April 7, 2017
Zengtao Chen, CEO
Chen Inc.
University of Alberta DICE 10-219
Edmonton, AB T6G 1H9

RE: Final Bid Proposal

Dear Dr. Zengtao Chen:

GreenRiggs Inc herewith submits the final bid proposal report for the design of a personal use, wind powered oil derrick. The pump jack will maximize wind and oil resources and assist the Government of Alberta during its impending energy crisis. This report contains Chen Inc.'s expectations of GreenRiggs, legal issues regarding the design, use of the equipment, financial aspects of the project, design methodology, and project scheduling.

After reviewing the time spent on conceptual design, full critical analysis of the design, and preparation of the final bid proposal, each member of GreenRiggs spent roughly 63 hours each. Each member earns $90/hour as a Junior Engineer, so, the estimated cost to pay the members of GreenRiggs will total $34020.

GreenRiggs Inc. ensures that its effort put into completing this project is done to the best of the abilities of its employees.

Regards,

Arief Muhayatsyah
General Manager of GreenRiggs Inc.

Bid No. 2

GreenRiggs Inc.

April 7th, 2017

Presented to the CEO of Chen Inc

Abstract

With an impending energy crisis, the Government of Alberta wishes to maximize two of its natural resources: wind and oil. Therefore, GreenRiggs Inc. has designed a wind powered pump jack meant for personal use in Alberta. This pump jack will be able to convert average wind forces in Alberta into a pumping action. The pump jack not industrial and designed so that two male young adults will be able to easily assemble and transport the pump jack within the province. After consulting with the CEO of Chen Inc. about the progress of this project, GreenRiggs had a final bid proposal ready by April 7, 2017. This bid proposal report will discuss the legal and business aspects of the design, what is expected by Chen Inc., the design methodology of all considered designs, and a full critical analysis of the selected design. After reviewing the time spent by each member of GreenRiggs, the estimated cost for this project is $34020. The figure below gives a general depiction of what the exterior of the pump jack will look like.

Wind Powered Pump Jack

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Word Count: 2498

1 Introduction

Due to the impending energy crisis, the Government of Alberta wishes to maximize its oil and wind resources. Therefore, GreenRiggs Inc. is designing a wind powered pump jack for personal use. From a family perspective, two adult males must be able to assemble, operate and transport the product without difficulty. Additionally, the cost of production, transportation, and assembly (if needed by the customer) must be reasonably priced.

The wind-powered pump jack can be bought by customers for private or commercial use. They are investing in equipment that would generate profit for them in the long run. Since customers are responsible for transporting the pump jack's components, they must fit in commercial heavy equipment haulers, such as semi-trucks. Also, this pump jack would attract more southern Albertans since the wind speeds there are faster than the rest of the province.

Though profit is important, the pump jack must be legally operable and transportable. The dimensions and weight of the pump jack must be within the semi-truck limits. In Canada, the truck weight and spatial dimension limits are 80 000 lbs and 14.63 m X 2.6 m X 2.6 m, respectively (Flatbed Trucking, 2017). Zoning, public hearings, building permits, site suitability are issues that need to be addressed before installation (CSA Guide to Canadian wind turbine codes and standards, 2008). Issues like the impact on water air and wildlife, acoustic noise, and worker safety must be met for the pump jack to be legally approved for production, sale, and operation.

Three design concepts were brought up during the first week: a horizontal, vertical and conventional wind-powered pump jack. The horizontal pump jack receives power from horizontal wind loads using a wind turbine, which transfers the rotational energy to the input shaft. The vertical pump jack is like the horizontal pump jack, but, it can catch wind from any angle using a relatively small wind turbine. The conventional pump jack has two structures that separate wind power and pumping, unlike the other two concepts. From a business and legal perspective, the three concepts are analyzed and ranked against each other.

2 Design Methodology

During the brainstorming sessions, GreenRiggs critiqued each design concept in hopes of determining the most viable concept. For all preliminary calculations, the change in wind speeds in southern Alberta in the years 1971-2000 were assumed to be negligible. The speed used was roughly 16 km/h (Annual mean Windspeed map, 2017). Also, the wind speed and the blade tips are assumed to be 30% higher than the average wind speed. Additionally, the effects of weather and climate change were not considered. The frictional forces were neglected and each part of the pump jack are assumed to be a rigid body.

Using these assumptions, some design specifications are found for each concept. For the horizontal pump jack, maximum rotational energy can be achieved if the wetted area (area covered when the blades spin), number of blades, length of the blades and material used for the turbine are optimized. Therefore, three 9 m long blades that are 120° apart will be used so that they did not interfere with other parts of the pump jack and are safe to operate. This maximizes the wetted area so less torque is needed to rotate the turbine. The turbine blades can also detach by having a loose collar connected to the shaft and blades using metal bars and hinges. This way, the blades and turbine are easier to transport by semi-truck. The loose collar dampens the turbine's vibration using a stiff spring and tight collar.

Ignoring the similarities in each of the designs – gears and gear types, gear box, slider crank, supports holding the pump jack – the advantages and disadvantages of each design were evaluated relative to each other. For the horizontal pump jack, it uses less parts, making it easier to transport. Its compact design and large propeller increases its efficiency and lower the maintenance cost. However, the large blades pose a threat to the environment and people nearby. The vertical pump jack can catch wind from any direction. It is better for wildlife and other people compared to the horizontal pump jack, mobile and easy to assemble. However, it will cost more to maintain it and is less efficient since it uses a smaller wind turbine. The conventional pump is easy to maintain because of the two structures that separate wind power and pumping. The pump power output is large and the design is very stable and can handle all kinds of weather. But, it is difficult to assemble and transport, not efficient and more expensive due to its size.

A decision matrix was implemented early in the project to select the best design. Various specifications were considered and given a weight based on their relative importance in the design, ranging from 0 to 100. Choices of 1,5,7, and 10 were used for poor, average, important, and critical consideration. The decision matrix ranked ease of assembly, mobility, cost, safety, and efficiency as a priority; mobility refers to the ease of transport. The environment and aesthetics were also considered. Estimated design cost (excluding material costs) is weighted highly due to the limited time to complete the project; it accounts for the complexity of the design and the expected total cost to pay the engineers. Safety is the most heavily weighted; Chen Inc. can minimize or completely avoid the potential lawsuits from the operators of the product while maintaining a competitive edge if safety is prioritized. Ease of assembly and mobility were weighted highly since customers must be able to transport the product effectively. Material cost was given a low weight because they are not as valuable as the time spent to build the product. Aesthetics weighted low because it does not affect the performance of the product. Efficiency was weighted lower than mobility because GreenRiggs believes the customers are unlikely to come visit the pump jack to investigate the remaining oil underground. A table showing the weight for each criterion is included in **Table M1**. Concepts 1 & 2 are the horizontal pump jack without and with a rotating body, respectively; concept 3 is the vertical pump; and concept 4 is the conventional pump jack. Based on the decision matrix, the horizontal pump jack design without a rotating body was chosen.

3 Full Critical Analysis of Horizontal Pump Jack

3.1 Material Selection

The pump jack uses a lifting mechanism containing a sucker rod and a crankshaft to extract the oil from the ground. When choosing the material for the gearbox, properties like fatigue, tensile and yield strength, along with hardness, toughness, creep and corrosion resistance.

There are three commonly used grade sucker rods: grades C, K, D and H. The grade K sucker rod was chosen for its corrosion resistance and, mainly, because its characteristics are within the scope of the project.

The crankshaft is made up of a shaft, crankpins and counterweights. The 300 series stainless steels tend be good resistors to corrosion and relatively cheap; red brass is highly machinable with a good fatigue strength. Therefore, stainless steel 316, 316L, 347 and red brass are being used.

An exterior truss structure will encase all the components of the pump jack, namely, the wind turbine, gearbox and crankshaft components. It will use plain carbon steel coated with epoxy paint to increase its corrosion resistance.

A further description of why each material was chosen, along with some material properties can be found in **Appendix A**.

3.2 Wind Turbine

The wind turbine provides the source of power for the pumpjack to operate. A MATLAB function was devised to calculate the torque, angular velocity and power using input parameters such as wind speed. The function is shown in **Appendix N.** Since the rotor tip is subject to higher speed than the rotor section closer to the hub, airfoil performance drastically changes with changing Reynolds number, causing the flow to separate. For this reason, the rotor is twisted (Lower Angle of attack is implemented close to the hub) and the vortex generators are put in place for early laminar to turbulent transition. See **Appendix P** for illustration.

3.3 Pumpjack

The pumping power for the pumpjack was derived and calculated as shown in **Appendix B**. The derivation relied on the assumption that oil was present immediately under the ground, which was why the displacement of fluid for a half-revolution of the crankshaft is equal to its diameter. A gear ratio of was calculated using the velocity of the crank shaft and turbine.

3.4 Gear Train

The design for the gear train has undergone several iterations. The purpose of the gearbox has not changed, only its components and dimensions so that safety factor requirements are fulfilled. Essentially, the gear system needs to increase angular velocity from the turbine shaft (input) for the pump jack shaft (output), which also reduces the torque. The turbine shaft is positioned horizontally and is at a certain height from the pump jack. 2 pairs of bevels would be required in the design. One pair would orient the shaft spin vertically from the turbine shaft and then travel down to the second pair where it changes back vertical axis orientation where it connects to the pump jack. The gear train calculations can be found in **Appendix D.**

3.5 Gears

Initially, 2 bevel pairs and a compound spur reducer was to be the design of the system, shown in **Figure C1**. However, this design required huge bevel pairs to handle large torques from the turbine shaft and a long train of compound spurs to provide enough reduction without the gear size difference being too large. Therefore, a planetary gear set was placed before the first bevel pair, reducing the torque imposed on the bevels, enabling it to be smaller, lighter, and have a more compact design. This in turn also reduced the length of the compound spur train. The final gear schematic is shown in **Figure C2**.

3.6 Shafts

The turbine shaft is the input shaft for the gear train, denoted shaft #1; the output shaft contains the meshing spur gears in the gearbox, denoted shaft #2. The maximum normal and shear stress are calculated by analyzing the forces applied by the turbine blades, bevel and spur gears, bearings and keyways. The cross sections of the shafts are circular.

All the stresses, forces, torques and lengths are known. Therefore, the shaft diameters were determined. Using singularity functions, shear force and bending moment diagrams, a shaft diameter greater than 36 mm will satisfy an angular deflection less than 2 degrees.
A detailed description of the analysis of the input turbine shaft and output gear shaft can be found **Appendix F**.

3.7 Bearings

Several bearings are put across the shafts to enable rotational movement while reducing friction and handling stress. They are also very easy to replace. Depending on the type of bearing chosen, it can also resist force in multiple directions. For the design that GreenRiggs developed, multiple rolling element bearings (REB) are implemented. With the small friction it provides, high speed and efficiency can be maintained. In general, noise can be an issue for REB's. This will not be an issue since the product will be used in the field with no people nearby. Another disadvantage is that REBs require more space in the radial direction. With the ample volume available in the base of the wind turbine, this is also not an issue. A multiple REB can be used for better load distribution. Please refer to **Appendix K** for bearing selection.

3.8 Truss Structure

A hollow, conical shape was used to design the truss since it provides more stability than a cylindrical shape. The truss is hollow so that most of the pump jack components can go inside it. The truss structure that holds the wind turbine is made of stainless steel 230 (SS230). SS230 has a yield strength, ultimate tensile strength, and elongation of 230 MPa, 210 MPa, and ±20 % respectively. After conducting a full stress analysis on the truss design, the conclusion was made that the truss would not collapse from the weight of the wind turbine, and bending stress due to the force applied by the wind is negligible. A detailed analysis of the truss structure can be seen in **Appendix L**.

3.9 Connections

Bolts are used to attach two temporary systems or mechanisms. For the pump jack design, the planetary gear system will need to be removable because it is really packed and needs to be maintained because of the short lifespans. There will be two bolts connecting the hub of the wind turbine to the planetary gear. The material is made of AISI 316 stainless steel because its corrosion resistance and strength values. **Appendix Q** explains how the bolts will be used.

4 Discussion of Design Compared to Specifications

After reviewing the design specifications, the horizontal pump jack will be able to meet all the legal requirements in Alberta. The pump jack is easy to manufacture, transport, safe, and efficient. Therefore, the horizontal pump will help the Government of Alberta during its impending energy crisis.

The project started January 20, 2017, during the first meeting with the CEO of Chen Inc. using the online program Liquid Planner, hours spent by each member of GreenRiggs were logged throughout the duration of this project. Estimates of the time to spend on each task were given by management. Mondays, Wednesdays and Fridays were the official working days for each group member. This does mean that each group member only worked on these days. If any member worked on the project on another day, management was notified in advance so that those hours would be included in the time management schedule. A Gantt chart was generated through Liquid Planner; it shows when the conceptual design, full critical analysis and final bid proposal was completed. The Gantt chart can be seen in **Figure M1**. After reviewing the project schedule, GreenRiggs could prepare the final bid proposal for the CEO of Chen Inc. by April 7, 2017, without any significant deviation from the initial schedule.

5 Estimated Costs

Each group member spent an average of 20 hours on the conceptual design phase; 25 hours on the full critical analysis phase; and 18 hours on preparing a final bid proposal, adding up 63 hours in total. All 6 members of GreenRiggs are Junior Engineers earning $90/hour. Excluding the cost of sourcing the materials and equipment needed for manufacturing, this project will cost a total of $5670 per member for the time spent to complete this project. Therefore, the estimated cost to pay GreenRiggs will amount to $34020.

6 Conclusion

GreenRiggs Inc. is designing a wind powered pump jack to help the Government of Alberta maximize its wind and oil resources during an energy crisis. Through decision matrices, brainstorming sessions, and business and legal research, GreenRiggs chose to design a horizontal wind powered pump jack that receives power from horizontal wind loads using a wind turbine which transfers the rotational energy to the input shaft.

After choosing to design a horizontal wind powered pump jack, a full critical analysis of the design took place. The materials used for different components of the pump jack were discussed in detail; the types of gears used for the gear train, along with the material were analyzed; and the necessary gear ratio was calculated. After the analysis, 300 series stainless steel would the main materials used for the gears and gearbox, with some brass being used for some of the crankshaft components. The gear train will consist of planetary, bevel, and spur gears. The diameters of the input and output shafts will be larger than 36 mm.

Management assigned roles to different members along with the number of hours to spend on each task. Using Liquid Planner, a Gantt chart was created showing the project schedule. Given each member is a Junior Engineer, the current schedule was deemed reasonable and achievable by each member. After completing this project, the total estimated cost was $34020.

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Appendix

A: Material Selection for Major Components of Pumpjack

The pump jack uses a lifting mechanism containing a sucker rod and a crankshaft to extract the oil from the ground. This lifting mechanism must be able to lift approximately 96.93 N. The lifting mechanism is subject to cyclic loading – it would require materials with high fatigue and tensile strengths for both the sucker rod and the crankshaft. When choosing the material for the gearbox, properties like fatigue, tensile and yield strength, along with hardness, toughness, creep and corrosion resistance.

There are three commonly used grade sucker rods: grades C, K, and D. Grade C sucker rods have moderate strength, and a certain degree of resistance to sulfide stress cracking, making it more suitable for sour environments. Grade D sucker rods are manufactured from high quality heat treated steel alloys, giving it high tensile and yield strengths. They can be used in non-corrosive or mildly corrosive environments. Grade K sucker rods are widely used in corrosive environments due to its corrosion resistance in H_2S, CO_2, and NaCl media. There are also high tensile (Grade H) sucker rods. They have extremely high tensile strengths and are ideal for large, deep wells. The materials used for these grade sucker rods are AISI 4138 high mechanical strength or 4330 steel. (Hengshui Haiwang Oil Thermal Recovery Equipment Co., Ltd., 2014) Sour natural gas wells are controversial and, thus, should not be mined. Also, using Grade H sucker rods are not necessary based on the scope of this project. Therefore, GreenRiggs recommended using a Grade K sucker rod for its good corrosion resistance.

The crankshaft of GreenRiggs' pump jack is like the ones used in vehicles. Since the pump jack moves at lower speeds than vehicles, the crankshaft used will have a lower fatigue stress. Steel, generally medium carbon steel alloys, are composed of iron and a small percentage of carbon. (AK Steel Corporation, 2007) Brass – red brass, leaded yellow brass and silicon brass – were also considered mainly for their machinability. Red brass is very machinable with a fatigue limit of around 76 MPa; it is typically used for hardware, buildings, industrially for pumps and valves, and plumbing. Leaded yellow brass has very good machinability, but mainly used for aesthetics industrially since it has no fatigue strength. Similarly, silicon brass has no fatigue strength but is very machinable. The crankshaft is made up of a shaft, crankpins and counterweights. Therefore, stainless steel 316, 316L, 347 and red brass will be used for the components of the crankshaft.

The main characteristics for the gearbox are the fatigue strength, corrosion resistance and machinability. For gearboxes, steel – stainless steel in particular – is typically used because of its high fatigue strength, corrosion resistance and ductility. (ASM International, 1990) The most common stainless steels are austenitic, ferritic, martensitic and duplex stainless steels. Austenitic stainless steels are highly formable, non-magnetic and have excellent corrosion resistance. They contain a mixture of chromium and nickel for surface quality. Austenitic stainless steels make up the 200 and 300-series, with the 300-series having better corrosion resistance. Ferritic steels are a group of 400-series that contain chromium, carbon and other alloying elements to increase its corrosion resistance even further than austenitic steels. Their downfall is their magnetism and inability to be heat treated. Martensitic stainless steels are another group in the 400-series. They contain higher levels of carbon than ferritic stainless steels and can be heat treated. Like ferrites, however, they are always magnetic. Duplex stainless have austenite and ferrite in their structure. Since they have characteristics of both austenite and ferrite, they have higher strength, ductility and are very non-corrosive. The main issue is that they are mainly used industrially and not very cost-effective. (AK Steel Corporation, 2007) Based on this, the best stainless steel to use is 316 steel. 316 stainless steel has a fatigue limit of approximately 270 MPa. (British Stainless Steel Association, 2017) Most of the materials used for the gearbox will also be used for gears, input shafts and keyways.

An exterior truss structure will encase all the components of the pump jack, namely, the wind turbine, gearbox and crankshaft components. It will use plain carbon steel coated with epoxy paint to increase its corrosion resistance. No further analysis was done since the truss is simply meant to contain all the pump jack components and be aesthetically pleasing.

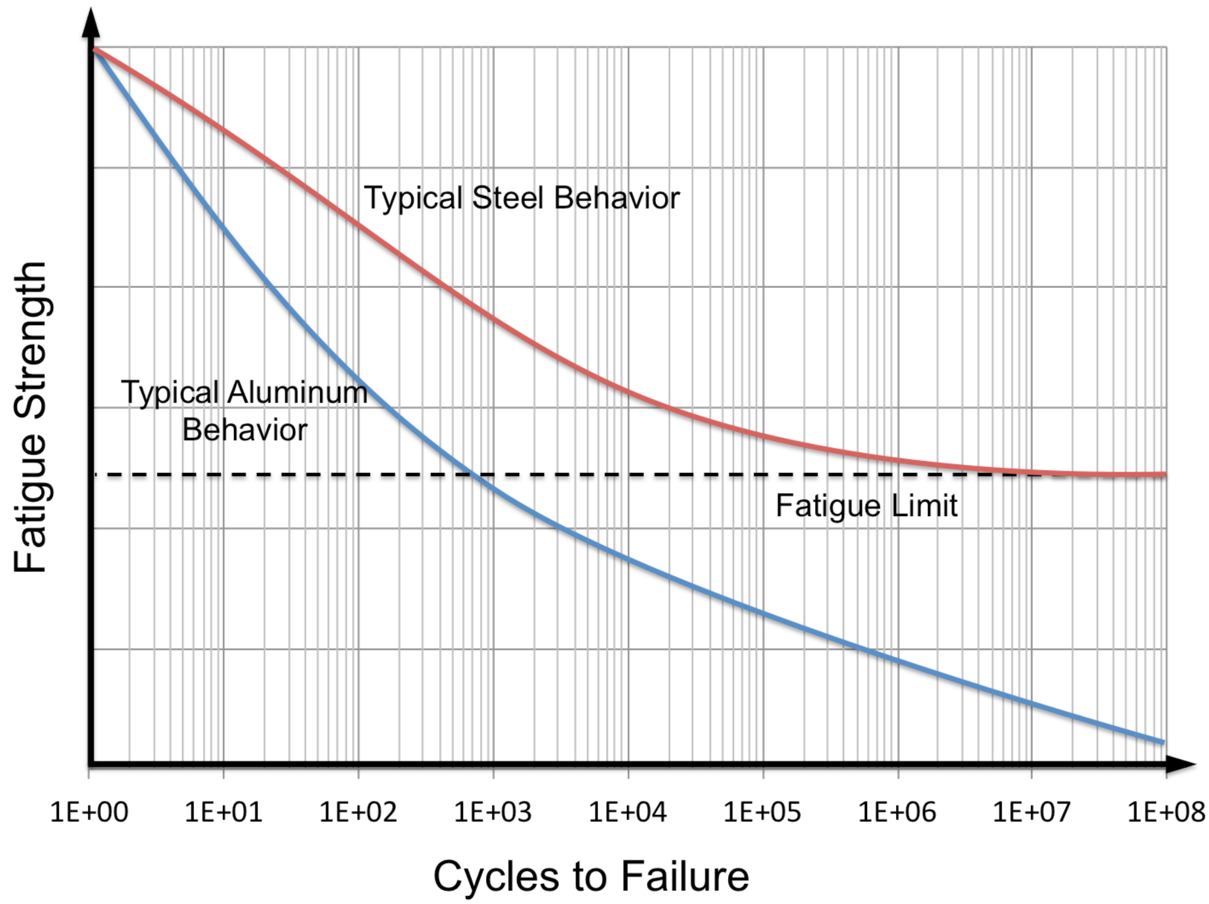


Figure A1: Typical SN-curve for steel and aluminum

Table A1: Mechanical sheet of the different grade sucker rod

Item	Grade	Tensile strength MPa	Yield strength MPa	Percentage elongation %	Contraction percentage of area %
ESRM-01	C	620 - 795	≥ 415	≥ 13	≥ 50
ESRM-02	K	620 - 795	≥ 415	≥ 13	≥ 60
ESRM-03	D	795 - 965	≥ 590	≥ 10	≥ 50
ESRM-04	KD	795 - 965	≥ 590	≥ 10	≥ 50
ESRM-05	HL	965 - 1195	≥ 795	≥ 10	≥ 45
ESRM-06	HY	965 - 1195	N/M	N/M	N/M

B: Pumping Power Requirement Calculation & Pumpjack Design Specifications

The **minimum pumping requirement** can be derived under the following assumptions:

- Weight of column of fluid is only being considered.
- Well pressures are negligible due to geology
- Density of fluid pumped is constant

The following parameters are given and some need to be solved for:

T_{in} = Torque into the gearbox (GB)

ω_{in} = angular velocity into the GB

G = Gear Ratio

T_{Out} = Torque output of gearbox

ω_{out} = angular velocity output from GB

P_{in} = Power into the gearbox

η_{gb} = Gearbox efficiency

P_{out} = Power output form gearbox

ρ_{crude} = Density of crude oil being pump

The radius of the crank r determines the height of one upstroke. The height of one lift stroke is given as

$$h = 2r$$

The diameter of the well column D can be used to calculate cross-sectional area of the volume of fluid.

$$A_f = \frac{\pi}{4} D^2$$

The volume of fluid that needs to be pumped is thus:

$$V_f = A_f h$$

The mass of this fluid is thus:

$$\rho = \frac{m}{V_f} \rightarrow m = \rho V_f$$

The work required to lift the column of fluid through its height is

$$W = mgh = \rho V_f gh$$

Given that the output angular velocity is given as ω_{out}, we can find the time taken for one lift stroke, i.e. half a revolution.

$$t = \frac{\theta}{\omega}$$

Thus, the power requirement for one revolution, assuming the returning stroke doesn't require any energy at all (since we are neglecting comparatively small viscous forces):

$$P_{pump} = \frac{\rho V_f g h}{2t}$$

We can also find the average mass flow rate for one lift stroke using:

$$\dot{m}_{avg} = \frac{m}{t}$$

The power supplied to the pump crank, which is the output power from the gearbox P_{out}, can be equated to the P_{pump} to solve for power and gearing requirements:

$$\boldsymbol{P_{out} = P_{pump}}$$

Recall that $P = T\omega$

$$P_{in} = \eta_{gb} P_{out}$$

$$T_{in}\omega_{in} = \eta_{gb} T_{out}\omega_{out} \qquad ; \qquad G = \frac{\omega_{in}}{\omega_{out}} = \frac{T_{out}}{T_{in}}$$

$$\boldsymbol{P_{out} = \frac{T_{in}\omega_{in}}{\eta_{gb}} = \frac{T_{in}\omega_{out}G}{\eta_{gb}} = \frac{\rho V_f g h}{2t}}$$

C: Gearbox Design & Selection

Gear Train Assembly – Skeletal Diagram

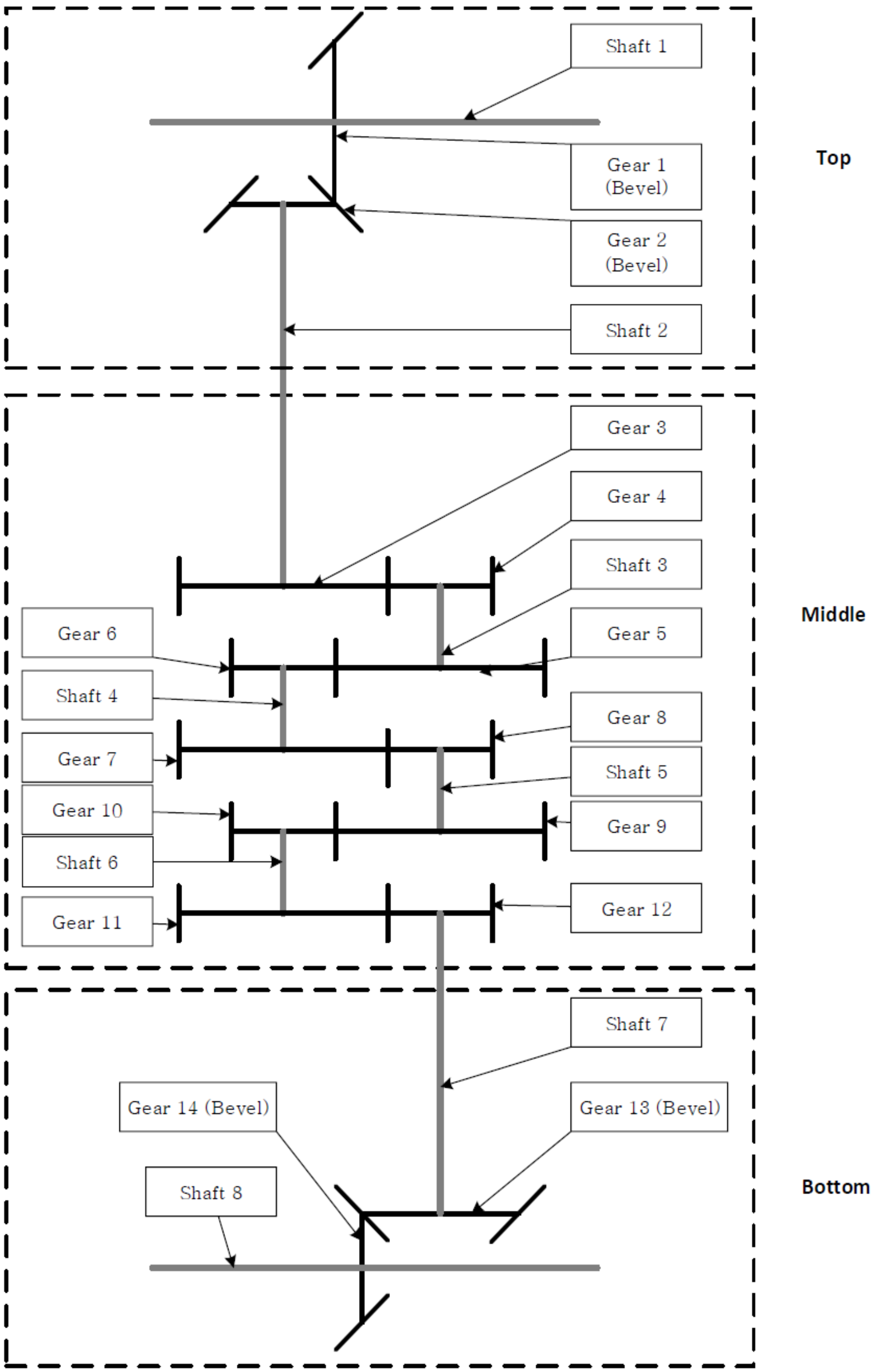


Figure C1: Earlier version of proposed gear system

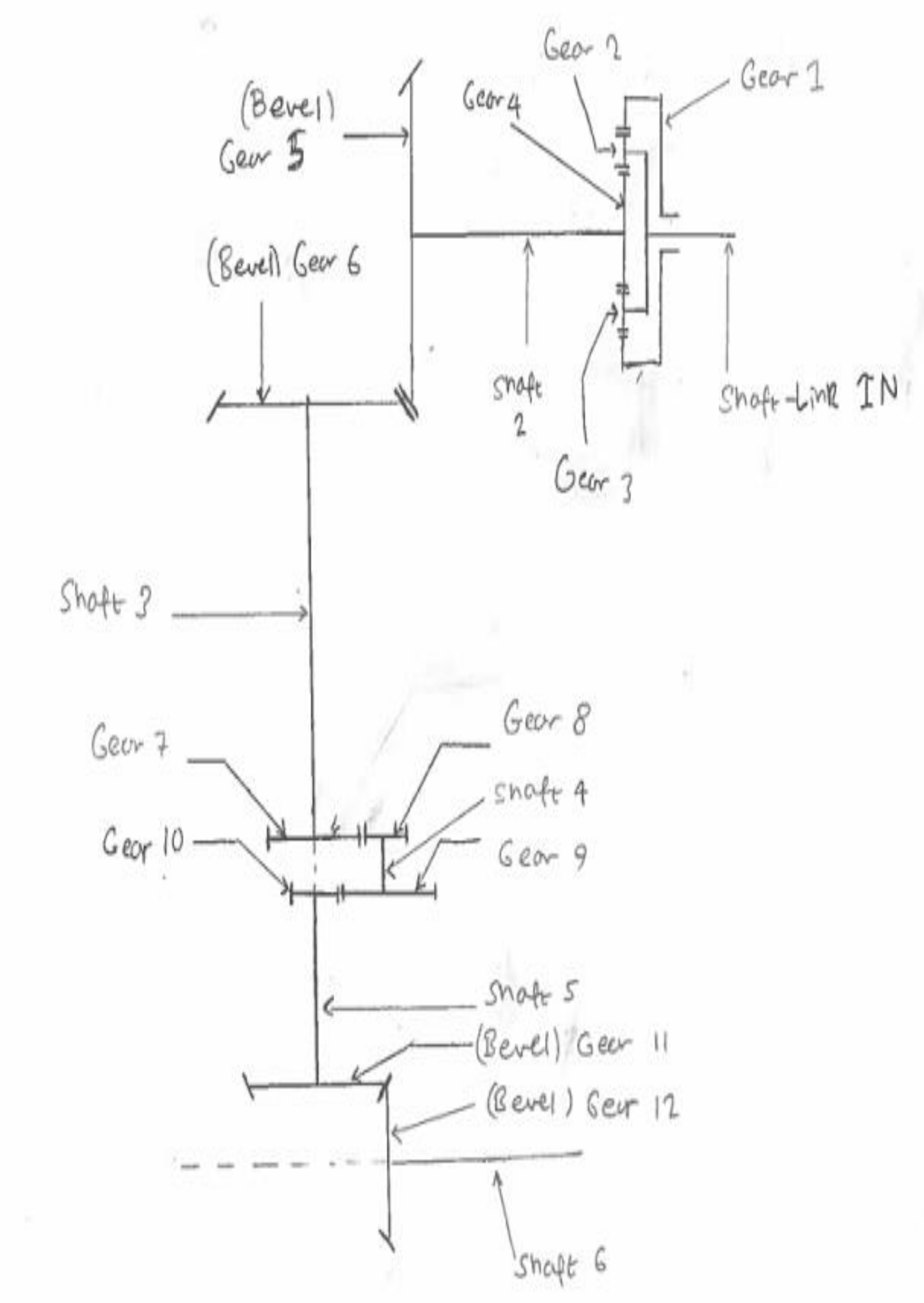


Figure C2: Schematic of final version of Gear system

D: Gear Train Calculations

Gear Train Design

Gear radii diameters are taken as pitch circle radii and diameters, repectively

$d_0 := 400\text{mm}$ $\quad r_0 := \frac{d_0}{2} = 0.2\,\text{m}$ $\quad \omega_0 := 0.72\,\frac{\text{rad}}{\text{s}}$ **This is the input Shaft-Link**

$d_1 := 500\text{mm}$ $\quad r_1 := \frac{d_1}{2} = 0.25\,\text{m}$ $\quad \omega_1 := 0\,\frac{\text{rad}}{\text{s}}$ **Fixed planetary ring gear**

$d_2 := 100\text{mm}$ $\quad r_2 := \frac{d_2}{2} = 0.05\,\text{m}$ $\quad \omega_2 := \frac{-r_0\,\omega_0}{r_2} = -2.88\,\frac{1}{\text{s}}$ **Planet gear**

$d_3 := 100\text{mm}$ $\quad r_3 := \frac{d_3}{2} = 0.05\,\text{m}$ $\quad \omega_3 := \omega_2 = -2.88\,\frac{1}{\text{s}}$ **Planet gear**

$d_4 := 300\text{mm}$ $\quad r_4 := \frac{d_4}{2} = 0.15\,\text{m}$ $\quad \omega_4 := -2\cdot\omega_3\left(\frac{r_3}{r_4}\right) = 1.92\,\frac{1}{\text{s}}$ **Sun Gear**

$d_5 := 500\text{mm}$ $\quad r_5 := \frac{d_5}{2} = 0.25\,\text{m}$ $\quad \omega_5 := \omega_4$ **Bevel**

$d_6 := 500\text{mm}$ $\quad r_6 := \frac{d_6}{2} = 0.25\,\text{m}$ $\quad \omega_6 := \frac{-\omega_5\cdot r_5}{r_6} = -1.92\,\frac{1}{\text{s}}$ **Bevel**

$d_7 := 240\text{mm}$ $\quad r_7 := \frac{d_7}{2} = 0.12\,\text{m}$ $\quad \omega_7 := \omega_6$ **Spur**

$d_8 := 100\text{mm}$ $\quad r_8 := \frac{d_8}{2} = 0.05\,\text{m}$ $\quad \omega_8 := \frac{-\omega_7\cdot r_7}{r_8} = 4.608\,\frac{1}{\text{s}}$ **Spur**

$d_9 := 240\text{mm}$ $\quad r_9 := \frac{d_9}{2} = 0.12\,\text{m}$ $\quad \omega_9 := \omega_8$ **Spur**

$d_{10} := 100\text{mm}$ $\quad r_{10} := \frac{d_{10}}{2} = 0.05\,\text{m}$ $\quad \omega_{10} := \frac{-\omega_9\cdot r_9}{r_{10}} = -11.059\,\frac{1}{\text{s}}$ **Spur**

$d_{11} := 400\text{mm}$ $\quad r_{11} := \frac{d_{11}}{2} = 0.2\,\text{m}$ $\quad \omega_{11} := \omega_{10}$ **Bevel**

$d_{12} := 400\text{mm}$ $\quad r_{12} := \frac{d_{12}}{2} = 0.2\,\text{m}$ $\quad \omega_{12} := \frac{-\omega_{11}\cdot r_{11}}{r_{12}} = 11.059\,\frac{1}{\text{s}}$ **Bevel**

$GR := \frac{\omega_{12}}{\omega_0} = 15.36$ **Gear Ratio set to slightly below 16 to ensure the torque to the pumpjack is enough to lift the crankshaft.** $\quad GR_{planet} := \frac{\omega_4}{\omega_0} = 2.667$

$N_1 := \text{Num}(d_1, mo_{plan}) = 100$

$N_2 := \text{Num}(d_2, mo_{plan}) = 20$

$N_3 := \text{Num}(d_3, mo_{plan}) = 20$

$N_4 := \text{Num}(d_4, mo_{plan}) = 60$

$N_5 := \text{Num}(d_5, mo_{bev}) = 25$

$N_6 := \text{Num}(d_6, mo_{bev}) = 25$

$N_7 := \text{Num}(d_7, mo_{helic}) = 48$

$N_8 := \text{Num}(d_8, mo_{helic}) = 20$

$N_9 := \text{Num}(d_9, mo_{helic}) = 48$

$N_{10} := \text{Num}(d_{10}, mo_{helic}) = 20$

$N_{11} := \text{Num}(d_{11}, mo_{bev}) = 20$

$N_{12} := \text{Num}(d_{12}, mo_{bev}) = 20$

Gear Modules

Modules:

$mo_{bev} := 20mm = 0.02\,m$ $\quad mo_{helic} := 5mm$ $\quad mo_{plan} := 5mm$

$Num(d, mo) := \frac{d}{mo}$ **Equation for Gear teeth calculation**

Gear Teeth

$N_1 := Num(d_1, mo_{plan}) = 100$

$N_2 := Num(d_2, mo_{plan}) = 20$

$N_3 := Num(d_3, mo_{plan}) = 20$

$N_4 := Num(d_4, mo_{plan}) = 60$

$N_5 := Num(d_5, mo_{bev}) = 25$

$N_6 := Num(d_6, mo_{bev}) = 25$

$N_7 := Num(d_7, mo_{helic}) = 48$

$N_8 := Num(d_8, mo_{helic}) = 20$

$N_9 := Num(d_9, mo_{helic}) = 48$

$N_{10} := Num(d_{10}, mo_{helic}) = 20$

$N_{11} := Num(d_{11}, mo_{bev}) = 10$

$N_{12} := Num(d_{12}, mo_{bev}) = 10$

Torque Calculations for Gears

Torque calculations for gears

Assuming 100% efficiency during interaction with each gear interface.

$T_0 := 509.9 N \cdot m$ $\quad Num_{planet} := 4$ **Number of planet gears around the sun gear**

$T_1 := 0$

$T_2 := \frac{T_0}{Num_{planet}} \cdot \left(\frac{\omega_0}{\omega_2}\right) = -31.869 \cdot N \cdot m$ **Torque is divided by the number of planets** $\quad T_3 := T_2 = -31.869 \cdot N \cdot m$

$T_{2a} := T_2 = -31.869\,J$ $\quad T_{3a} := T_2 = -31.869\,J$

These gears are the additional planets and will be similar to their derived gear in almostevery aspect

$T_4 := T_3 \cdot \left(\frac{\omega_3}{\omega_4}\right) = 47.803 \cdot N \cdot m$ $\quad T_5 := T_4 = 47.803 \cdot N \cdot m$

$T_6 := T_5 \cdot \left(\frac{\omega_5}{\omega_6}\right) = -47.803 \cdot N \cdot m$ $\quad T_7 := T_6 = -47.803 \cdot N \cdot m$

$T_8 := T_7 \cdot \left(\frac{\omega_7}{\omega_8}\right) = 19.918 \cdot N \cdot m$ $\quad T_9 := T_8 = 19.918 \cdot N \cdot m$

$T_{10} := T_9 \cdot \left(\frac{\omega_9}{\omega_{10}}\right) = -8.299 \cdot N \cdot m$ $\quad T_{11} := T_{10} = -8.299 \cdot N \cdot m$

$T_{12} := T_{11} \cdot \left(\frac{\omega_{11}}{\omega_{12}}\right) = 8.299 \cdot N \cdot m$

E: Gear Bending & Contact Stress Calculations

E1 Planetary Gears (0, 1, 2, 3, 4)

Planetary Gear Force Analysis: Gears 0, 1, 2, 3, 4

$\varphi_1 := 20°$ **Pressure Angles**

$mo_{plan} = 5 \times 10^{-3}\,m$ **Module**

$\psi_1 := 0°$ **Helix Angle (Straight)**

$W_{t1} := \frac{T_1}{r_1} = 0$ **Ring Gear** $W_{t2} := \frac{T_2}{r_2} = -637.375\,N$ **Planet Gear**

$W_{t3} := \frac{T_3}{r_3} = -637.375\,N$ **Planet Gear** $W_{t4} := \frac{T_4}{r_4} = 318.688\,N$ **Sun Gear**

The radial components are:

$W_{r1} := W_{t1} \cdot \tan(\varphi_1) = 0$ $W_{r2} := W_{t2} \cdot \tan(\varphi_1) = -231.986\,N$

$W_{r3} := W_{t3} \cdot \tan(\varphi_1) = -231.986\,N$ $W_{r4} := W_{t4} \cdot \tan(\varphi_1) = 115.993\,N$

Resultant force is:

$W_{R1} := \frac{W_{t1}}{\cos(\varphi_1)} = 0$ $W_{R2} := \frac{W_{t2}}{\cos(\varphi_1)} = -678.28\,N$

$W_{R3} := \frac{W_{t3}}{\cos(\varphi_1)} = -678.28\,N$ $W_{R4} := \frac{W_{t4}}{\cos(\varphi_1)} = 339.14\,N$

The spurs are straight and so will be assumed to have neglegible axial forces from the gears

**The calculations for the other planets gears 2a and 3a will have exactly the same calculations as gears 2 and 3 respectively. This assumption is carried forward with the rest of the planetary gear analysis with consistent notation used.

Bending Strength for Planetary Gears 0, 1, 2, 3, 4.

"Gear " 0 is the input shaft link and thus will have no teeth.

