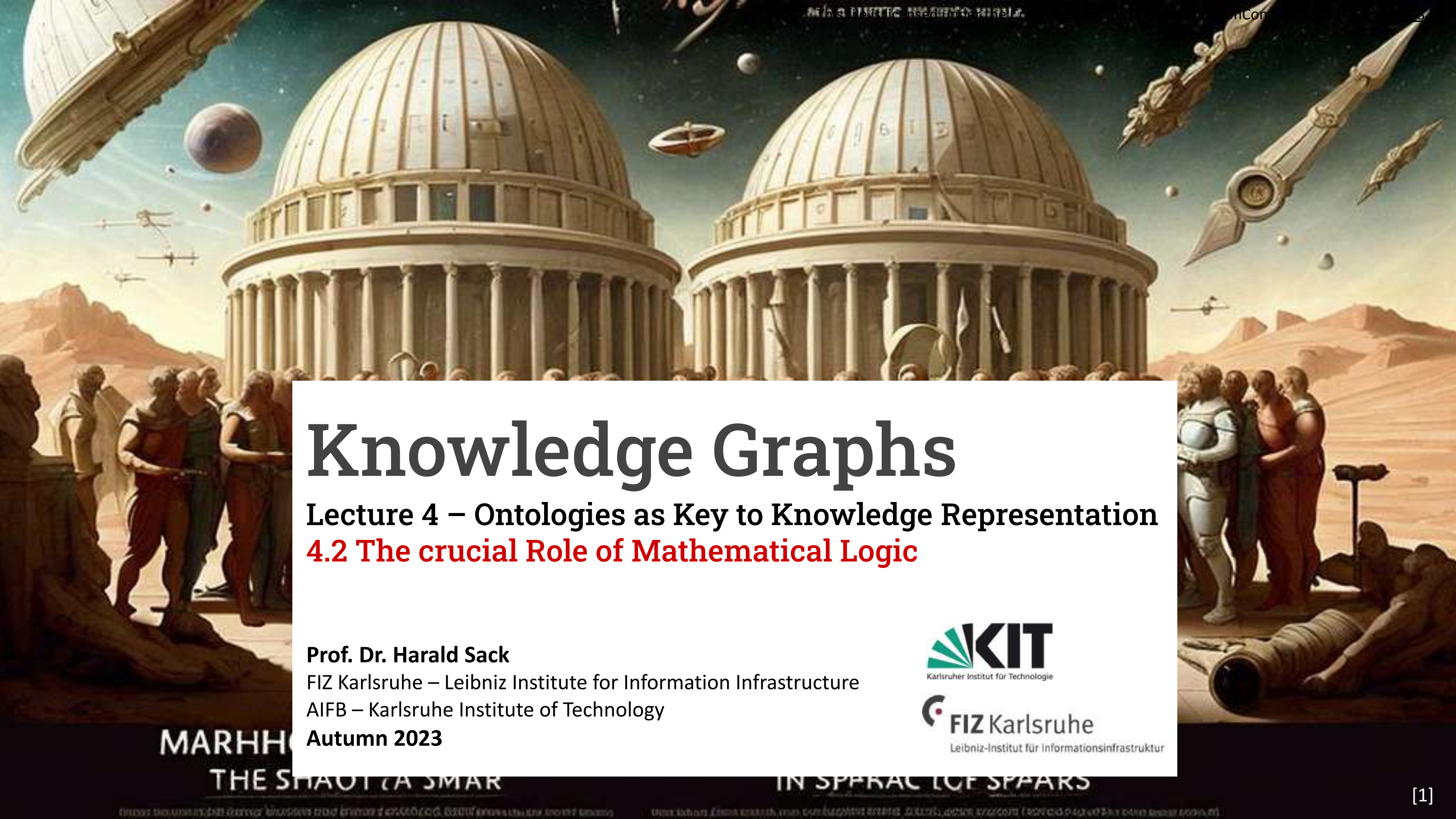

# Knowledge Graphs

## Lecture 4 – Ontologies as Key to Knowledge Representation

## 4.2 The crucial Role of Mathematical Logic


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**Autumn 2023**

[1]

# Knowledge Graphs

## Lecture 4: Ontologies as Key to Knowledge Representation





[1]

