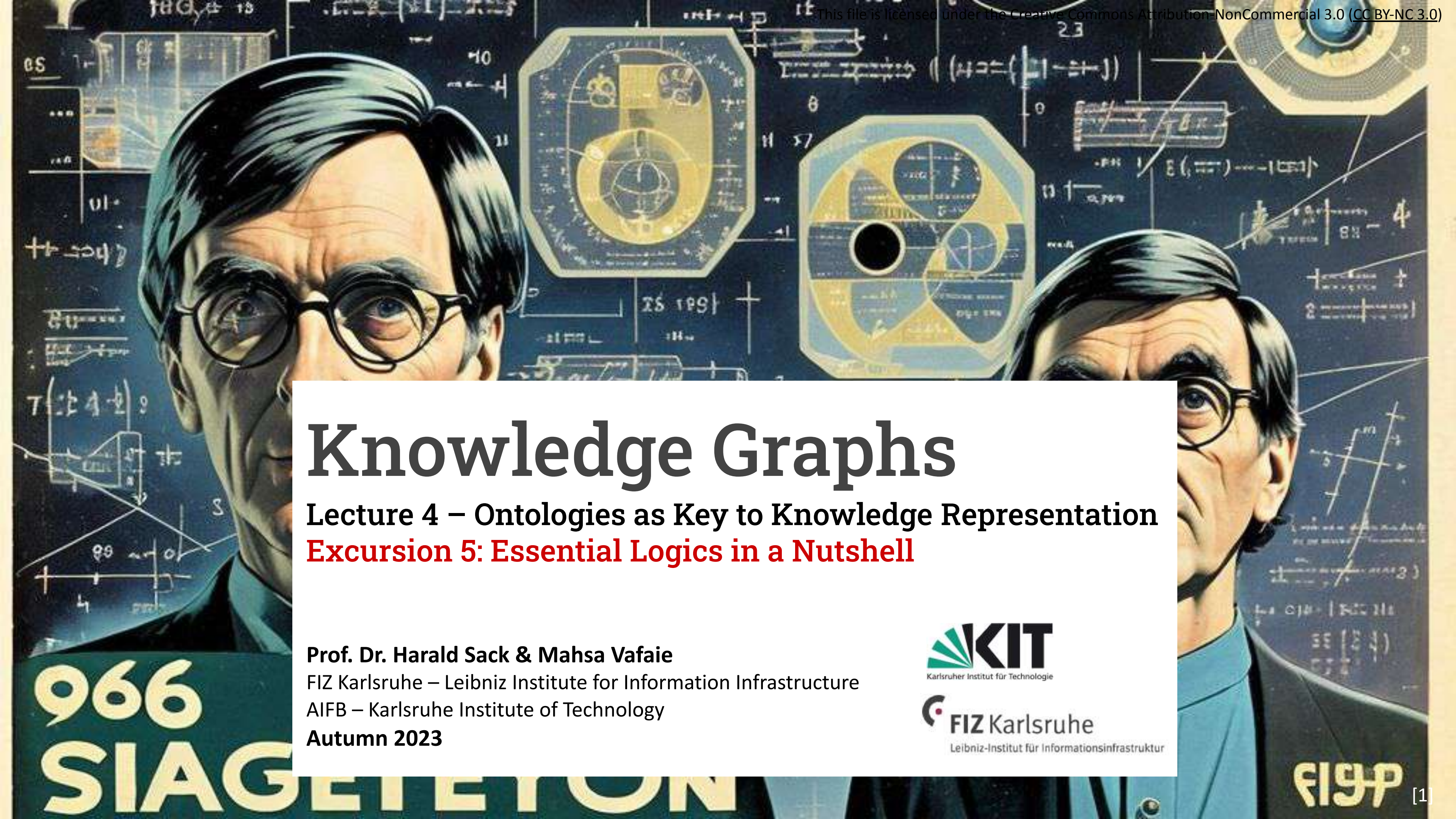

Knowledge Graphs

Lecture 4 – Ontologies as Key to Knowledge Representation

Excursion 5: Essential Logics in a Nutshell

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Knowledge Graphs

Lecture 4: Ontologies as Key to Knowledge Representation

[1]

Propositional Logic – PL

- In propositional logic the world consists simply of **facts** and nothing else (**statements** of **assertions**).
- Example for propositional logic assertions and deductions:
    - `If it rains, the road will get wet.`
    - `If the moon is made out of green cheese, then cows can fly.`
    - `If Oliver is in love, then he will be happy.`
- The world consists of objects and properties that distinguish one object from another.
- Between objects are relations. Some relations are unique, i.e. functions.

