

2006

A

$$\begin{array}{cccccccc} & & 20 & & 2 & & & \\ 1 & \frac{1}{3} & 2 & b \ 5, 0.5 & 3 & \frac{5}{9} & \frac{8}{9} & 4 \ 1 \ 3/2 \ 5 \quad ^2 \\ 6 & \frac{1}{2} & 7 & & 8 & & 0.98, 0.98 \ . \end{array}$$

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$$\begin{array}{ccccccc} A & " & & " & B_1, B_2, B_3 & & " \\ C_i & " & i & & " & i=1 & 2 \ 3 \end{array}$$

$$P(B_1) \ 0.4, \ P(B_2) \ 0.5, \ P(B_3) \ 0.7$$

$$C_1 \ B_1\overline{B_2}\overline{B_3} \ \overline{B_1}B_2\overline{B_3} \ \overline{B_1}\overline{B_2}B_3$$

$$P(C_1) \ P(B_1\overline{B_2}\overline{B_3}) \ P(\overline{B_1}B_2\overline{B_3}) \ P(\overline{B_1}\overline{B_2}B_3)$$

$$P(B_1)P(\overline{B_2})P(\overline{B_3}) \ P(\overline{B_1})P(B_2)P(\overline{B_3}) \ P(\overline{B_1})P(\overline{B_2})P(B_3)$$

$$0.4 \ 0.5 \ 0.3 \ 0.6 \ 0.5 \ 0.3 \ 0.6 \ 0.5 \ 0.7 \ 0.36$$

$$C_2 \ B_1B_2\overline{B_3} \ B_1\overline{B_2}B_3 \ \overline{B_1}B_2B_3 \quad P(C_2) \ 0.41$$

$$C_3 \ B_1B_2B_3 \quad P(C_3) \ 0.14$$

$$P(A) = \sum_{i=0}^3 P(C_i)P(A|C_i) \quad \text{12}$$

$$0.36 \ 0.2 \ 0.41 \ 0.6 \ 0.14 \ 1 \ 0.458 \quad \text{10}$$

14

$$(1) \ f_X(x) = \frac{1}{\sqrt{2}} e^{-\frac{x^2}{2}}$$

$$(2) \ X \ Y \quad F_X(x), F_Y(y).$$

$$F_Y(y) = P\{Y \leq y\} = P\{X^2 \leq y\}$$

$$y \leq 0 \quad F_Y(y) = 0.$$

$$y > 0 \quad F_Y(y) = P\{-\sqrt{y} \leq X \leq \sqrt{y}\} = F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

$$f_Y(y) = F_Y(y)$$

$$f_Y(y) = \frac{1}{2\sqrt{y}}[f_X(\sqrt{y}) - f_X(-\sqrt{y})], y > 0 \quad \frac{1}{\sqrt{2}\sqrt{y}}e^{-\frac{y}{2}}, y > 0$$

$$0, \quad 0,$$

$$(3) \quad (1) = 0.8413, \quad (2) = 0.9972$$

$$P\{Z \leq 1\} = P\{1 \leq X \leq 1\} = (1) - (-1)$$

$$2 - (1) = 1 - 2 = 0.8413 - 1 = 0.6826$$

$$P\{Z \leq 2\} = P\{2 \leq X \leq 1\} = P\{1 \leq X \leq 2\} = 2[(2) - (1)]$$

$$2 - (0.9972 - 0.8413) = 0.3118$$

$$P\{Z \leq 3\} = 1 - P\{Z \leq 1\} = P\{Z \leq 2\} = 1 - 0.6826 = 0.3118 = 0.0056$$

$Z \mid$	1	2	3
$P \mid$	0.6826	0.3118	0.0056

$$(1) \quad D \quad m(D) = 2, \quad , \quad (X, Y)$$

$$f(x, y) = \frac{1}{2} \quad (x, y) \in D,$$

$$0 \quad .$$

$$(2) \quad X \leq Y \quad f_X(x) = f_Y(y),$$

$$f_X(x) = \int_1^x f(x, y) dy = \frac{1}{2} dy = \frac{1}{2} (x - 1), \quad (1 \leq x \leq 3).$$

$$f_Y(y) = \int_y^3 f(x, y) dx = \frac{1}{2} dx = \frac{1}{2} (3 - y), \quad (1 \leq y \leq 3).$$

$$f(x, y) = f_X(x) f_Y(y), \quad X \leq Y$$

$$3 \quad f_Z(z) = \int_{-\infty}^{\infty} f_X(x, z) dx$$

$$\begin{aligned} &= \int_0^1 (1-x)^3 \cdot 1 \cdot x^3 dx \\ &= \int_0^1 (1-x)^3 x^3 dx \end{aligned}$$

$$2 \quad z=4 \quad f_Z(z) = \int_{-\infty}^{\infty} \frac{z-1}{2} dx = \frac{z}{4} - \frac{1}{2}$$

$$4 \quad z=6 \quad f_Z(z) = \int_{-\infty}^{\infty} \frac{3}{2} dx = \frac{z}{4} - \frac{3}{2}$$