# Why RDF(S) is not sufficient

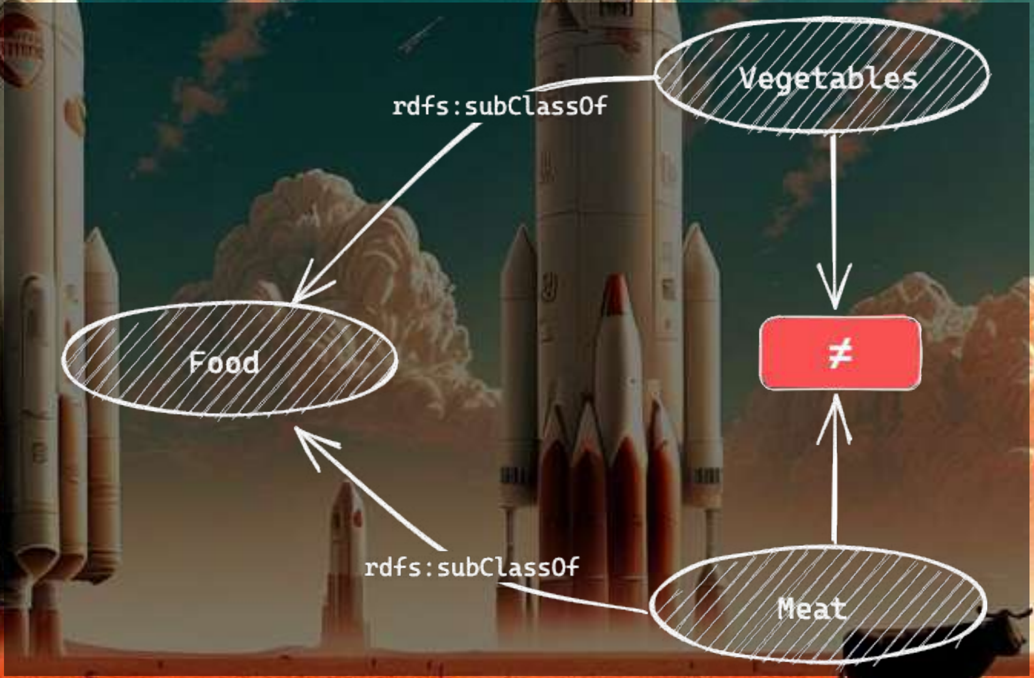
- Locality of Global Properties


- A Cow only eats vegetables
- Other animals only eat meat

[2]

