# 1. Title

Let’s talk about music.

Specifically about tunings.

A tuning defines the frequencies of tones.

# 2. Equal Temperement

On a normal piano, the 12 semi-tones are distributed evenly over the octave:

The frequency ratio between successive semi tones is constant. About 1.06.

The tuning is fixed by setting the frequency of A4 = 440Hz

At 440Hz the distance to the next semitone is about 26 Hz

# 3.

TuningSystems has the function equal\_tempered to generate an equally tempered scale of a given length.

The equal temperement is no natural.

Why are there any other tunings at all?

But why is ther 12 semi-tones?

# 4. Play

Before we look at that, lets play it.

# 5. Harmony

They sound nice!

# 6. Piano Harmony

The octave is on the piano, but how about the fifth?

It is slightly off, but only by 0.1%

Playing them in succession they are hard to tell apart.

But together, the frequency difference produces a ”beat tone” (see next slide)

# 7.

Beat tone of about 0.75 Hz between perfect fifth (3/2) and equal tempered fifth (2^(7/12))

# 8. Pythagorean tuning

So maybe the equal temerement is not so natural afterall.

But this might be the answer to the question about 12 semi-tones:

Going up 12 fifths is close to 7 full octaves.

Fifths are harmonic,

# 9.

Let’s hear them together.

The beat frequency is 6 Hz: the difference in frequencies of the pythagorean comma at 440 Hz

# 10. Cents and Pitch Class

Here we almost have the 12 semi-tones on the piano.

Likely the origin of the 12 semi-tones on the octave used in western music.

# 11. Pythagorean Tuning

In the actual Pythagorean tuning, we half the error and move it to the tritone (F#) by using 3/2 and 2/3 (fitfh down).

This is ”left” on the circle of fifths.

Max error 11.7 cents.

Note, we do only 5 down (2/3), but 6 up. This is normal practice (ref https://johncarlosbaez.wordpress.com/2023/10/07/pythagorean-tuning/)

F# pun.

# 12. TODO Play it!

# 13 Just Intonation TODO: Play

So equal temper is not perfect (fifths are off, thirds are worse, we come to those)

Pythagorean is not perfect: (the octave is off!)

What to do?

Just intonation gives 12 semi-tones all in simple fractions of multipla of 2, 3, and 5 (5-limit tuning).

# 14. Let’s compare

They look very similar.

Pythagorean uses only fractions of powers of 2 and 3 (3 limit tuning).

Just uses powers of 2, 3, 5 (5 limit tuning)

# 15. Harmonic Series

Strings and wind instruments naturally produce overtones. Here are the first 6.

Play it!

# 16. Harmonics and Triad

The first 5 overtones include form the major triad!.

Pythagorean has the fifth, but is off at the thrid.

12 tone equal temper (12TET) is also off.

# 17. More Harmonics

The 24 first harmonics gives 12 different tones.

But they are not equally spaced.

# 18. Subharmonics

Subharmonics play the same trick as with pythagorean tuning: Invert the fractions.

However,

overtones (intger multipla of fundamental frequency) occur spontaniously

undertones do not occur spontaniously

Note: The harmonics avoid the tritone (F#)

# 19. Just intonation and Triads

The whole notes of the just intonation are defined by requring the major triads of

Tonica

Dominant

Subdominant

are all in ratio 4:5:6

Play them!

# 20. Thank you!

This work was very much inspired by a series of blog posts by John Baez on different tuning systems. They are worth reading.

Shoutout to JuliaMusic (George Datseris of DrWatson.jl fame)

and WAV.jl by Daniel Casimiro

MusicTheory (by David P. Sanders) came out after I submitted abstract. Does not overlap (much)

Jeffrey A Fessler has a cource at University of Michgan on Music Signal Processing.

He also has a book coming up: https://github.com/JeffFessler/book-la-demo

https://www.cambridge.org/highereducation/books/linear-algebra-for-data-science-machine-learning-and-signal-processing/1D558680AF26ED577DBD9C4B5F1D0FED#overview