Gear 1 is the Ring Gear

Gears 2 and 3 are Planet Gears

Gear 4 is a Sun Gear

$N_{cyc1} := 10^{10}$ **; conservative number of cycles for operating lifetime (~10 yrs)**

$T_{f1} := 100$ **;operating temperatures should be around 100 F with atmospheric cooling**

$K_{L1} := K_{L'}(N_{cyc1}) = 0.8$

$S_{fb1'} := 280\text{MPa}$ **;nominal bending strength of AGMA A2 Grade 2 Steel**

$$K_{T1} := \begin{cases} 1 & \text{if } T_{f7} < 250 \\ \left[\left(\left(\dfrac{460 + T_{f7}}{620}\right)\right)\right] & \text{otherwise} \end{cases}$$

$K_{R1} := 1.25$ **;for 99.9% reliability**

$S_{fb1} := S_{fb}(K_{L1}, K_{T1}, K_{R1}, S_{fb1'}) = 179.208\cdot\text{MPa}$ **Bending Strength for Planetary Gears 1, 2, 3, 4**

Bending Stresses (Plantary Gears 1, 2, 3, 4)

The planetary gear set are **straight spur gears.**

$$\sigma_{b.spur}(W_t, K_a, K_m, K_s, K_B, K_I, w_{face}, mo, J_s, K_\nu) := \frac{W_t \cdot K_a \cdot K_m \cdot K_s \cdot K_B \cdot K_I}{w_{face} \cdot mo \cdot J_s \cdot K_\nu}$$

K-values:

$W_{t1} = 0$ $\quad$ $W_{t2} = -637.375\,N$

$K_{a1} := 1$ **Uniform turbine speed assumed.**

Given that $w_{face1} := 100mm$ $\quad K_{m1} := 1.65$ **Load distribution factor**

$K_s := 1$ **Always 1**

$K_{B1} := 1$ **Solid disk gear**

$K_{I1} := 1.42$ **Meshed to 4 gears, thus idler**

$K_{I2} = 1.42$ $\quad K_{I3} := K_{I2}$ **Meshed to two gears, thus idler (applies to 2a and 3a as well)**

$N_1 = 100$ $\quad N_2 = 20$ $\quad N_3 = 20$ $\quad N_4 = 60$

$J_1 := 0.38$ $\quad J_2 := 0.2$ $\quad J_3 := J_2$ $\quad J_4 := 0.37$ **From p.7-21 of course notes**

Assume good quality gears are used. $\quad Q_{v2} := 8$

$$B_2 := \frac{(12 - Q_{v2})^{\frac{2}{3}}}{4} = 0.63$$

$V_{t2'} := r_2 \cdot |\omega_2| = 0.144 \frac{m}{s}$ $\quad V_{t4'} := V_{t2'}$

$V_{t2} := V_t(V_{t2'}) = 28.346$ $\quad V_{t4} := V_t(V_{t4'}) = 28.346$

$AA2 := 50 + 56(1 - B_2) = 70.722$ $\quad AA4 := AA2$

$K_{\nu2} := K_\nu(AA2, V_{t2}, B_2) = 0.633$ $\quad K_{\nu4} := K_\nu(AA4, V_{t4}, B_2) = 0.633$

$$\sigma_{b2} := \sigma_{b.spur}(W_{t2}, K_{a1}, K_{m1}, K_s, K_{B1}, K_{I2}, w_{face1}, mo_{plan}, J_2, K_{\nu2}) = -23.578 \cdot MPa$$

$$\sigma_{b4} := \sigma_{b.spur}(W_{t4}, K_{a1}, K_{m1}, K_s, K_{B1}, K_{I2}, w_{face1}, mo_{plan}, J_4, K_{\nu4}) = 6.372 \cdot MPa$$

$V_{t3'} := r_3 \cdot |\omega_3| = 0.144 \frac{m}{s}$ $\quad V_{t3} := V_t(V_{t3'}) = 28.346$ $\quad V_{t1} := V_{t3}$

$AA3 := AA2$ $\quad K_{\nu3} := K_\nu(AA3, V_{t3}, B_2) = 0.633$ $\quad K_{\nu1} := K_\nu(AA4, V_{t1}, B_2) = 0.633$

$$\sigma_{b3} := \sigma_{b.spur}(|W_{t3}|, K_{a1}, K_{m1}, K_s, K_{B1}, K_{I1}, w_{face1}, mo_{plan}, J_3, K_{\nu3}) = 23.578 \cdot MPa$$

$$\sigma_{b1} := \sigma_{b.spur}(|W_{t2}|, K_{a1}, K_{m1}, K_s, K_{B1}, K_{I1}, w_{face1}, mo_{plan}, J_1, K_{\nu1}) = 12.41 \cdot MPa$$

The stresses for gears 2 and 3

Bending Saftey Factors for Spur Gears 1, 2, 3, 4

$SF_{bending1} := SF_{bending}(S_{fb1}, \sigma_{b1}) = 14.441$

$SF_{bending2} := SF_{bending}(S_{fb1}, \sigma_{b2}) = 7.601$

$SF_{bending3} := SF_{bending}(S_{fb1}, \sigma_{b3}) = 7.601$

$SF_{bending4} := SF_{bending}(S_{fb1}, \sigma_{b4}) = 28.122$

Surface Fatigue Strength: Spur Gears (1, 2, 3, 4)

$N_{cyc1} = 1 \times 10^{10}$

$C_{L1} := C_L(N_{cyc1}) = 0.679$ **Conservative number of cycles for whole lifetime (~10 yrs)**

$C_{H1} := 1$ **Assuming two gears are the same materials**

$S'_{fc1} := 720MPa$ **AGMA A2, Grade 1 Steel. Chosen for planetary to withstand the large torque input from the turbine.**

From Table 7-10
Assuming minimum surface hardness less than 180HB

$C_{T1} := 1$ K.T = 460 + T.F/620 Where Tf is in degrees F. for Tf higher than 250

$C_{R1} := 1.25$ **Realiability of 99.9%**

$$S_{fc1} := \frac{C_{L1} \cdot C_{H1} \cdot (S'_{fc1})}{C_{T1} \cdot C_{R1}} = 391.215 \cdot MPa$$

Surface Fatigue Strength: Spur Gears 1, 2, 3, 4

****The contact stresses for the planetary gears can be found in Appendix E3 along with the helical gears.**

E2 Bevel Gears (5, 6, 11, 12)

Bevel Gear analysis, Gears 5, 6, 11, 12.

Force analysis: tangential (t)

$$W_{t5} := \frac{T_5}{r_5} = 191.213\,N \qquad W_{t11} := \frac{T_{11}}{r_{11}} = -41.496\,N$$

$$W_{t6} := \frac{T_6}{r_6} = -191.213\,N \qquad W_{t12} := \frac{T_{12}}{r_{12}} = 41.496\,N$$

Pressure Angles

$$\varphi_5 := 20°$$

Cone angles:

$$\alpha_5 := \operatorname{atan}\left(\frac{r_5}{r_6}\right)\cdot\frac{180}{\pi} = 45 \qquad \alpha_6 := \operatorname{atan}\left(\frac{r_6}{r_5}\right)\cdot\frac{180}{\pi} = 45$$

$$\alpha_{11} := \operatorname{atan}\left(\frac{r_{11}}{r_{12}}\right)\cdot\frac{180}{\pi} = 45 \qquad \alpha_{12} := \operatorname{atan}\left(\frac{r_{11}}{r_{12}}\right)\cdot\frac{180}{\pi} = 45$$

Gear Geometry

$$L_{cone5.6} := \frac{r_6}{\sin\left(\alpha_6\cdot\frac{\pi}{180}\right)} = 0.354\,m \qquad w_{face5} := \frac{L_{cone5.6}}{3} = 0.118\cdot m$$

$$L_{cone11.12} := \frac{r_{12}}{\sin\left(\alpha_{12}\cdot\frac{\pi}{180}\right)} = 0.283\,m \qquad w_{face11} := \frac{L_{cone11.12}}{3} = 0.094\cdot m$$

The axial and radial components are in Bevel gears 5, 6, 11,:

All bevel sets will have same pressure angles.

$$W_{r5} := W_{t5}\cdot\tan(\varphi_5)\cdot\cos(\alpha_5) = 36.56\,N \qquad W_{r6} := W_{t6}\cdot\tan(\varphi_5)\cdot\cos(\alpha_6) = -36.56\,N$$

$$W_{a5} := W_{t5}\cdot\tan(\varphi_5)\,\sin(\alpha_5) = 59.219\,N \qquad W_{a6} := W_{t6}\cdot\tan(\varphi_5)\,\sin(\alpha_5) = -59.219\,N$$

Resultant forces are:

$$W_{R5} := \frac{W_{t5}}{\cos(\varphi_5)} = 203.484\,N \qquad W_{R6} := \frac{W_{t6}}{\cos(\varphi_5)} = -203.484\,N$$

The axial (a) and radial(r) components are in Bevel gears 11 and 12:

Pressure Angles $\varphi_{11} := 20°$

$$W_{r11} := W_{t11}\cdot\tan(\varphi_{11})\cdot\cos(\alpha_{11}) = -7.934\,N \qquad W_{r12} := W_{t12}\cdot\tan(\varphi_{11})\cdot\cos(\alpha_{12}) = 7.934\,N$$

$$W_{a11} := W_{t11}\cdot\tan(\varphi_{11})\,\sin(\alpha_{11}) = -12.851\,N \qquad W_{a12} := W_{t12}\cdot\tan(\varphi_{11})\,\sin(\alpha_{12}) = 12.851\,N$$

Resultant force is:

$$W_{R11} := \frac{W_{t11}}{\cos(\varphi_{11})} = -44.159\,N$$

Resultant force is:

$$W_{R12} := \frac{W_{t12}}{\cos(\varphi_{11})} = 44.159\,N$$

Bending Strength for Bevel Gears 5 and 6

$S_{fb}(K_L, K_T, K_R, S_{fb'}) := \dfrac{K_L \cdot S_{fb'}}{K_T \cdot K_R}$ **Bending Strength Equation**

$N_{cyc5} := 10^{10}$ **;conservative number of cycles for operating lifetime (~10 yrs)**

$T_{f5} := 100$ **;operating temperatures should be around 100 F with atmospheric cooling**

$K_{L'}(N_{cyc}) := 1.6831 \cdot N_{cyc}^{-0.0323}$ $K_{L5} := K_{L'}(N_{cyc5}) = 0.8$ **Life factor for Bending**

$S_{fb5'} := 35MPa$ **;nominal bending strength fatigue of AGMA Class 20 Cast Iron**

$K_{T5} := \begin{cases} 1 & \text{if } T_{f5} < 250 \\ \left[\left(\left(\dfrac{460 + T_{f5}}{620}\right)\right)\right] & \text{otherwise} \end{cases}$ **Temperature assumed to be lower than 250 F during operation**

$K_{R5} := 1.25$ **;for 99.9% reliability**

$S_{fb5} := S_{fb}(K_{L5}, K_{T5}, K_{R5}, S_{fb5'}) = 22.401 \cdot MPa$ **Bending strength for Gear 5**

$S_{fb6} := S_{fb}(K_{L5}, K_{T5}, K_{R5}, S_{fb5'}) = 22.401 \cdot MPa$ **Bending strength for Gear 6**

(same parameters as forGear 5)

Bending Strength for Bevel Gears 11 and 12

$N_{cyc11} := 10^{10}$**; conservative number of cycles for operating lifetime (~10 yrs)**

$T_{f11} := 100$ **;operating temperatures should be around 100 F with atmospheric cooling**

$K_{L11} := K_{L'}(N_{cyc11}) = 0.8$

$S_{fb11'} := (S_{fb5})$ **;Same material as Bevel Gear 5 and 6**

$K_{T11} := K_{T5}$ **;Temp. also assumed to be lower than 250 F during operation**

$K_{R11} := 1.25$ **;for 99.9% reliability**

$S_{fb11} := S_{fb}(K_{L11}, K_{T11}, K_{R11}, S_{fb11'}) = 14.337 \cdot MPa$ **Bending strength for Gear 11**

$S_{fb12} := S_{fb}(K_{L11}, K_{T11}, K_{R11}, S_{fb11'}) = 14.337 \cdot MPa$ **Bending strength for Gear 12**

Bending Stresses (Bevel Gears 5, 6, 11, 12)

$$\sigma_{b.bevel}(T, K_a, K_m, d, w_{face}, mo, J, K_\nu, K_x) := \frac{2 \cdot T \cdot K_a \cdot K_m}{d \cdot w_{face5} \cdot mo \cdot J \cdot K_\nu \cdot K_x}$$

Bending Stress Equation

K-values: $K_{a5} := 1$ **Uniform turbine speed assumed.**

$K_x := 1.15$ **For Bevels Gears**

$K_S := 1$ **This is always 1**

$w_{face5} = 0.118\,m$ $K_{m5} := 1.67$ **(Load distribution factor) from Table 7-5, pg. 7-23 of course notes. Obtained by interpolation.**

$w_{face11} = 0.094\,m$ $K_{m11} := 1.62$

Assume Medium quality gears are used. $Q_{v5} := 5$

$$V_{t5'} := r_5 \cdot \omega_5 = 0.48\,\frac{m}{s} \qquad V_{t11'} := r_{11} \cdot \omega_{11} = -2.212\,\frac{m}{s}$$

$$V_t(V_{t'}) := 196.85 \cdot \frac{V_{t'}}{m \cdot s^{-1}}$$

In ft/min

$$V_{t5} := V_t(V_{t5'}) = 94.488 \qquad V_{t11} := V_t(|V_{t11'}|) = 435.401$$

$$B5 := \frac{(12 - Q_{v5})^{\frac{2}{3}}}{4} = 0.915 \qquad B11 := \frac{(12 - Q_{v5})^{\frac{2}{3}}}{4} = 0.915$$

$$AA5 := 50 + 56(1 - B5) = 54.77 \qquad AA11 := 50 + 56(1 - B11) = 54.77$$

$$K_\nu(AA, V_t, B) := \left(\frac{AA}{AA + \sqrt{200 \cdot V_t}}\right)^B$$

When Q.v is less than or equal to 5, AA=50 and B=1

$$K_{\nu5} := K_\nu(50, V_{t5}, 1) = 0.267 \qquad K_{\nu11} := K_\nu(50, V_{t11}, 1) = 0.145$$

We can find J given: $N_5 = 25$ $N_6 = 25$ $N_{11} = 20$ $N_{12} = 20$

We can find that: $J_5 := 0.220$ $J_6 := 0.220$ $J_{11} := 0.200$ $J_{12} := 0.200$

Values of J obtained from Figure 7 -22, pg. 7-53 of course notes

$$\sigma_{b5} := \sigma_{b.bevel}(T_5, K_{a5}, K_{m5}, d_5, w_{face5}, mo_{bev}, J_5, K_{\nu5}, K_x) = 2.008 \cdot MPa$$

$$\sigma_{b6} := \sigma_{b.bevel}(|T_6|, K_{a5}, K_{m5}, d_6, w_{face5}, mo_{bev}, J_5, K_{\nu5}, K_x) = 2.008 \cdot MPa$$

$$\sigma_{b11} := \sigma_{b.bevel}(|T_{11}|, K_{a5}, K_{m5}, d_{11}, w_{face11}, mo_{bev}, J_{11}, K_{\nu11}, K_x) = 0.882 \cdot MPa$$

$$\sigma_{b12} := \sigma_{b.bevel}(T_{12}, K_{a5}, K_{m5}, d_{12}, w_{face11}, mo_{bev}, J_{12}, K_{\nu11}, K_x) = 0.882 \cdot MPa$$

Bending Saftey Factors for Bevel Gears 5, 6, 11, 12

$$SF_{bending}(S_{fb}, \sigma_b) := \frac{S_{fb}}{|\sigma_b|}$$

$$SF_{bending5} := SF_{bending}(S_{fb5}, \sigma_{b5}) = 11.157$$

$$SF_{bending6} := SF_{bending}(S_{fb6}, \sigma_{b6}) = 11.157$$

$$SF_{bending11} := SF_{bending}(S_{fb11}, \sigma_{b11}) = 16.251$$

$$SF_{bending12} := SF_{bending}(S_{fb12}, \sigma_{b12}) = 16.251$$

Although values are overly conservative, this is not the case for the calculations for contact stress of other gear components. This does satisfy the most important design requirement for saftey.

Surface Fatigue Strength: Bevel Gears 5, 6, 11, 12

$C_{L'}(N_{cyc}) := 2.466 \cdot N_{cyc}^{-0.056}$ **Conservative life factor equation for surface fatigue**

$N_{cyc5} = 1 \times 10^{10}$ **Conservative number of cycles for whole lifetime (~10 yrs)**

$$C_{L5} := C_{L'}(N_{cyc5}) = 0.679$$

$S_{fc'} := 340 \cdot MPa$ **;Nominal surface strength of AGMA Class 20 Grade 1 Cast Iron**

$C_{H5} := 1$ **Assuming two gears are the same materials**

$C_{R5} := K_{R5} = 1.25$ **Realiability of 99.9%**

$C_T := 1$ **Temp. assumed to be below 250 F during operation**

$S_{fc5} := \dfrac{C_{L5} \cdot C_{H5} \cdot S_{fc'}}{C_T \cdot C_{R5}} = 184.74 \cdot MPa$ **Surface Fatigue Strength for Bevel Gears 5, 6, 11, 12**

Contact Stresses for Bevel gears 5, 6, 11 and 12

$C_{md} := 1.5$ **Crowned Teeth. One gear is straddled while the other is Cantilevered.**

$C_{xc} := 1.5$ **Crowned teeth**

$C_a := K_{a1}$ **Application factor**

$C_s := K_S$ **Size factor**

$C_f := 1$ **Surface finish factor. Always 1.**

$C_{\nu.bev} := K_{\nu 1}$ **Dynamic Factor**

$C_b := 0.634$ **Bevel Gear stress factor**

$\nu_p := 0.3$
$\nu_g := 0.3$ **gear and pinion are of same material**

Geometric factors for surface stress:

$N_5 = 25$ $I_5 := 0.0670$

$N_6 = 25$ $I_6 := 0.0670$

$N_{11} = 20$ $I_{11} := 0.0625$

$N_{12} = 20$ $I_{12} := 0.0625$

Obtained values from figure 7-22 in pg. 7-53 of course notes.

$E_p := 2\cdot 10^5 \mathrm{MPa}$ $E_g := 2\cdot 10^5 \mathrm{MPa}$

$$C_{p.bev} := \sqrt{\frac{1}{\pi\cdot\left[\left(\frac{1-\nu_p^{\,2}}{E_p}\right)+\left(\frac{1-\nu_g^{\,2}}{E_g}\right)\right]}} = 1.87\times 10^5\,\frac{\mathrm{kg}^{0.5}}{\mathrm{m}^{0.5}\cdot\mathrm{s}}$$

$$\sigma_c\left(T_p, d, w_{face}, I_i, C_p\right) := C_p\cdot C_b\cdot\sqrt{\frac{2\cdot\left|T_p\right|\cdot C_a\cdot C_{md}\cdot C_s\cdot C_f\cdot C_{xc}}{w_{face}\cdot I_i\cdot d^2\cdot C_{\nu.bev}}}$$

$\sigma_{c5} := \sigma_c\left(T_5, d_5, w_{face5}, I_5, C_{p.bev}\right) = 49.184\cdot\mathrm{MPa}$

$\sigma_{c6} := \sigma_c\left(T_6, d_6, w_{face5}, I_6, C_{p.bev}\right) = 49.184\cdot\mathrm{MPa}$

$\sigma_{c11} := \sigma_c\left(T_{11}, d_{11}, w_{face11}, I_{11}, C_{p.bev}\right) = 29.653\cdot\mathrm{MPa}$

$\sigma_{c12} := \sigma_c\left(T_{12}, d_{12}, w_{face11}, I_{12}, C_{p.bev}\right) = 29.653\cdot\mathrm{MPa}$

Contact stress Saftey Factors

$$SF_{contact}\left(S_{fc}, \sigma_c\right) := \left(\frac{S_{fc}^{\,2}}{\sigma_c^{\,2}}\right)$$

$SF_{contact5} := SF_{contact}\left(S_{fc5}, \sigma_{c5}\right) = 14.108$

$SF_{contact6} := SF_{contact}\left(S_{fc5}, \sigma_{c6}\right) = 14.108$

$SF_{contact11} := SF_{contact}\left(S_{fc5}, \sigma_{c11}\right) = 38.813$

$SF_{contact12} := SF_{contact}\left(S_{fc5}, \sigma_{c12}\right) = 38.813$

E3 Helical Gears (7, 8, 9, 10)

Helical Gear Analysis Gears: 7, 8, 9, 10

$\varphi_7 := 20°$ **Pressure Angles**

$mo_{helic} = 5 \times 10^{-3}\,m$ **Module**

$\psi_7 := 30°$ **Helix Angle**

Tangential force components

$$W_{t7} := \frac{T_7}{r_7} = -398.359\,N \qquad W_{t8} := \frac{T_8}{r_8} = 398.359\,N$$

$$W_{t9} := \frac{T_9}{r_9} = 165.983\,N \qquad W_{t10} := \frac{T_{10}}{r_{10}} = -165.983\,N$$

Axial Force Components

$$W_a(W_t, \psi) := W_t \cdot \tan(\psi)$$

$$W_{a7} := W_a(W_{t7}, \psi_7) = -229.993\,N \qquad W_{a9} := W_a(W_{t9}, \psi_7) = 95.83\,N$$

$$W_{a8} := W_a(W_{t8}, \psi_7) = 229.993\,N \qquad W_{a10} := W_a(W_{t10}, \psi_7) = -95.83\,N$$

The radial components are:

$$W_{r7} := W_{t7} \cdot \tan(\varphi_7) = -144.991\,N \qquad W_{r9} := W_{t9} \cdot \tan(\varphi_7) = 60.413\,N$$

$$W_{r8} := W_{t8} \cdot \tan(\varphi_7) = 144.991\,N \qquad W_{r10} := W_{t10} \cdot \tan(\varphi_7) = -60.413\,N$$

Resultant forces on helical spurs are:

$$W_{R7} := \frac{W_{t7}}{\cos(\varphi_7)\cos(\psi_7)} = -489.507\,N \qquad W_{R9} := \frac{W_{t9}}{\cos(\varphi_7) \cdot \cos(\psi_7)} = 203.961\,N$$

$$W_{R8} := \frac{W_{t8}}{\cos(\varphi_7) \cdot \cos(\psi_7)} = 489.507\,N \qquad W_{R10} := \frac{W_{t10}}{\cos(\varphi_7) \cdot \cos(\psi_7)} = -203.961\,N$$

All helical spurs have same Helix angle and Pressure angle.

Bending Strength for Helical Spur Gears 7, 8, 9, 10

$N_{cyc7} := 10^{10}$ **;conservative number of cycles for operating lifetime (~10 yrs)**

;operating temperatures should be around 100 F with atmospheric cooling

$K_{L7} := K_{L'}(N_{cyc7}) = 0.8$

$S_{fb7'} := 210MPa$ **;nominal bending strength of AGMA A2 Grade 1 Steel**

$$K_{T7} := \begin{cases} 1 & \text{if } T_{f7} < 250 \\ \left[\left(\left(\frac{460 + T_{f7}}{620}\right)\right)\right] & \text{otherwise} \end{cases}$$

$K_{R7} := 1.25$

;for 99.9% reliability

$S_{fb7} := S_{fb}(K_{L7}, K_{T7}, K_{R7}, S_{fb7'}) = 134.406 \cdot MPa$ **Bending Strength for Spur gears 7, 8, 9, 10**

Bending Strength for Planetary Gears 0, 1, 2, 3, 4.

"Gear " 0 is the input shaft link and thus will have no teeth.

Gear 1 is the Ring Gear

Gears 2 and 3 are Planet Gears

Gear 4 is a Sun Gear

$N_{cyc1} := 10^{10}$ **; conservative number of cycles for operating lifetime (~10 yrs)**

$T_{f1} := 100$ **;operating temperatures should be around 100 F with atmospheric cooling**

$K_{L1} := K_{L'}(N_{cyc1}) = 0.8$

$S_{fb1'} := 280MPa$ **;nominal bending strength of AGMA A2 Grade 2 Steel**

$$K_{T1} := \begin{cases} 1 & \text{if } T_{f7} < 250 \\ \left[\left(\left(\frac{460 + T_{f7}}{620}\right)\right)\right] & \text{otherwise} \end{cases}$$

$K_{R1} := 1.25$

;for 99.9% reliability

$S_{fb1} := S_{fb}(K_{L1}, K_{T1}, K_{R1}, S_{fb1'}) = 179.208 \cdot MPa$ **Bending Strength for Planetary Gears 1, 2, 3, 4**

Bending Stresses (Helical Gears 7,8,9,10)

$$\sigma_{b.spur}(W_t, K_a, K_m, K_s, K_B, K_I, w_{face}, mo, J_s, K_\nu) := \frac{W_t \cdot K_a \cdot K_m \cdot K_s \cdot K_B \cdot K_I}{w_{face} \cdot mo \cdot J_s \cdot K_\nu}$$

K-values:

$W_{t7} = -398.359\,N$ $\qquad$ $W_{t8} = 398.359\,N$

$K_{a7} := 1$ **Uniform turbine speed assumed.**

Given that $\quad w_{face7} := 100mm \quad K_{m7} := 1.65$ **Load distribution factor**

$K_s := 1$ Always 1

$K_{B7} := 1$ **Solid disk gear**

$K_{I7} := 1.0$ **Only meshed to one gear, thus non-idler**

$K_{I8} := 1.0$ **Only meshed to one gear, thus non-idler**

Given also

$N_7 = 48 \qquad J_7 := 0.512 \cdot 0.985 = 0.504$

$N_8 = 20 \qquad J_8 := 0.46 \cdot 0.945 = 0.435$

$N_9 = 48 \qquad J_9 := J_7$

$N_{10} = 20 \qquad J_{10} := J_8$

From Figure 7-19, pg. 7-40 of course notes.

Assume medium quality gears are used. $\quad Q_{v7} := 6 \qquad B_7 := \frac{(12 - Q_{v7})^{\frac{2}{3}}}{4} = 0.825$

$V_{t7'} := r_7 \cdot |\omega_7| = 0.23\,\frac{m}{s} \qquad V_{t7} := V_t(V_{t7'}) = 45.354 \qquad V_{t8} := V_{t7}$

$AA7 := 50 + 56(1 - B_7) = 59.773$ **Same for the whole Compound Set**

$K_{\nu 7} := K_\nu(AA7, V_{t7}, B_7) = 0.455 \qquad K_{\nu 8} := K_\nu(AA7, V_{t8}, B_7) = 0.455$

$\sigma_{b7} := \sigma_{b.spur}(W_{t7}, K_{a7}, K_{m7}, K_s, K_{B7}, K_{I7}, w_{face7}, mo_{helic}, J_7, K_{\nu 7}) = -5.724 \cdot MPa$

$\sigma_{b8} := \sigma_{b.spur}(W_{t8}, K_{a7}, K_{m7}, K_s, K_{B7}, K_{I8}, w_{face7}, mo_{helic}, J_8, K_{\nu 8}) = 6.641 \cdot MPa$

$V_{t9'} := r_9 \cdot |\omega_9| = 0.553\,\frac{m}{s} \qquad V_{t9} := V_t(V_{t9'}) = 108.85 \qquad V_{t10} := V_{t9}$

$K_{\nu 9} := K_\nu(AA7, V_{t9}, B_7) = 0.358 \qquad K_{\nu 10} := K_\nu(AA7, V_{t10}, B_7) = 0.358$

$\sigma_{b9} := \sigma_{b.spur}(W_{t9}, K_{a7}, K_{m7}, K_s, K_{B7}, K_{I7}, w_{face7}, mo_{helic}, J_9, K_{\nu 7}) = 2.385 \cdot MPa$

$\sigma_{b10} := \sigma_{b.spur}(|W_{t10}|, K_{a7}, K_{m7}, K_s, K_{B7}, K_{I8}, w_{face7}, mo_{helic}, J_{10}, K_{\nu 8}) = 2.767 \cdot MPa$

Bending Saftey Factors for Helical Gears 7, 8, 9, 10

$$SF_{bending7} := SF_{bending}(S_{fb7}, \sigma_{b7}) = 23.48$$

$$SF_{bending8} := SF_{bending}(S_{fb7}, \sigma_{b8}) = 20.239$$

$$SF_{bending9} := SF_{bending}(S_{fb7}, \sigma_{b9}) = 56.352$$

$$SF_{bending10} := SF_{bending}(S_{fb7}, \sigma_{b10}) = 48.573$$

Surface Fatigue Strength: Helical Gears (7, 8, 9, 10)

$N_{cyc7} = 1 \times 10^{10}$

$C_{L7} := C_{L'}(N_{cyc7}) = 0.679$ **Life factor. Conservative number of cycles for whole lifetime (~10 yrs)**

$C_{H7} := 1$ **Assuming two gears are the same materials**

$S'_{fc7} := 720MPa$ **AGMA A2, Grade 1 Steel**

From Table 7-10
Assuming minimum surface hardness less than 180HB

$C_{T7} := 1$ K.T = 460 + T.F/620 Where Tf is in degrees F. for Tf higher than 250

$C_{R7} := 1.25$ **Realiability of 99.9%**

$$S_{fc7} := \frac{C_{L7} \cdot C_{H7} \cdot (S'_{fc7})}{C_{T7} \cdot C_{R7}} = 391.215 \cdot MPa$$

Surface Fatigue Strength: Helical Gears 7, 8, 9, 10

Contact Stress for Planetary Spurs 1, 2, 3, 4 and Helical 7 ,8 ,9 ,10

Taking Cp as 191Mpa^0.5 from steel on steel see table 7-9

$C_{p.spur} := 190\,(\mathrm{MPa})^{0.5}$ $\quad \varphi := 20°$ **Pressure angles are the same for all the gears.**

$N_g := 40 \qquad C_{m7} := K_{m7} \qquad C_{\nu.spur} := K_{\nu 1} = 0.633$

$$I_{ext}(N_p, N_g) := \sin(\varphi)\,\frac{\cos(\varphi)\left(\dfrac{N_g}{N_g + N_p}\right)}{2}$$

Addition of pinion teeth and gear teeth as gear mesh type is external.

$$I_{int}(N_p, N_g) := \sin(\varphi)\,\frac{\cos(\varphi)\left(\dfrac{N_g}{N_g - N_p}\right)}{2}$$

Subtraction of pinion teeth and gear teeth as gear mesh type is internal.

$I_1 := I_{int}(N_2, N_1) = 0.201$ $\qquad I_7 := I_{ext}(N_7, N_8) = 0.047$

$I_2 := I_{ext}(N_2, N_4) = 0.121$ $\qquad I_8 := I_{ext}(N_7, N_8) = 0.047$

$I_3 := I_{ext}(N_3, N_4) = 0.121$ $\qquad I_9 := I_{ext}(N_9, N_{10}) = 0.047$

$I_4 := I_{ext}(N_3, N_4) = 0.121$ $\qquad I_{10} := I_{ext}(N_9, N_{10}) = 0.047$

$C_{f1} := 1$ Surface finish $\qquad C_{m1} := K_{m1}$

$$\sigma_{c.spur}(W_t, w_{face}, C_m, I_i, d_i) := C_{p.spur}\cdot\sqrt{\frac{|W_t|\cdot C_a\cdot C_m\cdot C_s\cdot C_{f1}}{w_{face}\cdot I_i\cdot d_i\cdot C_{\nu.spur}}}$$

Equation appliacble to Helical gears

Uses torque from planet gears

$\sigma_{c1} := \sigma_{c.spur}(W_{t2}, w_{face1}, C_{m1}, I_1, d_1) = 77.254\cdot\mathrm{MPa}$

$\sigma_{c2} := \sigma_{c.spur}(W_{t2}, w_{face1}, C_{m1}, I_2, d_2) = 223.013\cdot\mathrm{MPa}$

$\sigma_{c3} := \sigma_{c.spur}(W_{t3}, w_{face1}, C_{m1}, I_3, d_3) = 223.013\cdot\mathrm{MPa}$

$\sigma_{c4} := \sigma_{c.spur}(W_{t4}, w_{face1}, C_{m1}, I_4, d_4) = 91.044\cdot\mathrm{MPa}$

$\sigma_{c7} := \sigma_{c.spur}(W_{t7}, w_{face7}, C_{m7}, I_7, d_7) = 181.733\cdot\mathrm{MPa}$

$\sigma_{c8} := \sigma_{c.spur}(|W_{t8}|, w_{face7}, C_{m7}, I_8, d_8) = 281.54\cdot\mathrm{MPa}$

$\sigma_{c9} := \sigma_{c.spur}(W_{t9}, w_{face7}, C_{m7}, I_9, d_9) = 117.308\cdot\mathrm{MPa}$

$\sigma_{c10} := \sigma_{c.spur}(W_{t10}, w_{face7}, C_{m7}, I_{10}, d_{10}) = 181.733\cdot\mathrm{MPa}$

Contact stress Safety Factors for Helical Gear 7 ,8 ,9 ,10

Compound Set

$SF_{contact7} := SF_{contact}(S_{fc7}, \sigma_{c7}) = 4.634$

$SF_{contact8} := SF_{contact}(S_{fc7}, \sigma_{c8}) = 1.931$

$SF_{contact9} := SF_{contact}(S_{fc7}, \sigma_{c9}) = 11.122$

$SF_{contact10} := SF_{contact}(S_{fc7}, \sigma_{c10}) = 4.634$

Contact stress Saftey Factors for Planetary Spurs 1, 2, 3, 4

$$SF_{.contact}(S_{fc}, \sigma_c) := \left(\frac{S_{fc}^{\ 2}}{\sigma_c^{\ 2}}\right)$$

Planetary Set

$SF_{contact1} := SF_{contact}(S_{fc1}, \sigma_{c1}) = 25.644$

$SF_{contact2} := SF_{contact}(S_{fc1}, \sigma_{c2}) = 3.077$

$SF_{contact.2a} := SF_{contact2} = 3.077$

$SF_{contact3} := SF_{contact}(S_{fc1}, \sigma_{c3}) = 3.077$

$SF_{contact.3a} := SF_{contact3} = 3.077$

$SF_{contact4} := SF_{contact}(S_{fc1}, \sigma_{c4}) = 18.464$

F: Shaft Sizing Calculations

F1 Shaft Sizing for Shaft Number One

Shaft Number 1

Given

Dimensions

$x_1 := 0$ $\quad x_2 := 0.25$ $\quad T_1 := 509.9$

torsional efficiency of bearings $\quad n_b := 0$ $\quad \varepsilon_{bearings} := 0.99^{n_b}$

Torsional inputs

$T_1 = 509.9$

$T_{bearings} := \left(1 - \varepsilon_{bearings}\right)$ $\qquad T_{bearings} = 0.01$

$T_{gear1} := T_1 - T_{bearings}$ $\qquad T_{gear1} = 509.89$

Forces at Y

$F_{turbine} := 650 \cdot 9.81 = 6.377 \times 10^3$

Moments

$M_{planet} := F_{turbine} \cdot x_2 = 2.232 \times 10^3$

Sum of the forces about y

$F_{turbine} + F_{planet} = 0$ $\qquad F_{planet} := -F_{turbine} = -6.377 \times 10^3$

Sum of the moment at x

$-T_{gear1} + T - T_{bearings} = 0$

Sum of the forces at x

$F_{gear1,x} - F_{planet,x} = 0$

Define singularity function

$$S(x,a,n) := \begin{cases} (x-a)^n & \text{if } (x-a) \ge 0 \wedge n \ge 0 \\ 0 & \text{if } (x-a) < 0 \wedge n \ge 0 \\ \infty & \text{if } (x-a) = 0 \wedge n \le -1 \\ 0 & \text{if } (x-a) \ne 0 \wedge n \le -1 \end{cases}$$

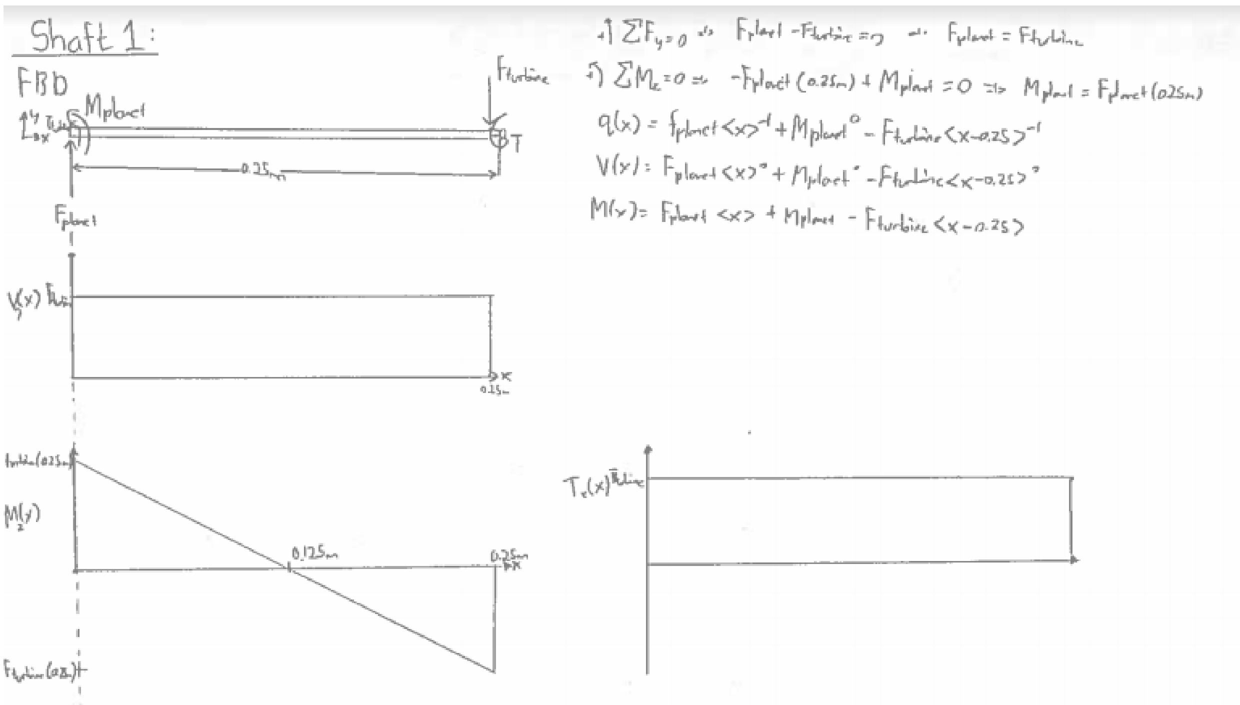


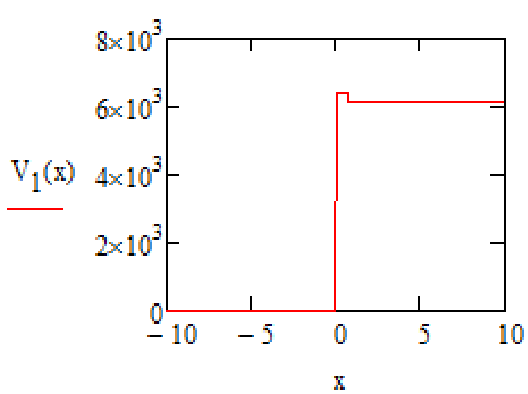
Figure F1: Different Diagrams for shaft number One

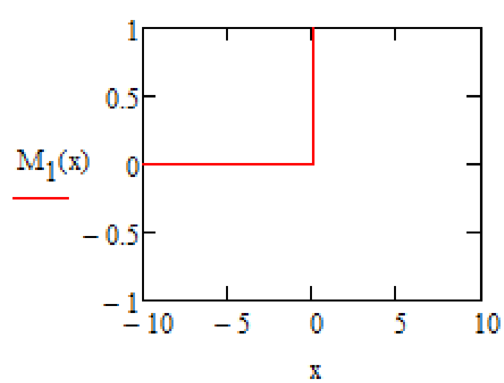
Singularity function

$$q_1(x) := F_{turbine} \cdot S(x, x_1, -1) - F_{gear1\_y} \cdot S(x, x_3, -1)$$

$$V_1(x) := F_{turbine} \cdot S(x, x_1, 0) - F_{gear1\_y} \cdot S(x, x_3, 0)$$

$$M_1(x) := F_{turbine} \cdot S(x, x_1, 1) - F_{gear1\_y} \cdot S(x, x_3, 0)$$





Geometric properties of the Shaft

$$I := \left(\frac{\pi \cdot d^4}{4 \cdot 2^4}\right)$$

Moment of inertia

$$J_o := \pi \cdot \frac{d^4}{32}$$

Polar moment of inertia

$$Q := \frac{4d \cdot \pi \cdot d^2 \pi \cdot d^2}{6 \cdot \pi 8 \cdot 8}$$

First moment of inertia

$$t := d$$

Thickness

Material AISI 316 Properties:

$$\rho := 7870 \ \frac{kg}{m^3}$$

$E := 205000000000$ $S_{ut} := 619838842$ $S_y := 415064497$ $G_{stainless} := 82000000000$

Fatigue Analysis
Calculated the minimum diameter needed for this application

Step 1. Calculate Alternating and Mean stresses

There is assumed to be no fluctuating torsion, so the mean and alternating torsion is zero
There is assumed to be no fluctuating torsion, so the mean and alternating torsion is zero

$M_a := F_{turbine} \cdot x_2 = 2.232 \times 10^3$ $\quad T_m := T_1 = 509.9$ $\quad T_a := 0$ $\quad M_m := 0$

Surface condition modification factor

$b := -0.265$ $\quad a := 4.51$

$$k_a := a \cdot S_{ut}^{\ b} = 0.021$$

The size modification factor which is a function of d

$$k_b(D) := \begin{cases} 1.24 \cdot d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51D^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

Load modification factor

$k_c := 1$ bending is dominant

Temperature modification factor

$k_d := 1$ no temperature effect

$k_e := 0.753$ 99.9% reliability

Miscellaneous-effects modification factor

$k_f := 1$ Excellent corrosion resistance

$S_e := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot 0.504 \cdot S_{ut} = 5.365 \times 10^6$

High Cycle Fatigue Domain (10000 to 10000000) cycles

The ultimate strength is close to 620 $f := 0.86$

$$a_h := \frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \qquad b_h := \frac{-1 \cdot \log\left(f \cdot \frac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}}\right)}{3}$$

$$S_f := a_h \cdot (10000000)^{b_h}$$

For the keyway, parallel is used because of the low cost and is able to sustain loads
The stress concentration factor for this type of key is:

$K_t := 2.2$ $K_{ts} := 3$

The corner radius is about 0.02 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$q := 0.72$ $q_s := 0.77$

$K_f := 1 + q \cdot (K_t - 1) = 1.864$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.54$

$$0.002794 \le D \le 0.058$$

The required safety factor

$n_f := 2.92$

General Diameter Equation for fatigue analysis

$$1 = 16\cdot\frac{n_f}{\pi\, d}\cdot\sqrt{\frac{4\cdot(K_f\cdot M_a)^2 + 3\cdot(K_{fs}\cdot T_a)^2}{S_f^{\,2}} + \frac{\left[4\cdot(K_f\cdot M_m)^2 + 3\cdot(K_{fs}\cdot T_m)^2\right]}{S_y^{\,2}}}$$

$$1 = 16\cdot\frac{n_f}{\pi\, d}\cdot\sqrt{\frac{4\cdot(K_f\cdot M_a)^2 + 3\cdot(K_{fs}\cdot T_a)^2}{\left[\frac{(f\cdot S_{ut})^2}{S_e\cdot 1.24\cdot d^{-0.107}}\cdot(10000000)^{\frac{-1\cdot\log\left(f\cdot\frac{S_{ut}}{S_e\cdot 1.24\cdot d^{-0.107}}\right)}{3}}\right]^2} + \frac{\left[4\cdot(K_f\cdot M_m)^2 + 3\cdot(K_{fs}\cdot T_m)^2\right]}{S_y^{\,2}}}$$

Find d using Excel goal seeking function, which is located in appendix

The diameter happens to be 0.0526865 meters: $D_1 := 0.0526865$

Mass will be:

$$m := \rho\cdot\pi\cdot\left(\frac{D_1}{2}\right)^2\cdot x_3 = 12.011\ \mathrm{kg}$$

Life expectancy:

$$\omega_1 := 0.72\ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$\mathrm{Life}_1 := \frac{2\cdot\pi\cdot 10000000}{\omega_1\cdot 3600} = 2.424\times 10^4\ \ \mathrm{hours}$$

$$\text{or}\qquad \mathrm{years} := \frac{\mathrm{Life}_1}{24\cdot 365} = 2.767$$

Check the twisting angle criterion

$$J_{D1} := \pi\cdot\frac{D_1^{\,4}}{32} = 7.565\times 10^{-7}$$

$$\phi_{per\_length} := \frac{T}{J_{D1}\cdot G_{stainless}}\cdot\frac{180}{\pi} = 0.118$$

$$\phi_{per\_length} < \phi_{max} \qquad \text{or} \qquad 0.351 < 2$$

Therefore the torsional deflection is satisfied

Shaft Design Criteria:
1.) The deflection at gear must be less than 0.000128 m to ensure proper mating
2.) Slope between the gear axis is less than 0.03 degrees
3.) Maximum angular deflection at bearings between 0.001-0.004 rad
4.) Shaft angular deflection is less than 0.00007 rad at non self aligning bearing

Integrating M to find θ and y

$$I_{D1} := \frac{\pi \cdot D_1^{\ 4}}{4 \cdot 2^4} = 3.782 \times 10^{-7} \qquad \theta = \int_0^x \frac{M(x)}{E \cdot I_D}\, dx$$ illustration only

Find θ_temp and y_temp to be a function of x and the unkonw integration constants C1 and C2

$$\theta := \frac{T \cdot (x_3 - x_2)^2}{2E \cdot I_{D1}} = 1.007 \times 10^{-4}$$ Which is below 0.001rad and meet crietrion 3

$$y = \int_0^x \theta\, dx + c_1 \cdot x + c_2$$

$$y := \frac{T \cdot (x_3 - x_2)^3}{3 \cdot 2E \cdot I_{D1}} = 1.175 \times 10^{-5}$$

Boundary conditions
-assumption: the deflection at the bearing is zero

$$y(x_1) = 0 = y(x_1) + c_1(x_1) + c_2$$

F2 Shaft Sizing for Shaft Number Two

Shaft Number 2

Given

Dimensions

$x_1 := 0$ $x_3 := 0.7$ $T := 127.475$

$x_2 := 0.35$

torsional efficiency of bearings $n_b := 1$ $\varepsilon_{bearings} := 0.99^{n_b}$

Torsional inputs

$T := 127.475$

$T_{bearings} := \left(1 - \varepsilon_{bearings}\right)$ $T_{bearings} = 0.01$

$T_{gear1} := T - T_{bearings}$ $T_{gear1} = 127.465$

Forces at Y

$F_{gear1\_y} := 239.015$ $F_{bearing1\_y} := 239.015$

$F_{planet\_y} := 0$

Sum of the forces about y

$F_{gear1\_y} + F_{bearing1\_y} + F_{planet} = 0$

Sum of the moment at x

$-T_{gear1} + T - T_{bearings} = 0$

Sum of the forces at x

$F_{gear1,x} - F_{planet,x} = 0$

Define singularity function

$$S(x,a,n) := \begin{cases} (x-a)^n & \text{if } (x-a) \geq 0 \wedge n \geq 0 \\ 0 & \text{if } (x-a) < 0 \wedge n \geq 0 \\ \infty & \text{if } (x-a) = 0 \wedge n \leq -1 \\ 0 & \text{if } (x-a) \neq 0 \wedge n \leq -1 \end{cases}$$