$$f_Z(z) = 0$$

$$\frac{z}{4} - \frac{1}{2}, \quad 2 \leq z \leq 4$$

$$f_Z(z) = \frac{z}{4} - \frac{3}{2}, \quad 4 \leq z \leq 6$$

10

$$(1) \quad X_1 \sim B(1, 0.8) \quad X_2 \sim B(1, 0.1)$$

$$EX_1 = 0.8, \quad DX_1 = 0.8 \cdot 0.2 = 0.16$$

$$EX_2 = 0.1, \quad DX_2 = 0.1 \cdot 0.9 = 0.09$$

$$X_1, X_2$$

		$X_2$	
		0	1
$X_1$	0	0.1	0.1
	1	0.8	0

$$\frac{1}{4}$$

$$P(X_1=1, X_2=0)=\frac{80}{100}=0.8$$

$$2-X_1X_2$$

$X_1X_2$	0	1
$P$	1	0

$$E(X_1X_2)=0\qquad\qquad\qquad\frac{1}{2}$$

$$\text{cov}(X_1,X_2)=EX_1X_2-EX_1EX_2\\=0-0.08\cdot 0.1=-0.08\qquad\qquad\qquad\frac{1}{2}$$

$$r_{X_1X_2}=\frac{\text{cov}(X_1,X_2)}{\sqrt{DX_1}\sqrt{DX_2}}=\frac{0.08}{\sqrt{0.16}\sqrt{0.09}}=\frac{2}{3}\frac{1}{2}$$

$$8$$

$$X_{n-1}\sim N(\mu,\sigma^2)\quad \bar{X}_n\sim N(\mu,\frac{\sigma^2}{n})\quad X_{n-1}-\bar{X}_n\sim N(0,\frac{n-1}{n}\sigma^2)$$

$$U=\frac{X_{n-1}-\bar{X}_n}{\sqrt{\frac{n-1}{n}}}-\frac{X_{n-1}-\bar{X}_n}{\sqrt{\frac{n}{n-1}}}\sim N(0,1)\qquad\frac{1}{3}$$

$$W=\frac{(n-1)S^2}{2}\sim \chi^2(n-1)\qquad\qquad\qquad\frac{1}{2}$$

$$X_{n-1}-\bar{X}_n\sim S^2\qquad\qquad\qquad U\sim W\qquad\qquad\qquad\frac{1}{4}$$

$$\frac{U}{\sqrt{W/(n-1)}}=\frac{X_{n-1}-\bar{X}_n}{\sqrt{\frac{n}{n-1}}}\bigg/\sqrt{\frac{(n-1)S^2/n}{n-1}}=\frac{X_{n-1}-\bar{X}_n}{S}\sqrt{\frac{n}{n-1}}\sim t(n-1)\frac{1}{2}$$

$$12$$

$$(1) \, E(X) = \int_0^{\infty} x f(x; \lambda) dx = \int_0^{\infty} x \frac{\lambda^2}{\sqrt{2}} e^{-\frac{x^2}{2}} dx = \frac{2}{\sqrt{2}} \left( -e^{-\frac{x^2}{2}} \right) \bigg|_0^{\infty} = \frac{\sqrt{2}}{\sqrt{1}}\qquad\frac{1}{3}$$

$$\frac{\sqrt{2}}{\sqrt{\phantom{x}}}\bar{X} \tag{12}$$

$$\ddot{\varrho}^{1/2}\frac{\bar{X}^2}{2} \tag{14}$$

$$2-L(\boldsymbol{x})=\prod_{i=1}^nf(x_i;\boldsymbol{\theta})=\prod_{i=1}^n\frac{2}{\sqrt{2}}e^{-\frac{x_i^2}{2}}=(0.5)^{\frac{n}{2}}e^{-\frac{\sum_{i=1}^nx_i^2}{2}} \tag{12}$$

$$\ln L(\boldsymbol{\theta})=-\frac{n}{2}\ln(0.5)-\frac{\sum_{i=1}^nx_i^2}{2} \tag{14}$$

$$\frac{d(\ln L(\boldsymbol{\theta}))}{d\boldsymbol{\theta}}=-\frac{n}{2}-\frac{\sum_{i=1}^nx_i^2}{2^2}=0 \tag{12}$$

$$\ddot{\varrho}^{1/2}\frac{\sum_{i=1}^nx_i^2}{n} \tag{14}$$

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$$H_0: \sigma^2=0.108^2, \quad H_1: \sigma^2>0.108^2; \tag{12}$$

$$\chi^2_{\frac{\alpha}{2}}=\frac{(n-1)S^2}{\frac{\sigma_0^2}{2}}=\chi^2_{\frac{\alpha}{2}}(n-1); \tag{12}$$

$$\frac{(n-1)S^2}{\frac{\sigma_0^2}{2}}=\chi^2_{\frac{\alpha}{2}}(n-1) \Rightarrow \frac{(n-1)S^2}{\frac{\sigma_0^2}{2}}=\chi^2_{1-\frac{\alpha}{2}}(n-1) \tag{13}$$

$$\chi^2_{0.025}(4)=11.143, \quad \chi^2_{0.975}(4)=0.484,$$

$$\frac{(n-1)s^2}{\frac{\sigma_0^2}{2}}=\frac{4-0.052}{0.108^2}=17.83>11.43, \tag{14}$$

$$H_0: \sigma^2=0.108^2. \tag{14}$$