# Why RDF(S) is not sufficient

- Disjunctive Classes

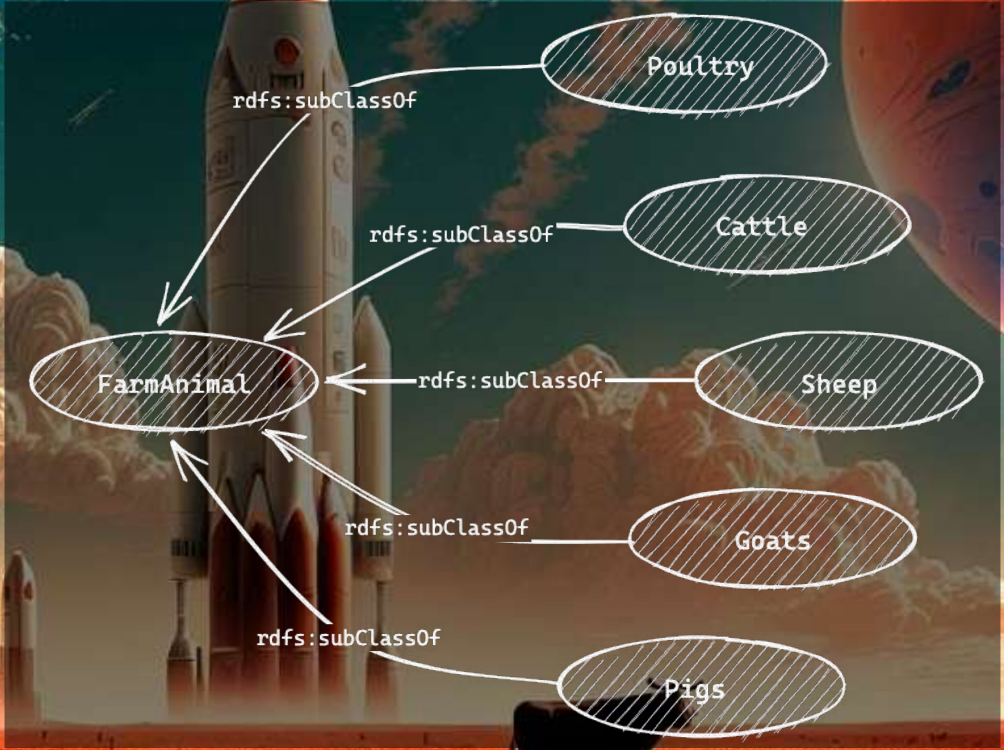



- RDFS Subclass relation cannot express disjunctive class (subclass) membership

[2]

# Why RDF(S) is not sufficient

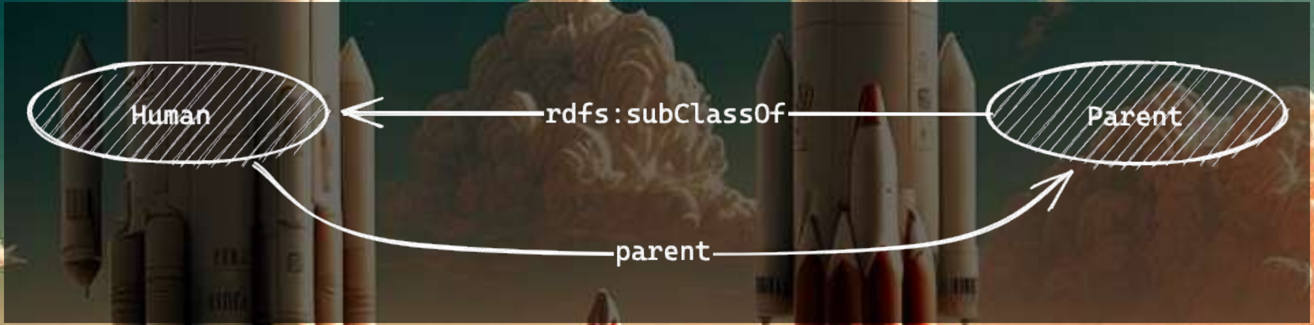
- **Class Combinations**
  - Combination of classes define a new class
  - A new class contains only members from given class combinations




[2]

# Why RDF(S) is not sufficient

- Cardinality Constraints




- Every human (usually) has two parents

# Why RDF(S) is not sufficient

## Special Property Constraints

- Transitivity (e.g. „is greater than“)
- Uniqueness (e.g. „is mother of“)
- Inversibility (e.g. „is parent of“ and „is child of“)

## General Problem of RDF(S)

RDF(S) does not have the possibility of **negation**

- `:harald rdf:type :Vegetarian .`
- `:harald rdf:type :NonVegetarian .`
- ...does not automatically generate a contradiction

# Why RDF(S) is not sufficient

An **ontology** is an **explicit**, **formal specification** **of a shared conceptualization**.

*acc. to Thomas R. Gruber: A Translation Approach to Portable Ontology Specifications.Knowledge Acquisition, 5(2):199–220, 1993.*