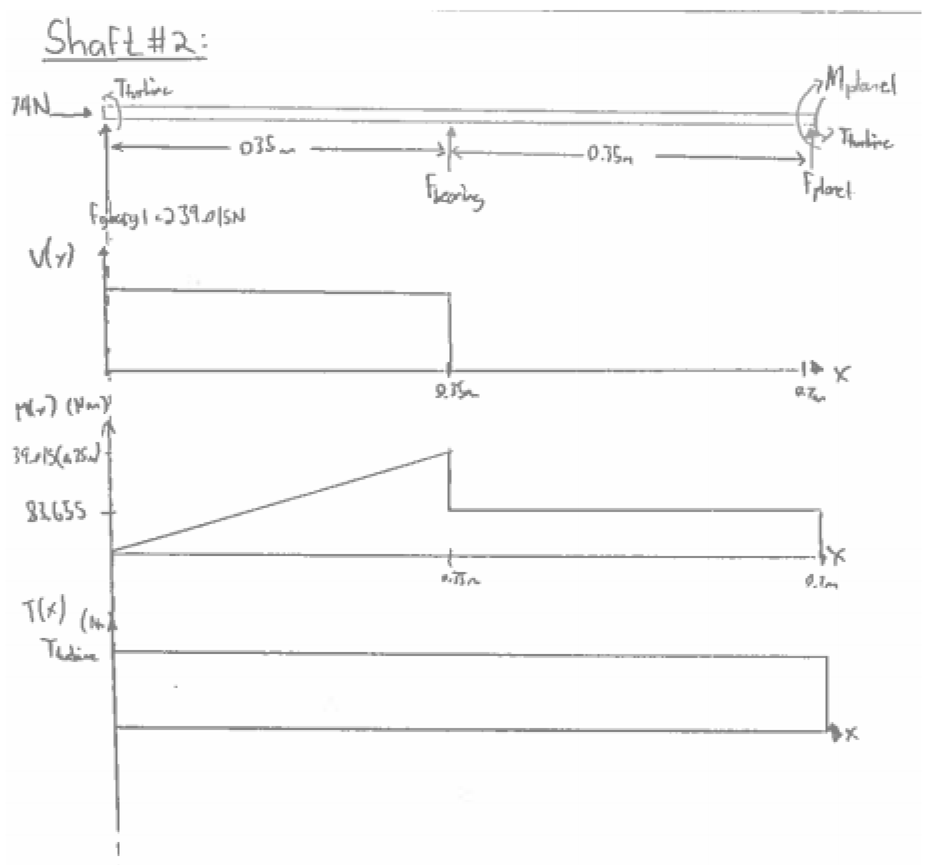


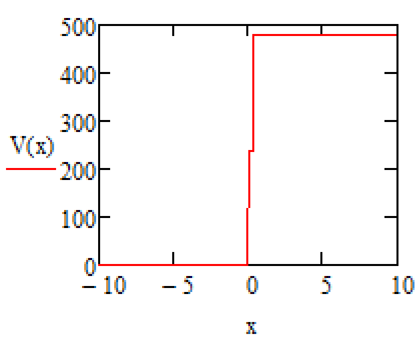
Figure F2: Different Diagrams of Shaft Number Two

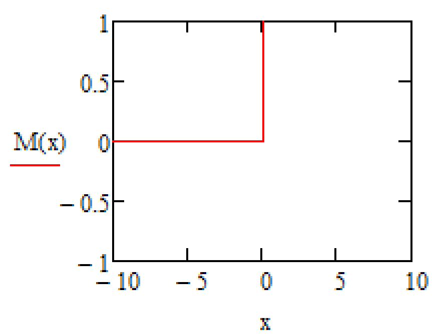
Singularity function

$$q(x) := F_{gear1\_y} \cdot S(x, x_1, -1) + F_{bearing1\_y} \cdot S(x, x_2, -1) + F_{planet\_y} \cdot S(x, x_3, -1)$$

$$V(x) := F_{gear1\_y} \cdot S(x, x_1, 0) + F_{bearing1\_y} \cdot S(x, x_2, 0) + F_{planet\_y} \cdot S(x, x_3, 0)$$

$$M(x) := F_{gear1\_y} \cdot S(x, x_1, 1) + F_{bearing1\_y} \cdot S(x, x_2, 1) + F_{planet\_y} \cdot S(x, x_3, 1)$$





Geometric properties of the Shaft

$$I := \left(\frac{\pi \cdot d^4}{4 \cdot 2^4}\right)$$ Moment of inertia

$$J_o := \pi \cdot \frac{d^4}{32}$$ Polar moment of inertia

$$Q := \frac{4d \cdot \pi \cdot d^2 \pi \cdot d^2}{6 \cdot \pi 8 \cdot 8}$$ First moment of inertia

$$t := d$$ Thickness

Material AISI 316 Properties:

$$\rho := 7870 \frac{kg}{m^3}$$

$E := 205000000000$ $\quad S_{ut} := 619838842$ $\quad S_y := 415064497$ $\quad G_{stainless} := 82000000000$

Fatigue Analysis

Calculated the minimum diameter needed for this application

Step 1. Calculate Alternating and Mean stresses

There is assumed to be no fluctuating torsion, so the mean and alternating torsion is zero

$M_a := F_{gear1\_y} \cdot x_2 = 83.655$ $\quad M_m := 0$

$T_m := T = 127.475$ $\quad T_a := 0$

Surface condition modification factor

$b := -0.265$ $\quad a := 4.51$

$$k_a := a \cdot S_{ut}^{\,b} = 0.021$$

The size modification factor which is a function of d

$$k_b(D) := \begin{cases} 1.24 \cdot d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51D^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

Load modification factor

$k_c := 1$ bending is dominant

Temperature modification factor

$k_d := 1$ no temperature effect

$k_e := 0.753$ 99.9% reliability

Miscellaneous-effects modification factor

$k_f := 1$ Excellent corrosion resistance

$$S_e := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot 0.504 \cdot S_{ut} = 4.963 \times 10^6$$

High Cycle Fatigue Domain (10000 to 10000000) cycles

The ultimate strength is close to 620 $f := 0.86$

$$a_h := \frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \qquad b_h := \frac{-1 \cdot \log\left(f \cdot \dfrac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}}\right)}{3}$$

$$S_f := a_h \cdot (10000000)^{b_h}$$

For the keyway, parallel is used because of the low cost and is able to sustain loads
The stress concentration factor for this type of key is:

$K_t := 2.2$ $K_{ts} := 3$

The corner radius is about 0.02 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$q := 0.72$ $q_s := 0.77$

$$K_f := 1 + q \cdot (K_t - 1) = 1.864$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.54$$

$$0.002794 \le D \le 0.058$$

The required safety factor

$$n_f := 2.92$$

General Diameter Equation for fatigue analysis

$$1 = 16 \cdot \frac{n_f}{\pi\, d} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{S_f^{\,2}} + \frac{\left\lfloor 4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2 \right\rfloor}{S_y^{\,2}}}$$

$$1 = 16 \cdot \frac{n_f}{\pi\, d} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left[\frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \cdot (10000000)^{\frac{-1 \cdot \log\left(f \cdot \frac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}} \right)}{3}} \right]^2} + \frac{\left\lfloor 4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2 \right\rfloor}{S_y^{\,2}}}$$

Find d using Excel goal seeking function, which is located in appendix

The diameter happens to be 0.0074096 meters: $D_2 := 0.007409$

Mass will be:

$$m := \rho \cdot \pi \cdot \left(\frac{D_2}{2} \right)^2 \cdot x_3 = 0.238 \quad \text{kg}$$

Life expectancy:

$$\omega_2 := 2.88 \quad \frac{\text{rad}}{\text{s}}$$

$$Life_2 := \frac{2 \cdot \pi \cdot 10000000}{\omega_2 \cdot 3600} = 6.06 \times 10^3 \quad \text{hours}$$

$$\text{or} \qquad years := \frac{Life_2}{24 \cdot 365} = 0.692$$

Check the twisting angle criterion

$$J_{D.2} := \pi \cdot \frac{D_2^{\,4}}{32} = 2.958 \times 10^{-10}$$

$$\phi_{per\_length} := \frac{T}{J_{D.2} \cdot G_{stainless}} \cdot \frac{180}{\pi} = 301.089$$

$\phi_{per\_length} < \phi_{max}$ or $301.089 > 2$

The Diameter should increase based on failing this criterion and also not meeting the minimum bearing diamter whichis 40 mm

$D_{22} := 0.04$

$$J_{D.22} := \pi \cdot \frac{D_{22}^{4}}{32} = 2.513 \times 10^{-7}$$

$$\phi_{per\_length.22} := \frac{T}{J_{D.22} \cdot G_{stainless}} \cdot \frac{180}{\pi} = 0.354$$

$\phi_{per\_length} < \phi_{max}$ or $0.354 < 2$

Therefore the torsional deflection is satisfied

Shaft Design Criteria:
1.) The deflection at gear must be less than 0.000128 m to ensure proper mating
2.) Slope between the gear axis is less than 0.03 degrees
3.) Maximum angular deflection at bearings between 0.001-0.004 rad
4.) Shaft angular deflection is less than 0.00007 rad at non self aligning bearing

Integrating M to find θ and y

$$I_{D.22} := \frac{\pi \cdot D_{22}^{4}}{4 \cdot 2^{4}} = 1.257 \times 10^{-7} \quad \theta = \int_0^x \frac{M(x)}{E \cdot I_D} \, dx$$ illustration only

Find θ_temp and y_temp to be a function of x and the unkonw integration constants C1 and C2

$$\theta := \frac{T \cdot (x_3 - x_2)^2}{2E \cdot I_{D.22}} = 3.031 \times 10^{-4}$$ Which is below 0.001rad and meet crietrion 3

$$y = \int_0^x \theta \, dx + c_1 \cdot x + c_2$$

Boundary conditions
-assumption: the deflection at the bearing is zero

$$y := \frac{T \cdot (x_3 - x_2)^3}{3 \cdot 2E \cdot I_{D.22}} = 3.536 \times 10^{-5}$$

$$y(x_1) = 0 = y(x_1) + c_1(x_1) + c_2$$

F3 Shaft sizing for Shaft 3

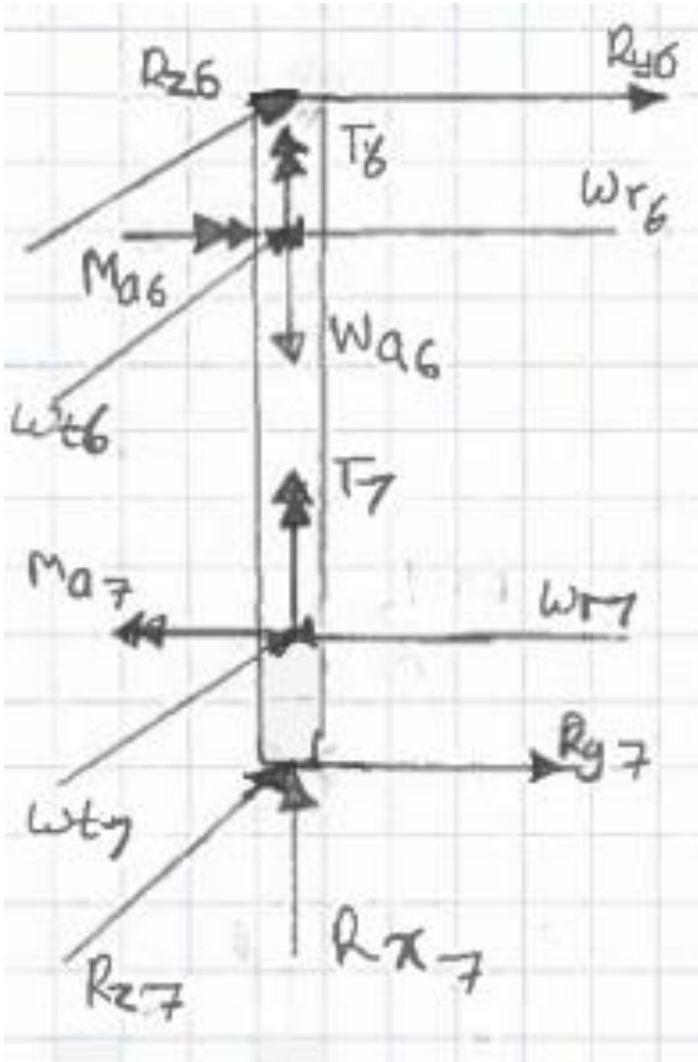


Figure F3: FBD for Shaft 3 (notation consistent with gears)

Shaft 3 Design Calculations

$a := 0.1\text{m} \quad L_3 := 7\text{m} \quad b_3 := L_3 - a = 6.9\,\text{m}$

$$R_{y3} := W_{r6} + W_{r7} - \frac{\left(W_{r6} \cdot a + W_{r7} \cdot b_3\right)}{L_3}$$

$S_{ut} := 619.83\text{MPa} = 6.198 \times 10^{8}\,\text{Pa} \quad S_y := 415\text{MPa} = 4.15 \times 10^{8}\,\text{Pa}$

$S'_e := 0.504 \cdot S_{ut} = 3.124 \times 10^{8}\,\text{Pa}$

The k factors:

Surface condiation modification factor

$$k_a := a_a \cdot \left(\frac{S_{ut}}{\text{Pa}}\right)^{b_a} \qquad a_a := 4.51 \qquad b_a := -0.265$$

Machined or Cold drawn

The size modification factor which is a function of d

$$k_b(d) := \begin{cases} 1.24d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51d^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

$k_c := 1$ **Load modification factor for when the combined loading is dominant**

$k_d := 1$ **Temperature modification factor**

$k_e := 0.753$ **Reliability factor for 99.9% reliability**

$k_f := 1$ **Miscellaneous-effects modification factor**

Here $S_e(d)$ is a funciton of d

$S_e(d) := k_a \cdot k_b(d) \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$

Use the DE-criterion (Von Mise stress) for the 3D stress state

$q := 0.85$

$q_s := q = 0.85$

$K_t := 2.2$

$K_{ts} := 3$

$K_{fs} := 1 + q_s \cdot \left(K_{ts} - 1\right) = 2.7$

$K_f := 1 + q \cdot \left(K_t - 1\right) = 2.02$

Shaft 3 Design Calculations

$a := 0.1\text{m} \quad L_3 := 7\text{m} \quad b_3 := L_3 - a = 6.9\,\text{m}$

$$R_{y3} := W_{r6} + W_{r7} - \frac{\left(W_{r6} \cdot a + W_{r7} \cdot b_3\right)}{L_3}$$

$S_{ut} := 619.83\text{MPa} = 6.198 \times 10^{8}\,\text{Pa} \quad S_y := 415\text{MPa} = 4.15 \times 10^{8}\,\text{Pa}$

$S'_e := 0.504 \cdot S_{ut} = 3.124 \times 10^{8}\,\text{Pa}$

The k factors:

Surface condiation modification factor

$$k_a := a_a \cdot \left(\frac{S_{ut}}{Pa}\right)^{b_a} \qquad a_a := 4.51 \qquad b_a := -0.265$$

Machined or Cold drawn

The size modification factor which is a function of d

$$k_b(d) := \begin{cases} 1.24d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51d^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

$k_c := 1$ **Load modification factor for when the combined loading is dominant**

$k_d := 1$ **Temperature modification factor**

$k_e := 0.753$ **Reliability factor for 99.9% reliability**

$k_f := 1$ **Miscellaneous-effects modification factor**

Here $S_e(d)$ is a funciton of d

$S_e(d) := k_a \cdot k_b(d) \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$

Use the DE-criterion (Von Mise stress) for the 3D stress state

$q := 0.85$

$q_s := q = 0.85$

$K_t := 2.2$

$K_{ts} := 3$

$K_{fs} := 1 + q_s \cdot \left(K_{ts} - 1\right) = 2.7$

$K_f := 1 + q \cdot \left(K_t - 1\right) = 2.02$

$$M_a := \frac{(R_{y3} \cdot a)}{2} + W_{a6} \cdot r_6 = -16.71\,\mathrm{J}$$

$$M_m := M_a$$

$$T_a := |T_6| + |T_7| = 95.606\,\mathrm{J}$$

$$T_m := T_a$$

$$n_f := 2$$

$$d := 50\mathrm{mm}$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.104\,\mathrm{m}$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.108\,\mathrm{m}$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.109\,\mathrm{m}$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.109\,\mathrm{m}$$

$$M_a := \frac{(R_{y3} \cdot a)}{2} + W_{a6} \cdot r_6 = -16.71\,J$$

$$M_m := M_a$$

$$T_a := |T_6| + |T_7| = 95.606\,J$$

$$T_m := T_a$$

$$n_f := 2$$

$$d := 50mm$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.104\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.108\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.109\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4 (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.109\,m$$

F4 Shaft sizing for Shaft 4

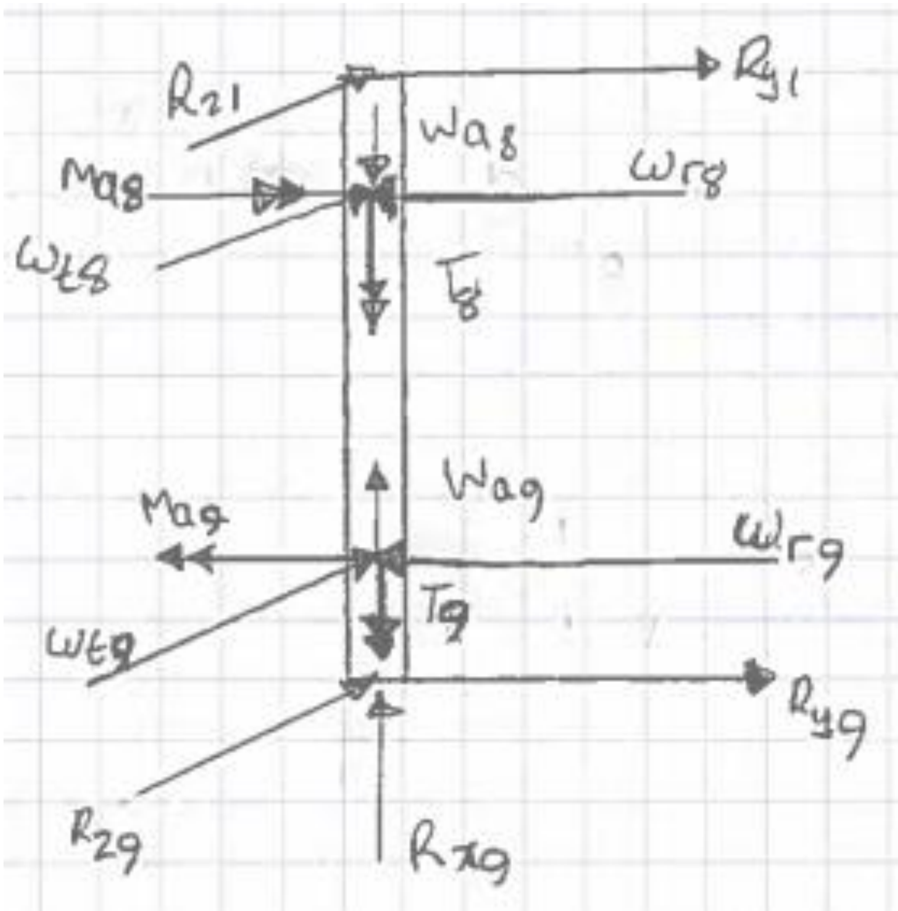


Figure F4: FBD for Shaft 4

$a := 0.1m \quad L_4 := 0.5m \quad b_4 := L_4 - a$

$$R_{y4} := W_{r8} + W_{r9} - \frac{(W_{r8} \cdot a + W_{r9} \cdot b_4)}{L_4}$$

$S_{ut} := 619.83MPa = 6.198 \times 10^8 Pa \quad S_y := 415MPa = 4.15 \times 10^8 Pa$

$S'_e := 0.504 \cdot S_{ut} = 3.124 \times 10^8 Pa$

The k factors:

Surface condition modification factor

$$k_a := a_a \cdot \left(\frac{S_{ut}}{Pa}\right)^{b_a} \quad a_a := 4.51 \quad b_a := -0.265$$

The size modification factor which is a function of d

$$k_b(d) := \begin{cases} 1.24d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51d^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

$k_c := 1$ **Load modification factor for when the combined loading is dominant**

$k_d := 1$ **Temperature modification factor**

$k_e := 0.753$ **Reliability factor for 99.9% reliability**

$k_f := 1$ **Miscellaneous-effects modification factor**

Here $S_e(d)$ is a funciton of d

$S_e(d) := k_a \cdot k_b(d) \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$

Use the DE-criterion (Von Mise stress) for the 3D stress state

$q := 0.82$

$q_s := q = 0.82$

$K_t := 2.2$

$K_{ts} := 3$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.64$

$K_f := 1 + q \cdot (K_t - 1) = 1.984$

$M_a := \frac{(R_{y4} \cdot a)}{2} + W_{a8} \cdot r_8 = 17.903\,J$

$M_m := M_a$

$T_a := |T_8| + |T_9| = 39.836\,J$

$T_m := T_a$

$n_f := 2$

$d := 50mm$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.079\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.081\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.081\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y{}^2}}\right]^{\frac{1}{3}} = 0.081\,m$$

F5 Shaft sizing for Shaft

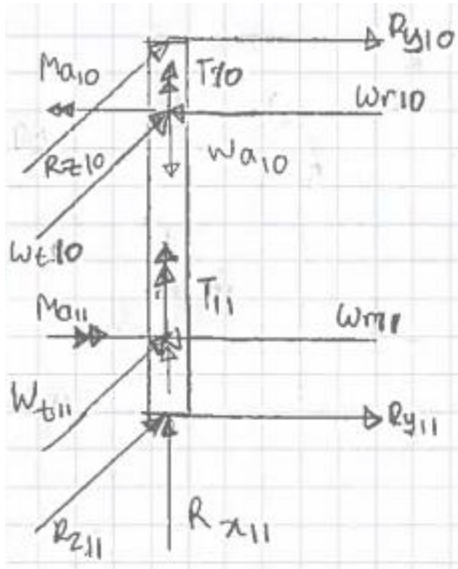


Figure F5: FBD for Shaft 5

Shaft Design Calculations 5

$a := 0.1\text{m} \quad L_5 := 2.8\text{m} \quad b := L_5 - a = 2.7\,\text{m}$

$$R_{y5} := W_{r10} + W_{r11} - \frac{(W_{r10} \cdot a + W_{r11} \cdot b)}{L_5}$$

$S_{ut} := 619.83\text{MPa} = 6.198 \times 10^8\,\text{Pa} \quad S_y := 415\text{MPa} = 4.15 \times 10^8\,\text{Pa}$

$S'_e := 0.504 \cdot S_{ut} = 3.124 \times 10^8\,\text{Pa}$

The k factors:

Surface condition modification factor

$$k_a := a_a \cdot \left(\frac{S_{ut}}{\text{Pa}}\right)^{b_a} \quad a_a := 4.51 \quad b_a := -0.265$$

The size modification factor which is a function of d

$$k_b(d) := \begin{cases} 1.24d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51d^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

$k_c := 1$ **Load modification factor for when the combined loading is dominant**

$k_d := 1$ **Temperature modification factor**

$k_e := 0.753$ **Reliability factor for 99.9% reliability**

$k_f := 1$ **Miscellaneous-effects modification factor**

Here $S_e(d)$ is a funciton of d

$$S_e(d) := k_a \cdot k_b(d) \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$$

Use the DE-criterion (Von Mise stress) for the 3D stress state

$$q := 0.82$$

$$q_s := q = 0.82$$

$$K_t := 2.2$$

$$K_{ts} := 3$$

$$K_{fs} := 1 + q_s \cdot \left(K_{ts} - 1\right) = 2.64$$

$$K_f := 1 + q \cdot \left(K_t - 1\right) = 1.984$$

$$M_a := \frac{\left(R_{y5} \cdot a\right)}{2} + W_{a10} \cdot r_{10} = -7.723\,\mathrm{J}$$

$$M_m := M_a$$

$$T_a := \left|T_{10}\right| + \left|T_{11}\right| = 16.598\,\mathrm{J}$$

$$T_m := T_a$$

$$n_f := 2$$

$$d := 50\mathrm{mm}$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.059\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.06\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.06\,m$$

$$d_{iter} := \left[\frac{16 \cdot n_f}{\pi} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left(S_e\left(\frac{d_{iter}}{mm}\right)\right)^2} + \frac{4(K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2}{S_y^{\,2}}}\right]^{\frac{1}{3}} = 0.06\,m$$

F6 Shaft Sizing for Shaft Number Six

Shaft Number 6

Given

Dimensions

$x_4 := 0$ $x_5 := 0.25$ $x_6 := 0.5$ $x_7 := 0.75$ $x_8 := 1$

$T_6 := 33.197$

torsional efficiency of bearings $n_b := 2$ $\varepsilon_{bearings} := 0.99^{n_b}$

Torsional inputs

$T_6 = 33.197$

$T_{bearings} := (1 - \varepsilon_{bearings})$ $T_{bearings} = 0.01$

$T_{pumpjack} := T_6 - T_{bearings}$ $T_{pumpjack} = 33.187$

Forces at Y

$F_{pumpjack} := -3629.7$ $R_{A6} := 3629.7$ $F_{gear12\_y} := -31.726$ $R_{B6} := 3629.7$

Moments

$M_{max6} := 910$

Geometric properties of the Shaft

$I := \left(\frac{\pi \cdot d^4}{4 \cdot 2}\right)$ Moment of inertia

$J_o := \pi \cdot \frac{d^4}{32}$ Polar moment of inertia

$Q := \frac{4d \cdot \pi \cdot d^2 \pi \cdot d^2}{6 \cdot \pi 8 \cdot 8}$ First moment of inertia

$t := d$ Thickness

Material AISI 316 Properties:

$\rho := 7870 \frac{kg}{m^3}$

$E := 205000000000$ $S_{ut} := 619838842$ $S_y := 415064497$ $G_{stainless} := 82000000000$

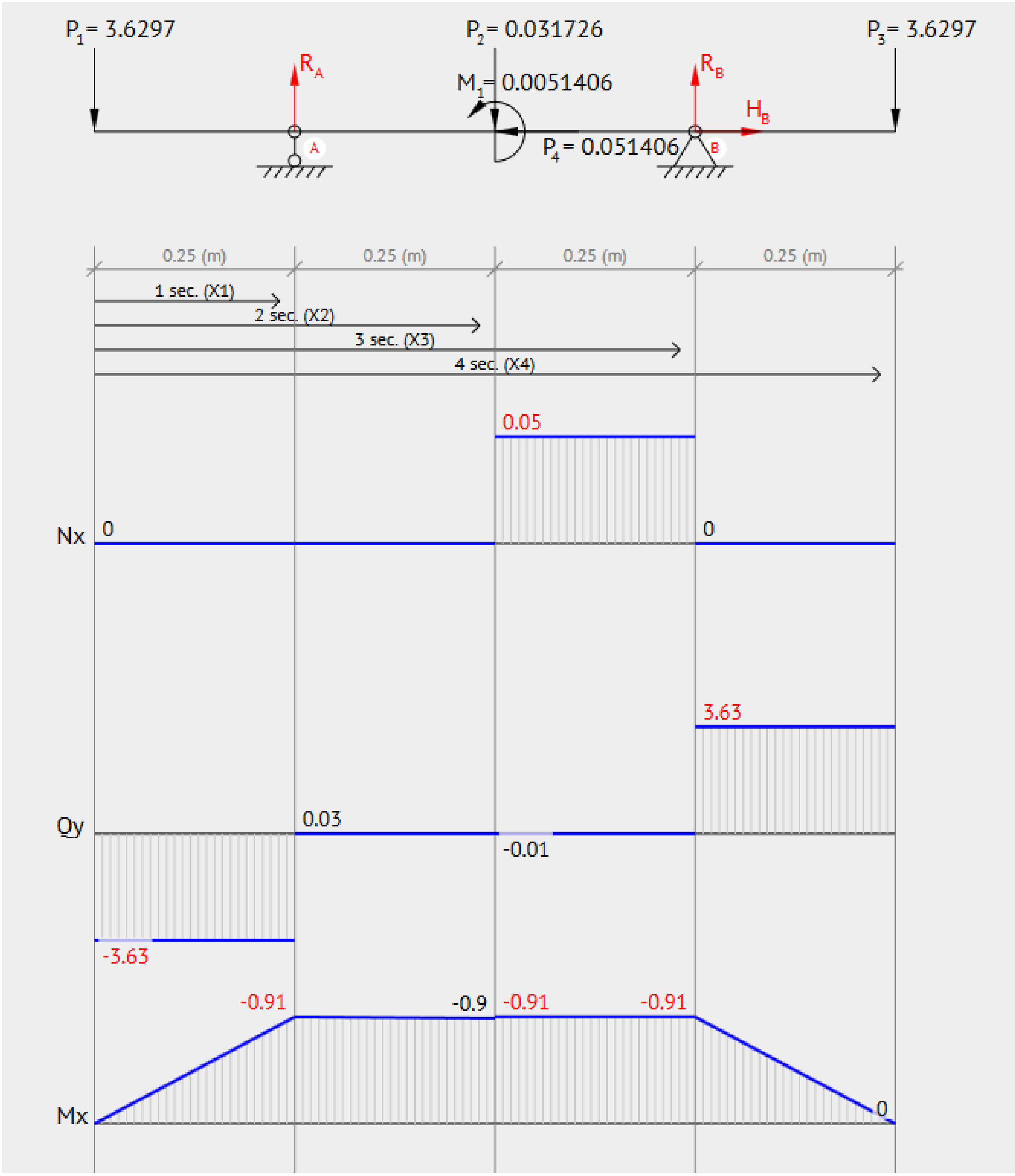


Figure F6: Different Diagrams of Shaft Number Six (kN vs. m)

Fatigue Analysis
Calculated the minimum diameter needed for this application

Step 1. Calculate Alternating and Mean stresses

There is assumed to be no fluctuating torsion, so the mean and alternating torsion is zero

$M_{a6} := M_{max6} = 910$ $\qquad M_m := 0$

$T_m := T_6 = 33.197$ $\qquad T_a := 0$

Surface condition modification factor

$b := -0.265 \qquad a := 4.51$

$k_a := a \cdot S_{ut}{}^{b} = 0.021$

The size modification factor which is a function of d

$$k_b(D) := \begin{vmatrix} 1.24 \cdot d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51D^{-0.157} & \text{if } 51 < d \le 254 \end{vmatrix}$$

Load modification factor

bending is dominant

Temperature modification factor

no temperature effect

$k_c := 1$

$k_d := 1$

Miscellaneous-effects modification factor

$k_e := 0.753$ $\qquad$ 99.9% reliability

$k_f := 1$ Excellent corrosion resistance

$S_e := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot 0.504 \cdot S_{ut} = 5.365 \times 10^6$

High Cycle Fatigue Domain (10000 to 10000000) cycles

The ultimate strength is close to 620 $f := 0.86$

$$a_h := \frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \qquad b_h := \frac{-1 \cdot \log\left(f \cdot \frac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}}\right)}{3}$$

$$S_f := a_h \cdot (10000000)^{b_h}$$

For the keyway, parallel is used because of the low cost and is able to sustain loads
The stress concentration factor for this type of key is:

$K_t := 2.2$ $K_{ts} := 3$

The corner radius is about 0.02 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$q := 0.72$ $q_s := 0.77$

$K_f := 1 + q \cdot (K_t - 1) = 1.864$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.54$

$0.002794 \le D \le 0.058$

The required safety factor

$n_f := 2.92$

General Diameter Equation for fatigue analysis

$$1 = 16 \cdot \frac{n_f}{\pi d} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{S_f^2} + \frac{\left[4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2\right]}{S_y^2}}$$

General Diameter Equation for fatigue analysis

$$1 = 16\cdot\frac{n_f}{\pi\, d}\cdot\sqrt{\frac{4\cdot(K_f\cdot M_a)^2 + 3\cdot(K_{fs}\cdot T_a)^2}{S_f^{\,2}} + \frac{\left[4\cdot(K_f\cdot M_m)^2 + 3\cdot(K_{fs}\cdot T_m)^2\right]}{S_y^{\,2}}}$$

$$1 = 16\cdot\frac{n_f}{\pi\, d}\cdot\sqrt{\frac{4\cdot(K_f\cdot M_a)^2 + 3\cdot(K_{fs}\cdot T_a)^2}{\left[\frac{(f\cdot S_{ut})^2}{S_e\cdot 1.24\cdot d^{-0.107}}\cdot(10000000)^{\frac{-1\cdot\log\left(f\cdot\frac{S_{ut}}{S_e\cdot 1.24\cdot d^{-0.107}}\right)}{3}}\right]^2} + \frac{\left[4\cdot(K_f\cdot M_m)^2 + 3\cdot(K_{fs}\cdot T_m)^2\right]}{S_y^{\,2}}}$$

Find d using Excel goal seeking function, which is located in appendix

The diameter happens to be 0.0185058 meters: $D_6 := 0.01850538$

Mass will be:

$$m_6 := \rho\cdot\pi\cdot\left(\frac{D_6}{2}\right)^2\cdot x_3 = 1.482\ \text{kg}$$

Life expectancy:

$$\omega_6 := 11.059\ \frac{\text{rad}}{\text{s}}$$

$$\text{Life}_6 := \frac{2\cdot\pi\cdot 10000000}{\omega_6\cdot 3600} = 1.578\times 10^3\ \text{hours}$$

$$\text{or} \qquad \text{years} := \frac{\text{Life}_6}{24\cdot 365} = 0.18$$

Check the twisting angle criterion

$$J_{D6} := \pi\cdot\frac{D_6^{\,4}}{32} = 1.151\times 10^{-8}$$

$$\phi_{per\_length} := \frac{T}{J_{D6}\cdot G_{stainless}}\cdot\frac{180}{\pi} = 7.736$$

$$\phi_{per\_length} > \phi_{max} \quad \text{or} \quad 7.736 > 2$$

Because the twisting angle criterion is not satisfied, choose a bigger diamter which also must be larger than the minimum bearing diamter size of 40 mm

$$D_{66} := 0.04$$

$$J_{D66} := \pi \cdot \frac{D_{66}^{\ 4}}{32} = 2.513 \times 10^{-7}$$

$$\phi_{per\_length.6} := \frac{T}{J_{D66} \cdot G_{stainless}} \cdot \frac{180}{\pi} = 0.354 \qquad \phi_{per\_length} < \phi_{max} \quad \text{or} \quad 0.354 > 2$$

Therefore the torsional deflection is satisfied

The new mass is then:

$$m_{66} := \rho \cdot \pi \cdot \left(\frac{D_{66}}{2}\right)^2 \cdot x_3 = 6.9\text{:}\ kg$$

Shaft Design Criteria:
1.) The deflection at gear must be less than 0.000128 m to ensure proper mating
2.) Slope between the gear axis is less than 0.03 degrees
3.) Maximum angular deflection at bearings between 0.001-0.004 rad
4.) Shaft angular deflection is less than 0.00007 rad at non self aligning bearing

Integrating M to find θ and y

$$I_{D66} := \frac{\pi \cdot D_{66}^{\ 4}}{4 \cdot 2^4} = 1.257 \times 10^{-7} \qquad \theta = \int_0^x \frac{M(x)}{E \cdot I_D}\, dx$$

illustration only

Find θ_temp and y_temp to be a function of x and the unkonw integration constants C1 and C2

$$\theta := \frac{T \cdot (x_3 - x_2)^2}{2E \cdot I_{D66}} = 3.031 \times 10^{-4}$$

Which is below 0.001rad and meet crietrion 3

$$y = \int_0^x \theta\, dx + c_1 \cdot x + c_2$$

$$y := \frac{T \cdot (x_3 - x_2)^3}{3 \cdot 2E \cdot I_{D66}} = 3.536 \times 10^{-5}$$

Deflection is less than 0.000128m

Boundary conditions
-assumption: the deflection at the bearing is zero

$$y(x_1) = 0 = y(x_1) + c_1(x_1) + c_2$$

The shaft needs to be modified in dimensions based on the bearing requirements. The shaft has been modified. For shaft one the diameter has been changed from 53 mm to 55 mm, shaft two changed from 7.4 mm to 17 mm. Shaft six has been redesigned because the pumpjack mass is too heavy and the bearings needed yielded to a large shaft diameter. The pumpjack will decrease in weight and the new shaft six will be recalculated based on the new pumpjack mass.

The new free body diagram for shaft 6:

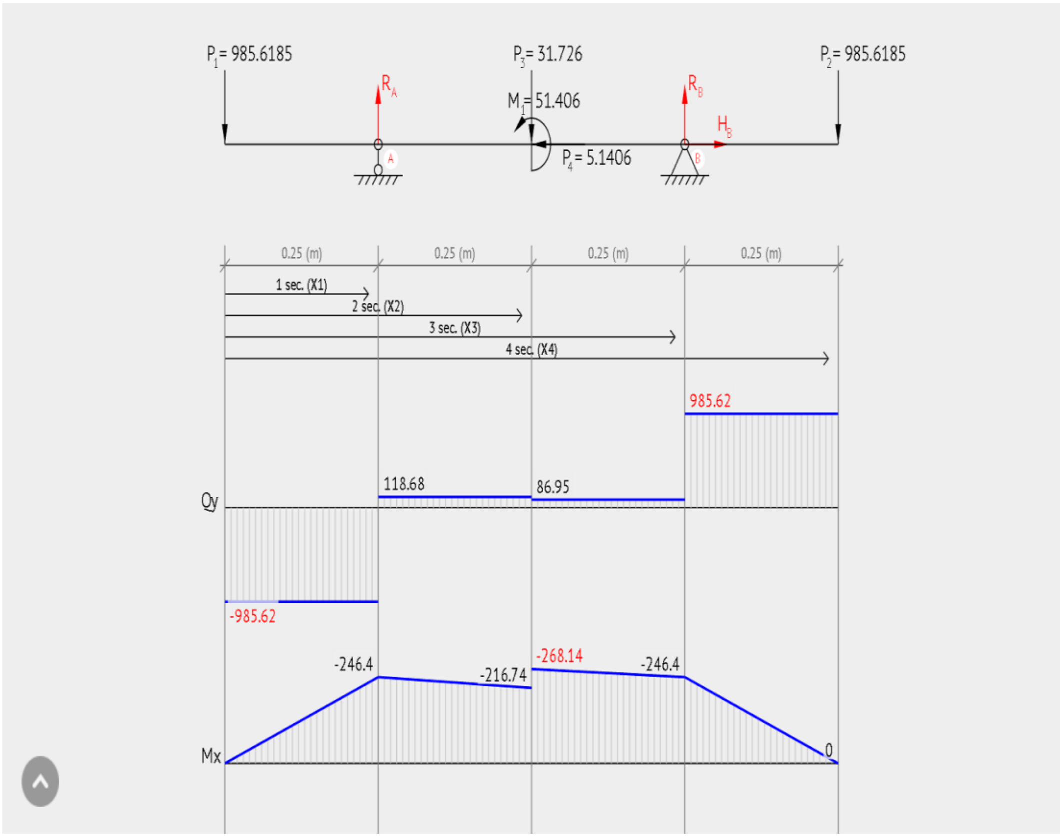


Figure F7: Different Diagrams of the New Shaft Number Six (N vs. m)

Calculations for the New Shaft Six

Shaft Number 6

Given

Dimensions

$x_4 := 0$ $x_5 := 0.25$ $x_6 := 0.5$ $x_7 := 0.75$ $x_8 := 1$

$T_6 := 33.197$

torsional efficiency of bearings $n_b := 2$ $\varepsilon_{bearings} := 0.99^{n_b}$

Torsional inputs

$T_6 = 33.197$

$T_{bearings} := (1 - \varepsilon_{bearings})$ $T_{bearings} = 0.01$

$T_{pumpjack} := T_6 - T_{bearings}$ $T_{pumpjack} = 33.187$

Forces at Y

$F_{pumpjack} := -3629.7$ $R_{A6} := 3629.7$ $F_{gear12\_y} := -31.726$ $R_{B6} := 3629.7$

Moments

$M_{max6} := 910$

Geometric properties of the Shaft

$$I := \left(\frac{\pi \cdot d^4}{4 \cdot 2}\right)$$ Moment of inertia

$$J_o := \pi \cdot \frac{d^4}{32}$$ Polar moment of inertia

$$Q := \frac{4d \cdot \pi \cdot d^2 \pi \cdot d^2}{6 \cdot \pi 8 \cdot 8}$$ First moment of inertia

$t := d$ Thickness

Material AISI 316 Properties:

$$\rho := 7870 \frac{kg}{m^3}$$

$E := 205000000000$ $S_{ut} := 619838842$ $S_y := 415064497$ $G_{stainless} := 82000000000$

Fatigue Analysis
Calculated the minimum diameter needed for this application

Step 1. Calculate Alternating and Mean stresses

There is assumed to be no fluctuating torsion, so the mean and alternating torsion is zero

$M_{max6} := 246.4$ $\qquad M_m := 0$

$T_m := T_6 = 33.197$ $\qquad T_a := 0$

Surface condition modification factor

$b := -0.265 \qquad a := 4.51$

$$k_a := a \cdot S_{ut}^{\,b} = 0.021$$

The size modification factor which is a function of d

$$k_b(D) := \begin{cases} 1.24 \cdot d^{-0.107} & \text{if } 2.79 \le d \le 51 \\ 1.51D^{-0.157} & \text{if } 51 < d \le 254 \end{cases}$$

Load modification factor

bending is dominant

Temperature modification factor

no temperature effect

$k_c := 1$

$k_d := 1$

Miscellaneous-effects modification factor

$k_e := 0.753$ 99.9% reliability

$k_f := 1$ Excellent corrosion resistance

$$S_e := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot 0.504 \cdot S_{ut} = 4.963 \times 10^{6}$$

High Cycle Fatigue Domain (10000 to 10000000) cycles

The ultimate strength is close to 620 $\quad f := 0.86$

$$a_h := \frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \qquad b_h := \frac{-1 \cdot \log\left(f \cdot \frac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}}\right)}{3}$$

$$S_f := a_h \cdot (10000000)^{b_h}$$

For the keyway, parallel is used because of the low cost and is able to sustain loads
The stress concentration factor for this type of key is:

$$K_t := 2.2 \qquad K_{ts} := 3$$

The corner radius is about 0.02 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$$q := 0.72 \qquad q_s := 0.77$$

$$K_f := 1 + q \cdot (K_t - 1) = 1.864$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.54$$

$$0.002794 \le D \le 0.058$$

The required safety factor

$$n_f := 2.92$$

General Diameter Equation for fatigue analysis

$$1 = 16 \cdot \frac{n_f}{\pi d} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{S_f^{\,2}} + \frac{\left[4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2\right]}{S_y^{\,2}}}$$

$$1 = 16 \cdot \frac{n_f}{\pi d} \cdot \sqrt{\frac{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2}{\left[\frac{(f \cdot S_{ut})^2}{S_e \cdot 1.24 \cdot d^{-0.107}} \cdot (10000000)^{\frac{-1 \cdot \log\left(f \cdot \frac{S_{ut}}{S_e \cdot 1.24 \cdot d^{-0.107}}\right)}{3}}\right]^2} + \frac{\left[4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2\right]}{S_y^{\,2}}}$$

Find d using Excel goal seeking function, which is located in appendix

The diameter happens to be 0.00263 meters: $D_6 := 0.00263$

Mass will be:

$$m_6 := \rho \cdot \pi \cdot \left(\frac{D_6}{2}\right)^2 \cdot x_3 = 0.03 \quad \text{kg}$$

Life expectancy:

$$\omega_6 := 11.059 \quad \frac{\text{rad}}{\text{s}}$$

$$\text{Life}_6 := \frac{2 \cdot \pi \cdot 10000000}{\omega_6 \cdot 3600} = 1.578 \times 10^3 \quad \text{hours}$$

$$\text{or} \qquad \text{years} := \frac{\text{Life}_6}{24 \cdot 365} = 0.18$$

Check the twisting angle criterion

$$J_{D6} := \pi \cdot \frac{D_6^{\,4}}{32} = 4.697 \times 10^{-12}$$

$$\phi_{per\_length} := \frac{T}{J_{D6} \cdot G_{stainless}} \cdot \frac{180}{\pi} = 1.896 \times 10^4$$

$\Phi_{per\_length} > \Phi_{max}$ or $1.896 \times 10^{4} < 2$

Because the twisting angle criterion is satisfied

Shaft Design Criteria:
1.) The deflection at gear must be less than 0.000128 m to ensure proper mating
2.) Slope between the gear axis is less than 0.03 degrees
3.) Maximum angular deflection at bearings between 0.001-0.004 rad
4.) Shaft angular deflection is less than 0.00007 rad at non self aligning bearing

Integrating M to find θ and y

$$I_{D6} := \frac{\pi \cdot D_6{}^4}{4 \cdot 2^4} = 2.349 \times 10^{-12} \qquad \theta = \int_0^x \frac{M(x)}{E \cdot I_D}\,dx \qquad \text{illustration only}$$

Find θ_temp and y_temp to be a function of x and the unkonw integration constants C1 and C2

$$\theta := \frac{T_6 \cdot (x_3 - x_2)^2}{2E \cdot I_{D6}} = 4.223$$

Which is fails to meet crietrion 3

D needs to be

$$D_{66} := \left[T_6 \cdot \frac{(x_3 - x_2)^2}{2 \cdot E \cdot 0.003 \cdot \frac{\pi}{4 \cdot 4}} \right]^{\frac{1}{4}} = 0.011$$

$$I_{D66} := \frac{\pi \cdot D_{66}{}^4}{4 \cdot 2^4} = 8.266 \times 10^{-10}$$

$$y = \int_0^x \theta\,dx + c_1 \cdot x + c_2$$

$$y := \frac{T_6 \cdot (x_3 - x_2)^3}{4 \cdot 2E \cdot I_{D66}} = 1.05 \times 10^{-3}$$

Deflection is less than 0.000128m

Boundary conditions
-assumption: the deflection at the bearing is zero $\quad y(x_1) = 0 = y(x_1) + c_1(x_1) + c_2$

The Diameter for shaft six is 0.011 m however the bearing requirements also needs to be satisfied which boosts the shaft to a diameter of 0.050 m.

