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Informational inefficiency of Bitcoin: A study based on high-frequency data

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ABSTRACT

The study reexamines the issue of informational efficiency of Bitcoin using data at different frequencies (15, 30, 60 and 120 min and daily data). In particular, we test the martingale hypothesis in Bitcoin returns using different variance ratio tests. We also examine the evolution of informational efficiency of Bitcoin using non-overlapping and overlapping moving window analysis. The study provides evidence of the presence of informational inefficiency in the Bitcoin market at higher frequency levels. The daily Bitcoin returns which appear to be following a memory-less stochastic process are in fact otherwise when we move to the higher frequencies of Bitcoin prices.

1. Introduction

Since its inception in Nakamoto (2008), Bitcoin is considerably growing and has become a new fascinating phenomenon in the financial markets. Bitcoin[1] is a complex peer-to-peer cryptocurrency and its understanding requires technical knowledge of algorithms and cryptography (Badev and Chen, 2014). Despite its complex nature, the market capitalization of Bitcoin overgrew from \$3.3 billion on 16 February 2015 (Fry and Cheah, 2016) to \$213.8 billion on 20 January 2018 (coinmarketcap.com). Bitcoin currently has an essential place in the financial markets (Dyhrberg, 2016), as it provides investors with a new instrument for portfolio allocation. The popularity of Bitcoin can be attributed to its novel features, transparency, simplicity (Urquhart, 2016) and low cost of foreign exchange (Kim, 2015).

Amidst the recognition of Bitcoin, many academic studies have exploredits economics and finance. Yermack (2015) highlights that the Bitcoin is more of a speculative investment than a currency as the market capitalization of Bitcoin is very high in comparison to the economic transactions it assists. Kristoufek (2013) examines the dynamic relationship between the investor attention in Bitcoin and its price. Rogojanu and Badea (2014) examine Bitcoin vis-à-vis other monetary systems, while Bornholdt and Sneppen (2014) discuss the dominance of Bitcoin over alternative cryptocurrencies. Fink and Johann (2014) study the market microstructure of Bitcoin and provide detailed insights related to it. Garcia et al. (2014) investigate the creation of the market bubble concerning Bitcoin. Bouoiyour et al. (2016) highlights that Bitcoin is one of the significant financial innovations of the recent times. Brandvold et al. (2015) and Ciaian et al. (2016) examine the price discovery in the Bitcoin market. Bouri et al. (2016) explore the role of trading volume in understanding the volatility in the Bitcoin market. Bariviera et al. (2017) study the long-range dependence and other

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[1] Understanding Bitcoin is not the main contribution of this study and therefore such details are not included in the main text. For a detailed discussion on Bitcoin mining, please refer to Bhaskar and Chuen (2015) and Kroll et al. (2013). For a deep understanding of Bitcoin-BlockChain principles and further technical details, please refer to Badev and Chen (2014).

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statistical properties of Bitcoin using daily and intraday prices. The authors find that even though the Bitcoin volatility is high, however, it has declined over time. The authors also find that the long-range dependence in Bitcoin not be related to its market liquidity. Very recently, Urquhart (2018) employ Google Trends data to study the attention of Bitcoin. The authors find that the high realized volatility and volume in the previous day significantly influences the attention of Bitcoin the next day.

To ensure a viable growth, Bitcoin needs to be stable and efficient so that different market participants can quickly enter and exit their positions. Also, for a wider acceptance as a financial instrument, the value of Bitcoin across different markets and exchanges needs to be the same, so that a Bitcoin transaction can yield equal value to the counterparties involved. Therefore, it becomes imperative to study the behavior of Bitcoin under the efficient market hypothesis, the foundation of which lies in the seminal works of Bachelier (1900); Samuelson (1965) and Fama (1970). According to the efficient market hypothesis, the current asset price reflects all information available which directly points to the martingale or random walk hypothesis. Testing market efficiency is important especially in case of a speculative market as it helps in understanding the market evolution regarding disclosures and transparency, which in turn has regulatory and policy implications.

Four recent studies have focused on examining the issue of informational efficiency concerning the Bitcoin market. Urquhart (2016) is the first to investigate the market efficiency of the Bitcoin market using a battery of five robust tests. The tests aim at studying the autocorrelations, serial dependence, and variance ratios in the daily Bitcoin returns. The findings of the study highlight the inefficient characteristics of the Bitcoin market over the full sample period. However, based on the non-overlapping window analysis, Urquhart (2016) argues that the Bitcoin market may become more efficient as it matures. Following this, Nadarajah and Chu (2017) use power transformations of the daily Bitcoin returns to show that the Bitcoin returns satisfy efficient market hypothesis. The authors highlight that the use power transformation does not lead to any loss of information. Bariviera (2017) study the time-varying behavior of long memory of Bitcoin returns and volatility using the Hurst exponent. The author advocates the use of De-Trended Fluctuation Analysis (DFA) to study the presence of long memory in the Bitcoin returns. The study concludes that the market is informationally efficient (since 2014) and volatility clustering is a key feature of the Bitcoin market. Most recently, Tiwari et al. (2018) reevaluate the issue of informational efficiency of Bitcoin by deploying a set of computationally efficient long-range dependence estimators. The study reports that the Bitcoin market is efficient, which is consistent with the findings of the previous three studies. The absence of empirical studies in analyzing the informational efficiency of high-frequency returns of Bitcoin is the main motivation for this study. The current study analyzes the informational efficiency of the Bitcoin market using high-frequency data.