↓

formal (machine readable) semantics

↓

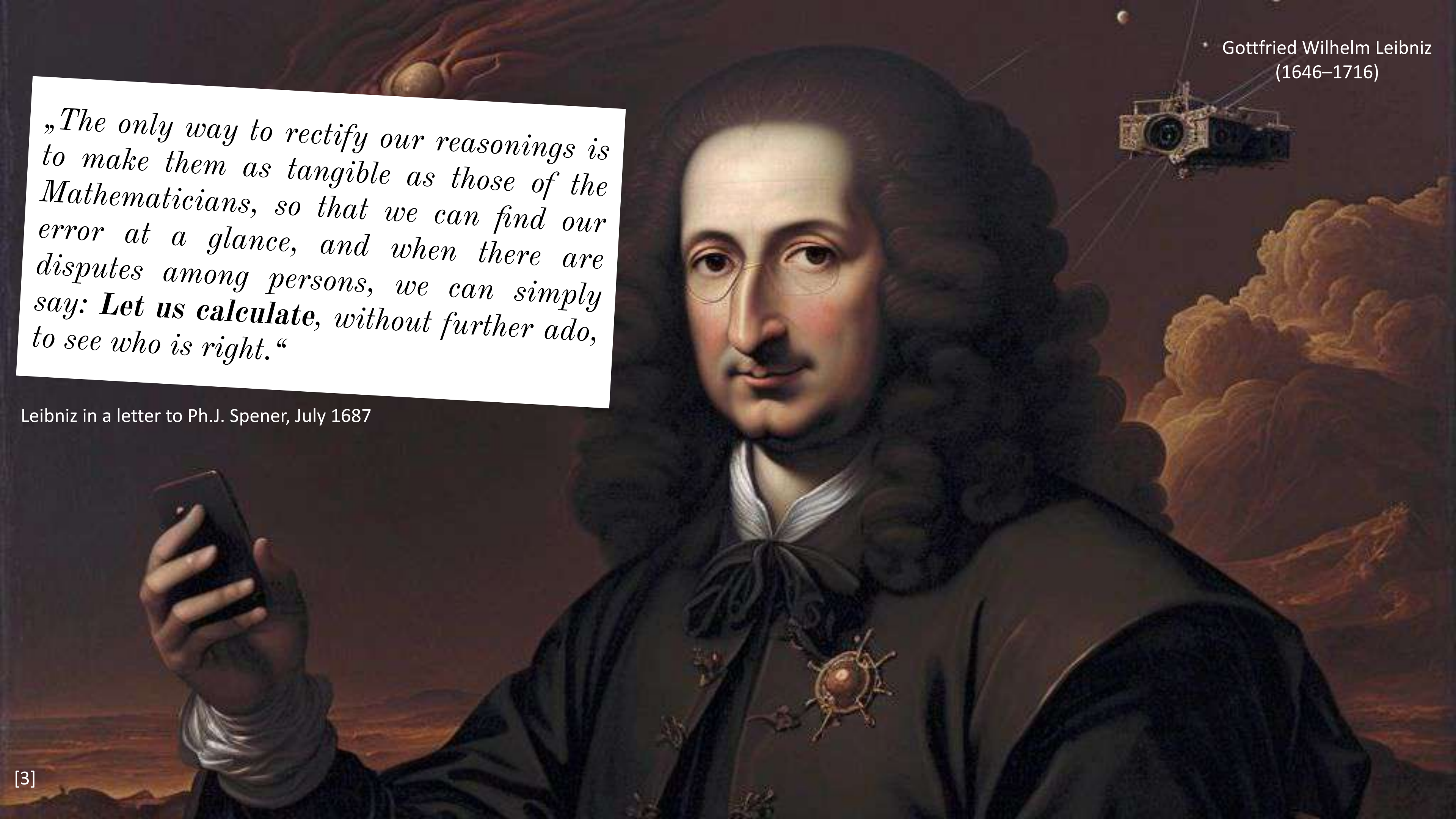
**mathematical logic**

# The Foundations of Logic

- Definition (for our lecture):
  ***Logic is the study of how to make correct formal deductions and inferences.***

- Why “formal logic”? ⇒ to enable automation

Gottfried Wilhelm Leibniz
(1646–1716)



*„The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say:* **Let us calculate**, *without further ado, to see who is right.“*

Leibniz in a letter to Ph.J. Spener, July 1687

[3]

# The Foundations of Logic

- **Syntax**: *symbols without meaning*
  defines rules, how to construct well-formed and valid sequences of symbols (strings)
- **Semantics**: *meaning of syntax*
  defines rules about how the meaning of complex sequences of symbols can be derived from atomic sequences of symbols

*Syntax*

```
If (i<0) then display ("negative account!")
```

*assignment of meaning*

*print the message "negative account!", if the account balance is negative*

# The Importance of Semantics

Why should I care about semantics?

*Well, from a philosophical POV, we need to specify the* ***relationship*** *between* ***statements in the logic*** *and the* ***existential phenomena*** *they describe.*


*Bertrand Russell (1872–1970)*

[4]

# The Importance of Semantics

- Why should I care about semantics?

*Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.*


*Bertrand Russell (1872–1970)*

- That's OK, but I don't get paid for philosophy.