Shaft Number 1				
Pi	3.141593		Goal Seek Function	
nf	2.92		d	diameter equation
Sut	6.2E+08		0.0526865	1.0001315
Sy	4.15E+08			
Kf	1.864			
Kfs	2.54			
f	0.86			
Ma	2232			
Mm	0			
Tm	509			
S.e	5.37E+06			

Table F1: Goal Seek Analysis for shaft 1 Iteration

Shaft Number 2				
Pi	3.141593		Goal Seek Function	
nf	1.89		d	diameter equation
Sut	6.2E+08		0.0007409	0.999920078
Sy	4.15E+08			
Kf	1.864			
Kfs	2.54			
f	0.86			
Ma	89.07			
Mm	0			
Tm	127.475			
S.e	5.37E+06			

Table F2: Goal Seek Analysis for shaft 2 Iteration

Shaft Number 6				
Pi	3.141593		Goal Seek Function	
nf	2.92		d	diameter equation
Sut	6.2E+08		0.0185058	0.999928582
Sy	4.15E+08			
Kf	1.864			
Kfs	2.54			
f	0.86			
Ma	910			
Mm	0			
Tm	33.197			
S.e	5.37E+06			

Table F6: Goal Seek Analysis for shaft 6 Iteration

Table F7: Goal Seek Analysis for the New Shaft Six

Shaft Number 6				
Pi	3.141593		Goal Seek Function	
nf	1.89		d	diameter equation
Sut	6.2E+08		0.0026347	1.000245157
Sy	4.15E+08			
Kf	1.864			
Kfs	2.54			
f	0.86			
Ma	264.4			
Mm	0			
Tm	343.333			
Se	5.37E+06			

G: Bearing Sizing & Calculations

Table G1: Safety Factor of Major Parts of the Oil Derrick

	nm	ns	ng	nf	nr	neff
Shaft 1	1.1	1.3	1	1.2	1.7	2.92
Shaft 2	1.1	1.3	1	1.2	1.7	2.92
Shaft 3	1.1	1.3	1	1.2	1.1	1.89
Shaft 4	1.1	1.3	1	1.2	1.1	1.89
Shaft 5	1.1	1.3	1	1.2	1.1	1.89
Shat 6	1.1	1.3	1	1.2	1.7	2.92
Bevel Gear	1.1	1.3	1	1.2	1.1	1.89
Planetary gears	1.1	1.3	1.2	1.2	1.1	1.89
Bolts	1.1	1.3	1	1.2	1.7	2.92
Keys	1.1	1.1	1	1.2	1.2	1.74
Truss	1.1	1.3	1	1.2	1.7	2.92

Justification:

The safety factor of the parts is grouped together due to the similarities and will be justified as a group.

Explanation for Shaft 1, 2,6:

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known.

Geometry: The shaft should be fitted properly with the gears and bearings

Failure Analysis: Failure analysis is extended to more complex cases such as friction losses

Reliability: It is important for the shafts to have good reliability due to the huge damage it may occur to maintenance workers when it collapses.

Explanation for Shaft 3, 4, 5

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known.

Geometry: The shaft should be fitted properly with the gears and bearings

Failure Analysis: Failure analysis is extended to more complex cases such as friction losses

Reliability: The reliability is not as importance because the damage it can cause to humans are minimal due to the casings.

Explanation for Bevel Gears

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known.

Geometry: The gears should be fitted properly with the gears because it is a manufactured part

Failure Analysis: Failure analysis is extended to more complex cases with impact forces

Reliability: Reliability is not required to be high even though that system will fail completely without this part but can cause minimal harm to human and the environment if this part is to fail.

Explanation for Planetary Gears

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known.

Geometry: The gears should be fitted properly with the gears because it is a manufactured part

Failure Analysis: Failure analysis is extended to more complex cases with impact forces

Reliability: Reliability is not required to be high even though that system will fail completely without this part but can cause minimal harm to human and the environment if this part is to fail.

Explanation for Bolts

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known and assumed to be zero.

Geometry: The bolt should be fitting properly because it is a manufacturer made part

Failure Analysis: Failure analysis is extended to more complex cases

Reliability: It is important for the shafts to have good reliability due to the huge damage it may occur to maintenance workers when it collapses.

Explanation for Keys

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but loads such as fluctuating torsion and shock during delivery are not known.

Geometry: The key should be fitted properly with the shaft because they are both a manufacturer made part.

Failure Analysis: failure analysis is simple to apply due to the basic shape

Reliability: the key way does not need a significant reliability because if the part does fail, it cannot cause much harm to the humans and the environment

Explanation for Truss

Materials: The materials are known through the manufacturer for the metal that we purchase.

Stress: Average loads are known but the geometry made it difficult to analyse without the use of finite elemental analysis.

Geometry: The truss should be dimensioned tightly because it is welded together.

Failure Analysis: Failure analysis is extended to more complex cases

Reliability: It is important for the shafts to have good reliability due to the huge damage it may occur to maintenance workers when it collapses.

H: Finite Element Analysis

Finite elemental analysis will be used as a testing mechanism for our design to analyse parts with difficult geometry and/or loadings to achieve more accurate results. Optimization based on finite elemental analysis will not be considered due to the difficulty. Finite elemental analysis is a computer simulation program that will provide the user a stress map for the geometry. Von Mises stress is a good indicator for the actual stress experienced because the metal used, carbon steel, is ductile and does not experience much ductile to brittle transition for the temperature range that we will be operating at.

For this project, Solid works will be used for the finite elemental analysis. Below are figures of the stress diagrams, displacement diagrams, and strain diagrams for different parts. Explanations will be followed for the boundary conditions, type of loadings, and mesh.

H1 Finite Elemental Analysis on the Pumpjack

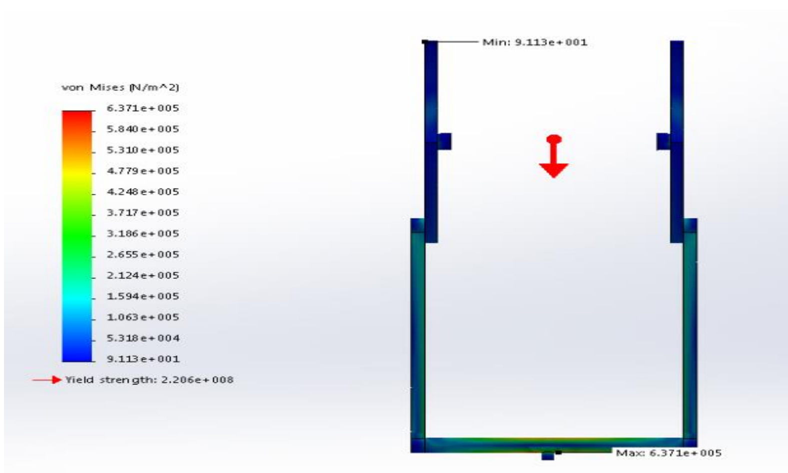


Figure H1: Stress Diagram of the Pumpjack using Von Mises Stress

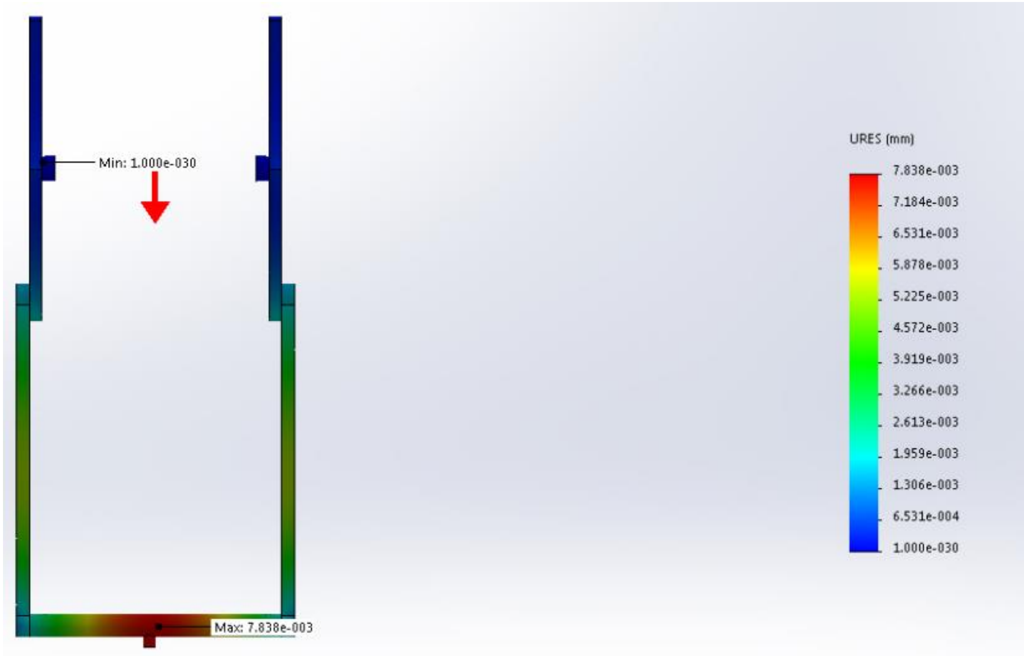


Figure H2: Displacement plot of the Pumpjack

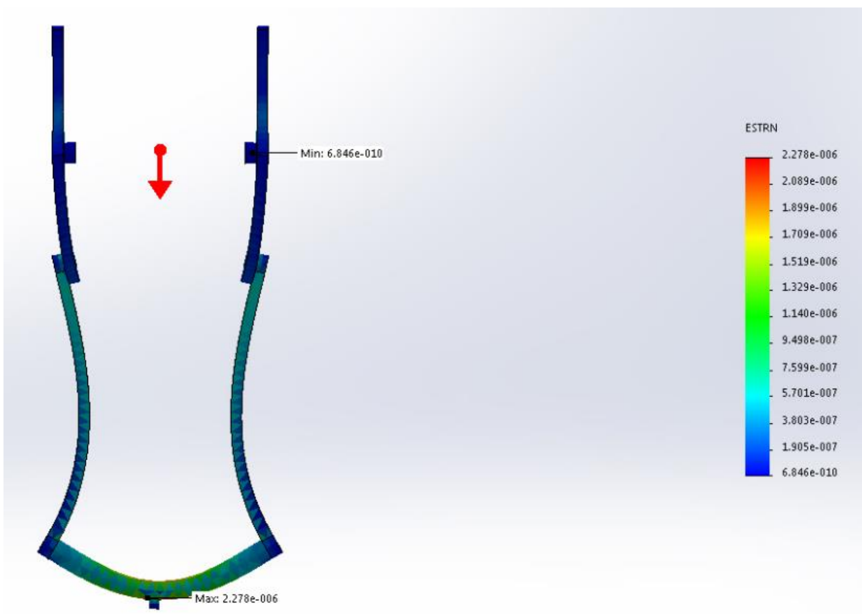


Figure H3: Strain Plot Diagram of the Pumpjack

Table H1: Boundary Conditions and Loading of the Pumpjack

Torsion at both sides (near center of gravity)	33.197 Nm
Gravitational Force	5886 N

Reactant Forces at both sides (near center)	5886 N

Table H2: FEA Properties of the Pumpjack

Number of Nodes	20745
Number of Elements	11573
Number of Degree of Freedom	61407

The stress diagram suggest that the part will not fail based on the dimensions of the solid model. This can be seen in figure H1, the maximum stress is under the yield stress with a safety factor of 355. The displacement is 7.83 mm at the bottom of the pumpjack but should not affect the performance because a high accuracy is not needed in that area.

H2 Finite Elemental Analysis on Shafts

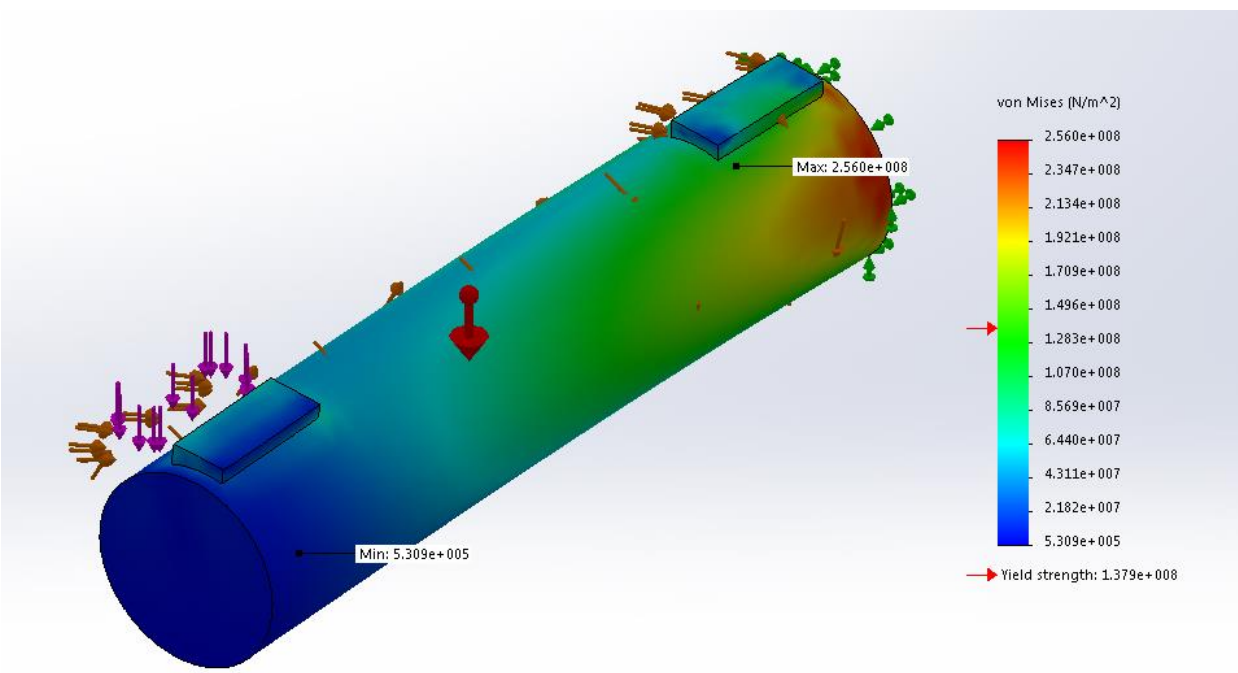


Figure H4: Stress Diagram of Shaft 1 with Keys using Von Mises Stress

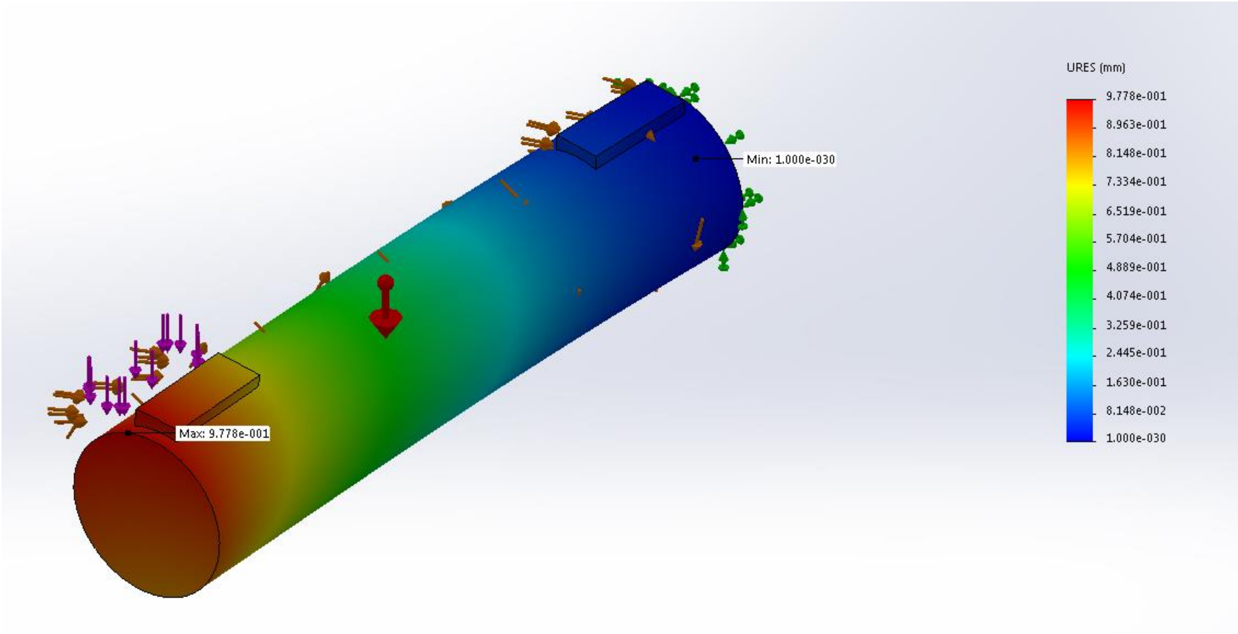


Figure H5: Displacement Diagram of Shaft 1 with Keys

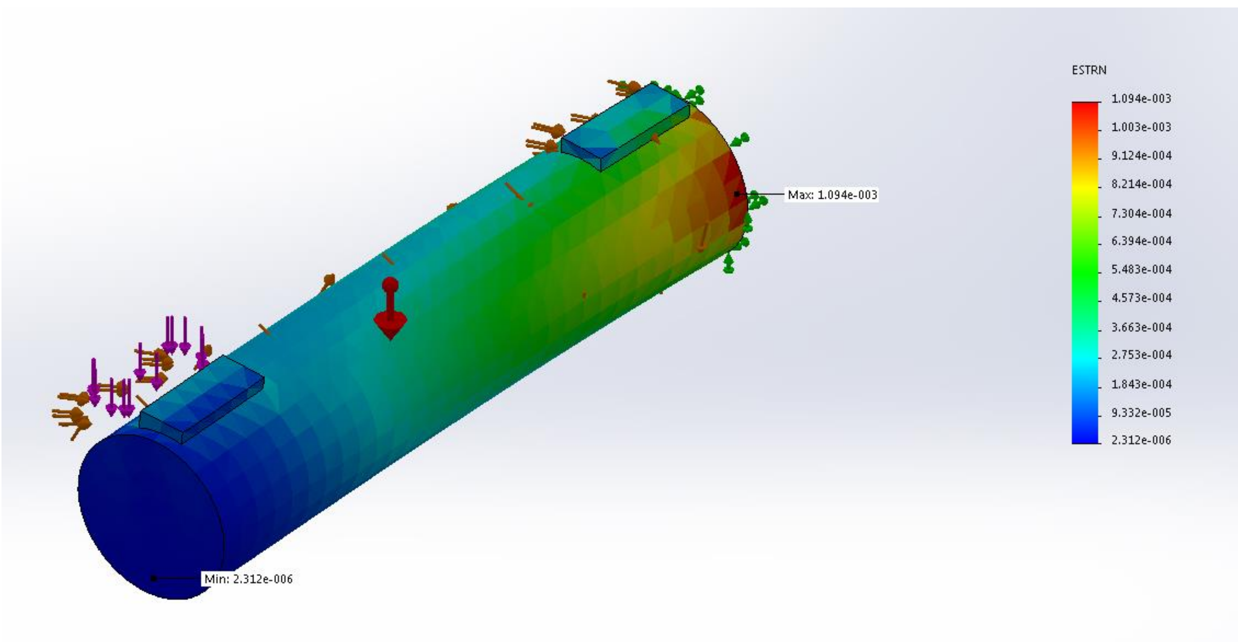


Figure H6: Strain Diagram of Shaft 1 with Keys

Table H3: Boundary Conditions and Loading of Shaft 1

Torsion	509.9 Nm
Gravitational Force	45.28 N
Turbine Gravitational force acting on the left	6376.5 N
Reactant Forces on the right	6421.78

Table H4: FEA Properties

Number of Nodes	20745
Number of Elements	11573
Number of Degree of Freedom	61407

The stress diagram suggest that the part will not fail based on the dimensions of the solid model. This can be seen in figure H4, the maximum stress is under the yield stress with a safety factor of 1.62. The displacement is not significant for this shaft because the accuracy is not important.

I: Calculations of the Keyways

I1 Calculations of the Keyways

Key ways

Shaft Number 1

Shaft diamter is about 5.2 cm
The recommended key way size has a dimension of: 16 x 10 mm
The stress concentration factor:

$$K_t := 2.2 \qquad K_{ts} := 3$$

The corner radius is about 0.157 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$$q := 0.81 \qquad q_s := 0.84$$

$$K_f := 1 + q \cdot (K_t - 1) = 1.972$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.68$$

Shear Failure

$$F_1 := 650 \cdot 9.81 = 6.377 \times 10^3$$

$$A_{shear1} := 0.016 \cdot 0.0254 \qquad \tau_{xy} := \frac{F_1}{A_{shear1}} = 1.569 \times 10^7$$

$$y := 0.052 \qquad M_1 := 650 \cdot 0.8 = 520$$

$$I_1 := \frac{\pi \cdot \left(\frac{y}{2}\right)^4}{4} = 3.589 \times 10^{-7}$$

$$\sigma_{x1} := \frac{K_t \cdot M_1 \cdot y}{I_1} = 1.657 \times 10^8 \qquad \sigma_{y1} := \frac{F_1}{\frac{0.016 \cdot 0.0254}{2}} = 3.138 \times 10^7$$

Torsion

$$T := 509$$

$$\tau_{torsion} := T \cdot \frac{\left(\frac{y}{2}\right)}{\frac{\pi \cdot y^4}{32}} = 1.844 \times 10^7$$

Von Mises

$$\sigma_{vm} := \frac{\sqrt{(\sigma_{x1} - \sigma_{y1})^2 + \sigma_{x1}^2 + \sigma_{y1}^2 + 6(\tau_{torsion} + \tau_{xy})^2}}{\sqrt{2}} = 1.636 \times 10^8$$

The Safety Factor

$$S_y := 415064497 \qquad SF := \frac{S_y}{\sigma_{vm}} = 2.538$$

Which is above 1.74

Shaft number 2

Shaft diamter is 7.4 mm
The recommended key way size has a dimension of: 3 x 3 mm
The stress concentration factor:

$$K_{t2} := 2.2 \quad K_{ts} := 3$$

The corner radius is about 0.0314 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$$q_{t2} := 0.77 \quad q_{s2} := 0.79$$

$$K_f := 1 + q_{t2} \cdot (K_t - 1) = 1.924$$

$$K_{fs} := 1 + q_{t2} \cdot (K_{ts} - 1) = 2.54$$

Shear Failure

$$F_2 := 239.015$$

$$A_{shear2} := 0.003 \cdot 0.003 \qquad \tau_{xy2} := \frac{F_2}{A_{shear2}} = 2.656 \times 10^7$$

$$y_2 := 0.04 \qquad M_2 := 39.015 \cdot 0.35 = 13.655 \qquad I_2 := \frac{\pi \cdot \left(\frac{y_2}{2}\right)^4}{4} = 1.257 \times 10^{-7}$$

$$\sigma_{x2} := \frac{K_t \cdot M_2 \cdot y_2}{I_2} = 9.563 \times 10^6 \qquad \sigma_{y2} := \frac{F_1}{\frac{0.012 \cdot 0.0254}{2}} = 4.184 \times 10^7$$

Torsion

$$T_2 := 127.475$$

$$\tau_{torsion2} := T_2 \cdot \frac{\left(\frac{y_2}{2}\right)}{\frac{\pi \cdot y_2^{\,4}}{32}} = 1.014 \times 10^7$$

Von Mises

$$\sigma_{vm2} := \frac{\sqrt{(\sigma_{x2} - \sigma_{y2})^2 + \sigma_{x2}^{\,2} + \sigma_{y2}^{\,2} + 6(\tau_{torsion2} + \tau_{xy2})^2}}{\sqrt{2}} = 7.405 \times 10^7$$

The Safety Factor

$$S_y := 415064497 \qquad SF := \frac{S_y}{\sigma_{vm2}} = 5.605$$

Which is above 1.74

Shaft number 6

Shaft diamter is about 18.5 mm
The recommended key way size has a dimension of: 6 x 6 mm
The stress concentration factor:

$K_{t6} := 2.2 \quad K_{ts} := 3$

The corner radius is about 0.0314 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$q := 0.77 \qquad q_s := 0.79$

$K_f := 1 + q \cdot (K_t - 1) = 1.972$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.68$

Shear Failure

$F_6 := 3630$

$A_{shear6} := 0.006 \cdot 0.006 \qquad \tau_{xy6} := \dfrac{F_6}{A_{shear6}} = 1.008 \times 10^8$

$y_6 := 0.04 \qquad M_6 := 910$

$$I_6 := \frac{\pi \cdot \left(\frac{y_6}{2}\right)^4}{4} = 1.257 \times 10^{-7}$$

$$\sigma_{x6} := \frac{K_{t6} \cdot M_6 \cdot y_6}{I_6} = 6.373 \times 10^8 \qquad \sigma_{y6} := \frac{F_6}{\frac{0.012 \cdot 0.0254}{2}} = 2.382 \times 10^7$$

Torsion

$T_6 := 33.197$

$$\tau_{torsion6} := T_6 \cdot \frac{\left(\frac{y_6}{2}\right)}{\frac{\pi \cdot y_6^{\,4}}{32}} = 2.642 \times 10^6$$

Von Mises

$$\sigma_{vm6} := \frac{\sqrt{(\sigma_{x6} - \sigma_{y6})^2 + \sigma_{x6}^{\,2} + \sigma_{y6}^{\,2} + 6\left(\tau_{torsion6} + \tau_{xy6}\right)^2}}{\sqrt{2}} = 6.508 \times 10^8$$

The Safety Factor

$S_y := 415064497 \qquad \dfrac{S_y}{\sigma_{vm6}} = 1.84$ Which is above 1.74

New Shaft number 6

Shaft diamter is about 50 mm
The recommended key way size has a dimension of: 14 x 9 mm
The stress concentration factor:

$$K_{t6} := 2.2 \quad K_{ts} := 3$$

The corner radius is about 0.0314 inches
The q, which is needed to determine the stress concentration factor is determined from the notch sesnsitivity graph

$$q := 0.77 \qquad q_s := 0.79$$

$$K_f := 1 + q \cdot (K_t - 1) = 1.924$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.58$$

Shear Failure

$$F_{66} := 985.62 + 118.68 = 1.104 \times 10^{3}$$

$$A_{shear6} := 0.014 \cdot 0.009 \qquad \tau_{xy66} := \frac{F_6}{A_{shear6}} = 2.881 \times 10^{7}$$

$$y_{66} := 0.04 \qquad M_{66} := 268.14$$

$$I_{66} := \frac{\pi \cdot \left(\frac{y_{66}}{2}\right)^4}{4} = 1.257 \times 10^{-7}$$

$$\sigma_{x66} := \frac{K_{t6} \cdot M_{66} \cdot y_{66}}{I_{66}} = 1.878 \times 10^{8} \qquad \sigma_{y66} := \frac{F_{66}}{\frac{0.012 \cdot 0.0254}{2}} = 7.246 \times 10^{6}$$

Torsion

$$T_6 := 33.197$$

$$\tau_{torsion66} := T_6 \cdot \frac{\left(\frac{y_{66}}{2}\right)}{\frac{\pi \cdot y_{66}^{\,4}}{32}} = 2.642 \times 10^{6}$$

Von Mises

$$\sigma_{vm66} := \frac{\sqrt{(\sigma_{x66} - \sigma_{y66})^2 + \sigma_{x66}^{\,2} + \sigma_{y66}^{\,2} + 6\,(\tau_{torsion66} + \tau_{xy66})^2}}{\sqrt{2}} = 1.921 \times 10^{8}$$

The Safety Factor

$$S_y := 415064497 \qquad \frac{S_y}{\sigma_{vm66}} = 2.16$$

Which is above 1.74

J: Tolerances for Keyways, Shafts & Holes

Table J1: Tolerances for the keyways

Shaft Number	**Shaft Diameter (mm)**	**Key size (mm) b x h**	**Fit**	**Width for Shaft (mm)**	**Width for Hub (mm)**	**Shaft Height (mm)**	**Hub height (mm)**
1	55	16 x 10	Normal	15.957-16	15.979-16.021	6-6.2	4.3-4.5
2	17	3 x 3	Normal	2.971-2.996	2.998-3.012	1.8-1.9	1.4-1.5
3	109	28 x 16	Normal	28 27.948	28.026 27.974	10-10.2	6.4-6.6
4	81	22 x 14	Normal	22 21.948	22.026 21.974	9-9.2	5.4-5.6
5	60	18 x 11	Normal	18 17.957	18.021 17.979	7-7.2	4.4-4.6
6	50	14 x 9	Normal	13.957-14.000	13.979-14.021	5.5-5.7	3.8-4

Table J2: Tolerances for the Holes

Hole	Basis	Fits	Description	Nominal (mm)	Limits (mm)
Gear 5	Shaft (G7/h6)	Clearance	Sliding fit	17	17.000-17.018
Gear 6	Shaft (G7/h6)	Clearance	Sliding fit	109	109.012-109.049
Gear 7	Shaft (G7/h6)	Clearance	Sliding fit	109	109.012-109.049
Gear 8	Shaft (G7/h6)	Interference	Press fit (ferrous)	81	80.941-80.976
Gear 9	Shaft (G7/h6)	Clearance	Sliding fit	81	81.012-81.049
Gear 10	Shaft (G7/h6)	Clearance	Sliding fit	60	60.010-60.040
Gear 11	Shaft (G7/h6)	Clearance	Sliding fit	60	60.010-60.040
Gear 12	Shaft (G7/h6)	Clearance	Sliding fit	50	50.010-50.040

K: Safety Factor

K1 Shaft 1

For shaft 1

w1=omega 0.202rad/s, 0.0321492985 rev/s> assuming 10 year life time,9810478446 cycles

Applied constant radial load $Wrt := 650 \cdot 9.81$ $Wr2 := 60.413$

Applied constant thrust load $Wat := 0$ $Wa2 := 95.83$

$K_R := 0.21$ Assume axial force from turbine is zero

$L_1 := 9810.478$ millions

$$L_{10} := \frac{L_1}{K_R} = 4.672 \times 10^4$$

for Bearing T

$$Fa := \left| |Wa2| - |Wat| \right| = 95.83$$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration Choose bearing 6311 first $C_{0\_6311} := 10000$

$$\frac{Fa}{C_{0\_6311}} = 9.583 \times 10^{-3}$$

$e < 0.19$

$$\frac{Fa}{v \cdot |Wrt|} = 0.015 \qquad 0.015 < e$$

Radial factor $X := 1$

Thrust factor $Y := 0$

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr8| + Y \cdot Fa) = 144.991$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 5.222 \times 10^3$$

$C_{10\_6311} := 12900$ $C_{10} < 12900$

Real fatigue life

$$L_{1\_6311} := K_R \cdot \left(\frac{C_{10\_6311}}{P_{eff}} \right)^3 = 1.479 \times 10^5 \qquad \text{millions}$$

For bearing #2

99%reliability under 9810478446 cycles 9810.47 millions $L_{10} = 4.672 \times 10^4$

First Iteration

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |Wr2| = 2.176 \times 10^3$$

Choose bearing 6311 first $C_{10\_6311} := 12900$

$C_{10} < 12900$

Real fatigue life

$$L_{1\_6311} := K_R \cdot \left(\frac{C_{10\_6311}}{|Wr2|}\right)^3 = 2.045 \times 10^6$$ millions

K2 shaft 2

For shaft 2

Ignore the weight of shaft

w2=omega 1.92rad/s, 0.30557749073 rev/s> assuming 10 year life time,1032144086 cycles

Applied constant radial load	$Wr5 := 36.56$	$Wr4 := 115.993$
Applied constant thrust load	$Wa5 := 59.219$	$Wa4 := 0$

Assume axial force from turbine is zero

$K_R := 0.21$

$L_1 := 1032.144$ millions

$$L_{10} := \frac{L_1}{K_R} = 4.915 \times 10^3$$

for Bearing 5

$$Fa := \left||Wa5| - |Wa4|\right| = 59.219$$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration

Choose bearing 6300 first $C_{0\_6300} := 850$

$$\frac{Fa}{C_{0\_6300}} = 0.07$$ e is between 0.26 and 0.28 $$\frac{Fa}{v \cdot |Wr5|} = 1.62$$

Radial factor $X := 0.56$

Thrust factor $Y := 1.63$ by interpolatin

Equivalnt Load calculation

$P_{eff} := (X \cdot v \cdot |Wr5| + Y \cdot Fa) = 117.001$

$C_{10\_6300} := 1400$

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 1.989 \times 10^{3}$

$C_{10} > 1400$

Second Iteration

Choose bearing 6301

$C_{0\_6301} := 1040$

$\frac{Fa}{C_{0\_6301}} = 0.057$

e is between 0.26 and 0.28

$\frac{Fa}{v \cdot |Wr5|} = 1.62$

Radial factor $X := 0.56$

Thrust factor $Y := 1.98$ by interpolatin

Equivalnt Load calculation

$P_{eff} := (X \cdot v \cdot |Wr5| + Y \cdot Fa) = 137.727$

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 2.342 \times 10^{3}$

Real fatigue life

$C_{10\_6301} := 1700$

$L_{1\_6301} := K_R \cdot \left(\frac{C_{10\_6301}}{P_{eff}} \right)^{3} = 394.918$ millions

Third Iteration

Choose bearing 6302

$C_{0\_6302} := 1200$

$\frac{Fa}{C_{0\_6302}} = 0.049$

e is between 0.22 and 0.26

$\frac{Fa}{v \cdot |Wr5|} = 1.62$

Radial factor $X := 0.56$

Thrust factor $Y := 1.78$ by interpolatin

Equivalnt Load calculation

$P_{eff} := (X \cdot v \cdot |Wr5| + Y \cdot Fa) = 125.883$

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 2.14 \times 10^{3}$

$C_{10\_6302} := 1930$

use multiple bearings instead

Fourth Iteration

Choose bearing 6303 $C_{0\_6303} := 1460$

use two bearings

$$\frac{Fa}{C_{0\_6303}} = 0.041$$

Radial factor $X := 0.56$ $\frac{Fa}{v \cdot |Wr5|} = 1.62$

Thrust factor $Y := 1.86$

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr5| + Y \cdot Fa) = 130.621$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 2.221 \times 10^{3}$$

$C_{10\_6303} := 2320$ $C_{10} < 2320$

Real fatigue life

$$L_{1\_6303} := K_R \cdot \left(\frac{C_{10\_6303}}{P_{eff}}\right)^{3} = 1.177 \times 10^{3} \quad \text{millions}$$

For bearing #4

99%reliability under 1032144086 cycles 1032.144 millions $L_{10} = 4.915 \times 10^{3}$

First Iteration

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |Wr4| = 1.972 \times 10^{3}$$

Choose bearing 6303 $C_{10\_6303} := 2320$

$C_{10} < 2320$

Real fatigue life

$$L_{1\_6303} := K_R \cdot \left(\frac{C_{10\_6303}}{|Wr4|}\right)^{3} = 1.68 \times 10^{3} \quad \text{millions}$$

K3 Shaft 3

for shaft 3

$$W_s := \pi \cdot \frac{d^2 \cdot 7 \cdot 9.81 \cdot 8050}{4} = 5.158 \times 10^3$$

shaft diameter $d := 0.109$

w6=w7=omega1.92rad/s, 0.30557749073 rev/s> assuming 10 year life time,1032144086 cycles

Applied constant radial load $Wr6 := -36.56$ $Wr7 := -144.991$

Applied constant thrust load $Wa6 := -59.219$ $Wa7 := -229.993$

for shaft 3, Wr6 and Wr7 work in the same direction

$$Fr := \left| |Wr6| + |Wr7| \right| = 181.551$$

$K_R := 0.21$

$L_1 := 1032$ millions

$$L_{10} := \frac{L_1}{K_R} = 4.914 \times 10^3$$

for Bearing #6

$$Fa := \left| |Wa7| - |Wa6| - W_s \right| = 4.988 \times 10^3$$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration

Choose bearing 6322 first $C_{0\_6322} := 32500$

$$\frac{Fa}{C_{0\_6322}} = 0.153$$

e between 0.30 and 0.34

$$\frac{Fa}{v \cdot |Wr6|} = 136.42$$

$136.42 > e$

Radial factor $X := 0.56$

Thrust factor $Y := 1.34966$ by interpolation

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr6| + Y \cdot Fa) = 6.752 \times 10^3$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 1.148 \times 10^5$$

$C_{10\_6322} := 32500$ $C_{10} > 32500$

Second Iteration

Choose bearing 6330

$C_{0\_6330} := 60000$

$$L_{1\_6322} := K_R \cdot \left(\frac{C_{10\_6322}}{P_{eff}}\right)^3 = 23.42$$

e is between 0.26 and 0.28

$\frac{Fa}{C_{0\_6330}} = 0.083$ $\frac{Fa}{v \cdot |Wr6|} = 136.42$ $136.42 > e$

Radial factor $X := 0.56$

Thrust factor $Y := 1.5557$ by interpolation

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr6| + Y \cdot Fa) = 7.78 \times 10^3$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 1.323 \times 10^5$$

$C_{10\_6322} := 32500$ $C_{10} > 32500$

dynamic load too big;
Use multiple bearings

$$Fa := \left| |Wa7| - |Wa6| - W_s \right| = 4.988 \times 10^3$$

third Iteration
assume 5 bearings

$Fa := \frac{Fa}{5} = 997.503$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

Choose bearing 6322 first $C_{0\_6322} := 32500$

e between 0.22 and 0.26

$\frac{Fa}{C_{0\_6322}} = 0.031$ $\frac{Fa}{v \cdot |Wr6|} = 27.284$ $27.284 > e$

Radial factor $X := 0.56$

Thrust factor $Y := 1.96$ by interpolation

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr6| + Y \cdot Fa) = 1.976 \times 10^3$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 3.359 \times 10^4$$

$C_{10\_6322} := 32500$ $C_{10} > 32500$

Fourth Iteration assume 6 bearings

$Fa := \dfrac{4988}{6} = 831.333$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

Choose bearing 6322 first $C_{0\_6322} := 32500$

$\dfrac{Fa}{C_{0\_6322}} = 0.026$ $\dfrac{Fa}{v \cdot |Wr6|} = 22.739$ e between 0.19 and 0.22 $22.739 > e$

Radial factor $X := 0.56$

Thrust factor $Y := 2.0342$ by interpolation

Equivalnt Load calculation

$P_{eff} := (X \cdot v \cdot |Wr6| + Y \cdot Fa) = 1.712 \times 10^{3}$

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 2.91 \times 10^{4}$

$C_{10\_6322} := 32500$ $C_{10} > 32500$

Real fatigue life

$L_{1\_6322} := K_R \cdot \left(\dfrac{C_{10\_6322}}{|Wr6|}\right)^{3} = 1.475 \times 10^{8}$ millions

Put 6 bearings across shaft 3

For bearing #7

99%reliability under 1032144086 cycles $L_{10} = 4.914 \times 10^{3}$

First Iteration 1032 millions

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |Wr7| = 2.465 \times 10^{3}$

Choose bearing 6322 first $C_{10\_6322} := 32500$

$C_{10} < 5000$

$L_{1\_6322} := K_R \cdot \left(\dfrac{C_{10\_6322}}{|Wr7|}\right)^{3} = 2.365 \times 10^{6}$ millions With previous bearing selection, radial force requirement satisfied

K4 Shaft 4

for Bearing #8

$Fa := ||Wa8| - |Wa9|| = 134.163$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration Choose bearing 6316 first $C_{0\_6316} := 18000$

$\frac{Fa}{C_{0\_6316}} = 7.454 \times 10^{-3}$ $e < 0.19$

$\frac{Fa}{v \cdot |Wr8|} = 0.925$ $0.925 > e$

Radial factor $X := 0.56$

Thrust factor $Y := 2.449$

Equivalnt Load calculation

$P_{eff} := (X \cdot v \cdot |Wr8| + Y \cdot Fa) = 409.76$

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 5.203 \times 10^3$

$C_{10\_6316} := 21200$ $C_{10} < 21200$

Real fatigue life

$L_{1\_6316} := K_R \cdot \left(\frac{C_{10\_6316}}{P_{eff}}\right)^3 = 2.908 \times 10^4$ millions

For bearing #9

99%reliability under 430060036 cycles 430 millions $L_{10} = 2.048 \times 10^3$

First Iteration

$C_{10} := L_{10}^{\frac{1}{3}} \cdot |Wr9| = 767.15$

Choose bearing 6316 first $C_{10\_6316} := 21200$

$C_{10} < 21200$

$L_{1\_6306} := K_R \cdot \left(\frac{C_{10\_6316}}{|Wr9|}\right)^3 = 9.075 \times 10^6$ millions

K5 Shaft 5

For shaft 5 w10=w11=omega 11.059rad/s, 1.76 rev/s> assuming 10 year life time,179194922 cycles

Applied constant radial load $Wr10 := 60.413$ $Wr11 := 7.934$

Applied constant thrust load $Wa10 := 95.83$ $Wa11 := 12.851$

$K_R := 0.21$ $L_1 := 179$ millions

$$L_{10} := \frac{L_1}{K_R} = 852.381$$

For Bearing #10

$$Fa := \left| |Wa10| - |Wa11| \right| = 82.979$$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration Choose bearing 6312 first $C_{0\_6312} := 10800$

$$\frac{Fa}{C_{0\_6312}} = 7.683 \times 10^{-3} \qquad e < 0.19$$

Thrust factor

$$\frac{Fa}{v \cdot |Wr10|} = 1.374 \qquad 1.374 > e$$

Radial factor $X := 0.56$ Thrust factor $Y := 2.4398$

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr10| + Y \cdot Fa) = 236.283$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 2.24 \times 10^3$$

$C_{10\_6312} := 14000$ $C_{10} < 14000$

Real fatigue life

$$L_{1\_6312} := K_R \cdot \left(\frac{C_{10\_6312}}{P_{eff}} \right)^3 = 4.368 \times 10^4 \quad \text{millions}$$

For bearing #11 430 millions

99%reliability under 430060036 cycles $L_{10} = 852.381$

First Iteration

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |Wr9| = 572.807$$

Choose bearing 6312 first $C_{10\_6312} := 14000$

$C_{10} < 14000$ Real fatigue life

$$L_{1\_6312} := K_R \cdot \left(\frac{C_{10\_6312}}{|Wr9|} \right)^3 = 2.613 \times 10^6 \quad \text{millions}$$

K6 Shaft 6

for shaft 6

w6=omega 11.059rad/s, 1.76009451565 rev/s> assuming 10 year life time,179194922 cycles

Applied constant radial load $Wr12 := 1104.3$ $Wrp := 1104.3$

Applied constant thrust load $Wa12 := 50$ $Wap := 0$

$K_R := 0.21$ Assume axial force from turbine is zero

$L_1 := 179.1949$ millions

$$L_{10} := \frac{L_1}{K_R} = 853.309$$

for Bearing 12

$$Fa := \left| |Wa12| - |Wap| \right| = 50$$

Rotation factor $v := 1$ Rotating inner ring(rotating shaft)

First Iteration Choose bearing 6304 first $C_{0\_6304} := 1930$

$$\frac{Fa}{C_{0\_6304}} = 0.026 \qquad \frac{Fa}{v \cdot |Wr12|} = 0.045 \qquad 0.045 < e$$

Radial factor $X := 1$

Thrust factor $Y := 0$

Equivalnt Load calculation

$$P_{eff} := (X \cdot v \cdot |Wr12| + Y \cdot Fa) = 1.104 \times 10^3$$

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 1.047 \times 10^4 \qquad C_{10\_6304} := 3000 \qquad C_{10} > 3000$$

second Iteration

Use 6 bearings $Fa := \frac{Fa}{6}$ $Wr12 := \frac{Wr12}{6}$

Radial factor Choose bearing 6304 first $C_{0\_6304} := 1930$

$$\frac{Fa}{C_{0\_6304}} = 4.318 \times 10^{-3} \qquad \frac{Fa}{v \cdot |Wr12|} = 0.045 \qquad 0.045 < e$$

$X := 1$ $Y := 0$

Equivalnt Load calculation $C_{10\_6304} := 3000$ $C_{10} < 3000$

$P_{eff} := (X \cdot v \cdot |Wr12| + Y \cdot Fa) = 184.05$

Real fatigue life

$$C_{10} := L_{10}^{\frac{1}{3}} \cdot |P_{eff}| = 1.746 \times 10^3 \qquad L_{1\_6304} := K_R \cdot \left(\frac{C_{10\_6304}}{|Wr12|}\right)^3 = 909.443 \text{ millions}$$

L: Truss Design & Analysis

L1 Truss design

The truss design is of a conical shape structure as compared to a complete cylindrical shape as its more stable. Since our base is larger as compared to the top, it provides a better support and decreases the moment on the overall design.