Consistent with the traditional approach in assessing the informational efficiency, the study contributes the literature in two important aspects. First, we expand the empirical studies by investigating the informational efficiency of Bitcoin using high-frequency data (15 min, 30 min, 60 min and 120 min high-frequency data). Second, we study the evolution of informational efficiency of Bitcoin across different periods using non-overlapping and overlapping moving window analysis. It allows testing of the adaptive market hypothesis in the Bitcoin market.

The rest of the study is organized as follows. Section 2 presents the methodology used. Section 3 describes the data used and its summary statistics. In Section 4, we present the empirical results and findings. Section 5 provides a discussion of the results. Finally, Section 6 provides some conclusions from our study.

2. Methodology

2.1. Martingale hypothesis

A time series is said to be a martingale difference sequence if the conditional mean of the series is independent of the information set based on its past values. Testing the martingale difference hypothesis helps in understanding whether a time series can be predicted or not, which in turn becomes a base for testing the weak-form efficiency of a market. A market is said to be weak-form efficient if it is not possible to predict the future price changes based on the historical data.

According to the martingale hypothesis, tomorrow's expected price is the same as today's price. Mathematically, if x_t represents the natural logarithm of the Bitcoin price, for x_t to follow a martingale,

$$E_t(x_t/\Omega_{t-1}) = x_{t-1} \tag{1}$$

where Ω_{t-1} is the information set containing all information up to time $t-1$.

The existing literature provides us with several approaches to examine the martingale behavior of tradable asset price series. The most common and widely used method is the variance ratio test[2], proposed by Lo and MacKinlay (1988).

2.2. Variance ratio (VR) test

As per the variance ratio test, for the Bitcoin price series to be a martingale, the variance of k-period Bitcoin returns must be equal to k times the variance of one-period Bitcoin return. Mathematically, if y_t represents the logarithmic return of Bitcoin at time t. The variance ratio of k-period Bitcoin return is defined as:

[2] For the detailed procedure on each method, readers are requested to refer to the respective references.

$$VR(y: k) = \frac{var(y_t + y_{t-1} + \ldots + y_{t-k+1})/k}{var(y_t)} \tag{2}$$

To investigate whether Bitcoin time series is informationally efficientor not, we make use of following three variants of the VR test:

2.2.1. Multiple variance ratio (MVR) test

Chow and Denning (1993) extend the methodology of Lo and MacKinlay (1988) by applying a multiple comparison statistical approach and propose a MVR statistics which then became the base for other MVR tests. MVR helps us to examine a vector of individual variance ratio tests for a respective vector of holding periods. Chow and Denning (1993) propose the following test statistics under the joint null hypothesis of $VR(k_i) = 1$ for $i = 1, \ldots, m$:

$$MVR = max_{1 \le i \le m} \ |M(y: k)| \tag{3}$$

The maximum of absolute value of the individual VR statistics is used to decide the rejection of the null hypothesis. The null hypothesis is rejected at the α level of significance if the MVR statistic is greater than the $(1-\alpha^*/2)th$ percentile of the standard normal distribution, where $\alpha^* = 1-(1-\alpha)^{1/l}$ (Stoline and Ury, 1979).

2.2.2. Automatic variance ratio (AVR) test

To find the optimal value of k, Choi (1999) propose another variant of the variance ratio test based on the frequency domain. Cochrane (1988) finds that the estimation of $VR(k)$ based on consistent estimators of variances is asymptotically equal to 2π times the normalized spectral density estimation at zero frequency based on the Bartlett kernel. Also, Andrews (1991) finds that the Quadratic Spectral kernel is optimal in the estimation of spectral density at zero frequency. Choi (1999) builds on the same idea and employs the Quadratic Spectral kernel to estimate variance ratio. Andrews (1991) method is used to optimally select the holding period k. The Choi's variance ratio estimator is defined as

$$VR(k) = 1 + 2\sum_{i=1}^{T-1} m(i/k)\hat{\rho}(i) \tag{4}$$

where

$$\hat{\rho}(i) = \frac{\sum_{t=1}^{T} (y_t - \hat{\mu})(y_{t+i} - \hat{\mu})}{\sum_{t=1}^{T} (y_t - \hat{\mu})^2}$$

and

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t \ and\ y_t = x_t - x_{t-1}$$

and

$$m(r) = \frac{25}{12\pi^2 r^2}\left[\frac{sin(\frac{6\pi r}{5})}{(\frac{6\pi r}{5})} - cos(\frac{6\pi r}{5})\right]$$

where *m(r)* is the quadratic spectral kernel. Under the null hypothesis of $2\pi f_y(0) = 1$, the Choi (1999) test statistic is:

$$AVR(k) = \frac{\sqrt{\frac{T}{k}}\,[VR(k) - 1]}{\sqrt{2}} \xrightarrow{d} N(0,1) \tag{5}$$

as $k \to \infty$, $T \to \infty$, $T/k \to \infty$. Being a two-tailed test, the critical values for the VR defined here are taken from both sides of the standard normal distribution. The $AVR(k)$ result holds when y_t is IID with a finite fourth moment. The AVR test defined here depends on the holding period or lag truncation point k. To select the holding point optimally, Choi (1999) employs Andrews (1991) method which is explained in the steps below:

Step 1: Choose an approximating autoregressive (AR) model for y_t by using model selection criteria.

Step 2: Use formula (6.2) in (Andrews, 1991) to estimate the optimal truncation point.