*From a practical POV, in order to specify, build and test (ontology-based) tools/systems we need to precisely define **relationships** (like entailment) **between logical statements** – this defines the **intended behaviour** of tools/systems.*

[4]

*Cordial thanks to [Ian Horrocks' seminar slides]!!*

# Variants of Semantics

e.g. programming languages

*Syntax*

```
FUNCTION
f(n:natural):natural;
BEGIN
     IF n=0 THEN f:=1
     ELSE f:=n*f(n-1);
END;
```

computation of the factorial

**intentional semantics**

- *“the meaning intended by the user”*
- *restricts the set of all possible models (meanings) to the meaning intended by the (human) user*

# Variants of Semantics

e.g. programming languages

*Syntax*

```
FUNCTION
f(n:natural):natural;
BEGIN
     IF n=0 THEN f:=1
     ELSE f:=n*f(n-1);
END;
```

computation of the factorial

$f{:}n \rightarrow n!$

**formal semantics**

*aims to express the meaning of symbol sequences (programs) in a* ***formal language****, in a way that assertions over the symbol sequences (programs) can be proven by the application of deduction rules.*

# Variants of Semantics

e.g. programming languages

*Syntax*

```
FUNCTION
f(n:natural):natural;
BEGIN
     IF n=0 THEN f:=1
     ELSE f:=n*f(n-1);
END;
```

computation of the factorial

$f{:}n \to n!$

behaviour of the program at execution

**procedural semantics**

*the meaning of a language expression (program) is the procedure that takes place internally, whenever the expression does occur.*

# Semantics and Mathematical Logic

How do I define the semantics of a mathematical logic?

*In mathematical logic we define the semantics in terms of **models** (a model theory). A model is supposed to be an analogue of (part of) the world being modeled.*


*Bertrand Russell (1872–1970)*

[4]

# Model-theoretic Semantics

Alfred Tarski
(1901–1983)

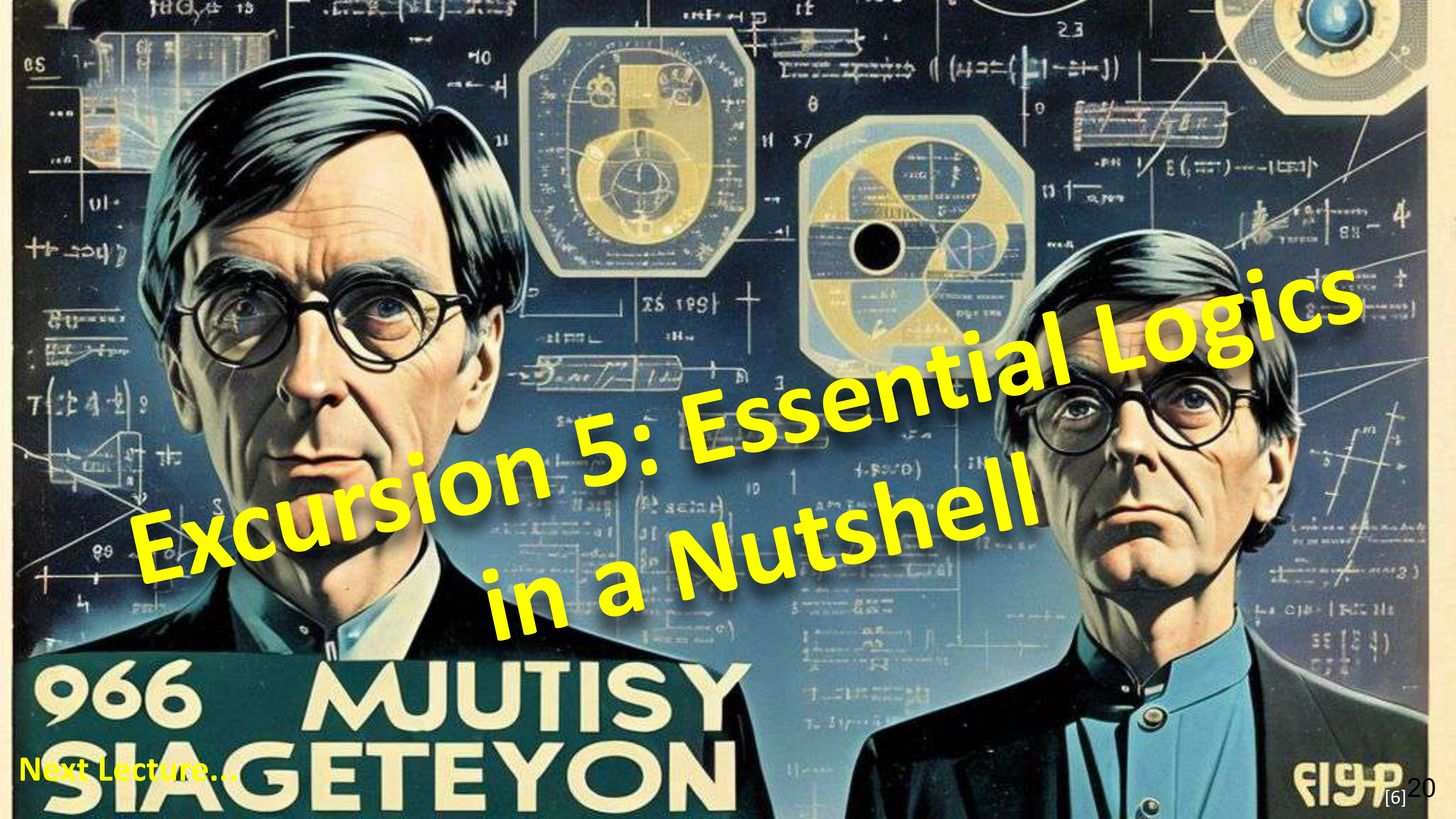
[5]