Truss has been cemented down on the ground rather than bolts as when we use bolts for this case, we technically should bolt down the truss on a solid base, and bolting it down to the ground will not provide a high stress bearing capacity for the bolts. It's because the ground is less dense as compared to cement. Thus, cementing down the truss would be a better idea. The truss design ensured no bending of the complete wind powered pump jack. Calculations regarding axial stress, bending moment, reaction forces are provided and proper meshing with the hub of the wind turbine.

The calculations can be found in L2 and complete material selection, maintenance, and corrosion protection can be found L3. The drawing can be found in Appendix N20.

L2 Truss detailed calculations

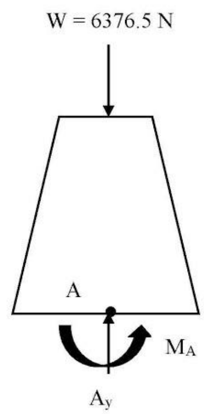
Assumptions:

- Wind turbine is treated as a concentrated force acting on the center of the truss
- The complete truss is treated as one rigid body
- Material used for truss design is plane carbon Steel
- The force due to the wind that acts on the turbine can be treated as a point force
- 200mm of truss design is being cemented down to the ground.

The truss will be cemented down, so the part of the truss that is in contact with the ground can be treated as a fixed support.

The mass of the wind turbine is 650 kg, meaning the force applied by the wind turbine is $W = 650(9.81) = 6376.5$ N

Therefore, the FBD is as follows:



A is the point at the center of the base circle. The reaction force A_x is equal and opposite to the weight of the turbine.

Since point A is at the center of the truss, the weight of the turbine is not causing any moment

$$\therefore A_y = 6376.5 \text{ N} \uparrow, M_A = 0$$

The axial stress applied by the wind turbine is the weight of the turbine divided by the cross-sectional area, such that

$$\sigma = \frac{W}{\pi d^2/4}$$

The axial stress increases when the diameter decreases, so the diameter at the top of the truss 1800mm will be used. The diameter at the top of the truss is $d =$ mm, so the axial compressive stress is calculated as follows:

$$\sigma_{MAX} = \frac{6376.5}{\pi \times (1800^2 - 1700^2)/4} = 231.9(10^{-4}) \text{ MPa} = 23.2 \text{ kPa}$$

Stainless steel 230 (SS230) is being used to build the truss, with a yield and ultimate tensile strength of $S_Y = 230$ MPa, $S_{UT} = 210$ MPa and the maximum stress due to the wind turbine is much less than this. Therefore, we can conclude that the weight of the turbine will not cause the truss to collapse.

The modulus of elasticity for SS230 is $E = 200$ GPa. In the elastic region, axial stress is defined as

$$\sigma = E\varepsilon$$

Where ε is the compression of the truss. Solving for ε, we get the following:

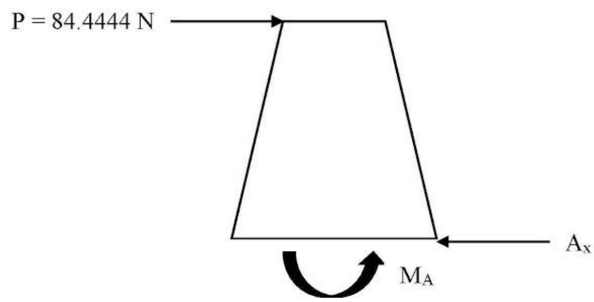
$$\varepsilon = \frac{23.2 \times 10^3}{200 \times 10^9} \times 100\% = 1.16(10^{-5})\ \%$$

For SS230, the maximum compression is 20%, which is much greater than compression by the turbine. Therefore, the compression caused the weight of the turbine is negligible.

The wind turbine receives a torque of 760 Nm from the wind. If the radius of one turbine blade is 9 m, the force applied by the wind can be calculated by dividing the torque by the radius, such that

$$P = \frac{760}{9} = 84.4444 \text{ N}$$

Therefore, a new FBD can be made to determine the bending stress acting on the truss

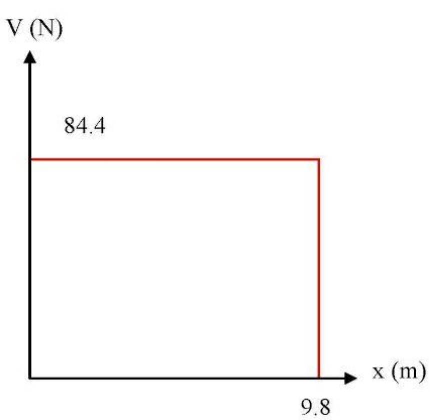


The reaction force A_x is equal and opposite to the wind force; the moment M_A is acting opposite to the moment applied by the wind force. The truss is 9.8 m tall, so the moment about A is force applied by the wind multiplied by the height of the truss.

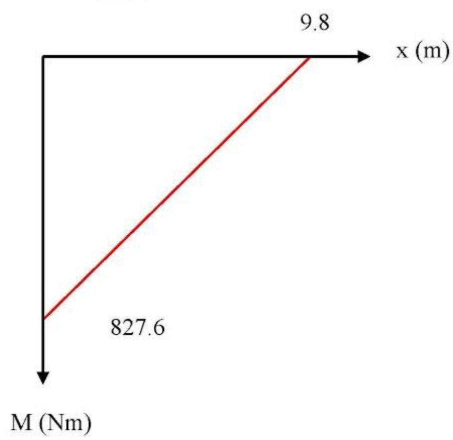
$$\therefore A_x = 84.4444 \text{ N} \leftarrow, M_A = 827.5556 \text{ Nm (CCW)}$$

The SFD and BMD can be made knowing all the reaction forces.

SFD:



BMD:



There are two cross-sections to consider for the truss: the circles at the top and bottom. The diameters at the top and bottom of the truss is 1800 mm and 3000 mm, respectively. The moment of inertia for a circular cross-section can be calculated using the following:

$$I = \frac{\pi(d_o^4 - d_i^4)}{64}$$

For the top part we have the outer diameter as 1800mm and the inner diameter as 1600mm.

For the bottom part we have the outer diameter as 3000mm and the inner diameter as 2800mm.

Therefore, the moments of inertia for the top and bottom using the above equation is

$$I_{TOP} = 1.936 \times (10^{11})\ \text{mm}^4, I_{BOTTOM} = 9.589 \times (10^{11})\ \text{mm}^4$$

The bending stress is calculated using the following:

$$\sigma = \frac{Mc}{I}$$

Where c is the furthest distance from the neutral axis (the furthest distance from the center of the cross section is the radius).

Thus the value for c for top part would be the radius of the circle on the top, which is 900mm.

Thus the value for c for bottom part would be the radius of the circle on the bottom, which is 1500mm.

The truss is a hollow design of uniform thickness of 100mm, thus the Inertia for the top and bottom part was found using

$$I = \frac{\pi(d_o^4 - d_i^4)}{64}$$

The bending stress at the top and bottom of the truss is

$$\sigma_{TOP} = \frac{827.5556(10^3) \times 900}{1.936 \times (10^{11})} = 3.847(10^{-3})\ \text{MPa} = 3.847\ \text{kPa}$$

$$\sigma_{BOTTOM} = \frac{827.5556(10^3) \times 1500}{9.589 \times (10^{11})} = 1.294(10^{-3})\ \text{MPa} = 1.294\ \text{kPa}$$

These stresses are much smaller than the yield and ultimate tensile strength for SS230, meaning that the bending of the truss due to the wind is negligible.

L3 Truss detailed analysis for material selection, legislation and corrosion protection

Truss Analysis

Truss is the framework, consisting of beams, rafters, posts and is supporting the wind turbine structure. The truss beams are connected together through welding processes or interlocked using bolts and nuts. To be able to support such a heavy load, we will look mainly into the Strength of the truss so that it can provide a strong and efficient structural support, material selection should be economical, stability and force balance of the truss.

Materials used for fabricating trusses are structural steel sections, seasoned wood, hollow or solid pipes, bamboo, etc. The selection of the material essentially depends upon the load factor, type of use, material availability etc. Truss Plates shall be manufactured from galvanized sheet steel conforming to or exceeding ASTM Standard A653/A653M "Standard specification for Sheet Steel, Zinc coated (Galvanized) by the Hot-Dip Process" and shall have the minimum properties specified in table below. For this project SS230 can be used as it fits our specifications and is the cheapest amongst the alternatives. (IDEAPARK, 2013).

GRADE	SS230	SS255	SS275	HSLAS340	HSLAS410
Ultimate Tensile Strength, MPa	310	360	380	410	480
Minimum Yield, MPa	230	255	275	340	410
Elongation (at failure) in 50 mm length, %	20	18	16	20	16

2) Corrosion resistant coating shall conform to ASTM A924, "Standard Specification for Steel Sheet, Zinc-Coated (Galvanized) by the Hot-Dip Process, General Requirements", Coating Designation G90, or ASTM A924, "Standards Specification for Electrolytic Zinc Coated Steel Sheets", Coating Class C, or such treatment as will give equivalent corrosion protection as applied to steel sheet before connector plates are stamped out. (IDEAPARK, 2013)

3) On metal connector plates there shall be provided some means such as holes, dimples, bosses, marked pattern, etc., to indicate location of any separately applied nails or fasteners so that nails or fasteners will not be spaced too closely together in the beam and cause excessive splitting.

The dimensions of a completed truss shall not exceed the differences shown in the table. These legislations are issued by Truss Plate Institute of Canada. (Truss Plate Institute of Canada, 2014)

Truss Dimensions	Maximum Difference between specified and measured dimensions
Length ≤ 9144 mm (30 feet)	6 mm (1/4")
Length > 9144 mm (30 feet)	13 mm (1/2")
Overall Height ≤ 1200 mm (4 feet)	3 mm (1/8")
Overall Height > 1200 mm (4 feet)	6 mm (1/4")
Left Heel/Stub Height	3 mm (1/8")
Right Heel/Stub Height	3 mm (1/8")
Left Overhang	3 mm (1/8")
Right Overhang	3 mm (1/8")

(Truss Plate Institute of Canada, 2014)

The metal connector plates used in unusual environmental conditions, or exposed to the weather, additional corrosion resistant protection for the metal connector plates shall be installed when such conditions are identified by the building designer/engineer.Galvanized G 90 metal connector plates subject to environments described above shall be painted with one coat of one of the following combinations (Truss Plate Institute of Canada, 2014):

a) Epoxy-Polyamide Primer (SSPC-Paint22)

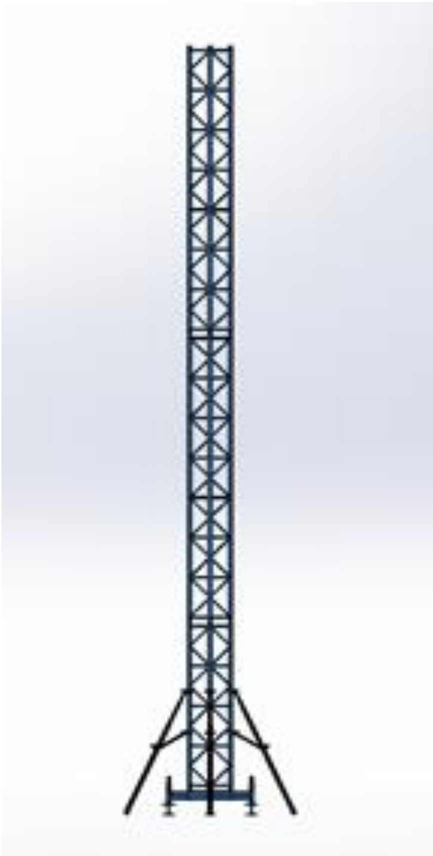
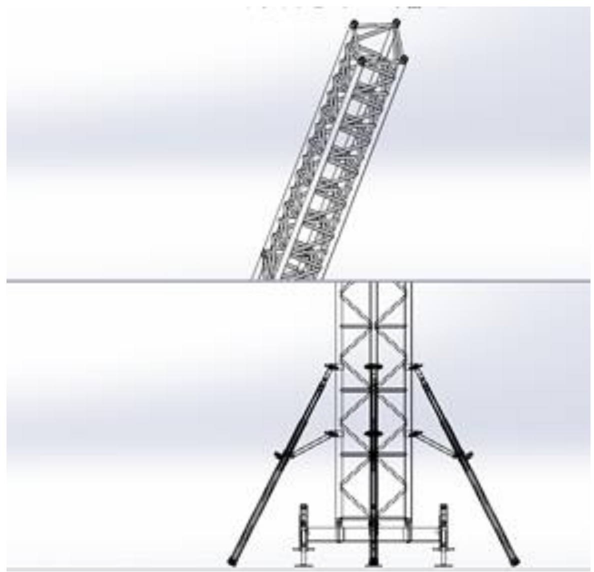
b) Coal-Tar Epoxy-Polyamide Black or Dark Red Paint (SSPC-Paint 16)

c) Basic Zinc Chromate-Vinyl Butyral Wash Primer (SSPC-Paint 27) and cold applied Asphaltic Mastic (Extra Thick Film) Paint (SSPC-Paint 12)

d) Any other coating or treatment acceptable to the building designer/engineer 3.

All coatings shall be brush applied to the embedded metal connector plates at the jobsite during or after truss installation. Embedded metal connector plates shall be free of dirt and oil prior to coating application. In addition, all of the Manufacturer's recommendations for application of products used must be followed implicitly (Steel Construction and Manufacturing, n.d.).

L4 Other designs for the truss

These pictures indicate the first truss design we thought of. This design was ignored because of various factors. The factors include that the truss design was not able to allocate the complete gear assembly within, as the dimension of the square structure of beams is just 350 mm and we needed at least 800mm (See picture below).

Moreover, the complete design had about 780 beams attached, thus making its very complex structure to design and adding on the weight of the overall object. The weight of the complete assembly was found to be 88432.27 Kg when we used the material as plain carbon steel for truss design. This value is more than 3 times higher as compared to the maximum value that can be transported (28 tons).

The design had to be supported through guy wires which add on more components to our complete assembly. Thus, adding more components meant more manufacturing cost, and it made our final product more expensive which is not viable.

Pictures for dimensions and top view and design are provided for better understanding.

M: Gantt Chart & Decision Matrix

Table M1: Decision Matrix for Wind Powered Pump Jack Concepts Consideration

	Ease of Assembly	Fan eff.	Material cost	Mobility	Maintenance	Gearbox efficiency	Environmental Consideration	Aesthetics	Estimated Design cost	Safety	TOTAL
Rating	90	80	70	90	80	80	50	30	90	100	
Design 1	7	5	7	7	7	7	3	5	7	5	470
Design 2	3	7	5	5	5	7	3	5	5	5	384
Design 3	5	3	7	7	5	5	7	5	3	7	408
Design 4	1	5	1	1	7	3	3	3	5	3	244

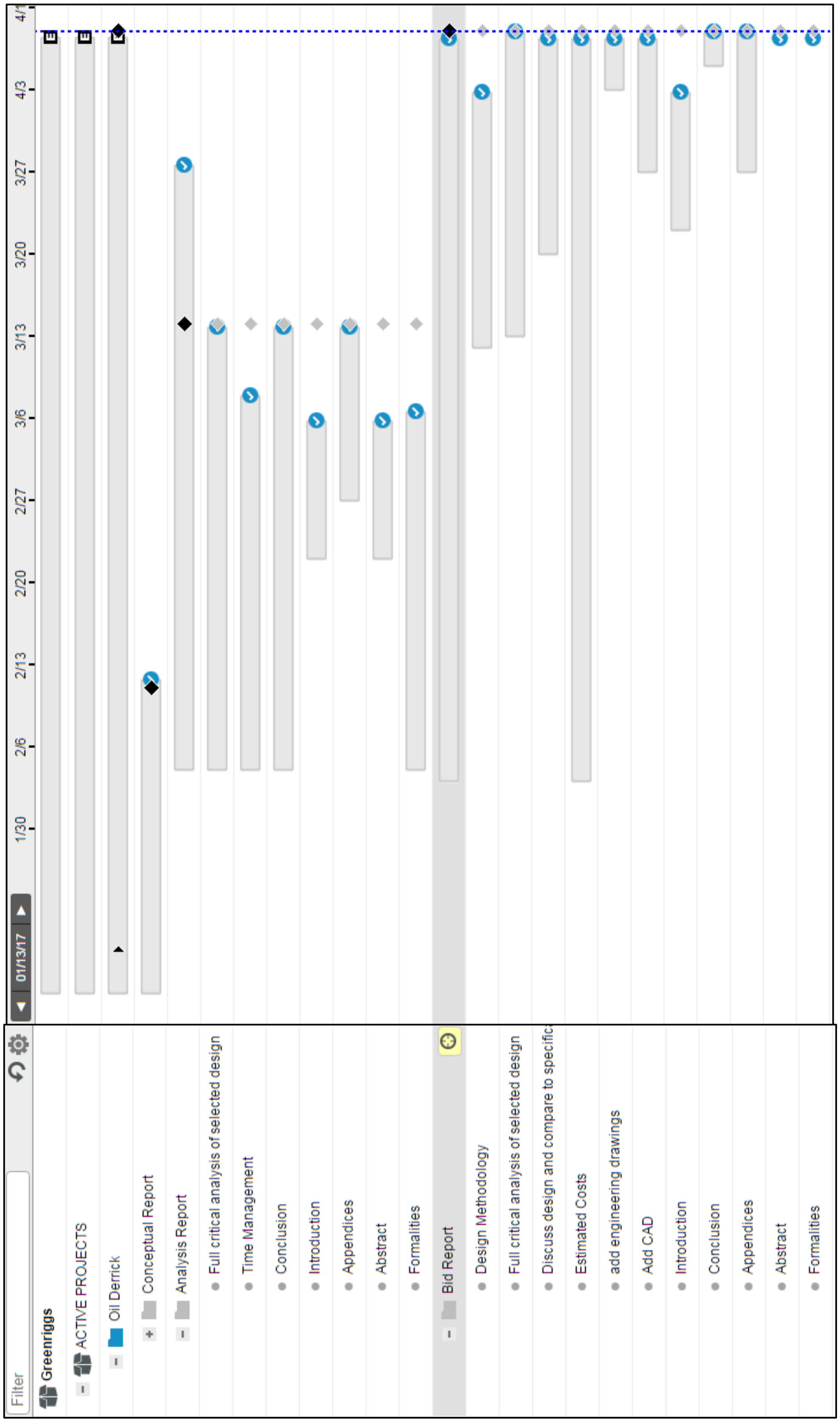


Figure M1: Gantt chart showing the dates from commencement to end of project

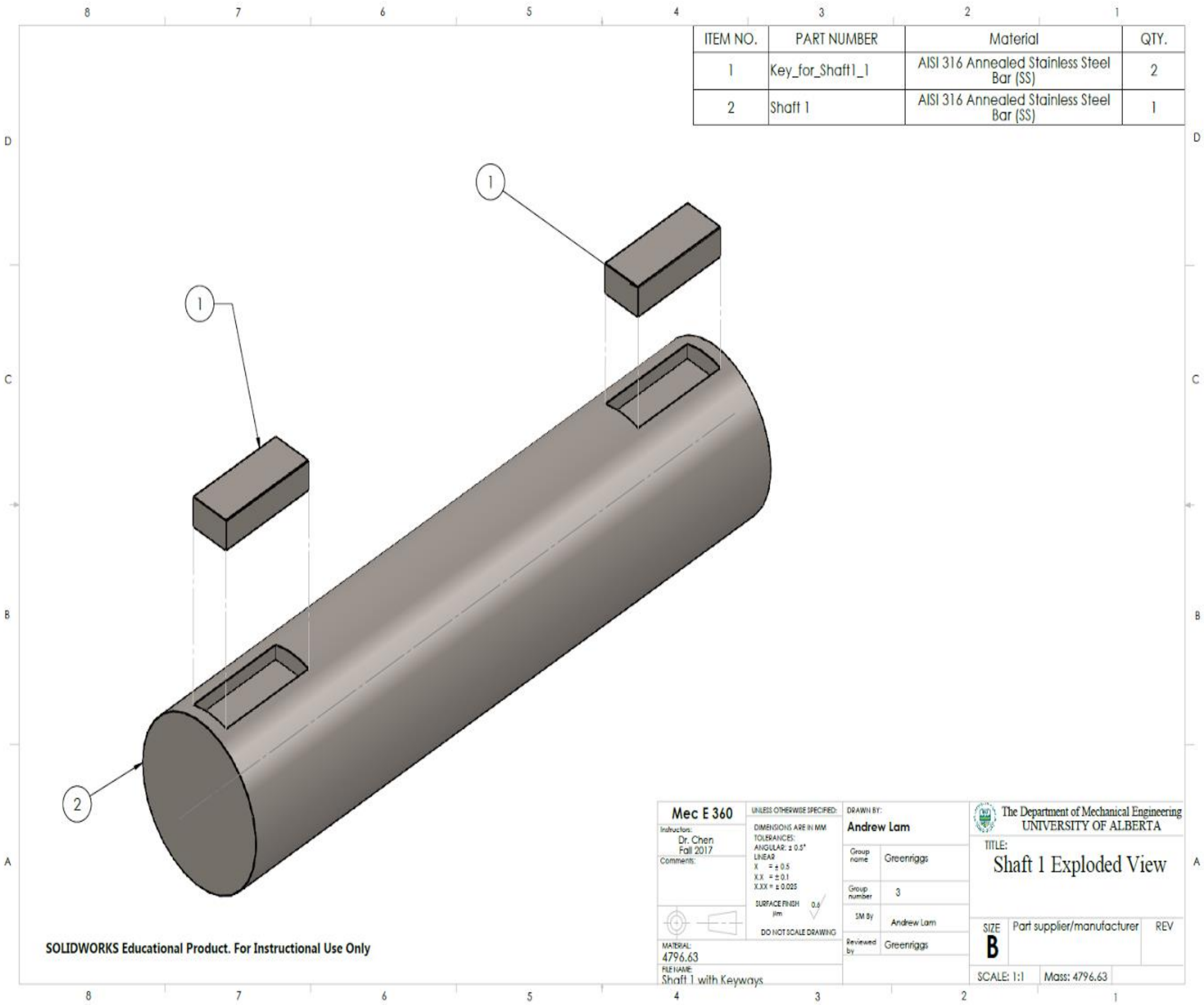
N: Wind Power Generation Calculation

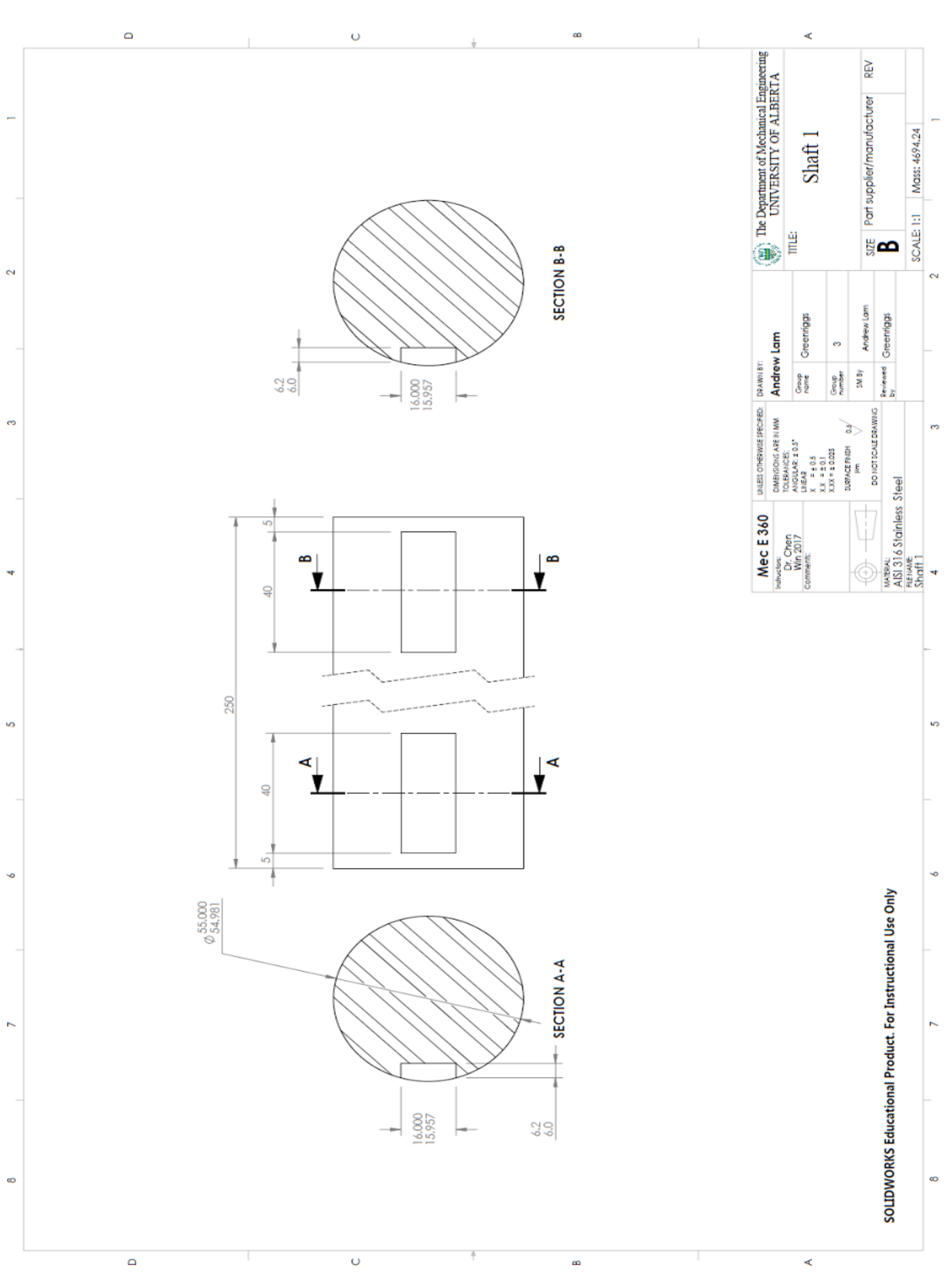
The calculation for the power generated by the wind can be calculated using following MATLAB function named, ‘calcp’, where the engineer inputs the air speed and blade diameter and receives an output of the maximum power generated by the wind.

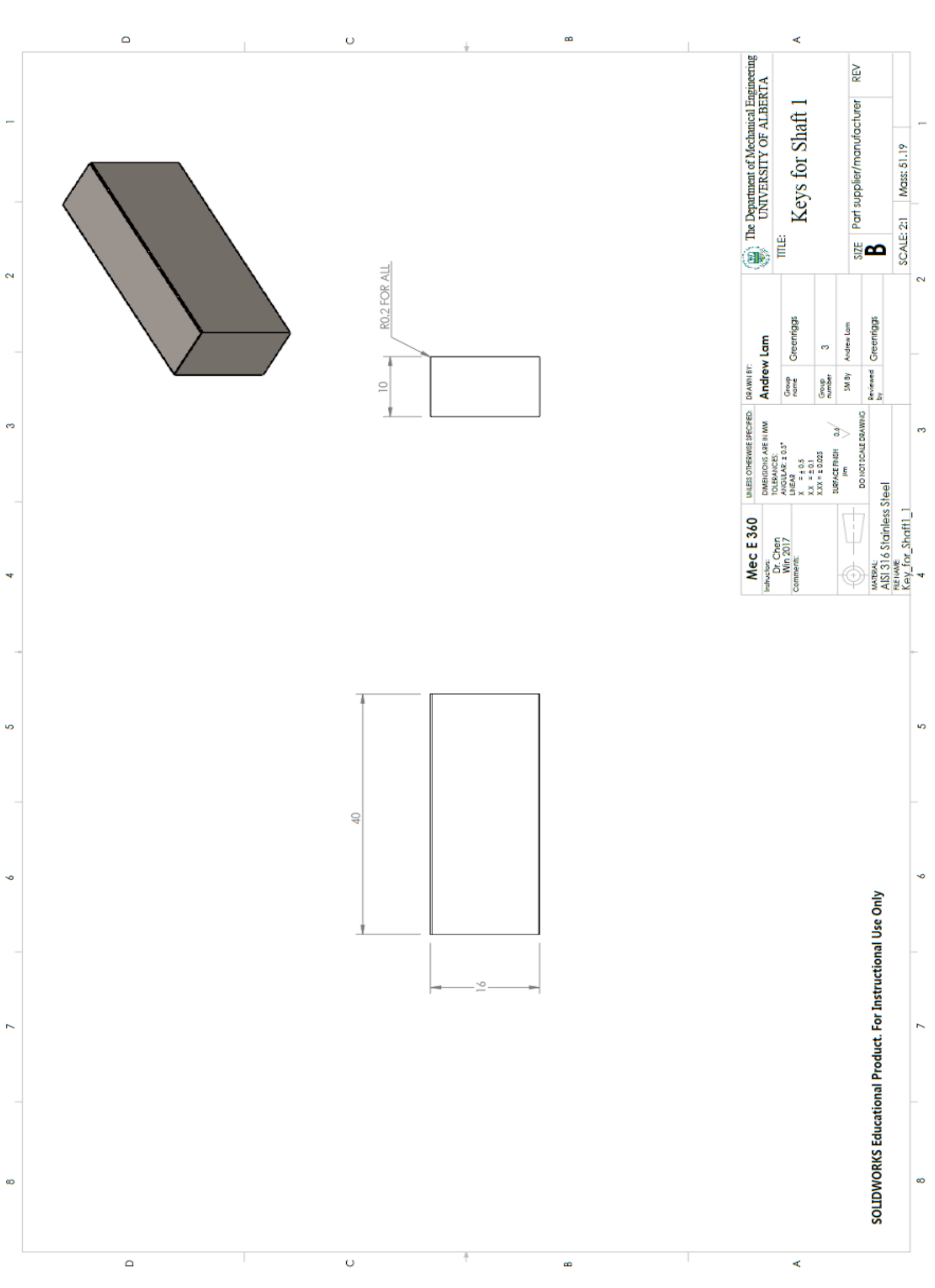
```
function maxpowergenerated = calcp(v,D)
% v = Free stream air speed (m/s)
% D = Blade diameter (m)

rho = 1.18 ;% (kg/m^3), Assume air at standard condition density
turbine_efficiency = 0.25 ;%In practice, the actual efficiency ranges between 20 and
40 percent and is about 35 percent for many wind turbines.
ke = ((v^2)/2)/1000 ;% (kJ/kg) kinetic energy of every unit mass of air flowing at
speed v
massflowrate = rho*(D^2)*pi/4*v;
maxpowergenerated = massflowrate*ke*turbine_efficiency ; %kW

fprintf('The Maximum Power Generated is%8.4f kW\n', maxpowergenerated )
end
```

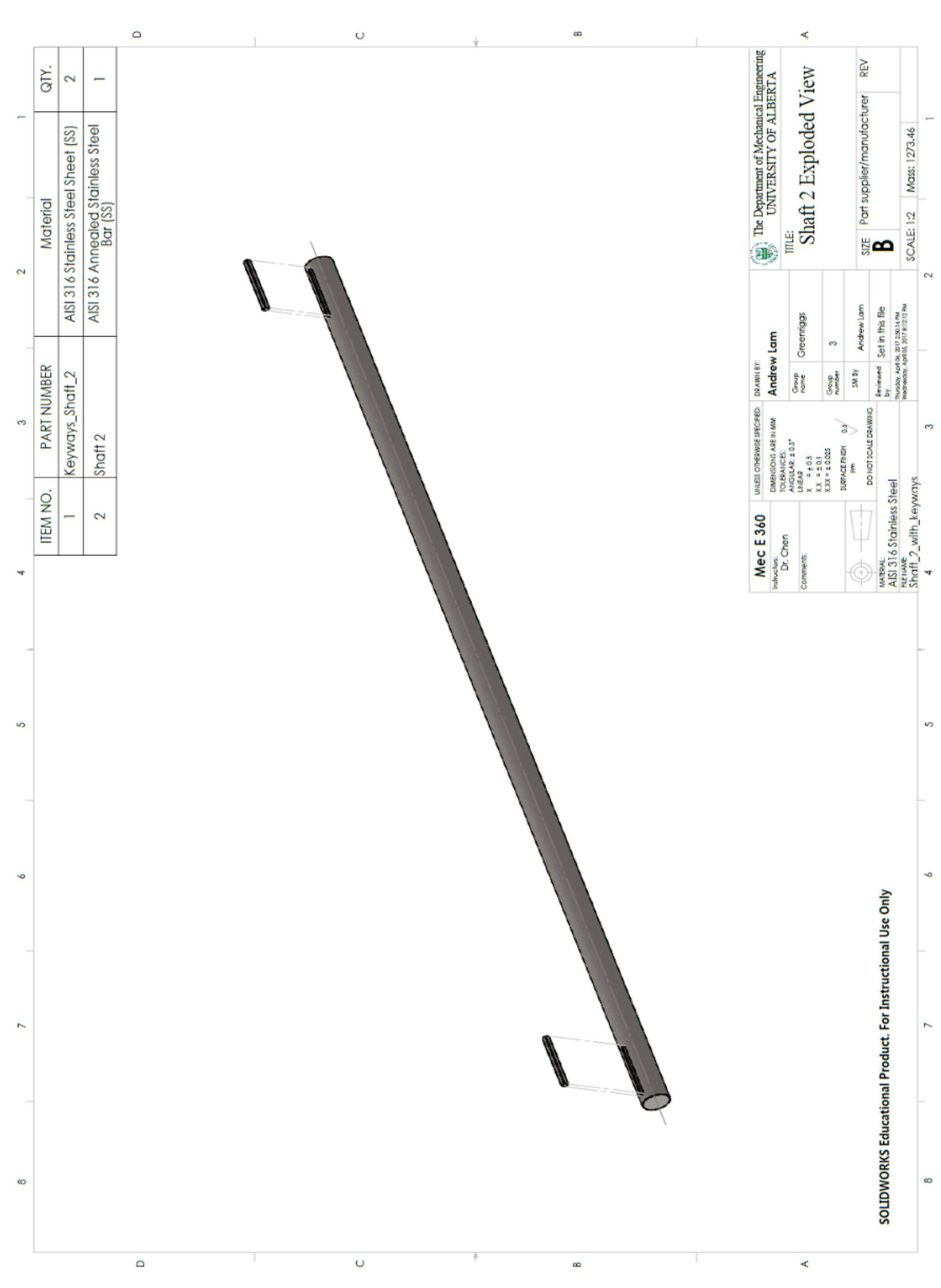
It must also be noted that the power generated here is not yet transmitted into the gearbox. Ideally, the power should be conserved through the gearbox due to the conservation of energy. Realistically there will be some losses and to account for it further calculations will use a gearbox efficiency η_{gb}. The purpose of the gearbox is mainly to change the angular velocity and torque to satisfy pumping requirements for the pump jack.

O: Drawing Package

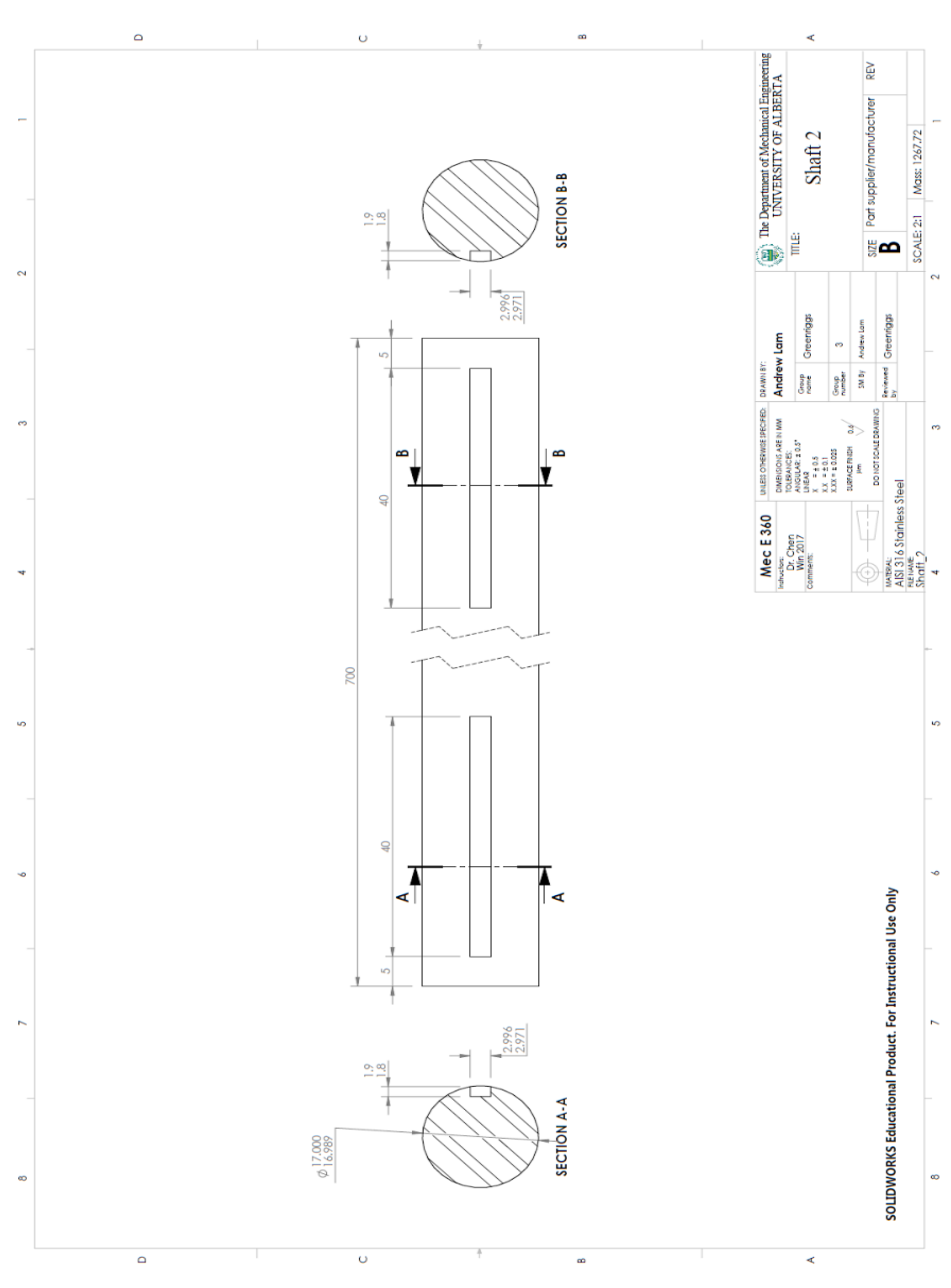



SECTION B-B
SECTION A-A
6.2
6.0
16.000
15.957
Ø 55.000
Ø 54.981
250
40
5
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Shaft 1
Part supplier/manufacturer
REV
SIZE
B
SCALE: 1:1
Mass: 4694.24
DRAWN BY:
Andrew Lam
Group name
Greenriggs
Group number
3
SW By
Andrew Lam
Reviewed by
Greenriggs
Mec E 360
Instructor:
Dr. Chen
Win 2017
Comments:
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
TOLERANCES:
ANGULAR: ± 0.5°
LINEAR
X = ± 0.5
XX = ± 0.1
XXX = ± 0.025
SURFACE FINISH μm
0.6
DO NOT SCALE DRAWING
MATERIAL:
AISI 316 Stainless Steel
FILE NAME:
Shaft 1
SOLIDWORKS Educational Product. For Instructional Use Only

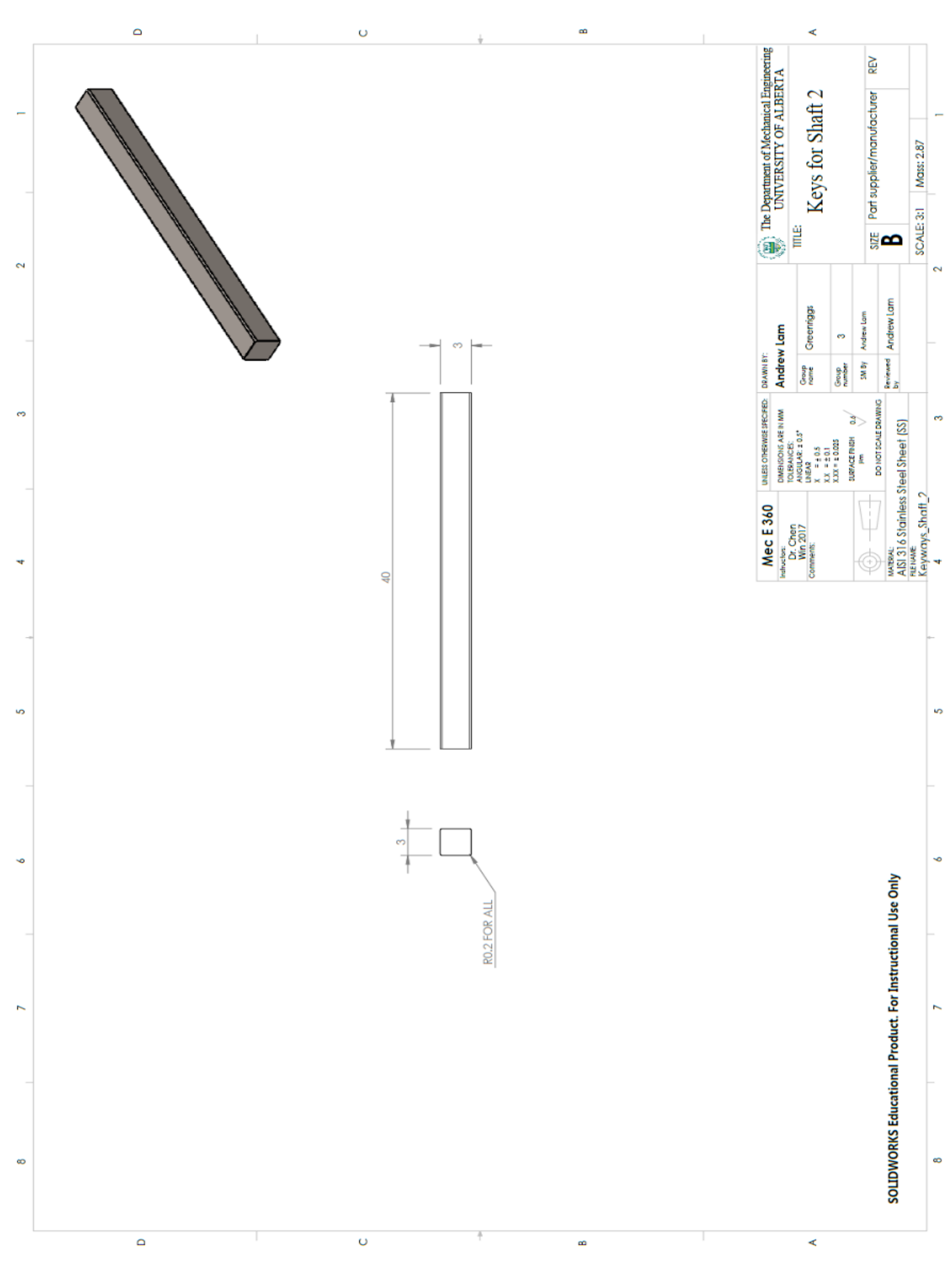

R0.2 FOR ALL
10
40
16
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Keys for Shaft 1
DRAWN BY:
Andrew Lam
Greenriggs
Mec E 360
Instructor:
Dr. Chen
Win 2017
Comments:
UNLESS OTHERWISE SPECIFIED:
DO NOT SCALE DRAWING
MATERIAL:
AISI 316 Stainless Steel
FILE NAME:
Key_for_Shaft1_1
SIZE
B
Part supplier/manufacturer
REV
SCALE: 2:1
Mass: 51.19
SOLIDWORKS Educational Product. For Instructional Use Only