Step 3: Use the so obtained optimal truncation point to calculate the variance ratio test.

The p–values are obtained using a weighted bootstrap procedure proposed by Wu (1986) to check the significance of the AVR test. Following are the steps that define the procedure:

1) In the first step, calculate normalized returns:

$$z_t = {(y_t - \overline{y})}/{\sigma(y)}$$

where, $\overline{y} = T^{-1}\sum_{t=1}^{T} y_t$, is the mean return and $\sigma(y) = \sqrt{T^{-1}\sum_{t=1}^{T} (y_t - \overline{y})^2}$, is the standard deviation of returns.

1) In the second step, for each t, draw a weighting factor, $z_t^*(t = 1,\ldots,T)$, with replacement from normalized returns, z_t.
2) In the next step, construct a bootstrap sample of T observations, $y_t^* = z_t^* y_t\,(t = 1,\ldots,T)$.
3) In the fourth step, calculate AVR test statistic, $AVR^*(k^*)$, the AVR test statistic obtained from $(y_1^*,\ldots,y_T^*)$.
4) In the last step, repeat steps 1–4, say m times, to construct a bootstrap distribution of the test statistic, $\{AVR^*(\hat{k}^*; j)\}_{j=1}^m$.

Finally, the two-tailed *p-values* of the AVR test are obtained as the proportion of absolute values of $\{AVR^*(\hat{k}^*; j)\}_{j=1}^m$ greater than the absolute values of *AVR(k)*.

2.2.3. Joint variance ratio (JVR) test

Chen and Deo (2006) propose a JVR test based on which the behavior of $VR(k)$ depends on the behavior of the periodogram values $I(\theta_j)$. Hence, the VR statistic based on the periodogram, for the diff ;erencing period k; is given by:

$$VR_p(k) = \frac{1}{1-(k/T)}\frac{4\pi}{T\hat{\sigma}^2}\sum_{j=1}^{[(T-1)/2]} W_k(\theta_j) I_y(\theta_j) \tag{6}$$

where $I_Y(\theta_j) = (2\pi T)^{-1}|\sum_{t=1}^{T}(y_t-\hat{\mu})exp(-i\theta_j t)|^2$, $\hat{\sigma}^2 = (T-1)^{-1}\sum_{t=1}^{T}(y_t-\hat{\mu})$, $\theta_j = 2\pi j/T$; and $W_k(\theta) = k^{-1}\{sin(0.5k\theta)/sin(0.5\theta)\}^2$ is a weighting function.

W. W. Chen and Deo (2006) show that when k is not too large, the power transformed the VR statistic $VR_p^{\beta k}(k)$ takes care of the skewness problem and thus affords a better approximation by the normal distribution in finite samples compared to the original $VR_p(k)$. W. W. Chen and Deo (2006) also show that the distribution of the then obtained power transformed VR statistic is robust to conditional heteroscedasticity.

$$\beta_k = 1-\frac{2}{3}\frac{\left(\sum_{j=1}^{[(T-1)/2]} W_k(\theta_j)\right)\left(\sum_{j=1}^{[(T-1)/2]} W_k^3(\theta_j)\right)}{\left(\sum_{j=1}^{[(T-1)/2]} W_k^2(\theta_j)\right)^2} \tag{7}$$

As per Theorem 5 of W. W. Chen and Deo (2006), suppose $k_1 < k_2 < \ldots\ldots, < k_l < n$ (representing holding periods) be positive integers, then as $k_1 \to \infty$, $k_l n^{-1} \to 0$, $k_i k_j \to a_{ij}$, for $1 \le i \le j \le l$. As y_t follows a conditionally heteroskedastic martingale difference series, W. W. Chen and Deo (2006) showed that:

$$V_{p,\beta} = (VR_p^{\beta_1}(k_1), \ldots\ldots\ldots\ldots, VR_p^{\beta_1}(k_l))' \approx N(\mu_\beta, \Sigma_\beta) \tag{8}$$

Based on this, W. W. Chen and Deo (2006) proposed a JVR test based on their individual power transformed VR statistic. The test statistic is:

$$JVR(k) = (\mathbb{V}_{p,\beta}(k)-\mu_\beta)' \sum\nolimits_\beta^{-1} (\mathbb{V}_{p,\beta}-\mu_\beta) \tag{9}$$

where $\mathbb{V}_{p,\beta}$ is a column vector sequence of VR statistic. The $JVR(k)$ statistic follows a χ^2 distribution with k degrees of freedom under the null hypothesis of a random walk.

2.3. Kuan and Lee (KL) (2004) test

Based on the moment conditions derived by Bierens (1982), Kuan and Lee (2004) propose a different approach for testing the martingale behavior of any time series. The authors report superior size and power properties of their test when compared with other alternative tests of the martingale difference hypothesis. In addition to the different variants of variance ratio tests, we also make use of KL test to study the informational efficiency of Bitcoin.