- **Model-theoretic semantics** performs the semantic interpretation of artificial and natural languages by "***identifying meaning with an exact and formally defined interpretation with a model***"
- = **formal interpretation with a model**
- e.g. model-theoretic semantics of **propositional logic**
  - assignment of truth values ***"true"*** and ***"false"*** to atomic assertions and
  - description of logical connectives with ***truth tables***

# Model-theoretic Semantics

- Any logic **L := ( S , ⊨ )** consists of
  (1) a **set of statements S** and
  (2) an **entailment relation ⊨**
- Let $\Phi \subseteq S$ and $\varphi \in S$ : $\Phi \vDash \varphi$

  „*φ is a logical consequence of Φ*" or
  „*from the assertions of Φ follows the assertion φ*"
- If for 2 assertions $\varphi,\psi \in S$
  both $\{\varphi\} \vDash \psi$ and $\{\psi\} \vDash \varphi$,
  then both assertions φ and ψ are logically equivalent: $\varphi \equiv \psi$




# Excursion 5: Essential Logics in a Nutshell

Next Lecture...

[6]

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- Ludlow, Peter, "*Descriptions*", The Stanford Encyclopedia of Philosophy (Winter 2022 Edition), Edward N. Zalta & Uri Nodelman (eds.).
- Hodges, Wilfrid, "*Model Theory*", The Stanford Encyclopedia of Philosophy (Spring 2022 Edition), Edward N. Zalta (ed.).
- Aidan Hogan (2020), *The Web of Data*, Springer.
  Chap. 7.1 Shape Constraint Language - SHACL, pp. 453–500.


## Picture References:


[1] "A Scifi movie poster depicting Raphael's "School of Athens" with all the important classical Philosophers including their significant tools set into a retro futuristic urban environment of planet Mars with spaceships in the sky.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot

[2] "A Scifi movie poster depicting a cow grazing on the vaste red prairies of Mars in a retro futuristic rural environment of planet Mars. A rocket ship is starting in the background far away leaving contrails behind.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot

[3] "A baroque painting with a closeup of Gottfried Wilhelm Leibniz texting with his smartphone. Leibniz is wearing a huge dark baroque allonge wig. We see a mechanical computer on Leibniz's writing desk.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot

[4] James Francis Horrabin, "Bertrand Russell". The Masses: 37, 1917, [Public Domain], https://commons.wikimedia.org/wiki/File:Bertrand_Russell,_by_J._F._Horrabin.jpg

[5] George Bergman, Alfred Tarski, The Oberwolfach photo collection, 1968, [GFDL], https://commons.wikimedia.org/wiki/File:AlfredTarski1968.jpeg

[6] "In this 1960s pulp cover picture, in the waning days of a future Galactic Empire, the mathematician Hari Seldon spends his life developing a theory of psychohistory, a new and effective mathematics of sociology. Using statistical laws of mass action, it can predict the future of large populations.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot


[6]