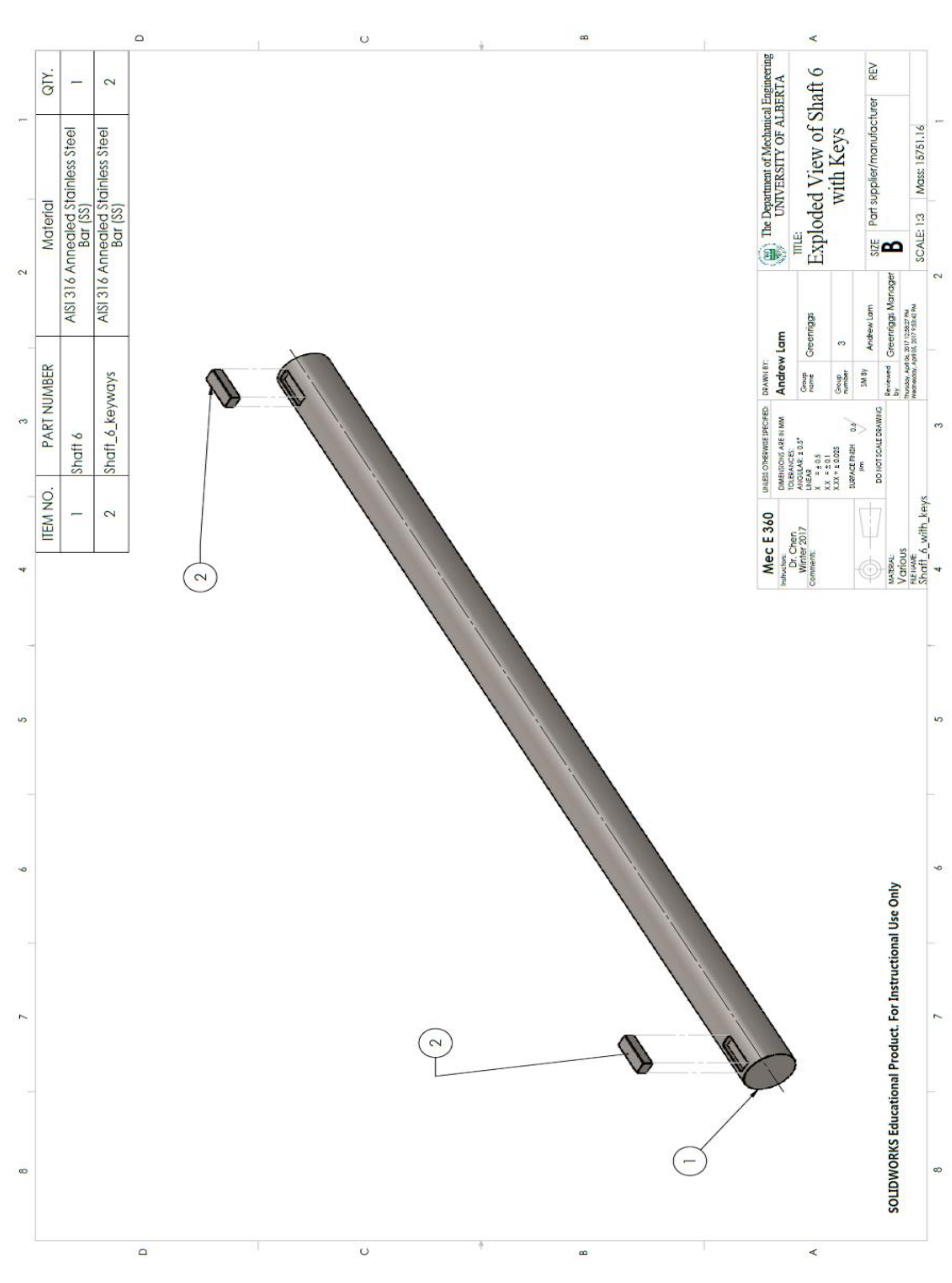


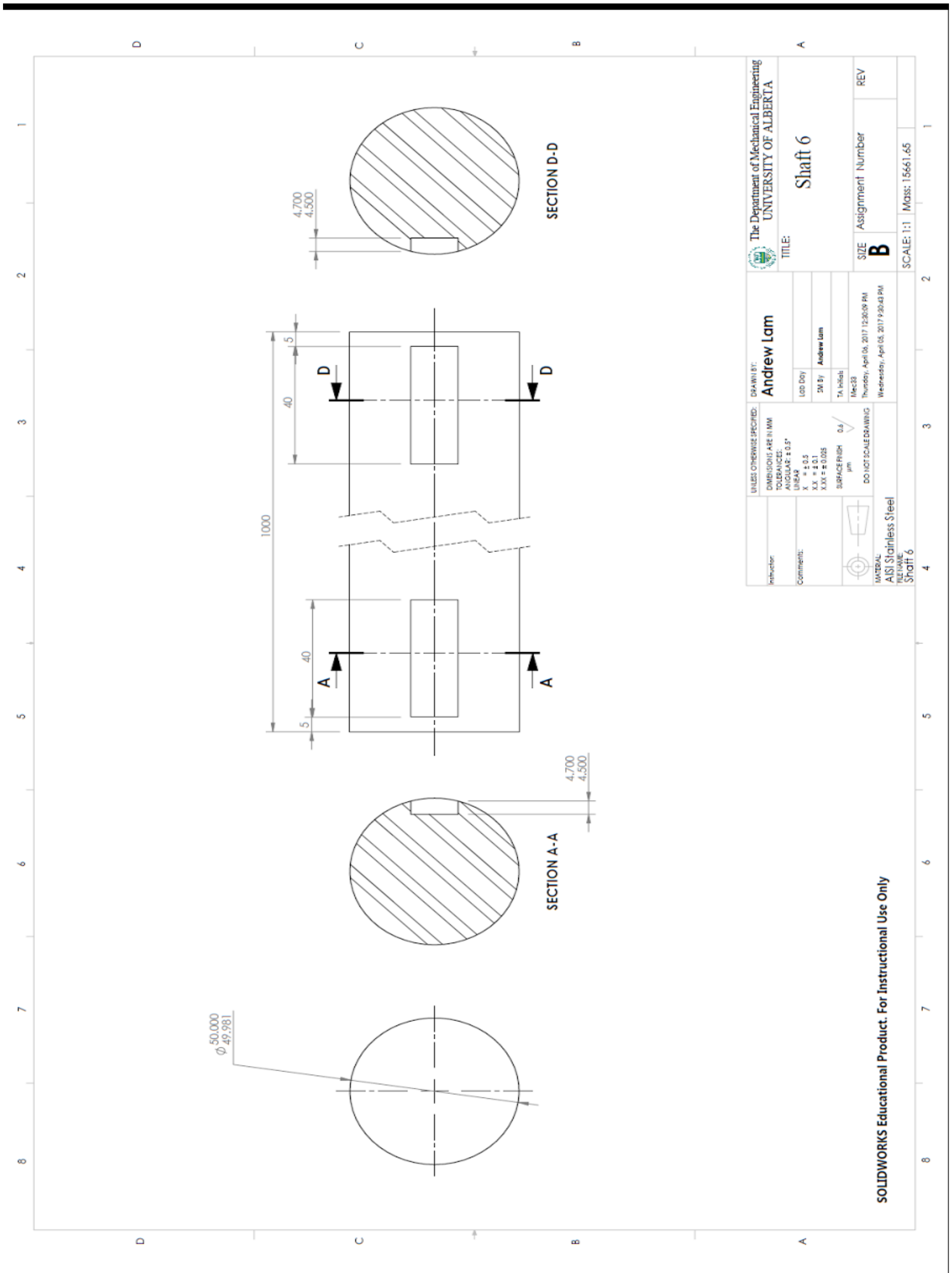
ITEM NO.
PART NUMBER
Material
QTY.
1
Keyways_Shaft_2
AISI 316 Stainless Steel Sheet (SS)
2
2
Shaft 2
AISI 316 Annealed Stainless Steel Bar (SS)
1
SOLIDWORKS Educational Product. For Instructional Use Only
Mec E 360
Instructor: Dr. Chen
Comments:
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
DO NOT SCALE DRAWING
DRAWN BY: Andrew Lam
Group name Greenriggs
Group number 3
SM By Andrew Lam
Reviewed by Set in this file
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Shaft 2 Exploded View
SIZE B
Part supplier/manufacturer
REV
SCALE: 1:2
Mass: 1273.46
MATERIAL: AISI 316 Stainless Steel
FILE NAME: Shaft_2_with_keyways

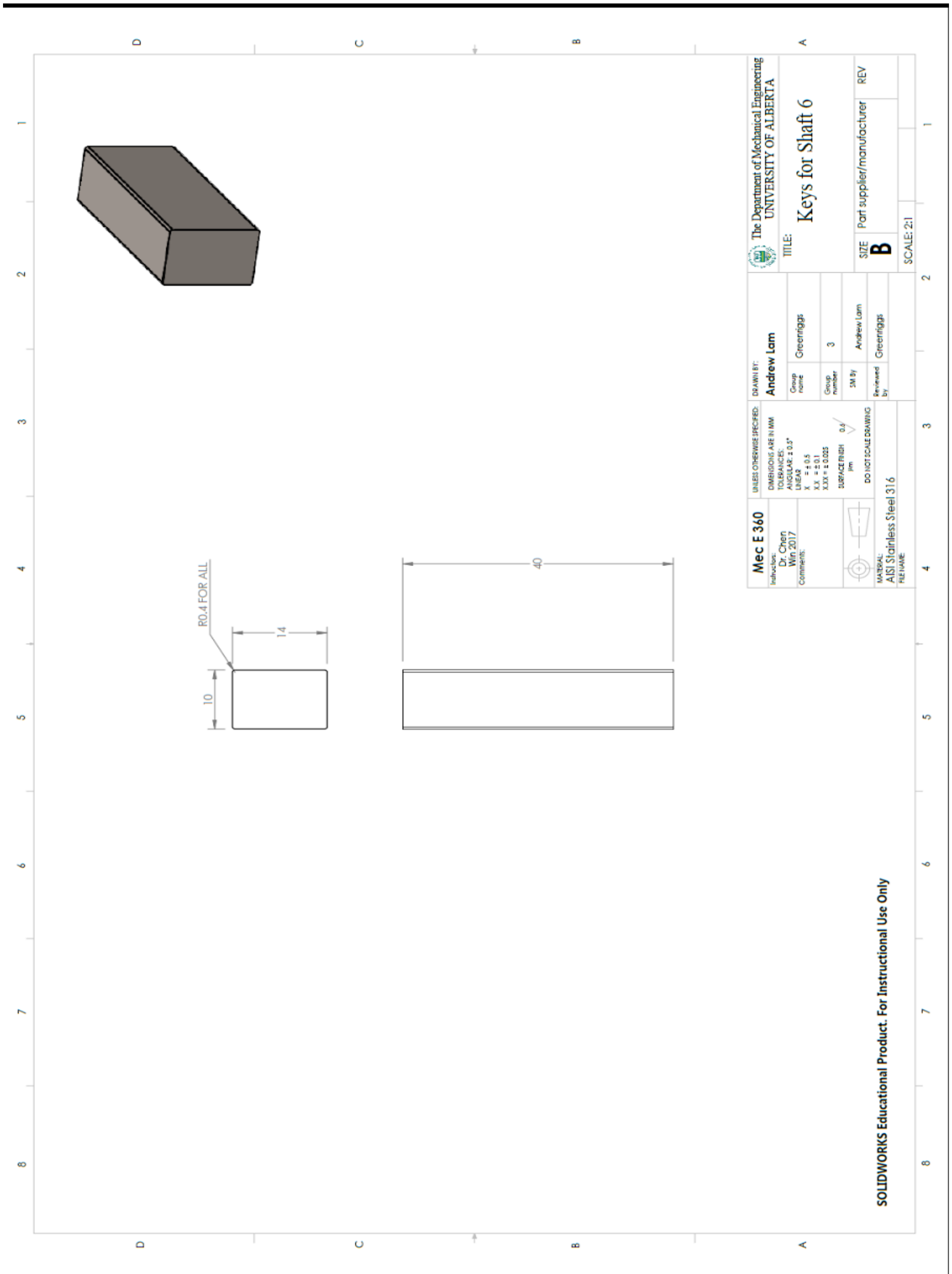


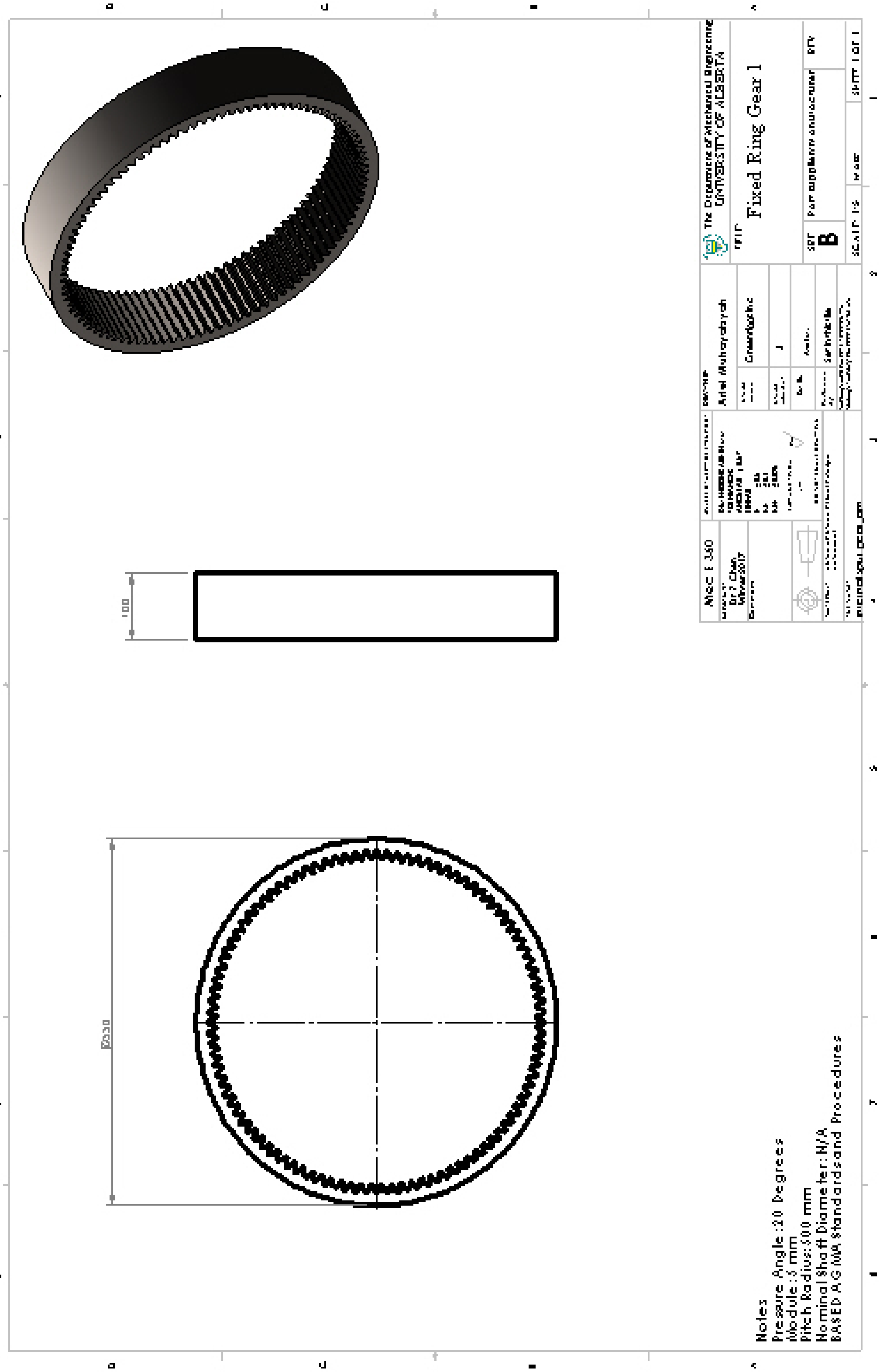
SECTION B-B
SECTION A-A
1.9
1.8
2.996
2.971
5
40
700
A
B
ϕ 17.000
16.989
SOLIDWORKS Educational Product. For Instructional Use Only
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Shaft 2
DRAWN BY:
Andrew Lam
Greenriggs
3
Mec E 360
Dr. Chen
Win 2017
MATERIAL:
AISI 316 Stainless Steel
FILE NAME:
Shaft_2
SIZE
B
SCALE: 2:1
Mass: 1267.72
Part supplier/manufacturer
REV
DO NOT SCALE DRAWING

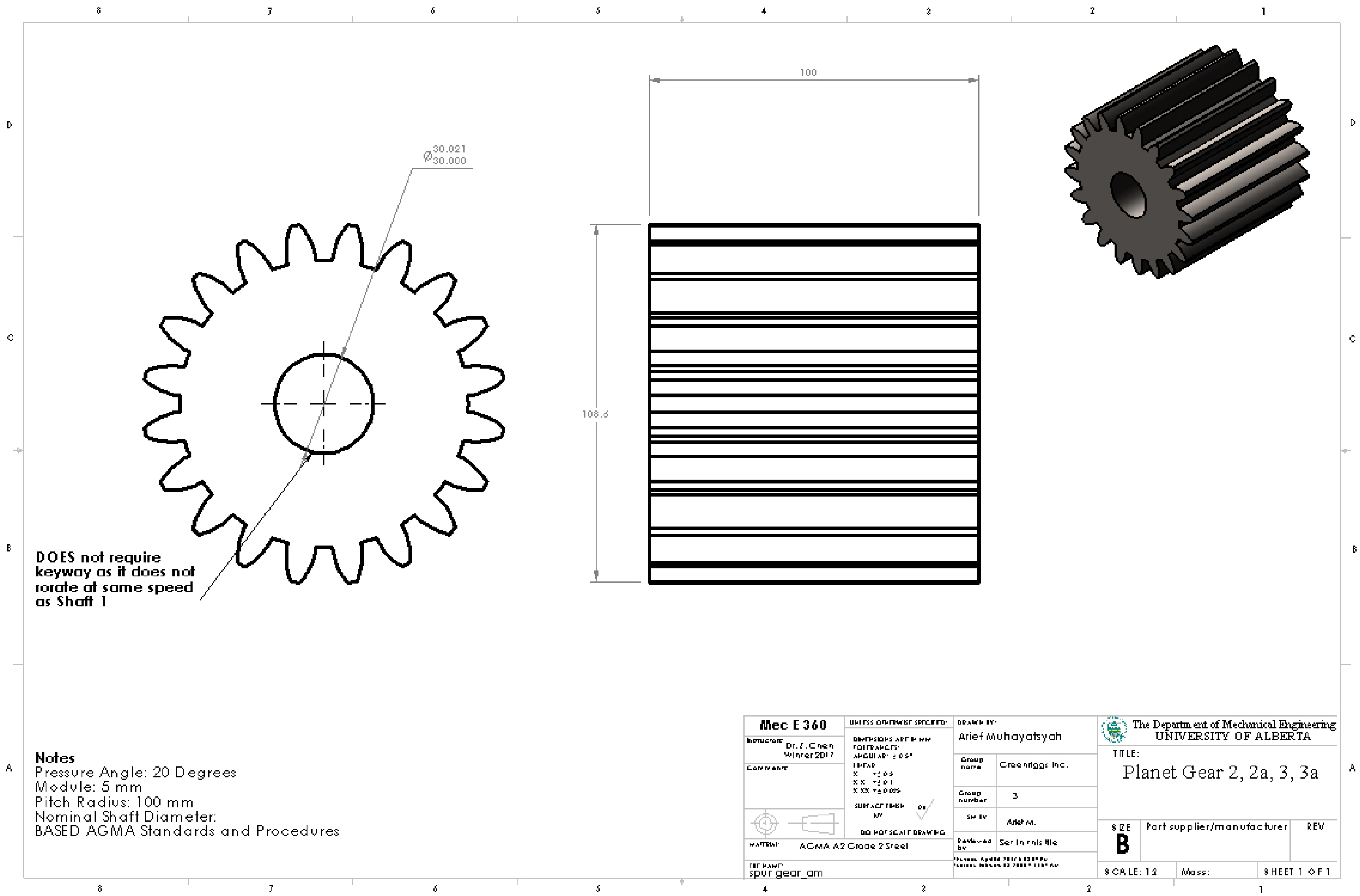

Mec E 360
Instructor: Dr. Chen Win 2017
Comments:
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
DRAWN BY:
Andrew Lam
Group name
Greenriggs
Group number
3
SM By
Andrew Lam
Reviewed by
Andrew Lam
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Keys for Shaft 2
SIZE
B
Part supplier/manufacturer
REV
SCALE: 3:1
Mass: 2.87
DO NOT SCALE DRAWING
MATERIAL:
AISI 316 Stainless Steel Sheet (SS)
FILE NAME:
Keyways_Shaft_2
40
3
3
R0.2 FOR ALL
SOLIDWORKS Educational Product. For Instructional Use Only


ITEM NO.
PART NUMBER
Material
QTY.
1
Shaft 6
AISI 316 Annealed Stainless Steel Bar (SS)
1
2
Shaft_6_keyways
AISI 316 Annealed Stainless Steel Bar (SS)
2
SOLIDWORKS Educational Product. For Instructional Use Only
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Exploded View of Shaft 6 with Keys
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:3
Mass: 15751.16
DRAWN BY:
Andrew Lam
Group name
Greenriggs
Group number
3
SM By
Andrew Lam
Reviewed by
Greenriggs Manager
Mec E 360
Instructor:
Dr. Chen
Winter 2017
Comments:
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
DO NOT SCALE DRAWING
MATERIAL:
Various
FILE NAME:
Shaft_6_with_keys

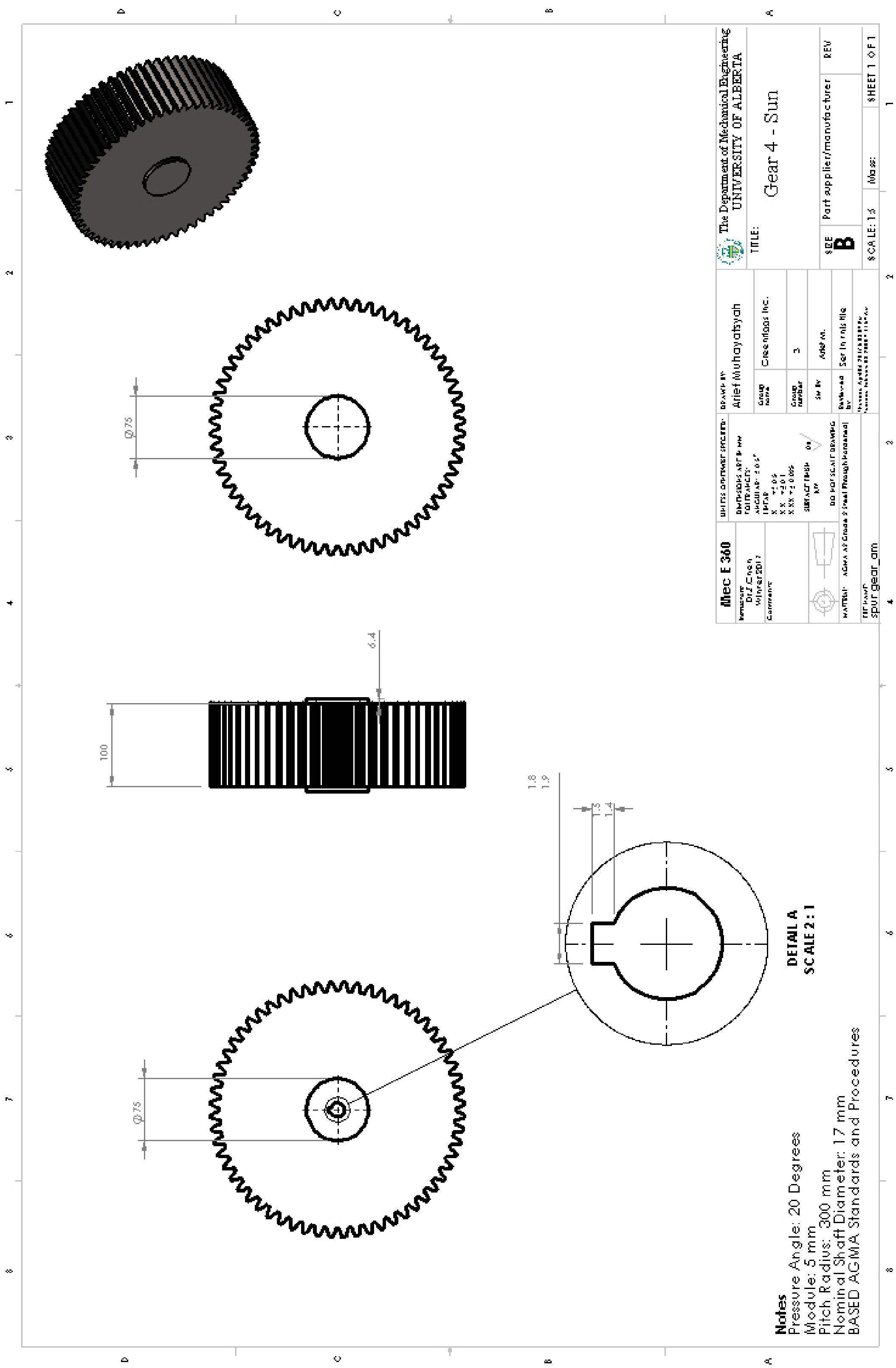

SECTION D-D
SECTION A-A
4.700
4.500
1000
40
5
50.000
49.981
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Shaft 6
Andrew Lam
Assignment Number
SIZE
B
REV
SCALE: 1:1
Mass: 15661.65
MATERIAL
AISI Stainless Steel
FILENAME
Shaft 6
DO NOT SCALE DRAWING
SOLIDWORKS Educational Product. For Instructional Use Only

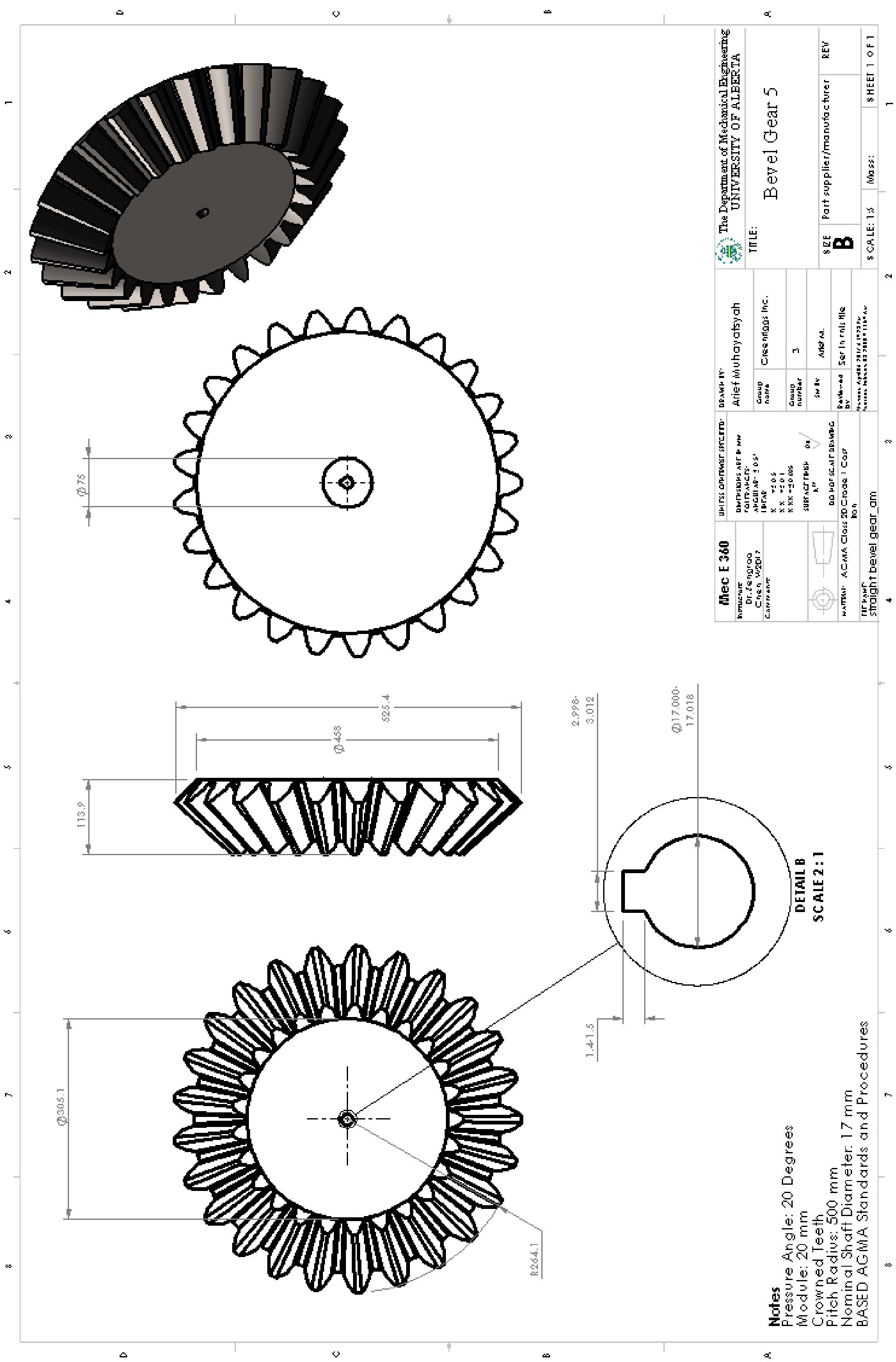

R0.4 FOR ALL
14
10
40
Mec E 360
Dr. Chen
Win 2017
Andrew Lam
Greenriggs
AISI Stainless Steel 316
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Keys for Shaft 6
Part supplier/manufacturer
REV
SIZE
B
SCALE: 2:1
SOLIDWORKS Educational Product. For Instructional Use Only

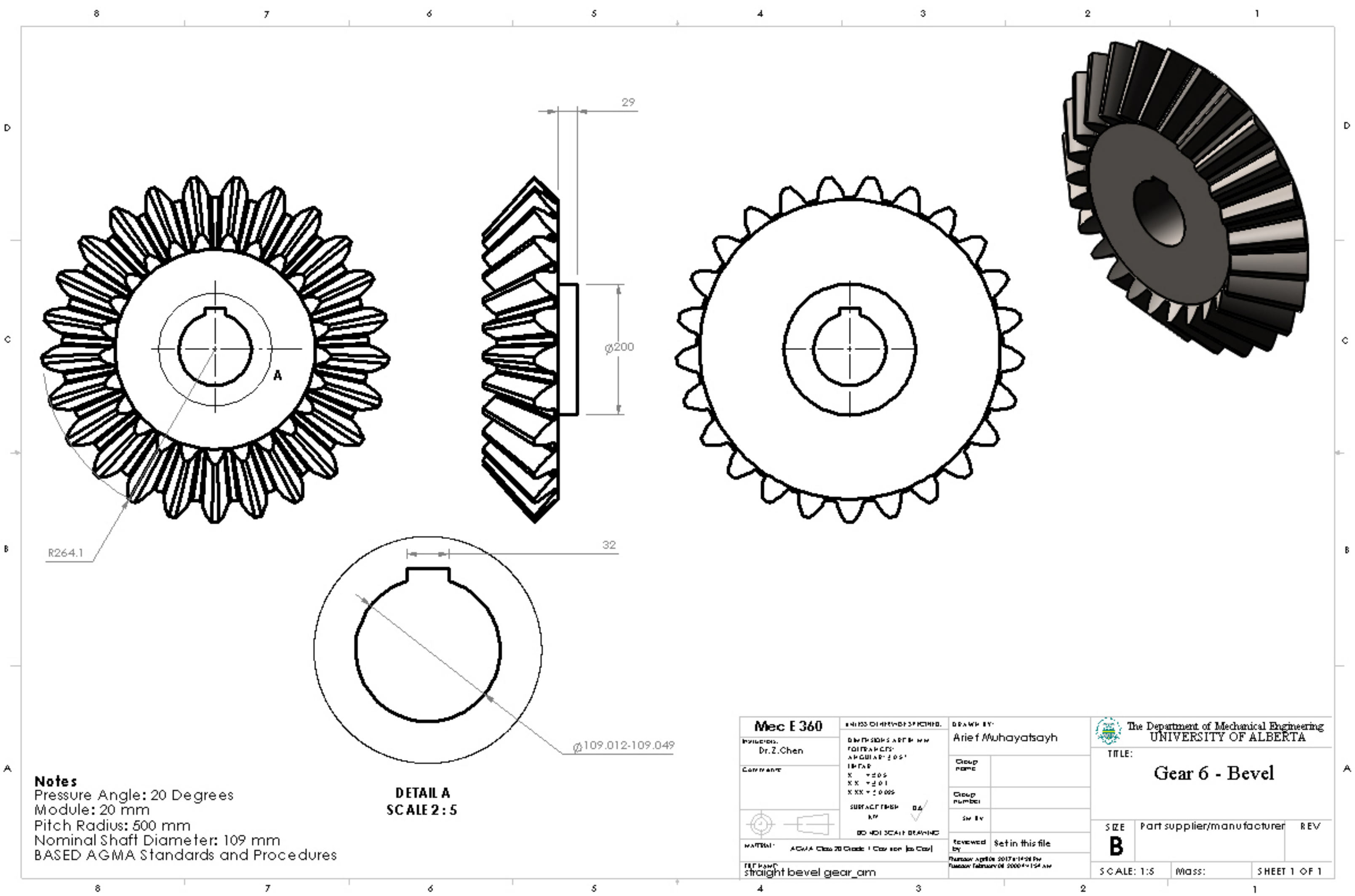

Notes
Pressure Angle :20 Degrees
Module :3 mm
Pitch Radius:300 mm
Nominal Shaft Diameter: N/A
BASED AGMA Standardsand Procedures
UNIVERSITY OF ALBERTA
Fixed Ring Gear 1
MecE 360

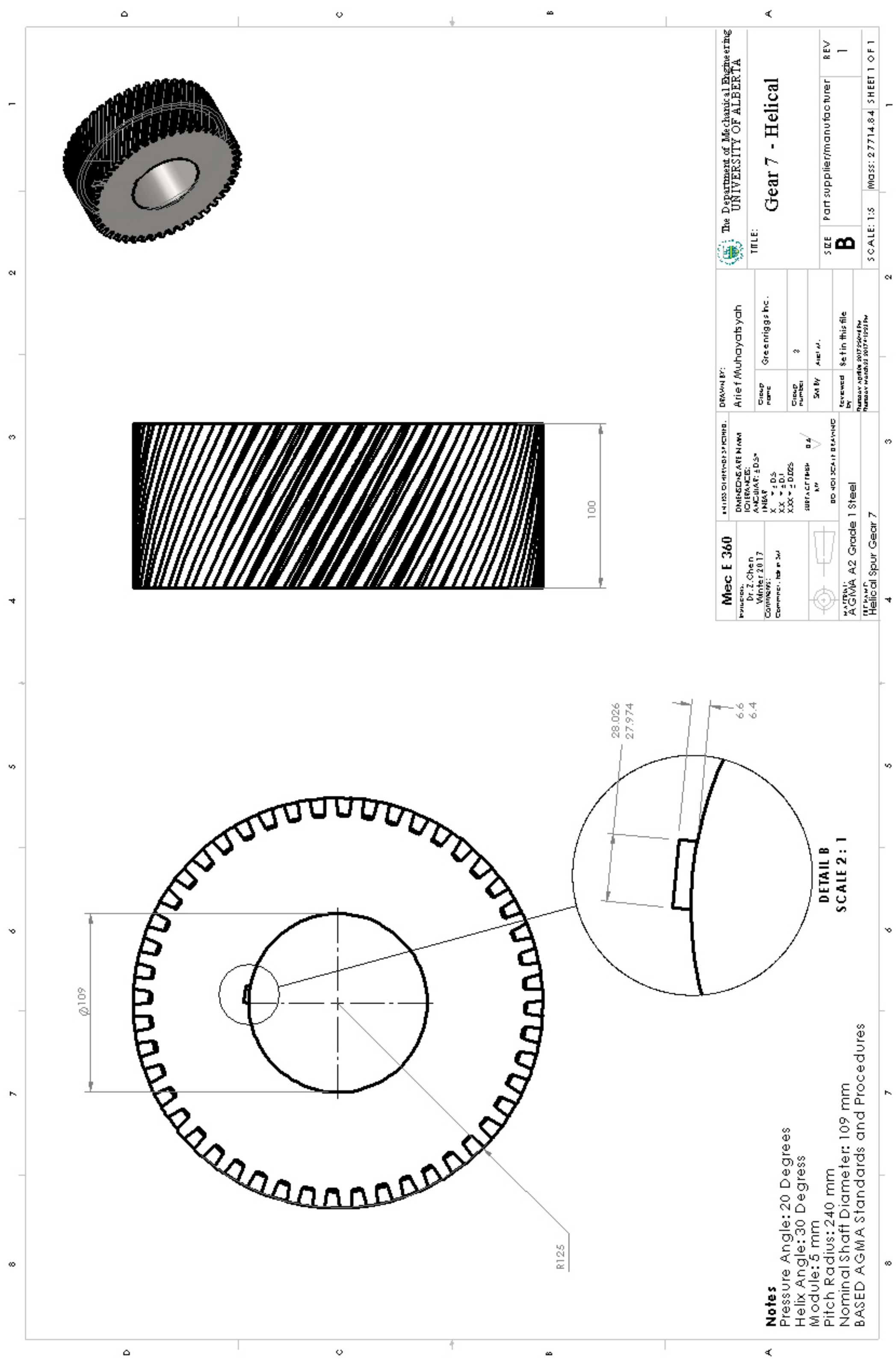


Ø30.021 30.000
100
108.6
DOES not require keyway as it does not rotate at same speed as Shaft 1
Notes
Pressure Angle: 20 Degrees
Module: 5 mm
Pitch Radius: 100 mm
Nominal Shaft Diameter:
BASED AGMA Standards and Procedures
Mec E 360
Dr. Z. Chen
Winter 2017
DRAWN BY:
Arief Muhayatsyah
Group name
Greenriggs Inc.
Group number
3
AGMA A2 Grade 2 Steel
DO NOT SCALE DRAWING
spur gear_am
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Planet Gear 2, 2a, 3, 3a
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:2
Mass:
SHEET 1 OF 1

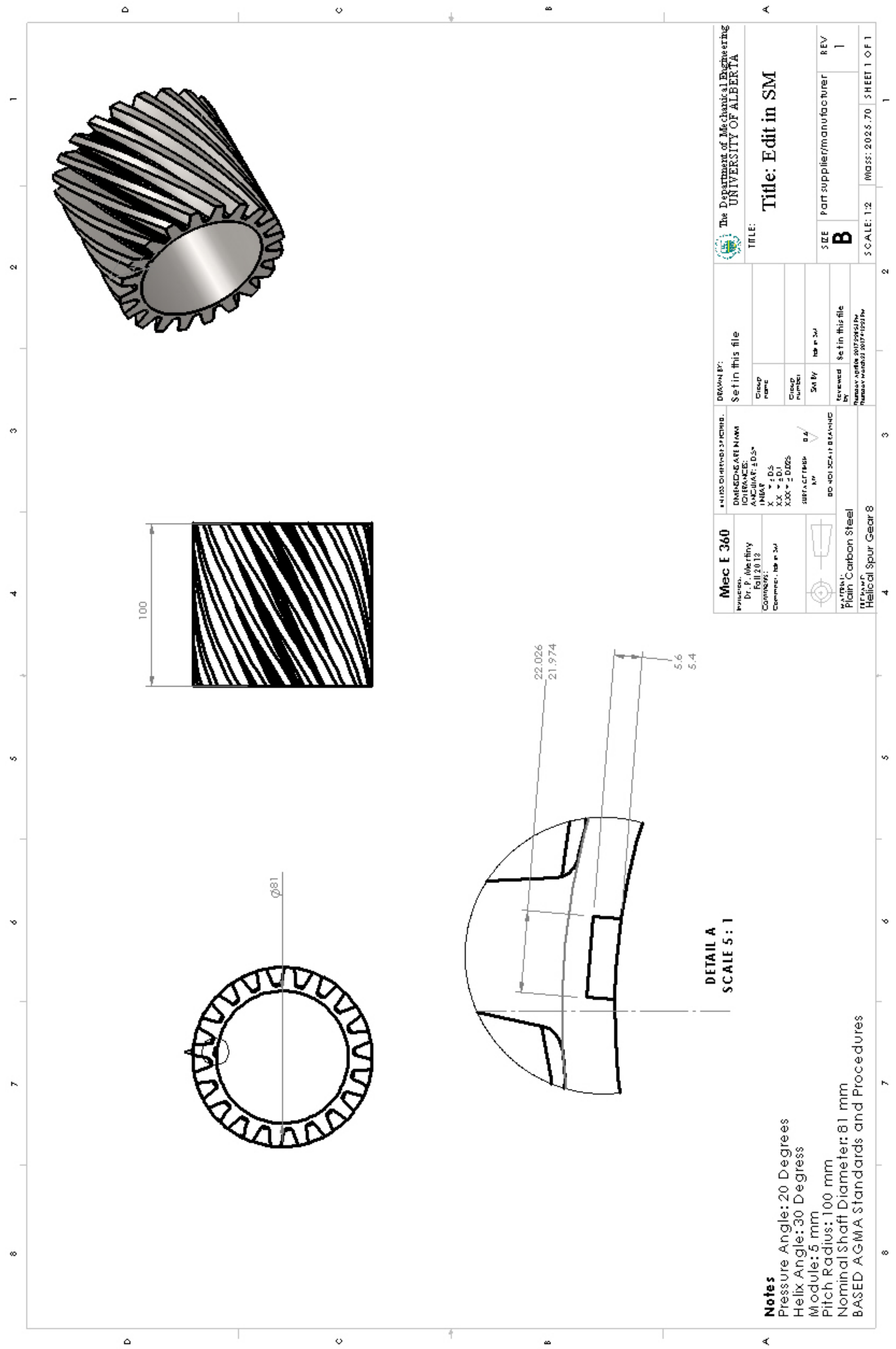

The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 4 - Sun
Ariel Muhayatyan
Greendogs Inc.
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:5
Mass:
SHEET 1 OF 1
Mec E 360
spur_gear_am
DETAIL A
SCALE 2 : 1
Ø75
100
6.4
Notes
Pressure Angle: 20 Degrees
Module: 5 mm
Pitch Radius: 300 mm
Nominal Shaft Diameter: 17 mm
BASED AGMA Standards and Procedures


The Department of Mechanical Engineering UNIVERSITY OF ALBERTA
TITLE:
Bevel Gear 5
Mec E 360
Arief Muhayatsyah
Part supplier/manufacturer
SIZE B
SCALE: 1:5
SHEET 1 OF 1
straight bevel gear_am
Ø75
525.4
Ø458
113.9
2.998-3.012
Ø17.000-17.018
DETAIL B
SCALE 2 : 1
1.4-1.5
Ø305.1
R264.1
Notes
Pressure Angle: 20 Degrees
Module: 20 mm
Crowned Teeth
Pitch Radius: 500 mm
Nominal Shaft Diameter: 17 mm
BASED AGMA Standards and Procedures