Developing on Y.-T. Chen et al. (2000) and Bierens (1982); Kuan and Lee (2004) propose the following test statistic for testing the martingale difference hypothesis. The test statistic for a given lag k is given as follows:

$$J_g = \frac{T-k}{\hat{\sigma}_{c,g}^2\hat{\sigma}_{s,g}^2-\hat{\sigma}_{cs,g}^2}[\hat{\sigma}_{s,g}^2\bar{\psi}_{c,g}^2 + \hat{\sigma}_{c,g}^2\bar{\psi}_{s,g}^2 - 2\hat{\sigma}_{cs,g}\bar{\psi}_{c,g}\bar{\psi}_{s,g}] \tag{10}$$

where

$$\bar{\psi}_{j,g} = \frac{1}{T-k}\sum_{t=k+1}^{T}\psi_{j,g}(y_t, y_{t-1,k}),\ j = c, s,$$

$$\hat{\sigma}_{j,g}^2 = \frac{1}{T-k}\sum_{t=k+1}^{T}\psi_{j,g}(y_t, y_{t-1,k})^2,\ j = c, s,$$

$$\hat{\sigma}_{cs,g}^2 = \frac{1}{T-k}\sum_{t=k+1}^{T}\psi_{c,g}(y_t, y_{t-1,k})\psi_{s,g}(y_t, y_{t-1,k}),$$

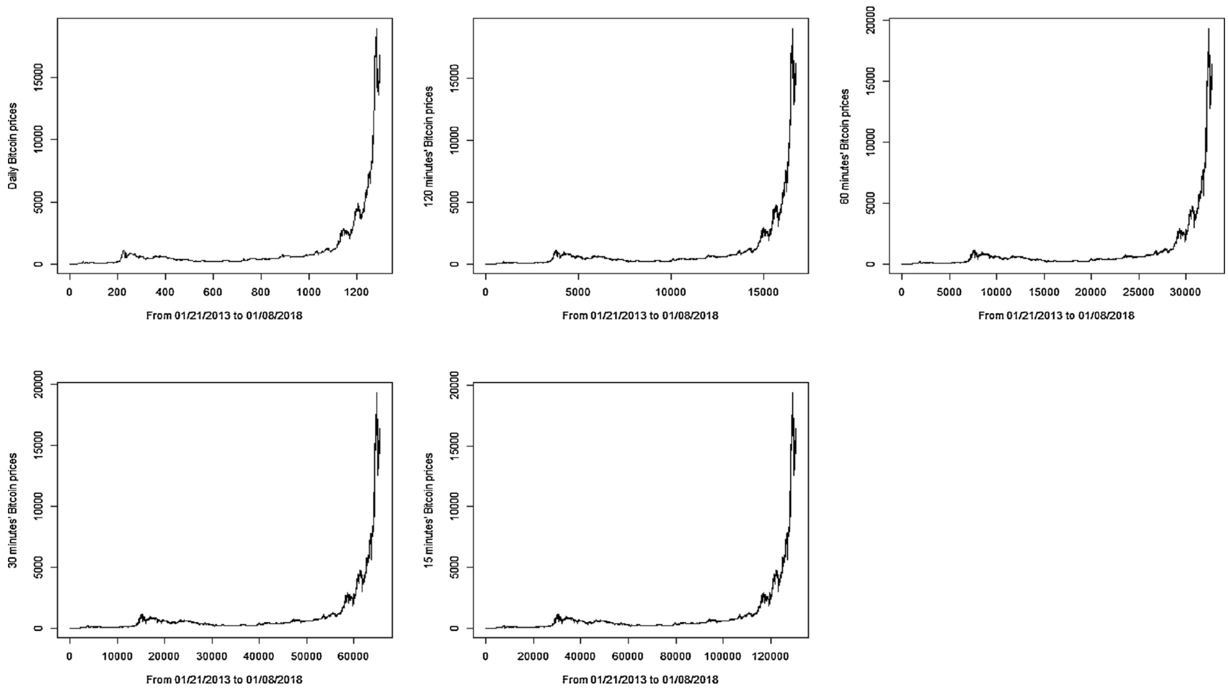

Fig. 1. Bitcoin price evolution.
The figure plots Bitcoin prices at different frequencies on Bitstamp exchange over the period 01/21/2013 to 01/08/2018. The frequencies considered are daily, 120 min, 60 min, 30 min and 15 min.

$$\psi_{j,g}(y_t, y_{t-1,k}) = y_t \varphi_{j,g}(y_{t-1,k}),\ j = c,\ s,$$

$$\varphi_{c,g}(y_{t-1,k}) = \int_{\mathbb{R}^{k+}} \cos(\omega' y_{t-1,k}) g(\omega) d\omega$$

$$\varphi_{s,g}(y_{t-1,k}) = \int_{\mathbb{R}^{k+}} \sin(\omega' y_{t-1,k}) g(\omega) d\omega$$

$$y_{t-1,k} = \{y_{t-1},\ \ldots\ldots, y_{t-k}\}$$

where T represents the sample size, c and s represent the real and imaginary part, respectively. Kuan and Lee (2004) find that under the null hypothesis J_g follows a chi-square distribution and is given as follows:

$$J_g \xrightarrow{D} \chi^2 \tag{11}$$

3. Data

We collect Bitcoin price data from Bloomberg database over the period 21 January 2013 to 08 January 2018. Bloomberg provides real-time and historical Bitcoin exchange rates relative to different currencies and across different exchanges. The data consist of daily, 120 min, 60 min, 30 min, and 15 min high-frequency prices of Bitstamp. The Bitstamp is the largest Bitcoin exchange based in Luxembourg, UK. It is considered to be the safest Bitcoin exchange by market participants across the world. We use the value of Bitcoin relative to the US dollar. The evolution of Bitcoin price across different frequencies are shown in Fig. 1, and the corresponding Bitcoin returns are shown in Fig. 2. The first look at Fig. 1 tells us that the prices were quite low in the beginning and have seen a considerable boost in the coming years. However, it is also visible that the Bitcoin prices are dropping towards the end.