Propositional Logic – PL

Syntax:

- **Logical connectives**: Op={ ¬, ∧, ∨, →, ↔, (,) }
- a set of symbols Σ
- with Σ∩Op=∅ and {`true, false`}
- **Production rules** for propositional formulas (propositions):
  - all atomic formulas are propositions (*all elements of Σ*)
  - if φ is a proposition, then also ¬φ
  - if φ and ψ are propositions, then also φ∧ψ, φ∨ψ, φ→ψ, φ↔ψ
- **Priority**: ¬ prior to ∧, ∨ prior to →, ↔

connective	name	intentional meaning
¬	negation	“not”
∧	conjunction	“and”
∨	disjunction	“or”
→	implication	“if - then”
↔	equivalence	“if, and only if, then”

Propositional Logic – PL

How to model facts

Simple Assertion	Modeling
The Moon is made of green cheese	g
It rains	r
The street is getting wet	n

Composed Assertion	Modeling
If it rains, then the street will get wet.	$r \rightarrow n$
If it rains and the street does not get wet, then the moon is made of green cheese.	$(r \land \neg n) \rightarrow g$

Propositional Logic – PL

Model-theoretic semantics

- **Interpretation `I`**:
  Mapping of all atomic propositions to `{t,f}`.

- If F is a formula and `I` an interpretation, then `I(F)` is a truth value computed from F and `I` via **truth tables**.

I(p)	I(q)	I(¬p)	I(p∨q)	I(p∧q)	I(p→q)	I(p↔q)
f	f	t	f	f	t	t
f	t	t	t	f	t	f
t	f	f	t	f	f	f
t	t	f	t	t	t	t

Propositional Logic – PL

Model-theoretic semantics

- We write $I \models F$, if $I(F)=t$,
  and call interpretation I a **Model** of formula F.

- Rules of Semantics:
  - I is a model of ¬φ, iff I is not a model of φ
  - I is a model of (φ∧ψ), iff I is a model of φ AND of ψ
  - ...

- Basic concepts:
  - tautology
  - satisfiable
  - refutable
  - unsatisfiable (contradiction)

Propositional Logic – PL

Model-theoretic semantics

- **Decidability**

  All **true entailments** can be found, and all **false entailments** can be refuted, as long as you spend enough time.

  ⇒ there always exist terminating automatic theorem provers.

- Another useful property:

  semantic entailment/inference

  - $\{\varphi_1, \ldots, \varphi_n\} \vDash \varphi$ holds, iff
    $(\varphi_1 \land \ldots \land \varphi_n) \rightarrow \varphi$ is a tautology

    syntactic entailment/inference

  - The decision, if an assertion is a tautology, can be made via truth tables.
  - In principle this equals the evaluation of all possible interpretations.

First Order Logic – FOL

quantifier	name	intentional meaning
∃	existential quantifier	“it exists”
∀	universal quantifier	“for all”

- **Operators (logical connectives)** as in propositional logic
- **Variables**, e.g., X, Y, Z, …
- **Constants**, e.g., a, b, c, … (i.e. a named object from the domain of discourse)
- **Functions**, e.g., f, g, h, … (incl. arity)
- **Relations / Predicates**, e.g., p, q, r, … (incl. arity)

Example of a FOL formula: $(\forall X)(\exists Y)\ ((p(X) \lor \neg q(f(X),Y)) \rightarrow r(X))$

First Order Logic – FOL

FOL Syntax

- "Correct" formulation of **Terms** from Variables, Constants and Functions:

  `f(X), g(a,f(Y)), s(a), .(H,T), x_location(Pixel)`

- "Correct" formulation of **Atoms** (or **Atomic Formulas**) from Predicates with Terms as arguments:

  `p(f(X)),q(s(a),g(a,f(Y))),add(a,s(a),s(a)),`
  `greater_than(x_location(Pixel),128)`

- "Correct" formulation of (composed) **Formulas** from Atomic Formulas, Operators and Quantifiers:

  `(∀Pixel)(greater_than(x_location(Pixel),128) → red(Pixel))`

- If in doubt, use brackets
- All variables should be quantified

First Order Logic – FOL

How to model Facts

- “All kids love ice cream.”
  $\forall X\colon \mathtt{Child}(X) \rightarrow \mathtt{lovesIceCream}(X)$

- “The father of a person is its male parent.”
  $\forall X\ \forall Y\colon \mathtt{isFather}(X,Y) \leftrightarrow (\mathtt{Male}(X) \land \mathtt{isParent}(X,Y))$

- “There are (one or more) interesting lectures.”
  $\exists X\colon \mathtt{Lecture}(X) \land \mathtt{Interesting}(X)$

- “The relation ‘isNeighbor’ is symmetric.”
  $\forall X\ \forall Y\colon \mathtt{isNeighbor}(X,Y) \rightarrow \mathtt{isNeighbor}(Y,X)$

First Order Logic – FOL

How to model Facts

- **"All kids love ice cream."**
  $\forall X: \mathrm{Child}(X) \rightarrow \mathrm{lovesIceCream}(X)$

- "The father of a person is its male parent."
  $\forall X\ \forall Y: \mathrm{isFather}(X,Y) \leftrightarrow (\mathrm{Male}(X) \land \mathrm{isParent}(X,Y))$

- **"There are (one or more) interesting lectures."**
  $\exists X: \mathrm{Lecture}(X) \land \mathrm{Interesting}(X)$

- "The relation 'isNeighbor' is symmetric."
  $\forall X\ \forall Y: \mathrm{isNeighbor}(X,Y) \rightarrow \mathrm{isNeighbor}(Y,X)$

First Order Logic – FOL

How to model Facts

There is a significant difference:

"All kids love ice cream."

∀X: Child(X) → lovesIceCream(X)

It is possible that X is not a kid but (nevertheless) loves ice cream.

∀X: Child(X) ∧ lovesIceCream(X)

There are only kids in the universe of discourse and they love ice cream.

∀X: Child(X)	∀X: lovesIcecream(x)	∀X: Child(X) → lovesIceCream(X)	∀X: Child(X) ∧ lovesIceCream(X)
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic – FOL

How to model Facts

There is a significant difference:

"There are (one or more) interesting lectures."

$\exists X\text{: Lecture}(X) \land \text{Interesting}(X)$

$\exists X\text{: Lecture}(X) \rightarrow \text{Interesting}(X)$

∃X: Lecture(X)	∃X: Interesting(X)	∃X: Lecture(X) → Interesting(X)	∃X: Lecture(X) ∧ Interesting(X)
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic – FOL

Model-theoretic Semantics

- **Structure**:
  - Definition of a **domain** D.
  - **Constant symbols** are mapped to elements of D.
  - **Function symbols** are mapped to functions in D.
  - **Relation symbols** are mapped to relations over D.

- **Then**:
  - **Terms** will become elements of D.
  - **Relation symbols** with arguments will become `true` or `false`.
  - **Logical connectives** and **quantifiers** are treated likewise.

First Order Logic – FOL

The Penguin Example

```
Penguins are Black and White.
Some old TV shows are Black and White.
Therefore, some Penguins are Old TV Shows.
```

Logic: another thing that Penguins aren't very good at...