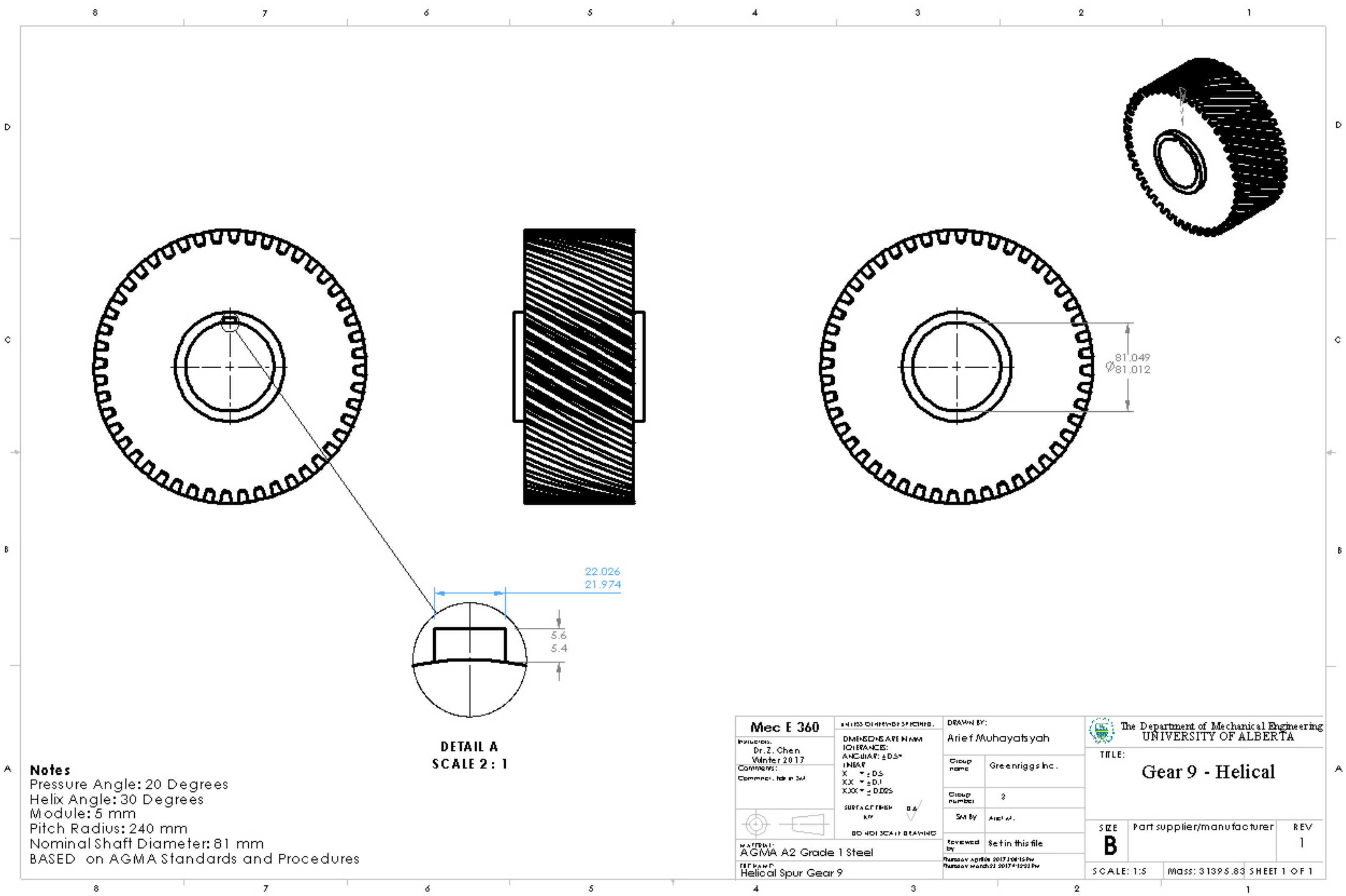

29
⌀200
32
R264.1
⌀109.012-109.049
A
DETAIL A
SCALE 2 : 5
Notes
Pressure Angle: 20 Degrees
Module: 20 mm
Pitch Radius: 500 mm
Nominal Shaft Diameter: 109 mm
BASED AGMA Standards and Procedures
Mec E 360
Dr. Z. Chen
DRAWN BY:
Arief Muhayatsayh
Set in this file
straight bevel gear_am
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 6 - Bevel
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:5
Mass:
SHEET 1 OF 1

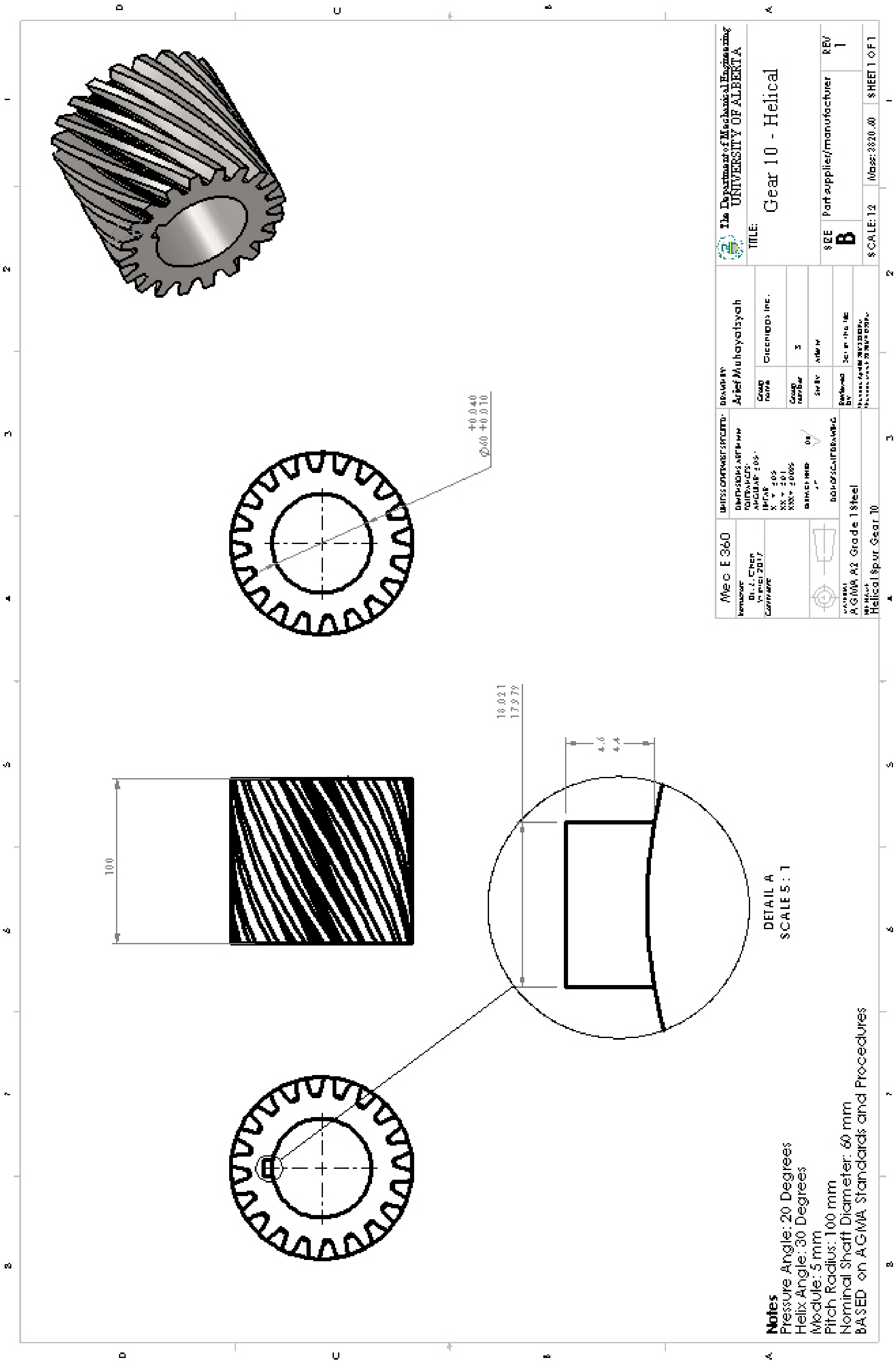


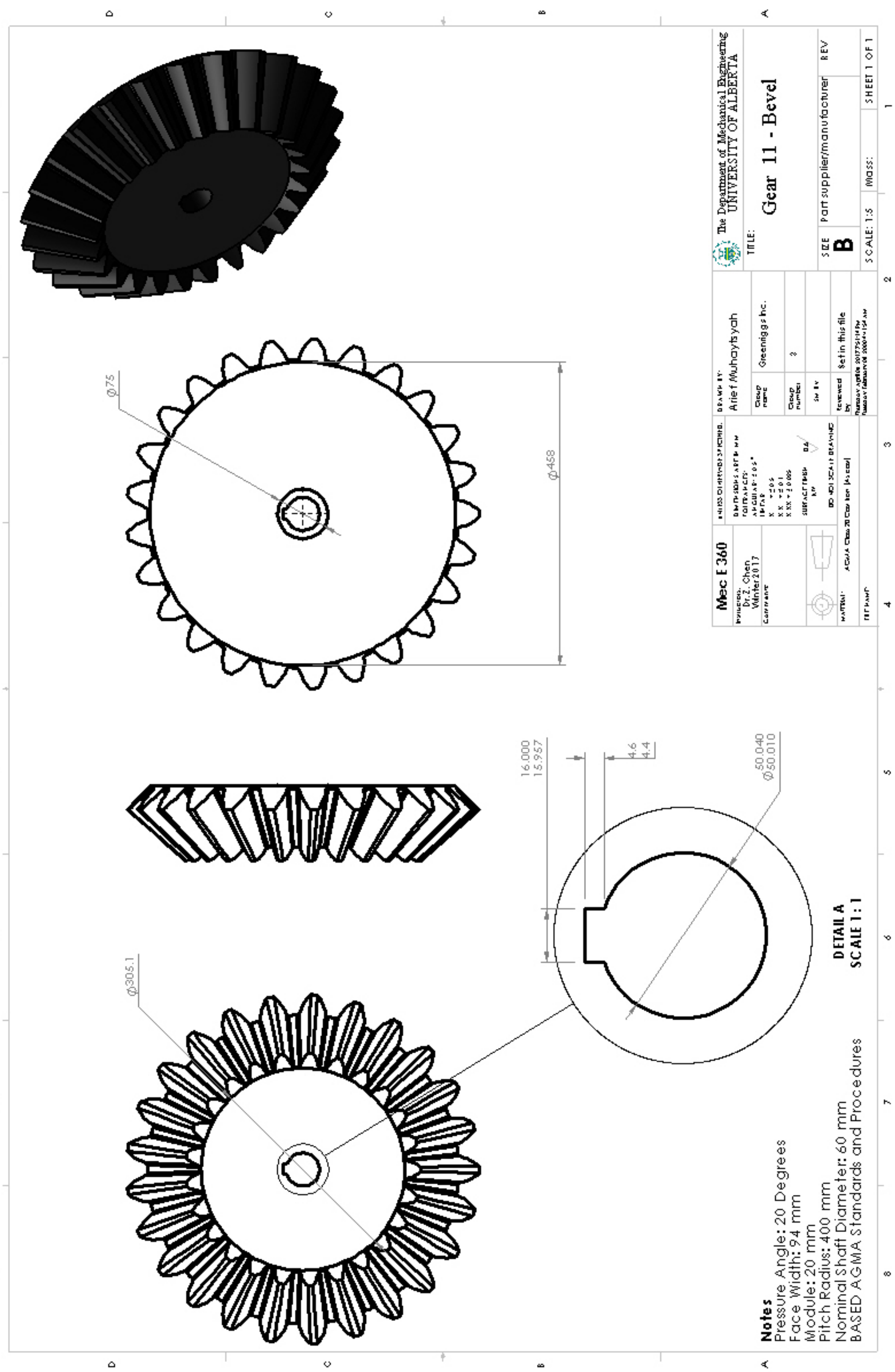
N12: Gear 8 - Helical

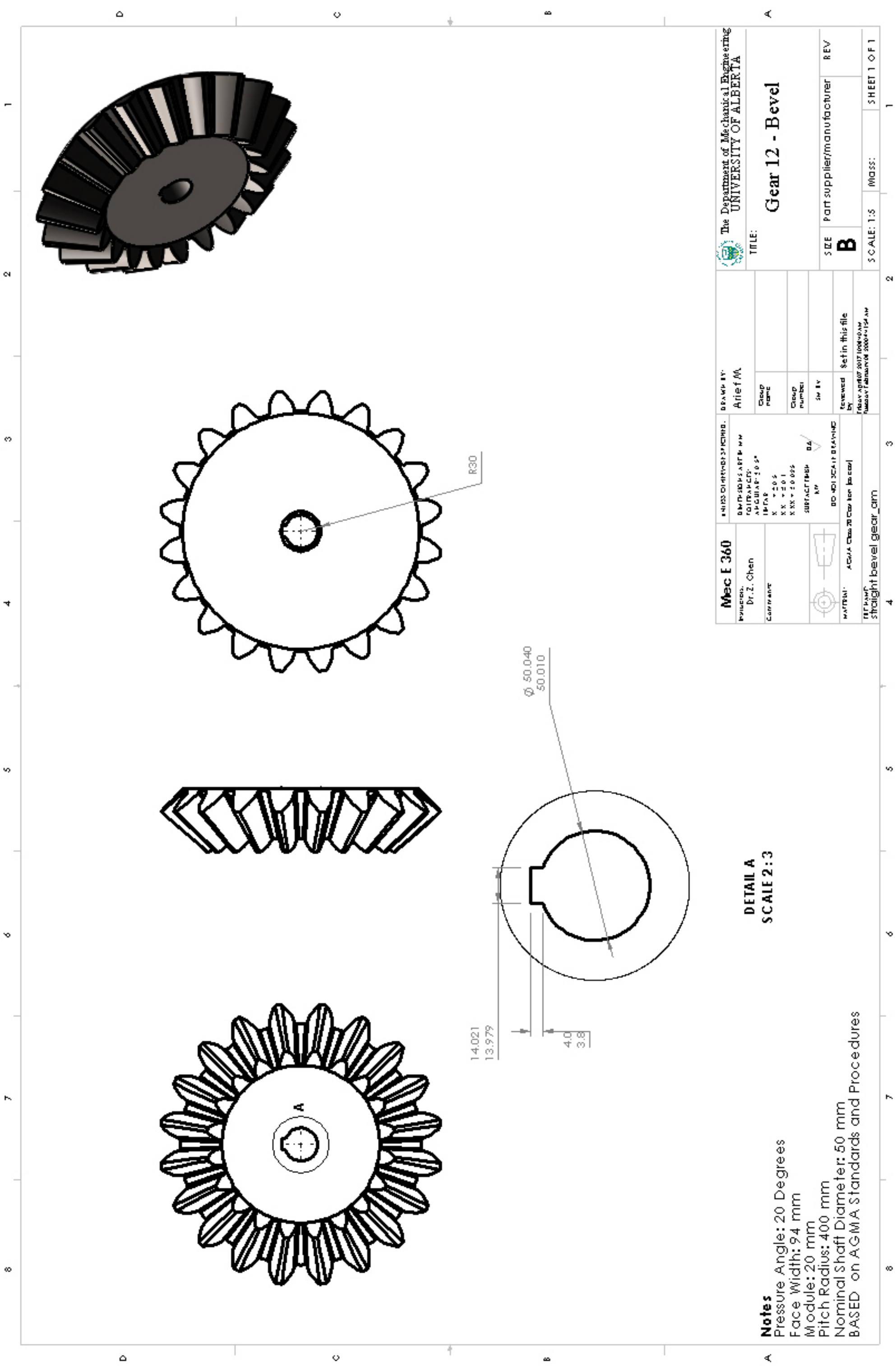


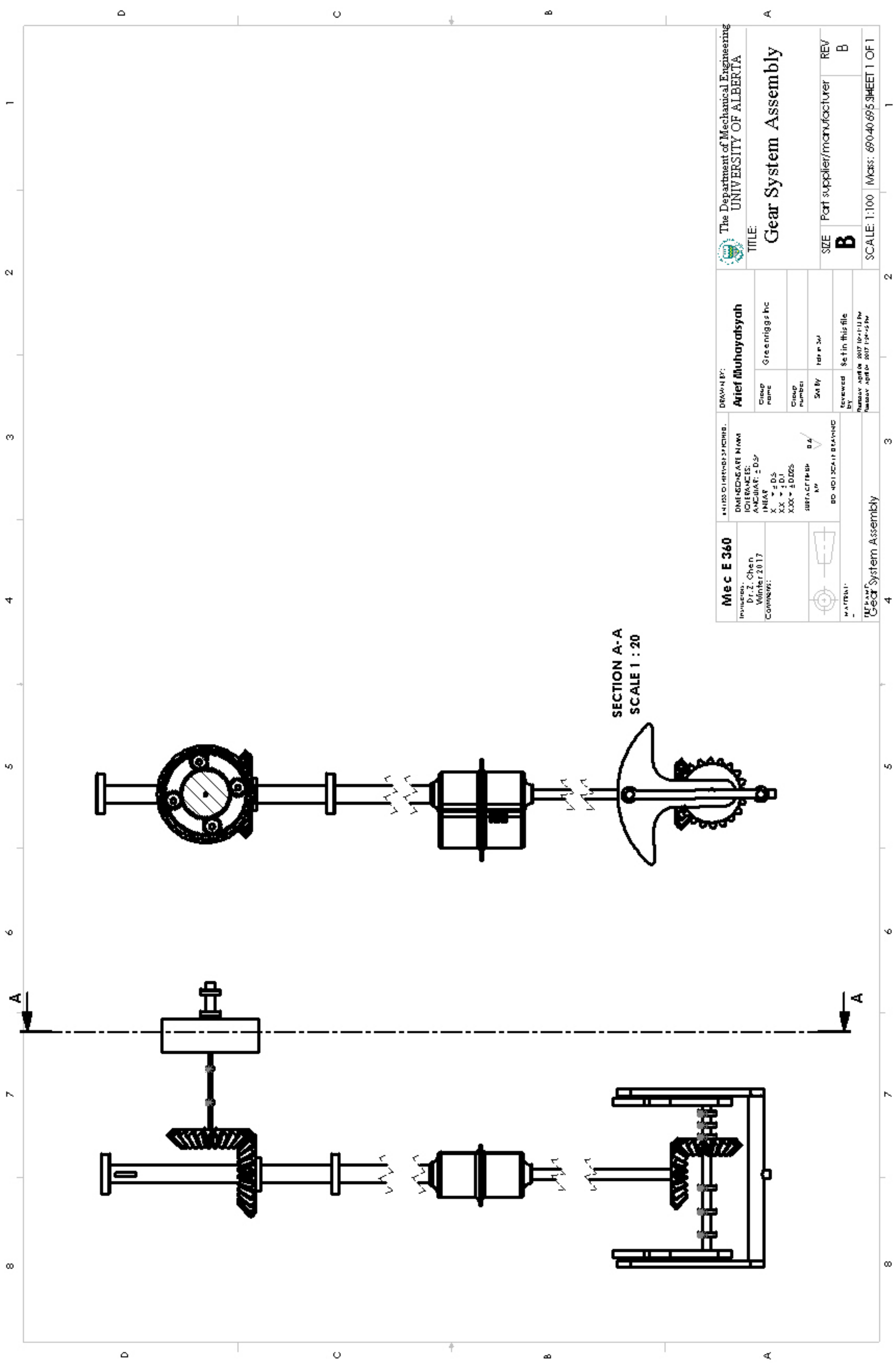
100
22.026
21.974
5.6
5.4
Ø81
DETAIL A
SCALE 5 : 1
Notes
Pressure Angle: 20 Degrees
Helix Angle: 30 Degress
Module: 5 mm
Pitch Radius: 100 mm
Nominal Shaft Diameter: 81 mm
BASED AGMA Standards and Procedures
Mec E 360
Dr. P. Mertiny
Fall 2012
DRAWN BY:
Set in this file
Plain Carbon Steel
Helical Spur Gear 8
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Title: Edit in SM
Part supplier/manufacturer
SIZE
B
REV
1
SCALE: 1:2
Mass: 2025.70
SHEET 1 OF 1


81.049
Ø81.012
22.026
21.974
5.6
5.4
DETAIL A
SCALE 2 : 1
Notes
Pressure Angle: 20 Degrees
Helix Angle: 30 Degrees
Module: 5 mm
Pitch Radius: 240 mm
Nominal Shaft Diameter: 81 mm
BASED on AGMA Standards and Procedures
Mec E 360
Dr. Z. Chen
Winter 2017
DRAWN BY:
Arief Muhayatsyah
Greenriggs Inc.
Set in this file
MATERIAL:
AGMA A2 Grade 1 Steel
Helical Spur Gear 9
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 9 - Helical
SIZE
B
Part supplier/manufacturer
REV
1
SCALE: 1:5
Mass: 31395.83
SHEET 1 OF 1

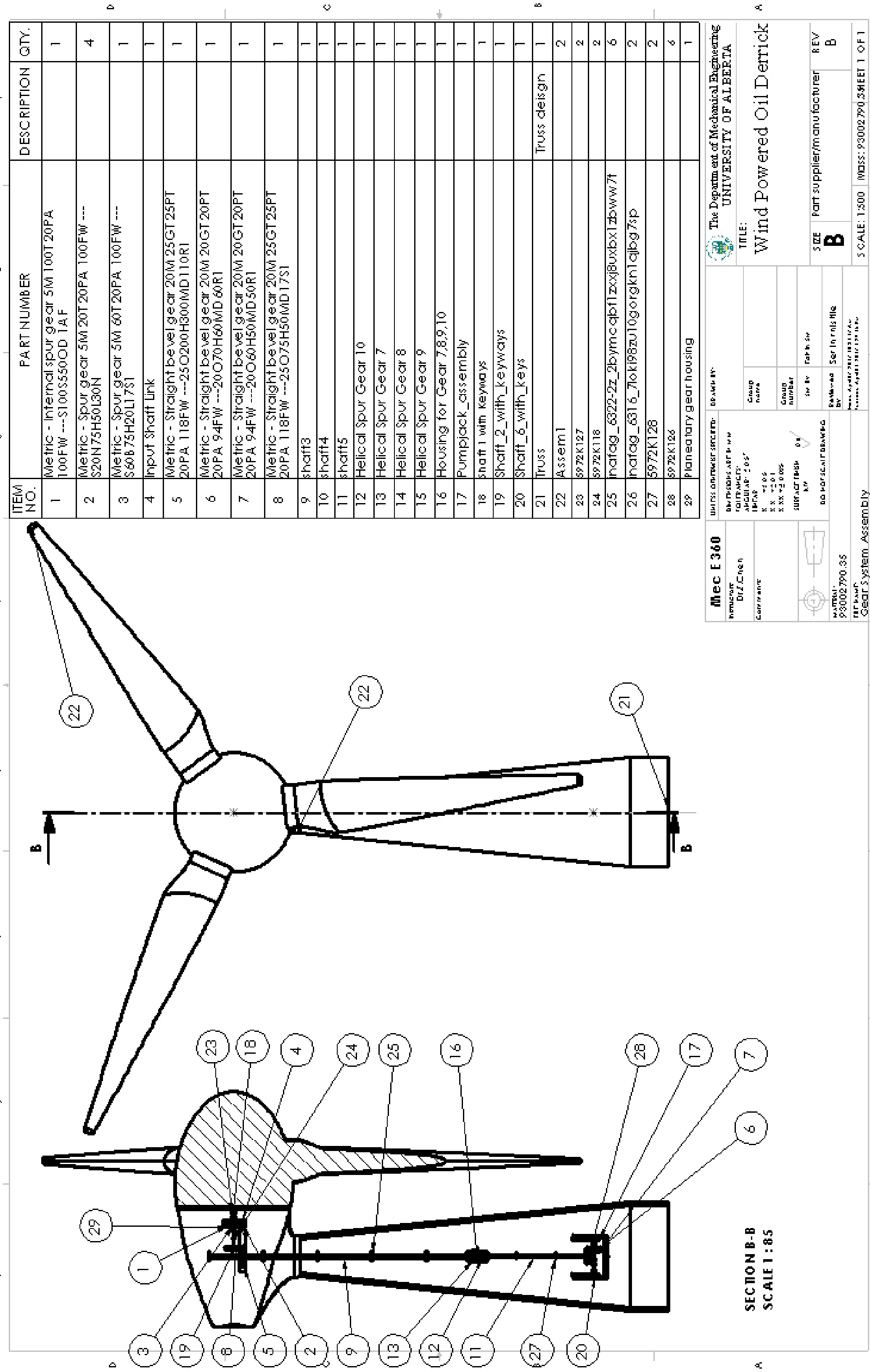

100
18.021
17.979
4.6
4.4
+0.040
Ø60 +0.010
DETAIL A
SCALE 5 : 1
Notes
Pressure Angle: 20 Degrees
Helix Angle: 30 Degrees
Module: 5 mm
Pitch Radius: 100 mm
Nominal Shaft Diameter: 60 mm
BASED on AGMA Standards and Procedures
Mec E 360
Arief Muhayatsyah
AGMA A2 Grade 1 Steel
Helical Spur Gear 10
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 10 - Helical
SIZE
B
Part supplier/manufacturer
REV
1
SCALE: 1:2
SHEET 1 OF 1


Ø305.1
Ø75
Ø458
16.000
15.957
4.6
4.4
Ø50.040
Ø50.010
DETAIL A
SCALE 1 : 1
Notes
Pressure Angle: 20 Degrees
Face Width: 94 mm
Module: 20 mm
Pitch Radius: 400 mm
Nominal Shaft Diameter: 60 mm
BASED AGMA Standards and Procedures
Mec E 360
Dr. Z. Chen
Winter 2017
DRAWN BY:
Arief Muhaytsyah
Greenriggs Inc.
Set in this file
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 11 - Bevel
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:5
Mass:
SHEET 1 OF 1

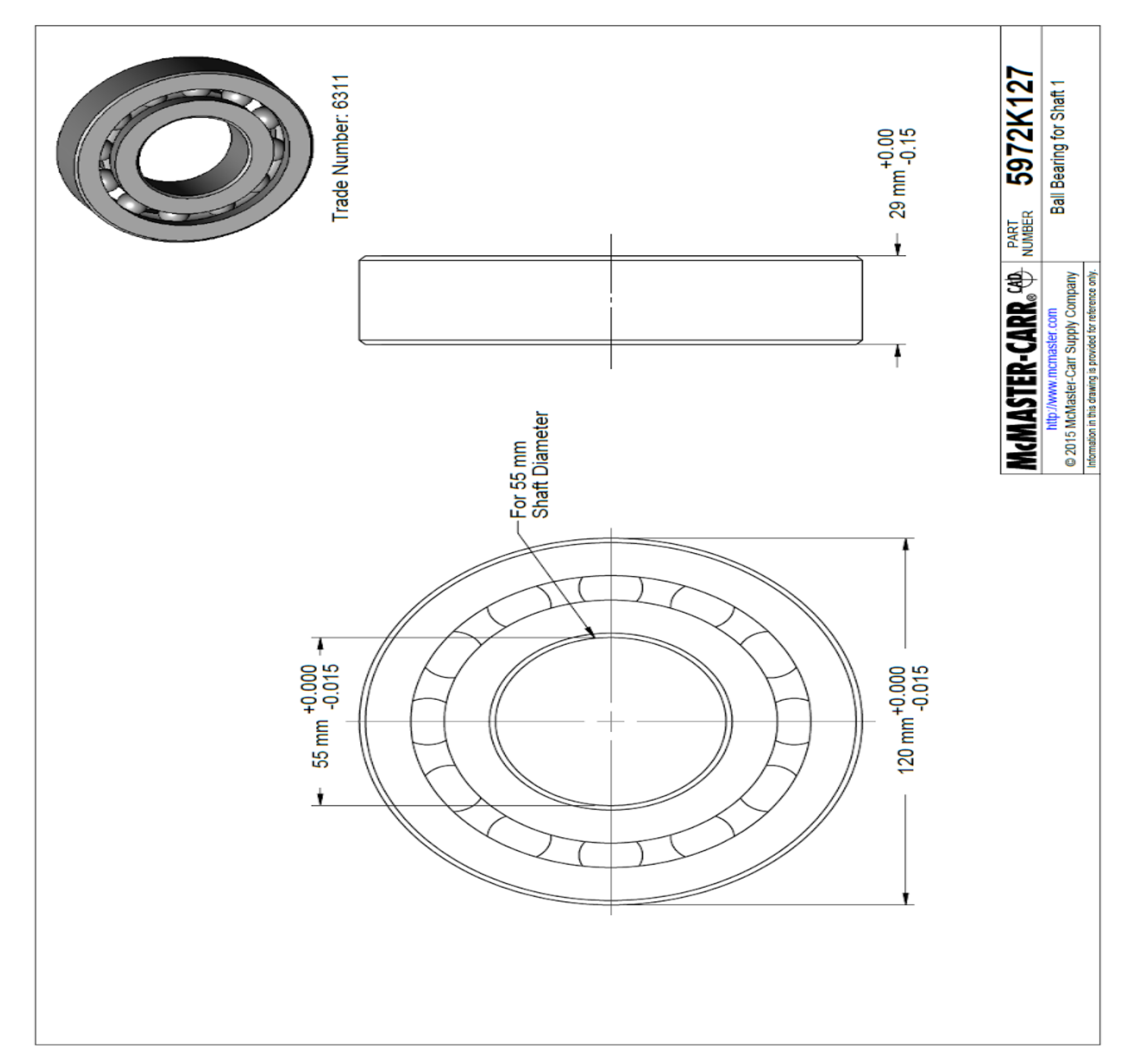

R30
Ø 50.040
50.010
14.021
13.979
4.0
3.8
A
DETAIL A
SCALE 2 : 3
Notes
Pressure Angle: 20 Degrees
Face Width: 94 mm
Module: 20 mm
Pitch Radius: 400 mm
Nominal Shaft Diameter: 50 mm
BASED on AGMA Standards and Procedures
Mec E 360
Dr. Z. Chen
DRAWN BY:
Arief M.
Set in this file
straight bevel gear_am
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear 12 - Bevel
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:5
Mass:
SHEET 1 OF 1

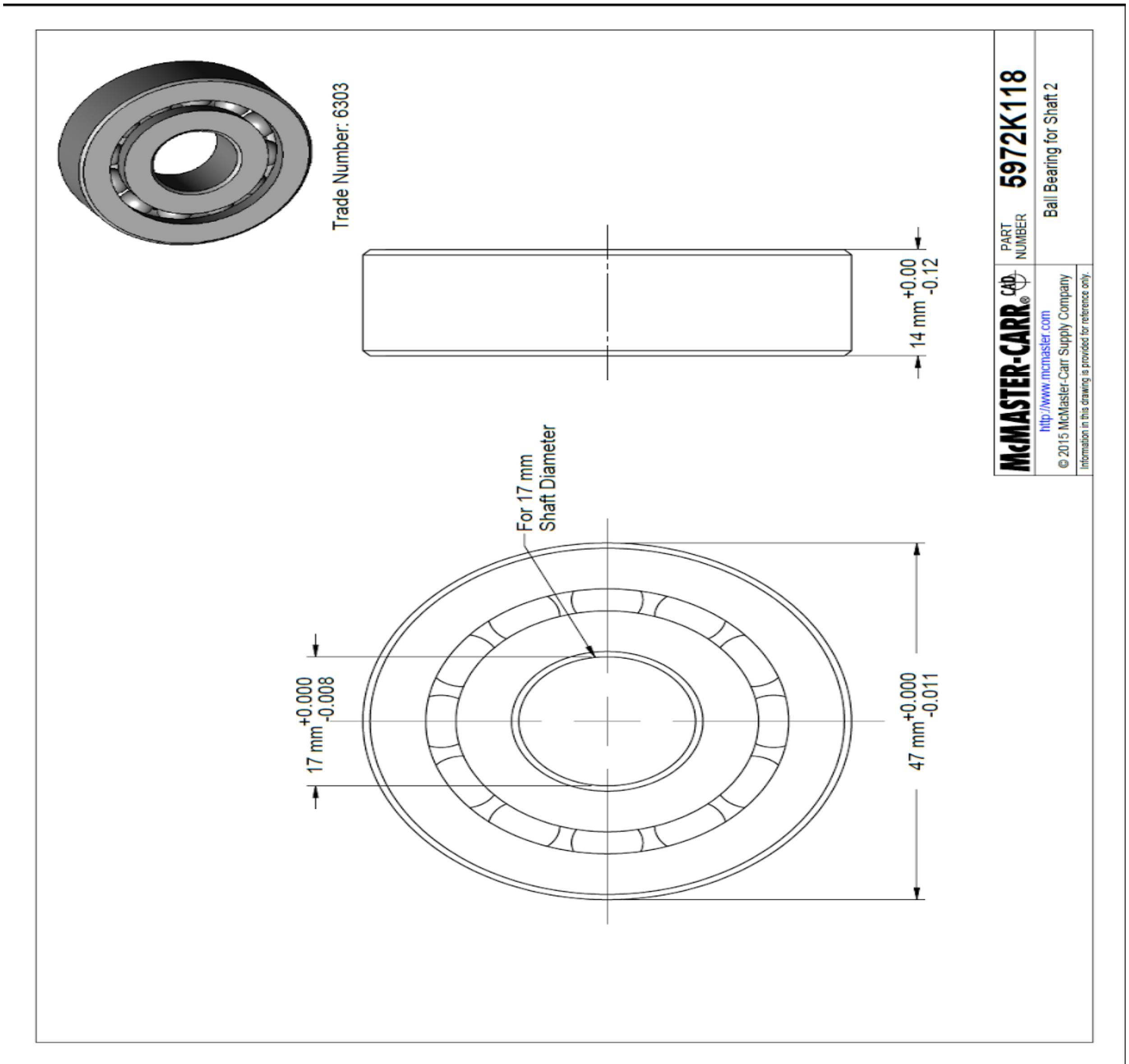


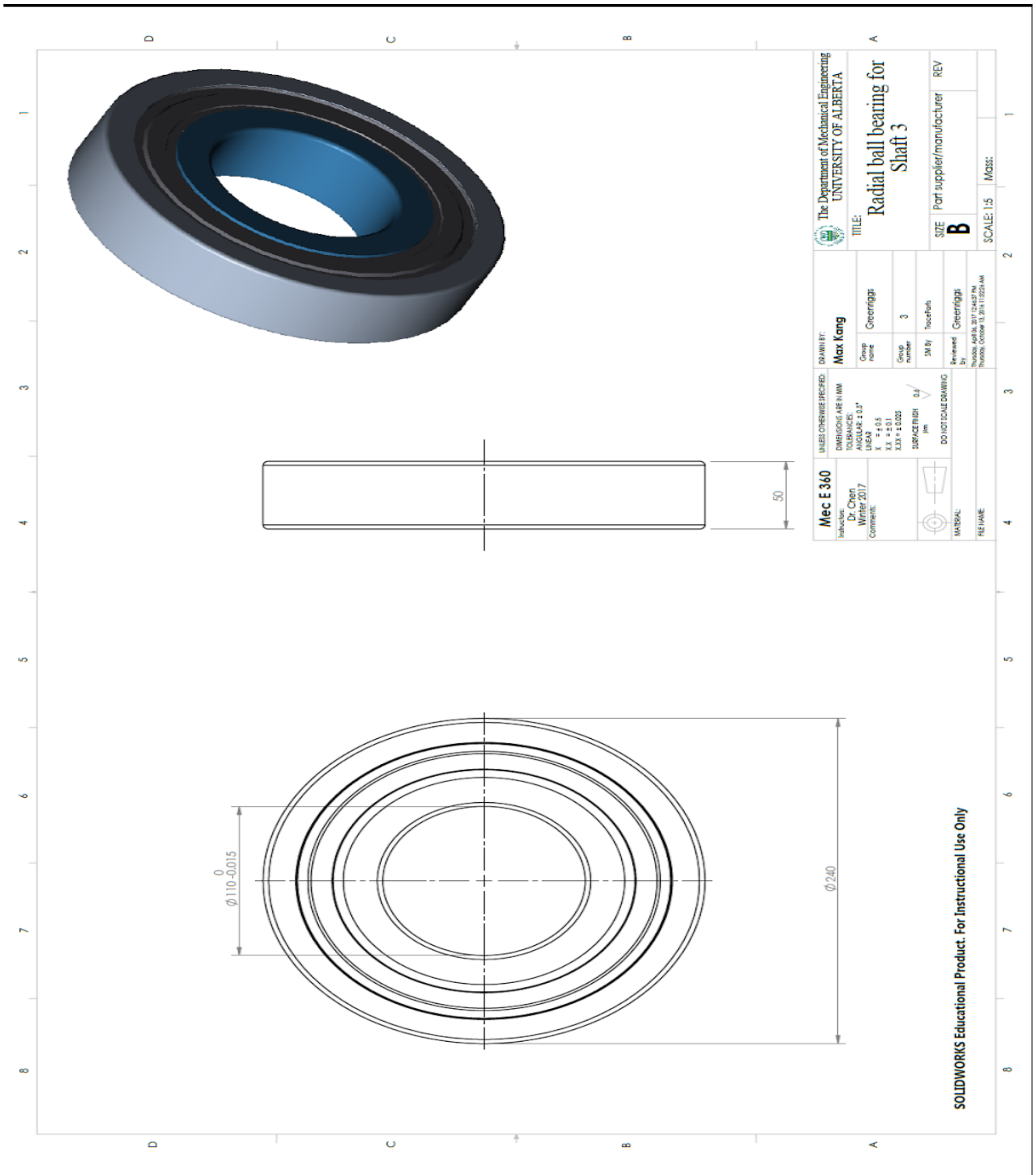
SECTION A-A
SCALE 1 : 20
A
A
Mec E 360
Dr. Z. Chen
Winter 2017
DRAWN BY:
Arief Muhayatsyah
Greenriggs Inc
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Gear System Assembly
SIZE
B
Part supplier/manufacturer
REV
B
SCALE: 1:100
SHEET 1 OF 1
DO NOT SCALE DRAWING
Gear System Assembly

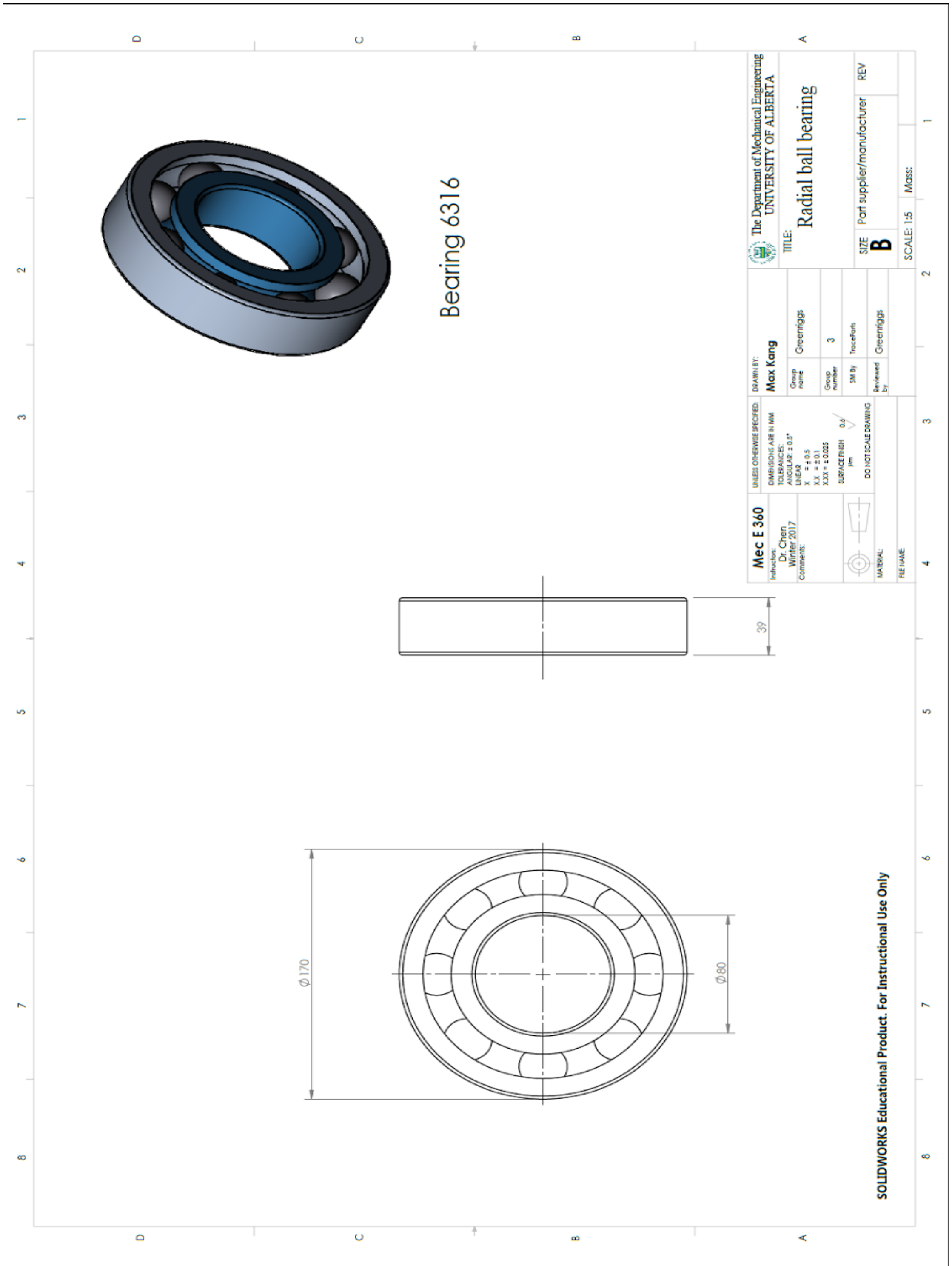

ITEM NO. | PART NUMBER | DESCRIPTION | QTY.
1 | Metric - Internal spur gear 5M 100T 20PA 100FW ---S100S550OD 1A F | | 1
2 | Metric - Spur gear 5M 20T 20PA 100FW ---S20N75H50L30N | | 4
3 | Metric - Spur gear 5M 60T 20PA 100FW ---S60B75H20L17S1 | | 1
4 | Input Shaft Link | | 1
5 | Metric - Straight bevel gear 20M 25GT 25PT 20PA 118FW ---25O200H300MD110R1 | | 1
6 | Metric - Straight bevel gear 20M 20GT 20PT 20PA 94FW ---20O70H60MD60R1 | | 1
7 | Metric - Straight bevel gear 20M 20GT 20PT 20PA 94FW ---20O60H50MD50R1 | | 1
8 | Metric - Straight bevel gear 20M 25GT 25PT 20PA 118FW ---25O75H50MD17S1 | | 1
9 | shaft3 | | 1
10 | shaft4 | | 1
11 | shaft5 | | 1
12 | Helical Spur Gear 10 | | 1
13 | Helical Spur Gear 7 | | 1
14 | Helical Spur Gear 8 | | 1
15 | Helical Spur Gear 9 | | 1
16 | Housing for Gear 7,8,9,10 | | 1
17 | Pumpjack_assembly | | 1
18 | Shaft 1 with Keyways | | 1
19 | Shaft_2_with_keyways | | 1
20 | Shaft_6_with_keys | | 1
21 | Truss | Truss deisgn | 1
22 | Assem1 | | 2
23 | 5972K127 | | 2
24 | 5972K118 | | 2
25 | inafag_6322-2z_2bymcqbf1zxxj8uxbx1zbww7t | | 6
26 | inafag_6316_7loki98zu10gorgkn1qjbg7sp | | 2
27 | 5972K128 | | 2
28 | 5972K126 | | 6
29 | Planeatary gear housing | | 1
SECTION B-B
SCALE 1 : 85
B
B
Mec E 360
Instructor: Dr.Z.Chen
Comments:
MATERIAL: 93002790.35
FILE NAME: Gear System Assembly
DO NOT SCALE DRAWING
Group name
Group number
Reviewed by
See in this file
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Wind Powered Oil Derrick
SIZE
B
Part supplier/manufacturer
REV
B
SCALE: 1:500
Mass: 93002790.35
SHEET 1 OF 1