If C_{t-1} and C_t represent the closing prices of Bitcoin at two consecutive time intervals, the Bitcoin return is calculated as:

$$r_t = \ln\left(\frac{C_t}{C_{t-1}}\right) \tag{12}$$

Table 1 reports the descriptive statistics of the Bitcoin returns across different frequencies. The log returns are mean positive and negatively skewed (except for 15 min). Also, the log returns at all frequencies are leptokurtic (fat-tailed) with kurtosis higher than that of the normal distribution. The results from the Jarque-Bera test also confirm the non-normality in Bitcoin returns at all frequencies. Furthermore, the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests show that the Bitcoin returns across different frequencies are stationary.

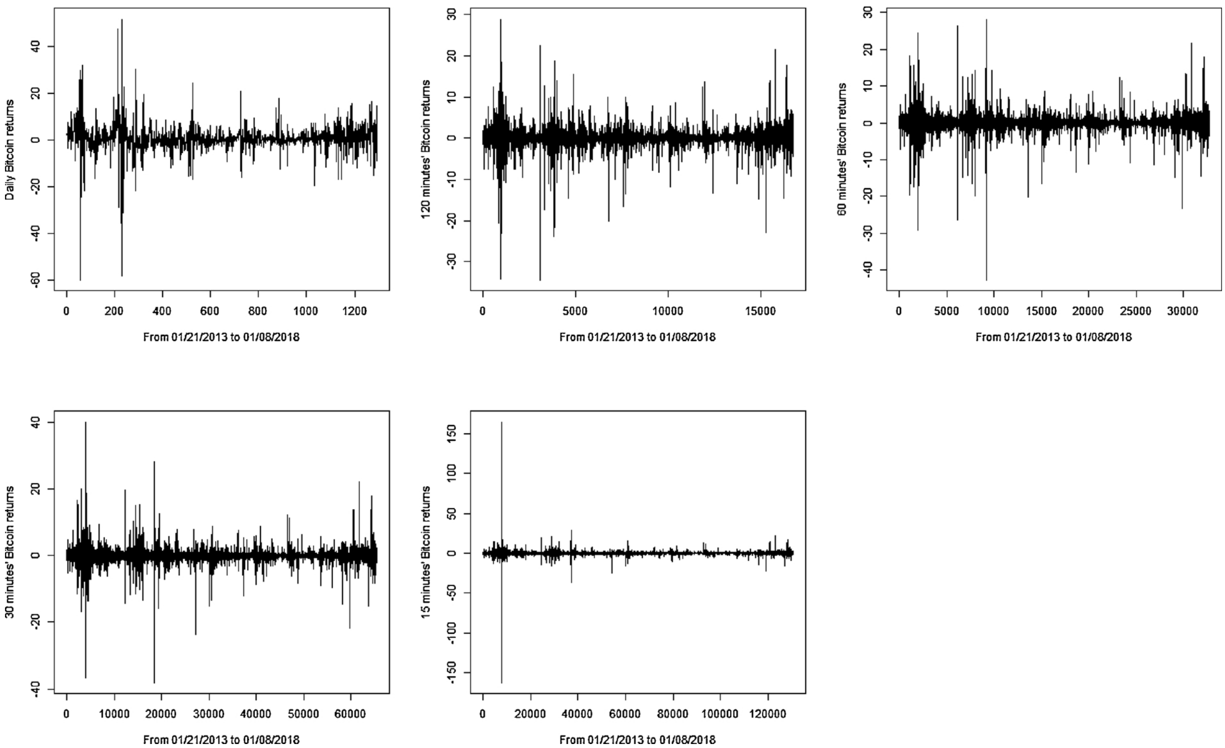

Fig. 2. Bitcoin returns.
The figure plots Bitcoin returns at different frequencies on Bitstamp exchange over the period 01/21/2013 to 01/08/2018. The frequencies considered are daily, 120 min, 60 min, 30 min and 15 min.

Table 1
Descriptive statistics.

	Daily	120 min	60 min	30 min	15 min
Mean	0.527	0.041	0.021	0.010	0.005
Standard deviation	6.151	1.753	1.361	1.062	1.046
Skewness	−0.595	−1.863	−1.575	−0.350	1.336
Kurtosis	22.768	59.611	96.360	193.350	9401.139
Jarque-Bera	28102#	2,490,474#	12,679,537#	101,867,451#	481,041,692,921#
ADF	−9.693#	−26.17#	−32.93#	−37.64#	−51.65#
KPSS	0.314	0.326	0.323	0.318	0.328

represent significance at 1% level. The table reports the summary statistics of Bitcoin returns at different frequencies on Bitstamp exchange over the period 01/21/2013 to 01/08/2018. The frequencies considered are daily, 120 min, 60 min, 30 min and 15 min.

4. Empirical results

First, we provide the full-sample estimates of MVR, AVR and JVR statistics of daily, 120 min, 60 min, 30 min and 15 min Bitcoin returns. In the same manner, we also provide the p-values of KL test statistic for the whole sample. It can help us to analyze the efficiency characteristics of the Bitcoin returns at different frequencies in the whole sample. Next, we undertake the non-overlapping and overlapping moving window analysis to analyze the evolution of the market efficiency in Bitcoin and analyze its implications concerning the adaptive market hypothesis (AMH).

