. The significant negative correlations indicate that Bitcoin is a strong hedge against these indices.

Table 5
Constant Conditional Correlation (CCC) bivariate GARCH (1,1) estimation results for Bitcoin daily, weekly and monthly returns.

VARIABLES	GSPC	N225	SSE	GSPTSE	STOXX
	Panel A: daily returns				
BTC_{t-1}	0.017	0.018	0.016	0.015	0.013
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
$Index\_Return_t$	−0.111	−0.012	0.055	0.037	0.216
	(0.197)	(0.110)	(0.084)	(0.216)	(0.137)
$Index\_Return_{t-1}$	0.118	0.068***	0.009	−0.043*	0.083
	(0.117)	(0.026)	(0.061)	(0.025)	(0.078)
ρ_{12}	0.003***	0.001	0.000	0.001**	0.001***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
Constant	0.015	0.001***	0.001***	0.003***	0.000
	(0.041)	(0.000)	(0.000)	(0.000)	(0.001)
log likelihood	9162	8335	8452	9331	8494
	Panel B: weekly returns				
BTC_{t-1}	0.232***	0.010	0.001	0.246***	−0.009
	(0.063)	(0.064)	(0.063)	(0.062)	(0.064)
$Index\_Return_t$	−0.089	−0.023	0.142	−0.409	−0.057
	(0.358)	(0.206)	(0.161)	(0.346)	(0.268)
$Index\_Return_{t-1}$	0.346	0.054	−0.066	−0.049	0.130
	(0.239)	(0.132)	(0.055)	(0.055)	(0.158)
ρ_{12}	0.011***	0.000	−0.049***	0.001	0.000
	(0.004)	(0.001)	(0.000)	(0.076)	(0.004)
Constant	0.079	0.001***	0.011***	0.010**	0.011***
	(0.079)	(0.000)	(0.000)	(0.004)	(0.000)
log likelihood	1259	1089	1111	1285	1109
	Panel C: Monthly returns				
BTC_{t-1}	0.286***	0.276***	−0.015***	−0.004***	−0.002
	(0.000)	(0.006)	(0.000)	(0.000)	(0.149)
$Index\_Return_t$	−2.350***	−2.219***	−0.573***	−3.476**	−1.501***
	(0.000)	(0.000)	(0.000)	(1.420)	(0.382)
$Index\_Return_{t-1}$	0.471***	0.027	0.252*	1.223	1.428***
	(0.000)	(0.120)	(0.133)	(2.295)	(0.349)
ρ_{12}	0.154***	0.136	0.265***	0.134	0.062
	(0.000)	(0.000)	(0.107)	(0.129)	(0.007)
Constant	0.071	0.012**	0.108***	0.003***	0.001
	(0.108)	(0.000)	(0.000)	(0.000)	(0.122)
log likelihood	110.6	72.98	52.34	127.5	89.66

BTC, GSPC, N225, SSE, GSPTSE, and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX indices respectively, calculated as the logarithmic differences of price. BTC_{t-1} is the lagged Bitcoin return. $Index\_Return_t$ and $Index\_Return_{t-1}$ are the contemporaneous and lagged index returns. ρ_{12} is the correlation between the two error terms in the CCC model. Standard errors are reported in parentheses. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

5.3. Frequency dependent regression model

Both GARCH and CCC models indicate that the correlation between Bitcoin and market index returns varies across frequencies. Therefore, we next use the frequency dependence regression model proposed by Ashley and Verbrugge (2009) to further investigate the dynamics of Bitcoin's hedging abilities. Instead of assuming the coefficients on each coefficient to be constant for each chosen frequency, the frequency dependent model allows the value of these coefficients to vary over time. The values of the coefficients at any given time depend on the recent history of the explanatory variable they multiply with because of the variation with frequency.

We conducted frequency dependence analysis using Bitcoin's daily return data with a one year rolling window to perform the one-sided filtering. Following Ashley and Verbrugge (2009), the data set is augmented with two weeks of data with an AR(2) process for each index. We use the filter of over one month, one week to one month, and less than a week as the cutoff for low, medium, and high frequencies respectively. The regression results are reported in Table 6.

Consistent with the GARCH and CCC models, the high frequency coefficients in Table 6 show that the high frequency returns (D_3^*) are uncorrelated with the Bitcoin returns, so Bitcoin can only be

Table 6
Frequency dependence estimation results for Bitcoin daily returns.

VARIABLES	GSPC	N225	SSE	GSPTSE	STOXX
BTC_{t-1}	0.001	0.001	−0.004	−0.004	−0.003
	(0.026)	(0.025)	(0.025)	(0.025)	(0.025)
Index_Return$_{t-1}$	0.217*	0.094	−0.054	0.002	0.162**
	(0.125)	(0.078)	(0.067)	(0.129)	(0.082)
D_1^* (one month or over)	0.334	0.228	−0.572*	−0.260	−0.255
	(0.602)	(0.432)	(0.340)	(0.623)	(0.477)
D_2^* (one week to a month)	−0.865**	−0.372	−0.128	−0.644	−0.664**
	(0.367)	(0.294)	(0.250)	(0.414)	(0.307)
D_3^* (1 - 6 days)	−0.061	−0.044	0.013	0.061	0.077
	(0.094)	(0.070)	(0.043)	(0.109)	(0.071)
Constant	0.000***	0.000***	0.004***	0.000***	0.003***
	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Observations	1,575	1,575	1,575	1,575	1,575
AIC	−5280	−5277	−5280	−5276	−5281
BIC	−5232	−5229	−5232	−5228	−5232
Chi^2 test	6.870	2.420	2.850	2.870	6.130
	[0.08]	[0.49]	[0.42]	[0.41]	[0.11]
log likelihood	2649	2648	2649	2647	2649