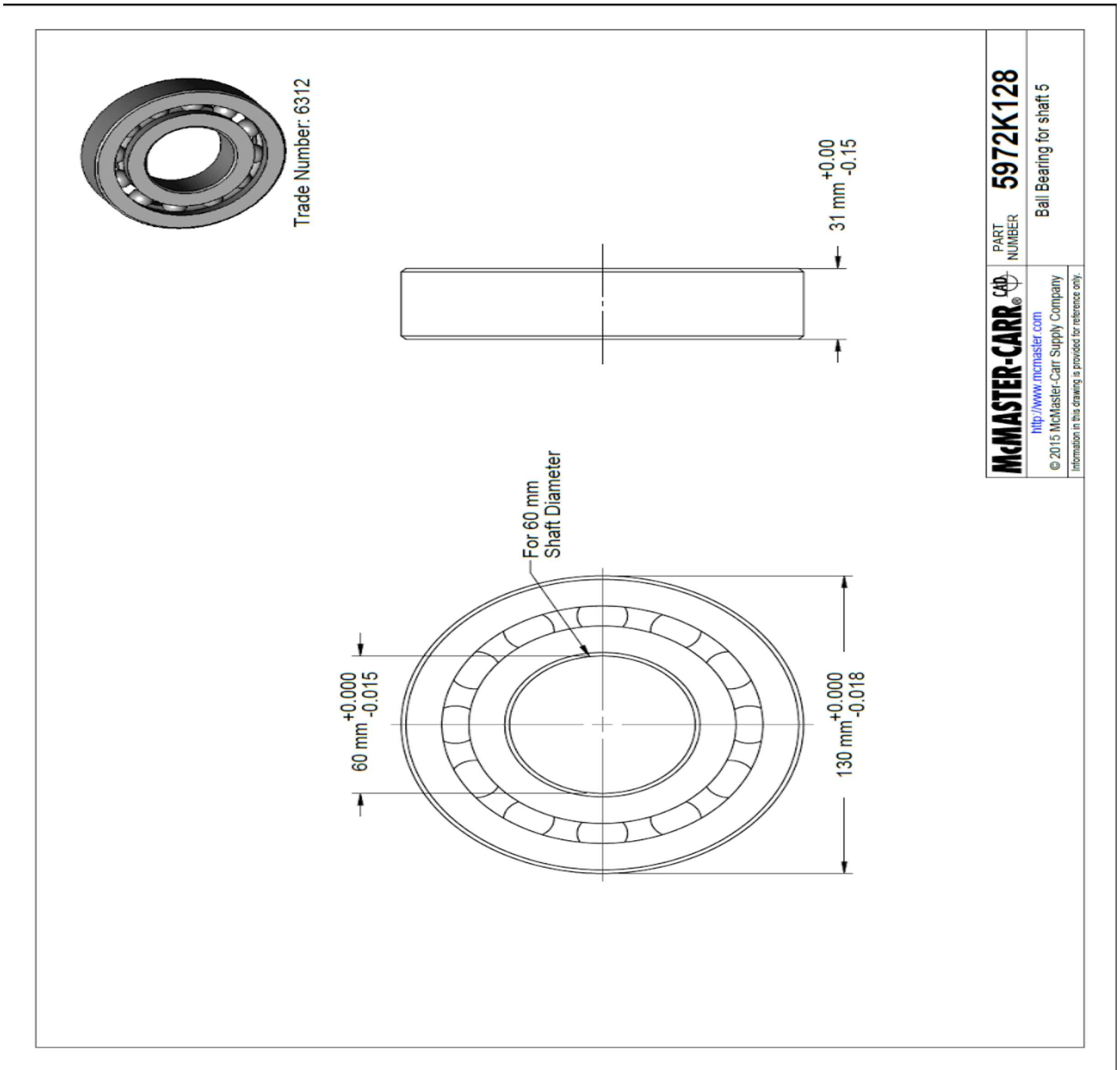
L19:

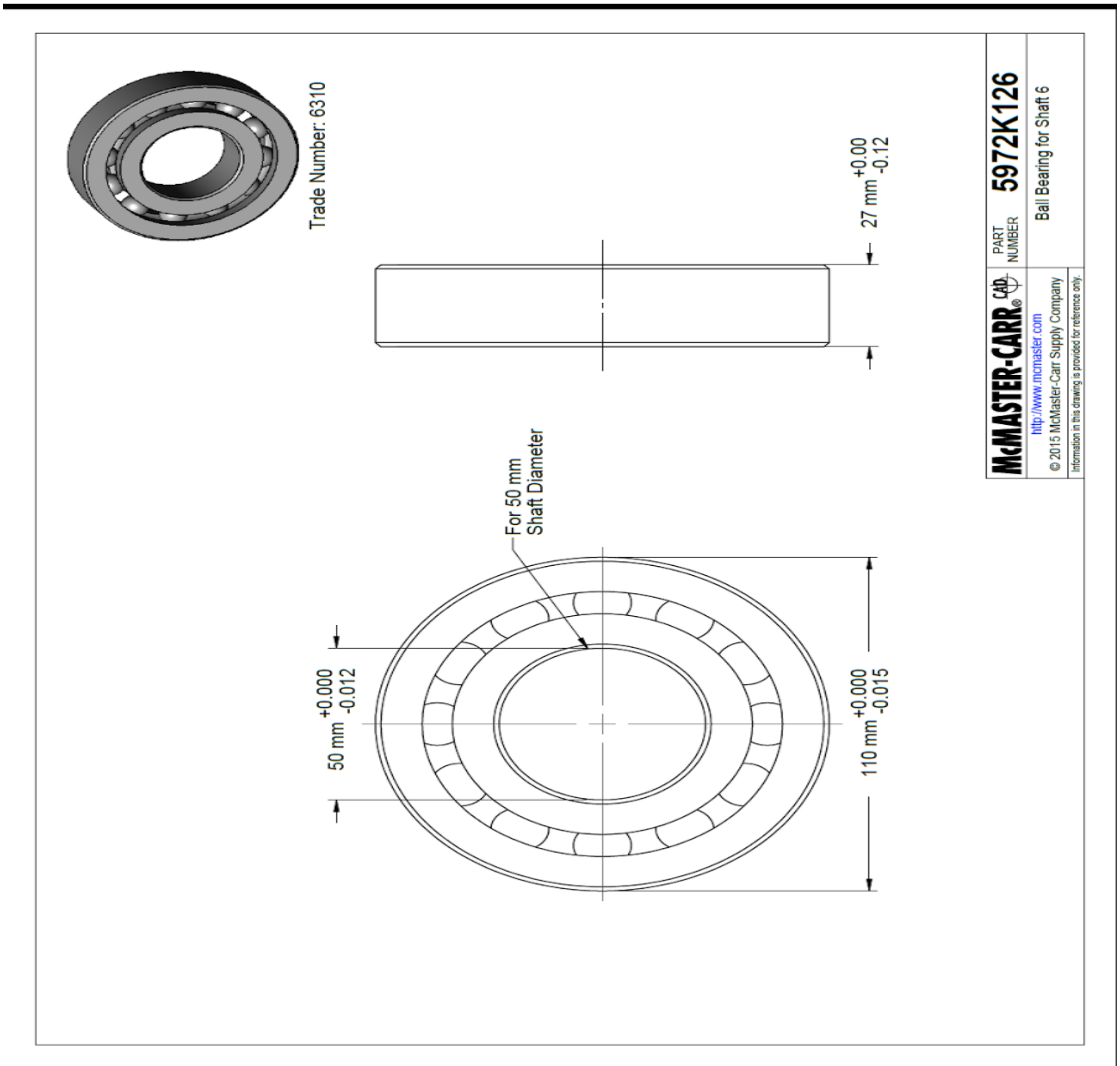

Trade Number: 6311
29 mm +0.00 -0.15
For 55 mm Shaft Diameter
55 mm +0.000 -0.015
120 mm +0.000 -0.015
McMASTER-CARR® CAD
PART NUMBER
5972K127
http://www.mcmaster.com
Ball Bearing for Shaft 1
© 2015 McMaster-Carr Supply Company
Information in this drawing is provided for reference only.

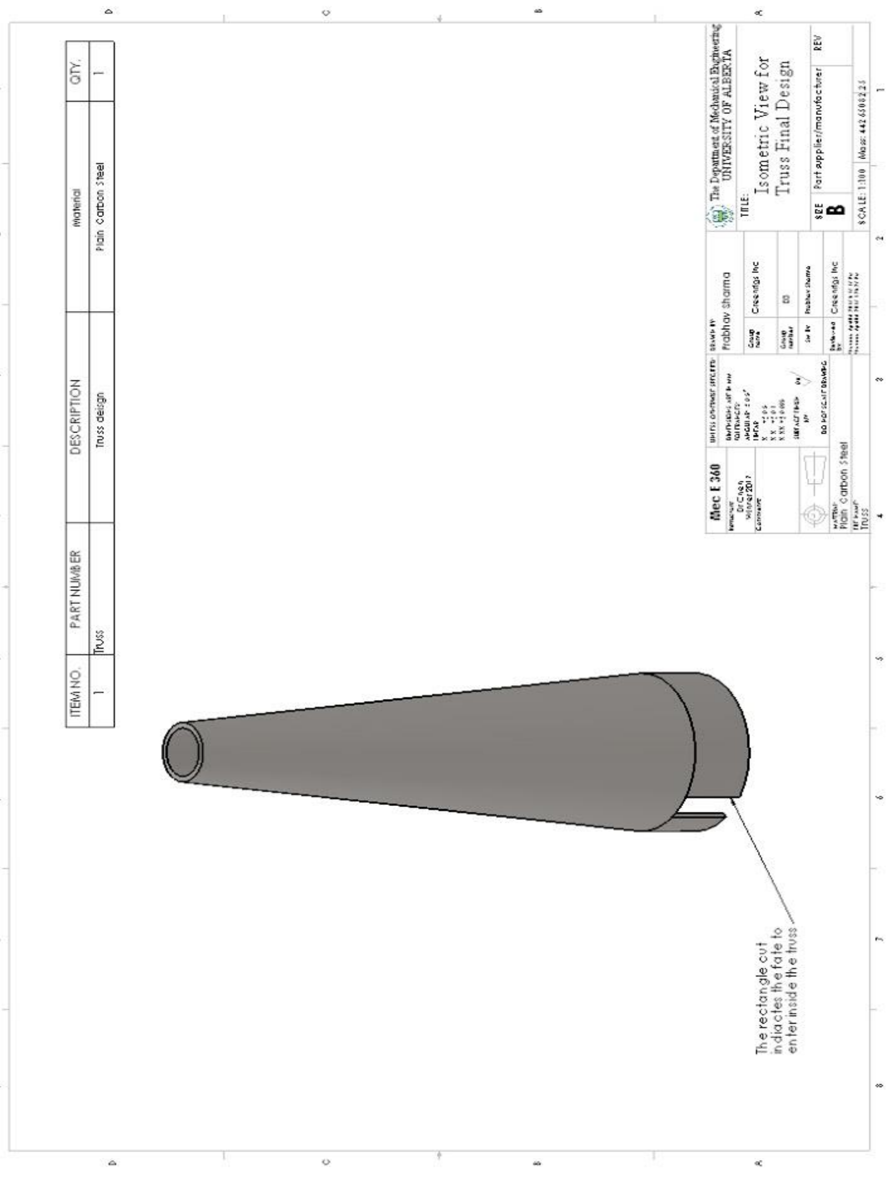

Trade Number: 6303
For 17 mm
Shaft Diameter
17 mm +0.000 -0.008
47 mm +0.000 -0.011
14 mm +0.00 -0.12
McMASTER-CARR®
CAD
PART NUMBER
5972K118
Ball Bearing for Shaft 2
http://www.mcmaster.com
© 2015 McMaster-Carr Supply Company
Information in this drawing is provided for reference only.

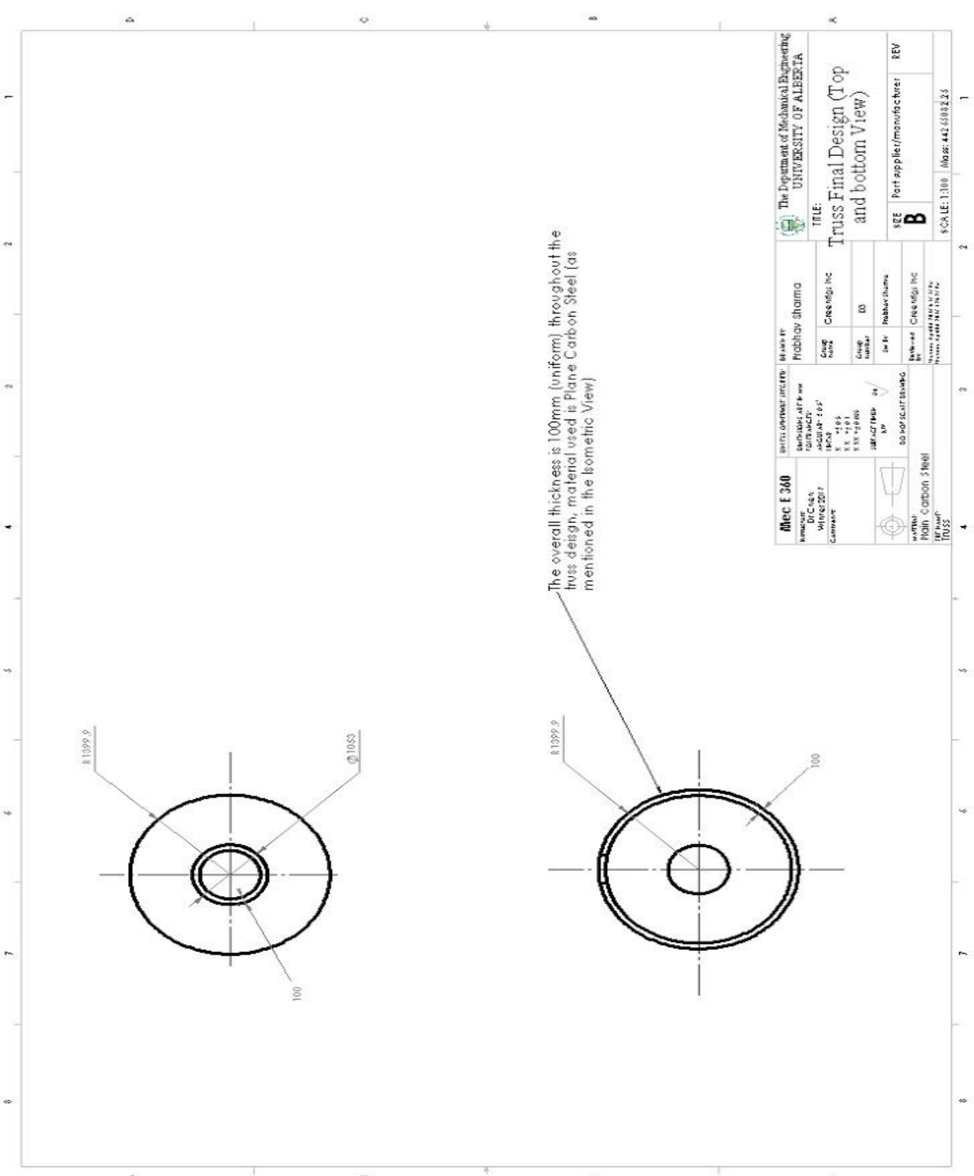

The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Radial ball bearing for Shaft 3
DRAWN BY:
Max Kang
Greenriggs
Mec E 360
Dr. Chen
Winter 2017
SIZE
B
SCALE: 1:5
Part supplier/manufacturer
REV
Mass:
50
Ø240
Ø110-0.015
SOLIDWORKS Educational Product. For Instructional Use Only

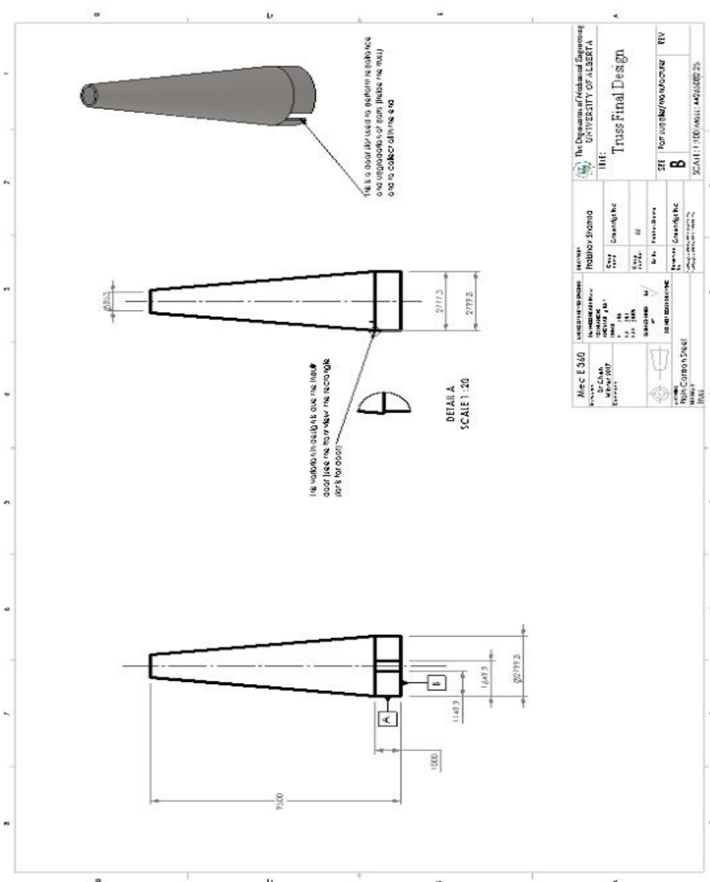

Bearing 6316
The Department of Mechanical Engineering UNIVERSITY OF ALBERTA
TITLE:
Radial ball bearing
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:5
Mass:
DRAWN BY:
Max Kang
Group name
Greenriggs
Group number
3
SM By
TraceParts
Reviewed by
Greenriggs
Mec E 360
Instructor:
Dr. Chen
Winter 2017
Comments:
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
DO NOT SCALE DRAWING
MATERIAL:
FILE NAME:
39
Ø170
Ø80
SOLIDWORKS Educational Product. For Instructional Use Only

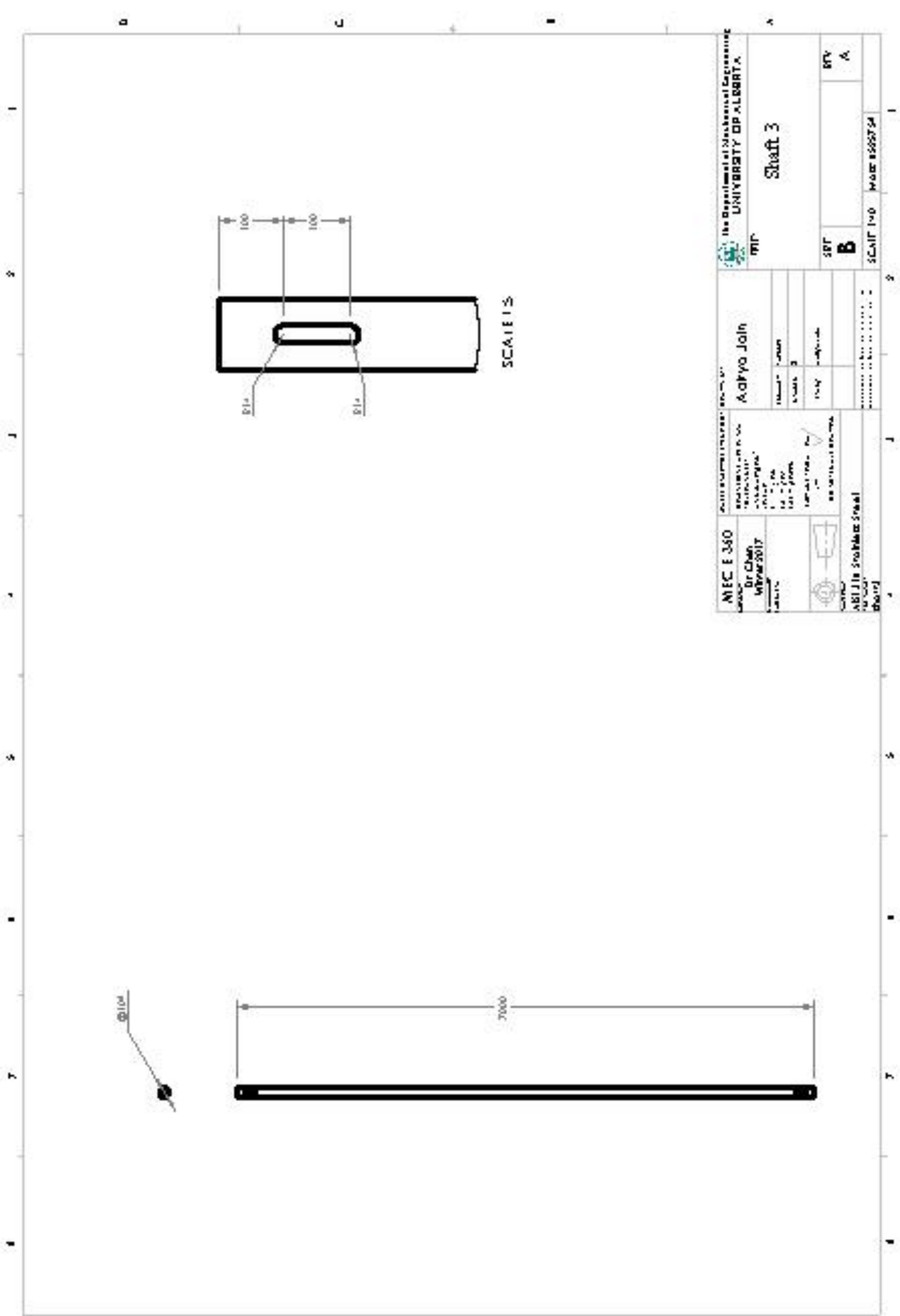

Trade Number: 6312
31 mm +0.00 -0.15
For 60 mm Shaft Diameter
60 mm +0.000 -0.015
130 mm +0.000 -0.018
McMASTER-CARR
PART NUMBER
5972K128
Ball Bearing for shaft 5
http://www.mcmaster.com
© 2015 McMaster-Carr Supply Company
Information in this drawing is provided for reference only.

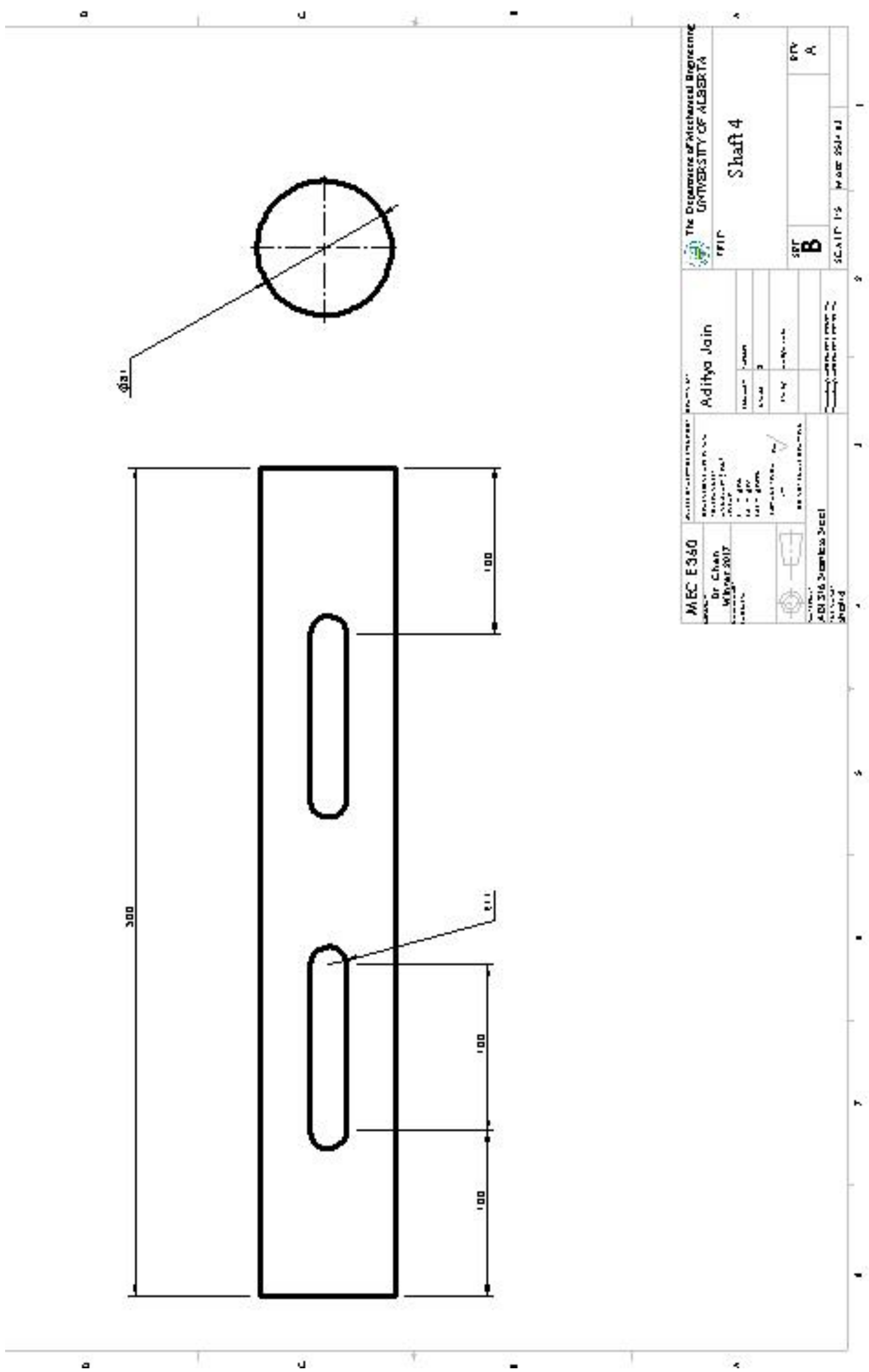

Trade Number: 6310
27 mm +0.00 -0.12
For 50 mm Shaft Diameter
50 mm +0.000 -0.012
110 mm +0.000 -0.015
McMASTER-CARR® CAD
PART NUMBER
5972K126
http://www.mcmaster.com
© 2015 McMaster-Carr Supply Company
Ball Bearing for Shaft 6
Information in this drawing is provided for reference only.

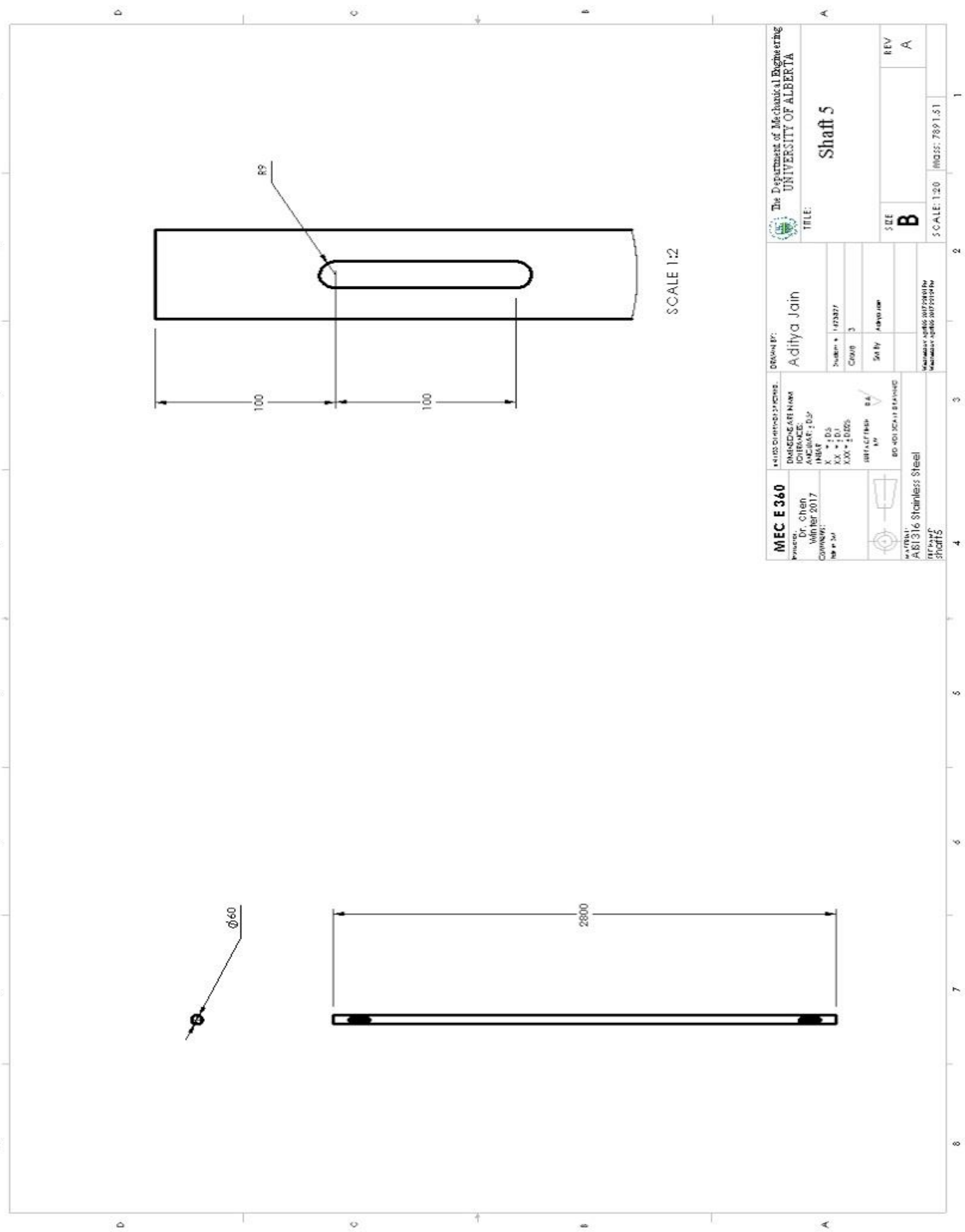

ITEM NO.
PART NUMBER
DESCRIPTION
Material
QTY.
1
Truss
Truss design
Plain Carbon Steel
1
The rectangle cut indicates the fate to enter inside the truss
Mec E 360
Dr Chen
Winter 2017
Comment
DO NOT SCALE DRAWING
Prabhav sharma
Group name
Creendigs Inc
Group number
Prabhav Sharma
Creendigs Inc
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Isometric View for Truss Final Design
SIZE
B
Part supplier/manufacturer
REV
SCALE: 1:100
Plain Carbon Steel
Truss


The overall thickness is 100mm (uniform) throughout the truss deisgn, material used is Plane Carbon Steel (as mentioned in the Isometric View)
R1399.9
100
The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Truss Final Design (Top and bottom View)
Mec E 360
Prabhav Sharma
Plain Carbon Steel
SIZE
B
REV

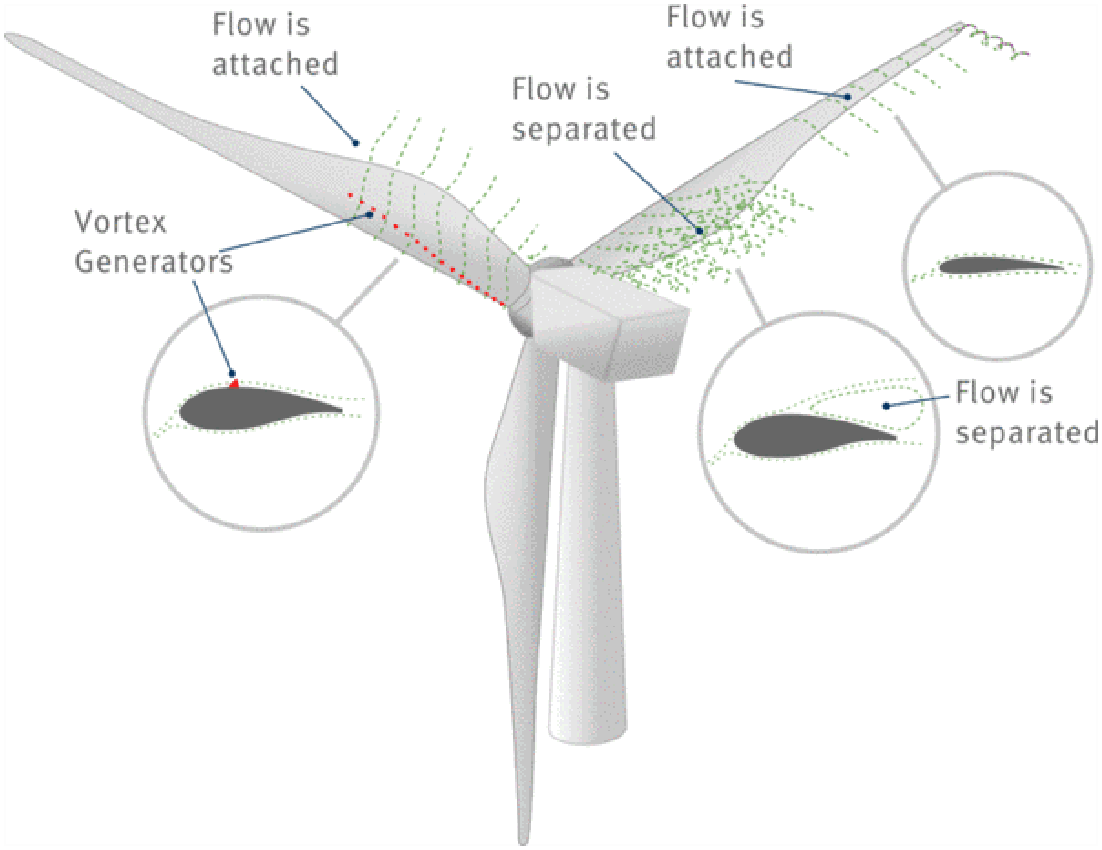

Truss Final Design
UNIVERSITY OF ALBERTA
Mec E 360
DETAIL A
SCALE 1:20
B


UNIVERSITY OF ALBERTA
Shaft 3
Aarya Jain
MEC E 360
100
100
7000


The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
Shaft 4
Aditya Jain
MEC E360
Dr. Chan
Winter 2017
AISI 316 Stainless Steel
B
A
300
100
100
100


The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
Shaft 5
SCALE 1:2
R9
100
100
2800
⌀60
MEC E 360
Dr. Chen
Winter 2017
Aditya Jain
AISI316 Stainless Steel
Shaft5
SIZE
B
REV
A
SCALE: 1:20

P: Vortex Generator



Q: Bolt Connections

Bolt Design

Tensiel load in the bolt

$$F_b = F_i + \frac{K_b \cdot P}{k_m + k_b}$$

Compressive Force in the Materials

$$F_m = F_i - \frac{k_m + P}{k_m + k_b}$$

Assume major diameter is 0.073 in

$$d := 0.073$$

$$A_b := \pi \cdot \frac{d^2}{4}$$

The tensile area of the bolt

$$A_t := 0.0026$$

The bolt that will be used is AISI 316 Stainless Steel

$E_b := 29007547.546$ $\quad S_p := 36000$

Bolt thread length

$l_{thd} := 2 \cdot d + 0.25 = 0.396$ $\quad l_{bolt} := 1.771$ $\quad l := 1.57$

$$l_s := l_{bolt} - l_{thd} = 1.375$$

$$l_t := l - l_s = 0.195$$

$$k_m := d \cdot E_b \cdot 0.78715 \cdot \exp\left(0.62873 \cdot \frac{d}{l}\right)$$

$$k_b := \frac{\left(A_t \cdot A_b \cdot E_b\right)}{A_t \cdot l_s + A_b \cdot l_t} = 7.189 \times 10^4$$

$$F_p := S_p \cdot A_t = 93.6$$

$F_i := 0.75 \cdot F_p = 70.2$ $\quad P := 0$

$$F_b := F_i + \frac{k_b \cdot P}{k_m + k_b}$$

Load on Bolt

$$\sigma_{bolt} := \frac{F_b \cdot 6894.76}{A_t} = 1.862 \times 10^8 \quad \text{Pa}$$

Force on the Material

$$F_m := F_i - \frac{k_m \cdot P}{k_m + k_b} = 70.2$$

The weight of the planetary gears and housing is:

$W := 30 \cdot 9.81 = 294.3$

$\sigma_{normal} := \dfrac{W}{A_t} = 1.132 \times 10^5$ Pa

Total Force acting on the bolt is

$\sigma_{vm} := \dfrac{1}{\sqrt{2}} \sqrt{\left(\sigma_{normal} - \sigma_{bolt}\right)^2 + \sigma_{bolt}{}^2 + \sigma_{normal}{}^2} = 1.861 \times 10^8$

The yield strength $S_y := 290000000$ Mpa

The safety factor becomes

$n := \dfrac{S_y}{\sigma_{vm}} = 1.558$

The diameter is too small to satisfy the safety factor which needs to be above 2.92

Redimension the diameter

Bolt Design for New Diameter

Tensiel load in the bolt

$F_b = F_i + \dfrac{K_b \cdot P}{k_m + k_b}$

Compressive Force in the Materials

$F_m = F_i - \dfrac{k_m + P}{k_m + k_b}$

Assume major diameter is 0.099 in

$d := 0.25$

$A_b := \pi \cdot \dfrac{d^2}{4}$

The tensile area of the bolt

$A_t := 0.0318$

The bolt that will be used is AISI 316 Stainless Steel

$E_b := 29007547.546$ $S_p := 36000$

Bolt thread length

$l_{thd} := 2 \cdot d + 0.25 = 0.75$ $l_{bolt} := 1.771$ $l := 1.57$

$l_s := l_{bolt} - l_{thd} = 1.021$

$l_t := l - l_s = 0.549$

$k_m := d \cdot E_b \cdot 0.78715 \cdot \exp\left(0.62873 \cdot \frac{d}{l}\right)$

$k_b := \frac{\left(A_t \cdot A_b \cdot E_b\right)}{A \cdot l + A \cdot l} = 7.621 \times 10^5$

$F_p := S_p \cdot A_t = 1.145 \times 10^3$

$F_i := 0.4 \cdot F_p = 457.92$ $\qquad P := 0$

$$F_b := F_i + \frac{k_b \cdot P}{k_m + k_b}$$

Load on Bolt

$$\sigma_{bolt} := \frac{F_b \cdot 6894.76}{A_t} = 9.928 \times 10^7 \quad \text{Pa}$$

Force on the Material

$$F_m := F_i - \frac{k_m \cdot P}{k_m + k_b} = 457.92$$

The weight of the planetary gears and housing is:

$W := 30 \cdot 9.81 = 294.3$

$$\sigma_{normal} := \frac{W}{A_t} = 9.255 \times 10^3 \text{ Pa}$$

Total Force acting on the bolt is

$$\sigma_{vm} := \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{normal} - \sigma_{bolt}\right)^2 + \sigma_{bolt}^{\ 2} + \sigma_{normal}^{\ 2}} = 9.928 \times 10^7$$

The yield strength $\qquad S_y := 290000000 \quad \text{Mpa}$

The safety factor becomes

$$n := \frac{S_y}{\sigma_{vm}} = 2.921$$

Which satisfies the bolts minimal safety factor

In conclusion

Grade #	Diameter (in.)	Material	Length (mm)	Length of each metal sheet (mm)
3	0.25	Low or medium carbon	45	20

One bolt is on the top and another one is at the bottom.

R: Energy Efficiency for Pump Jack

Energy efficiency targets for pumpjack operations are provided in Table 1.1. These targets identify the minimum efficiency that should be achieved on all pumpjacks.

(2-Course%20schedule.pdf)

Table 1.1
Target Mechanical Efficiencies

Sources	Efficiency Targets
Engine	>20%
Surface	>95%
Subsurface	>85%
Overall	>17%

The largest source of inefficiency in pumpjack operation is due to the use of inefficient gas powered engines. Efficiency loss in gas engines is not within the scope of this module.

Other inefficiencies in the operation of pumpjacks are a result of mechanical and lifting efficiencies in the surface and sub-surface operation of pumpjacks. These inefficiencies are a result of wearing parts, improper designs, changing well conditions and ultimately aging equipment. The efficiency of a pumpjack system can be defined as follows: Efficiency (η) = Useful horsepower / Input horsepower Where: $\eta = \eta_{lift} \times \eta_{mech} \times \eta_{eng}$

Where η_{lift} = Lifting Efficiency

η_{mech} = Mechanical Efficiency

η_{eng} = Engine Efficiency

The surface mechanical efficiency, η_{mech}, and the engine efficiency, η_{mot}, vary in quite narrow ranges. At the same time, their values are not easy to improve upon.

Lifting efficiency should be considered as the governing factor since it varies in a broad range depending on the pumpjack setup and operating conditions. Therefore, considerable improvements on the pumping system's overall energy efficiency can only be realized by achieving a maximum of lifting efficiency. horsepower is the energy which actually goes into moving the oil out of the ground. The useful horsepower delivered by a pumping system can be defined as:

Useful Horsepower = Ldyn x Q x SG / 136,000

Where Q = Liquid Production Flowrate, bpd

SG = Specific Gravity

Ldyn = Dynamic liquid level in the well, ft

variations in efficiencies should be a major focus in the replacement and design of rod strings and pump sizes as well as in the choice of operational parameters of polished rod stroke length and pumping speed.

S: Pump Jack Legislation & Safety Standards

The pump jack legislation by Government of Canada are provided in the table below

Pumping Modes with the Best and Worst Lifting Efficiencies for Lifting 500 bpd from 6,000 ft.

Pumping Mode	Best	Worst
API Rod No.	86	85
Pump Size	2 ½"	1 ¼"
Stroke Length	120"	144"
Pumping Speed	8.3 SPM	18.4 SPM
PRHP	23.5 HP	58.3 HP
Lifting Efficiency	94.1%	37.9%
Pumping Unit Size	C-912D-305-168	

S1 Occupational health and safety legislation for the complete Assembly

(ohs.ca/safetystandards/pumpjacks)

(1) Prior to starting a task, a hazard assessment must be completed. This encourages workers to be aware of the hazards and potential serious outcomes of not eliminating or controlling the hazards. The hazard assessment asks the question "What if?" [OHS Code Section 7]

(2) Hazardous energy must be controlled by rendering equipment inoperative if it is being serviced, or if this is not possible, the employer must develop and implement procedures and controls that ensure the equipment can be serviced safely. [OHS Code Section 212]

(3) Where there is a foreseeable danger of head injury, workers must use protective headwear. [OHS Code Section 234]

(4) Safeguards must be provided where it is possible for a worker to come into contact with moving parts of machinery. [OHS Code Section 310

(5) the employer must ensure that all reasonably practicable steps are taken to keep a worker's exposure to the harmful substance as low as reasonably practicable. [OHS Code Section 16] Methane is such a substance. It is listed in Schedule 1, Table 2 with no set OEL. According to the Material Safety Data Sheet (MSDS) for methane, wellhead gas (natural gas) is approximately 87 percent methane along with some other components such as ethane, propane and butane. Methane is colourless and odourless, therefore difficult to recognize. Methane is highly

flammable, can act as an asphyxiant and may also have a mild narcotic effect. Any release of wellhead gas must be done in a manner that does not unnecessarily expose workers to this highly flammable product or that may also disorient a worker through its narcotic effect. An alternate method of removing wellhead gas when troubleshooting pumpjacks may have to be considered. Many facilities scavenge or conserve wellhead gas. AL038 — Alerts 3 February 2010 AL038 — Alerts 4 February 2010

(6) Work processes must not create fire and explosion hazards. Containers used for the transfer of flammable or combustible products must be grounded to control static electricity. The plastic pail used in the incident described above was not suitable for trying to capture wellhead fluids and was not grounded. Blowing the wellhead gas into the pail concentrated the flow towards the faces of the two workers completing the task and put them at risk. The operating pumpjack was a potential source of ignition of the flammable vapours being released. [OHS Code Section 165]

(7) If a worker may be exposed to a harmful substance at a work site, the employer must establish procedures to minimize worker exposure, ensure workers are trained in the procedures and inform workers of the health hazards associated with the harmful substance. [OHS Regulation Section 15]

S2 Assembly (Wind powered oil derrick) Maintenance

Practices followed when servicing well sites with pump jacks need to be reviewed by employers.

Good maintenance practices can also impact energy efficiency of the pumpjack system. Maintenance activities include:

• replacing bearings and worn parts, inspecting and lubricating gearboxes,

• adjusting the seal of the packing head,

• inspection, tightening or replacement of worn belts, adjusting stroke length as indicated,

• ensuring fit between the stuffing and the polished rod is not too tight, and

• the overall unit should be inspected and lubricated as required.