4.1. Whole-sample results

Table 2 reports the test statistic of MVR, AVR and JVR tests across different frequencies of Bitcoin returns for the whole sample. All the three test statistics are not significant for daily and 120 min Bitcoin returns, indicating that the efficient market hypothesis cannot be rejected in these two cases. It is reasonable to say that the Bitcoin returns at lower frequencies follow a memoryless stochastic process. These findings for daily data are consistent with results of earlier studies of Bariviera (2017) and Urquhart (2016). However, when we turn to the results about higher frequency, the findings point to the fact that the daily data do not reflect all the available information about prices for Bitcoin, as there is some evidence of inefficiency as we move to higher frequencies. In

Table 2
Variance ratio test statistics (Whole sample).

	Daily	120 min	60 min	30 min	15 min
MVR	0.770	0.690	1.445	1.665	2.585*
AVR	−0.608	−4.336	−13.55*	−26.73*	−55.99*
JVR	2.785	3.285	4.650	5.378	9.582*

* represent significance at 5% level. The table reports MVR, AVR and JVR test statistics for Bitcoin returns at daily, 120 min, 60 min, 30 min and 15 min, frequencies. The significance of MVR and JVR test statistics is tested based on the Chi-square critical values at 10%, 5% and 1%. In the case of AVR, the weighted bootstrap procedure is used to check for the significance. The CD_2 statistic is reported for MVR test.

Table 3
Kuan and Lee test p-values (Whole sample).

	Daily	120 min	60 min	30 min	15 min
KL (p-value)	0.372	0.012	0.055	0.000	0.000

The table reports p-values of KL test for Bitcoin returns at daily, 120 min, 60 min, 30 min and 15 min, frequencies. Significant p-values imply inefficiency in the Bitcoin returns at a particular level of significance. The p-values related to KL test statistic are less than 0.05 (5% level of significance) for 120 min, 30 min and 15 min Bitcoin returns, and for 60 min Bitcoin returns the p-value is less than 0.1 (10% level of significance).

particular, we observe inefficiency in 15 min Bitcoin returns for all the three tests. Moreover, the AVR test exhibits some evidence of inefficiency in 60 min and 30 min Bitcoin returns. Furthermore, the KL test in Table 3 reports inefficiency in 120 min, 30 min and 15 min Bitcoin returns at 5% level of significance, and in 60 min Bitcoin returns at 10% level of significance. The results are interesting and intriguing as they tell us how the Bitcoin market, which is efficient at the daily level of data, shows some signs of inefficiency when we move to the higher frequencies. Hence, the Bitcoin returns do not always necessarily follow a memoryless stochastic process as it may be possible for a high-frequency trader to gain extra returns over time through speculation. The same fact is highlighted in Fig. 3 based on the plots of variance ratios against holding period at different frequencies. The plots exhibit that the variance ratios are outside the 95% confidence bands when we move from daily and 120 min to 60 min, 30 min and 15 min Bitcoin returns. The results for daily data are consistent with the findings of Nadarajah and Chu (2017) for the variance ratio plots. This observation further strengthens our argument that the Bitcoin market exhibits some inefficiency at the high-frequency level for the whole sample and therefore needs a more in-depth investigation.

4.2. Non-overlapping rolling window analysis

To investigate the issue further and to get more insights into the evolution of informational efficiency in the Bitcoin market, we undertake a non-overlapping rolling window analysis of the VR tests. Table 4 reports the year-wise (2013–2017) VR statistics for the data under study based on the non-overlapping rolling window analysis. The MVR test provides evidence of inefficiency in 15 min (for the years 2013– 2015) and 30 min (for the year 2013) data. Similarly, the JVR test provides evidence of inefficiency in 15 min' data for the years 2013 and 2014. These results are in synchronization with the prediction made by Urquhart (2016) that the Bitcoin market may become more efficient over time. However, the results of the AVR test indicate inefficiency in the high-frequency data for all the given periods.

Not only the AVR test, but the KL test in Table 5 also signals towards the same fact as it reports informational inefficiency in high frequency as well as daily Bitcoin returns even in the year 2017. Since AVR and KL tests have desirable size properties and higher power than the MVR and JVR test, we advocate for the presence of informational inefficiency in the Bitcoin market at higher frequencies of return data, even in the most recent years. The results of non-overlapping window analysis reveal some interesting insights. The Bitcoin market which looks informationally efficient when we consider the whole sample, has experienced many incidents of inefficiency in the past and continues to do so even in 2017 if we consider high-frequency Bitcoin returns.

4.3. Overlapping rolling-window analysis

Fig. 4 plots the rolling window estimates of the MVR and JVR tests statistics and their respective critical values at 5% level, and the p-values based on the rolling window (260 observations) estimates of the AVR test and the KL test[3]. The moving window estimates of the MVR, JVR and AVR tests vary substantially, however, are not significant for daily Bitcoin returns. Except for the AVR test, which exhibits some signs of inefficiency at the end of 2014, we mainly observe daily Bitcoin returns to be informationally efficient, which is consistent with the earlier studies. Moreover, the KL test is better able to explain the underlying dynamics which the other three tests are not able to highlight. The KL tests report incidents of market inefficiency in the Bitcoin market across years

[3] The plots are only provided for the daily Bitcoin returns; same plots can be provided for other frequencies upon request.