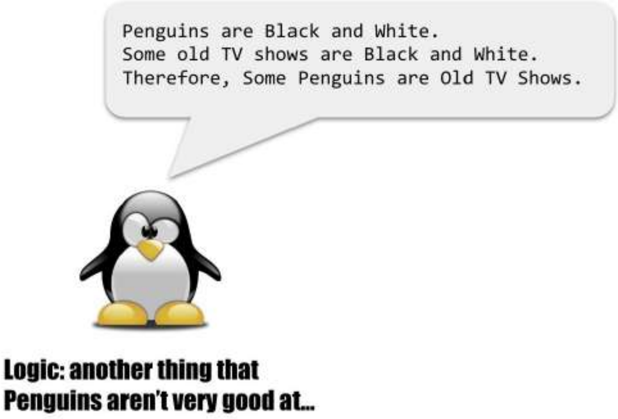
[2]

First Order Logic – FOL

The Penguin Example

$$F = (((\forall X)(penguin(X) \rightarrow blackandwhite(X)) \land (\exists X)(oldTVshow(X) \land blackandwhite(X))) \rightarrow (\exists X)(penguin(X) \land oldTVshow(X))$$

What is the intentional semantics?



[2]

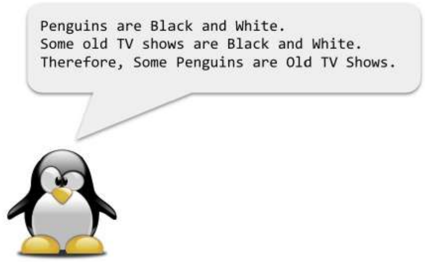
First Order Logic – FOL

The Penguin Example

$$
\begin{aligned}
\mathtt{F} = (\ & ((\forall \mathtt{X})(\mathtt{penguin(X)} \rightarrow \mathtt{blackandwhite(X)}) \wedge \\
& (\exists \mathtt{X})(\mathtt{oldTVshow(X)} \wedge \mathtt{blackandwhite(X)})\ ) \\
& \rightarrow (\exists \mathtt{X})(\mathtt{penguin(X)} \wedge \mathtt{oldTVshow(X)}\ )
\end{aligned}
$$

Interpretation $\mathtt{I}$:

- **Domain**: a set M, containing elements $\mathtt{a}, \mathtt{b}$.
- … no constants or function symbols …
- We show: the formula is *refutable* (i.e. it is not a tautology)
- If $\mathtt{I(penguin)(a)}$, $\mathtt{I(blackandwhite)(a)}$, $\mathtt{I(oldTVshow)(b)}$, $\mathtt{I(blackandwhite)(b)}$ is true , $\mathtt{I(oldTVshow)(a)}$ is false,
- then the formula $\mathtt{F}$ with Interpretation $\mathtt{I}$ is false, i.e. $\mathtt{I} \nvDash \mathtt{F}$