BTC, GSPC, N225, SSE, GSPTSE, and STOXX are returns for Bitcoin, S&P 500, Nikkei, Shanghai A-Share, TSX, and Euro STOXX indices respectively, calculated as the logarithmic differences of price. BTC_{t-1} is lagged Bitcoin return. Index_Return_t-1 is the lagged index return. D_1^*, D_2^* and D_3^* are dummy variables representing low frequency, medium frequency, and high frequency fluctuations in the corresponding index return. Chi^2 tests report the test statistics for the null hypothesis of no frequency dependence (all $Dj^*=0$) and their corresponding p values are in the squared brackets. Standard errors are reported in parentheses. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

a weak hedge for these indices in high frequency. For medium frequency returns (D_2*), the frequency dependence model produces two negative significant coefficients against the S&P and Euro index. Because the length of the medium frequency is in between weekly and monthly, these significant results might be driven by the combination of cyclical components in both weekly and monthly frequencies. For the low frequency returns (D_1^*), the only significant coefficient is the correlation with the Shanghai A-share, indicating strong hedge ability. The other four coefficients are insignificant. The results reveal that frequency dependent correlation between Bitcoin and various market indices are not exactly the same as the ones from the GARCH and CCC models. One possible explanation is that the frequency dependence model captures the nonlinear relationship between Bitcoin and market indices. Furthermore, the frequency components are different from the daily, weekly, and monthly data in the other models, so these results reflect the differences in time domain. Another possible explanation is that the hedging results are sensitive to model specifications and assumptions about model parameters. Despite differences in results, the findings that Bitcoin could be used to hedge S&P, TSX, Euro, Shanghai A-Share, and Nikkei indices are supported by the frequency dependence model as well.

6. Conclusion

We use three different time series models, GARCH, CCC and the frequency dependence model to analyze whether Bitcoin can hedge risk against a number of equity markets, and determine whether these abilities change under different investment horizons. The GARCH (1, 1) model provides compelling evidence that Bitcoin can be used as a strong hedge against the Euro-Index, Shanghai A-Share, S&P 500, Nikkei, and the TSX Index for monthly returns. Moreover, Bitcoin weekly and daily returns also exhibit risk-mitigating abilities by being a weak hedge against these indices.

Allowing interdependence between Bitcoin and the market index, the CCC model confirms similar weak hedging abilities for Bitcoin's daily and weekly returns. Bitcoin's strong hedging abilities in monthly frequency stay robust under the CCC model. Similarly, the frequency dependence model shows that the high frequency returns could only weak hedge the market indices. The daily Bitcoin return could be a strong hedge against medium frequency returns of the S&P and Euro index, and a weak hedge against Nikkei, Shanghai, and TSX index. The low frequency return shows a strong hedge against Shanghai A-Share and a weak hedge against the other four indices.

These findings suggest that holding Bitcoin could provide hedging benefits for investors. Moreover, the longer-term returns have stronger hedging abilities than the short term returns. Therefore, holding Bitcoin longer may benefit investors by providing risk management abilities to their equity portfolios.

References

Ashley, R., & Verbrugge, R. (2009). Frequency dependence in regression model coefficients: An alternative approach for modeling Nonlinear dynamic relationships in time series. *Econometric Reviews*, *28*, 4–20.

Bariviera, A. F., Basgall, M. J., Hasperué, W., & Naiouf, M. (2017). Some stylized facts of the Bitcoin market. *Physica A*, *484*, 82–90.

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *The Financial Review*, *45*, 217–229.

Blau, B. M. (2017). Price dynamics and speculative trading in Bitcoin. *Research in International Business and Finance*, *41*, 493–499.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics*, *72*, 498–505.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters*, *20*, 192–198.

Cheah, E., & Fry, J. (2015). Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters*, *130*, 32–36.

Coindesk. (2017). *Bitcoin closing price* from. https://www.coindesk.com/price/

Dyhrberg, A. H. (2016a). Bitcoin, gold and the dollar – A GARCH volatility analysis. *Finance Research Letters*, *16*, 85–92.

Dyhrberg, A. H. (2016b). Hedging capabilities of bitcoin. Is it the virtual gold? *Finance Research Letters*, *16*, 139–144.

Luther, W. J., & Salter, A. W. (2017). Bitcoin and the bailout. *The Quarterly Review of Economics and Finance*, *66*, 50–56.

Russo, C., & Migliozzi, B. (2017). *This is where people are buying bitcoin all over the world*. Bloomberg. https://www.bloomberg.com/graphics/2017-bitcoin-volume/

Tan, H. B., & Ashley, R. (1999). Detection and modeling of regression parameter variation across frequencies: With an application to testing the permanent income hypothesis. *Macroeconomic Dynamics*, *3*(1), 69–83.

Tully, E., & Lucey, B. (2007). A power GARCH examination of the gold market. *Research in International Business and Finance*, *21*(2), 316–325.

Yermack, D. (2013). *Is Bitcoin a real currency? An economic appraisal* Retrieved from. National Bureau of Economic Research. http://www.nber.org/papers/w1974