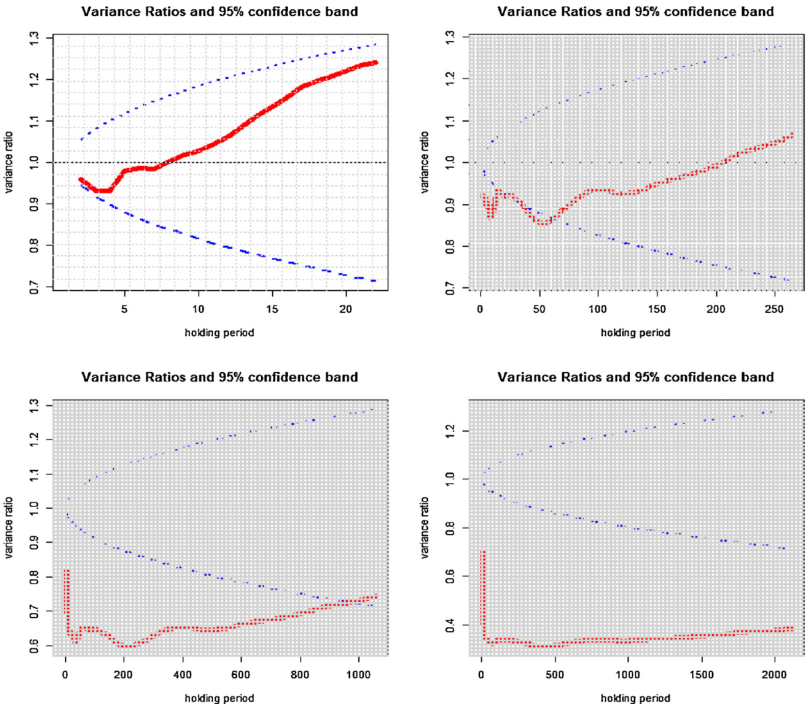

Fig. 3. VR statistic versus holding period. Top row: left (daily), middle (120 min) and right (60 min). Bottom row: left (30 min) and middle (15 min). The figure plots unstandardized variance ratios against holding periods with conventional 95 percent confidence band. It makes use of standard errors from iid returns.

Table 4
Variance ratio test (Non-overlapping window).

	Year	Daily	120 minutes	60 minutes	30 minutes	15 minutes
MVR	2013	0.244	0.615	1.017	2.393**	2.567*
	2014	1.206	1.551	1.599	1.874	2.991*
	2015	0.690	0.606	0.747	0.738	2.328**
	2016	0.835	0.274	0.345	0.340	0.392
	2017	0.396	0.795	0.857	0.948	1.014
AVR	2013	−0.008	−2.411	−8.514*	−17.987*	−36.353*
	2014	−1.353	−3.942*	−10.209*	−14.066*	−18.981*
	2015	−0.629	−0.998	−3.029	−1.933	−6.885*
	2016	−0.574	−0.961	−2.984	−2.232	−3.860*
	2017	0.606	−2.951*	−2.649*	−2.214**	−2.119**
JVR	2013	1.501	1.888	2.232	3.215	9.531*
	2014	2.251	3.049	4.250	6.138	9.877*
	2015	1.257	0.341	0.375	0.380	0.758
	2016	2.077	0.398	0.231	0.230	0.203
	2017	1.625	0.836	1.702	1.835	1.986

*, ** represent significance at 5% and 10% level respectively. The table reports MVR, AVR and JVR test statistics year wise for Bitcoin returns at daily, 120 min, 60 min, 30 min and 15 min, frequencies. The years considered are 2013, 2014, 2015, 2016 and 2017.

Table 5
Kuan and Lee test p-values (Non-overlapping window).

	Daily	120 min	60 min	30 min	15 min
2013	0.128	0.022	0.553	0.000	0.000
2014	0.470	0.326	0.147	0.291	0.000
2015	0.166	0.363	0.050	0.049	0.000
2016	0.072	0.093	0.088	0.029	0.229
2017	0.024	0.004	0.011	0.004	0.041

The table reports the p-values of KL test year wise for Bitcoin returns at daily, 120 min, 60 min, 30 min and 15 min, frequencies. The years considered are 2013, 2014, 2015, 2016 and 2017. The rolling window considered is 260 observations.