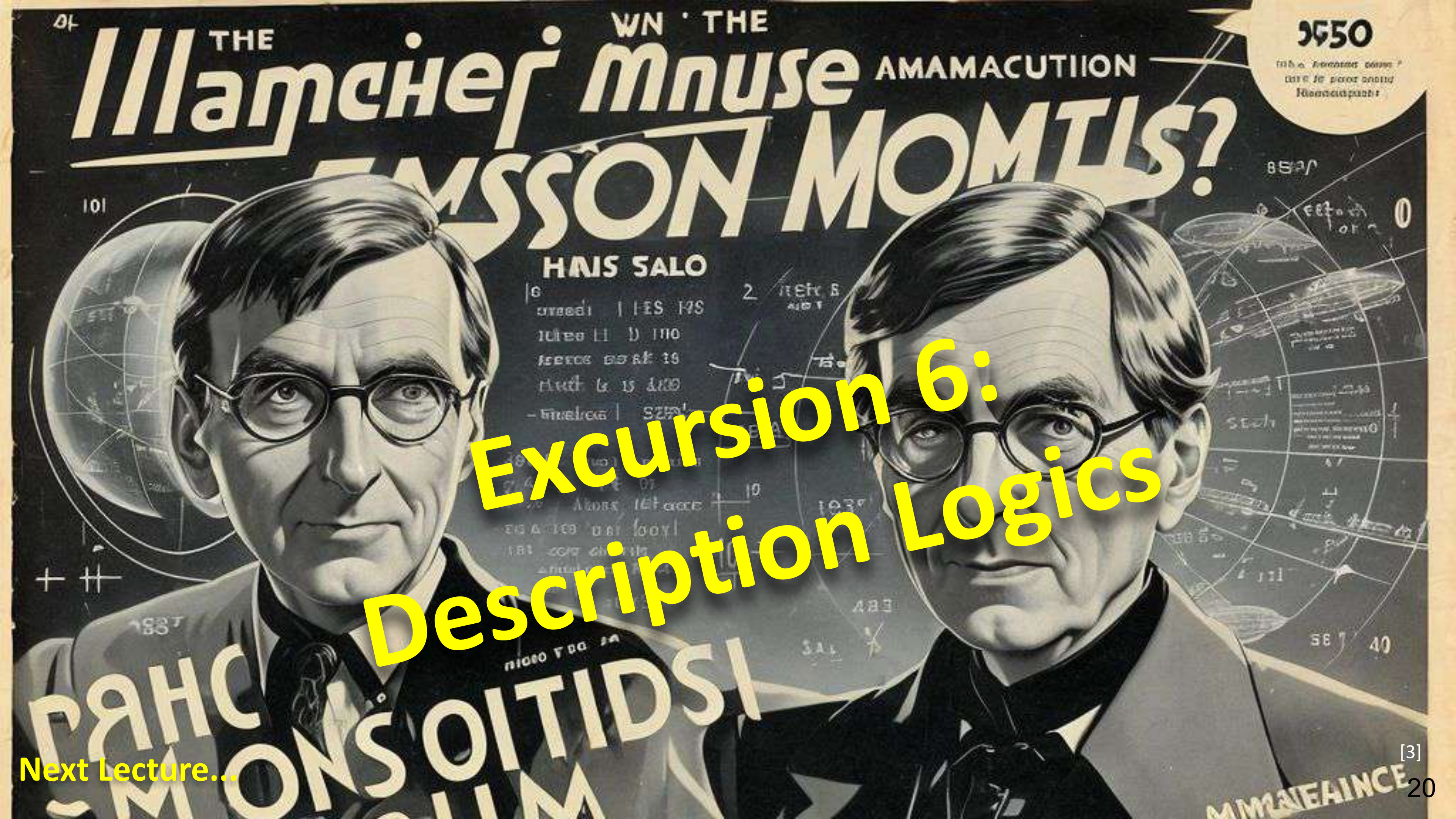
Logic: another thing that Penguins aren't very good at...

[2]

First Order Logic – FOL

Logical Entailment

- A **theory** T is a set of formulas.
- An interpretation $\mathcal{I}$ is a **model** of T, iff $\mathcal{I} \vDash G$ for all formulas G in T.
- A formula F is a **logical consequence** of T, iff all models of T are also models of F.
- Then we write $T \vDash F$.
- Two formulas F, G are called **logically equivalent**, iff $\{F\} \vDash G$ and $\{G\} \vDash F$.
- Then we write $F \equiv G$



Excursion 6: Description Logics

Next Lecture...

[3]

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- *Logic for Computer Science*. (2022, April 4). Wikibooks, The Free Textbook Project. Retrieved 17:01, February 17, 2023 from https://en.wikibooks.org/w/index.php?title=Logic_for_Computer_Science&oldid=4045328 .

Picture References:

[1] "In this 1960s pulp cover picture, in the waning days of a future Galactic Empire, the mathematician Hari Seldon spends his life developing a theory of psychohistory, a new and effective mathematics of sociology. Using statistical laws of mass action, it can predict the future of large populations.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot

[2] Penguin, pixabay [Public Domain], https://pixabay.com/vectors/penguin-tux-animal-bird-cute-158551/

[3] "In this 1950s pulp cover picture, in the waning days of a future Galactic Empire, the mathematician Hari Seldon spends his life developing a theory of psychohistory, a new and effective mathematics of sociology. Using statistical laws of mass action, it can predict the future of large populations.", created via ArtBot, Deliberate, 2023, [CC-BY-4.0], https://tinybots.net/artbot