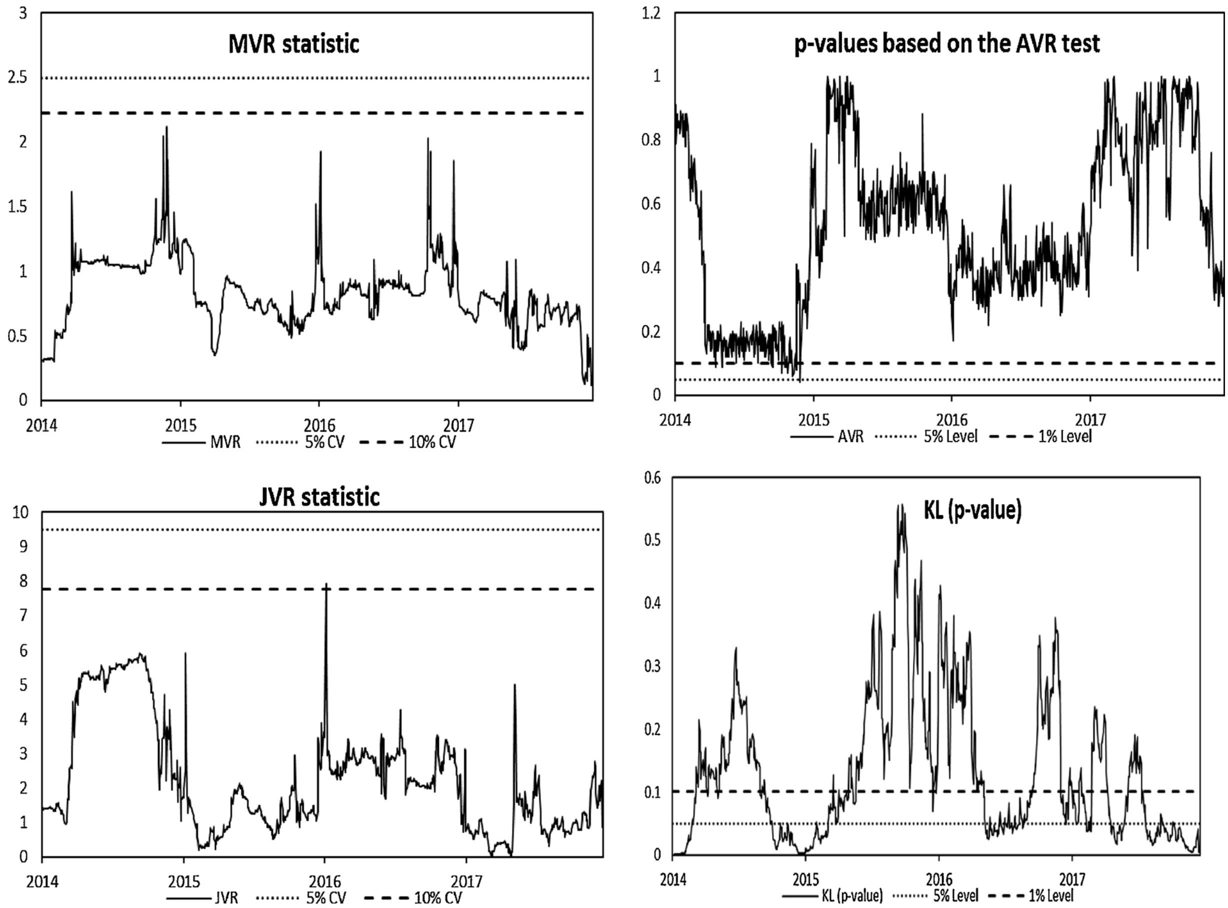

Fig. 4. Rolling window estimates of MVR, JVR, AVR and KL.
The figure plots the rolling window estimates of MVR test statistic, AVR test p-values, JVR test statistic and KL test p-values for Bitcoin returns at daily, 120 min, 60 min, 30 min and 15 min, frequencies.

even at the daily level of data.

5. Discussion

In this section, we discuss the possible reasons for our findings. The cryptocurrency markets, in general, are still emerging and there is still a scarcity of reliable information. Moreover, as a matter of course, the discussion will appear to be a bit speculative. The results presented in this study strongly advocate that the Bitcoin market is not informationally efficient, especially at the higher frequencies. The origin of such inefficiency can be attributed to the following two reasons:

i Endogenous factors of an emerging, non-mature market, and
ii The non-existence of fundamental traders.

Because of these reasons, the price is mainly decided by pure speculation. Cheah and Fry (2015) provide a discussion and empirical evidence that the fundamental value of Bitcoin is zero. Given its fundamental value to be zero, the fundamental traders have no basis to trade. The market participants invest without a clear trading strategy. The authors also suggest that the Bitcoin market may be subject to irrational bubbles because most of the investors resort to heuristics and are driven by psychological factors. Furthermore, an emerging non-mature Bitcoin market, intuitively promising higher returns, can attract many speculative investors with different appetites for trading. Such a combination can, in turn, drive a market to a situation far from efficiency. Our findings highlight the inefficient behavioral characteristics of the Bitcoin market at higher frequencies and the market participants can further exploit such inefficiencies. The Bitcoin market is yet to attract big institutional investors and there does not exist any alternative asset through which the arbitrageurs can keep the prices stable. However, some exchanges are beginning to come up with derivatives on Bitcoin and assets that replicate the future cash flows of Bitcoin. Such products related to Bitcoin can increase the depth of the Bitcoin market to a greater extent, thereby leading the market to price stability and increased efficiency.

6. Conclusion

Bitcoin market has recently shown an enormous growth. As Bitcoin is mainly used for transferring value globally, examining its informational efficiency is of high importance. Tiwari et al. (2018), Bariviera (2017) and Nadarajah and Chu (2017) advocate that the Bitcoin market is random at the daily level. The existing studies argue that the Bitcoin market is either efficient or moving towards efficiency. However, our findings from the full sample, non-overlapping window, and overlapping moving window analysis point to the randomness of the Bitcoin market at the daily level. The KL test advocates for the presence of inefficiency in the Bitcoin market even at the daily level. The results about the higher frequencies of Bitcoin prices indicate a consistent departure from the random behavior. Therefore, we cannot find a reason to ignore the presence of informational inefficiency in the Bitcoin market, especially at higher frequencies. Since, the Bitcoin market is yet to be regulated by the formal agencies, it is apparent that the market efficiency may not be a possibility under current operating conditions. The Bitcoin market participants being anonymous and pseudonymous pose a significant challenge to introduce regulations present in the traditional financial markets. The future evolution of Bitcoin remains an exciting topic and with our study using only up to 15 min of high-frequency Bitcoin returns, it would be interesting for future researchers to test the informational efficiency of the Bitcoin market by incorporating more information (i.e., using 10 min and 5 min Bitcoin prices).

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