FTC Assignment 2, 15 points

Part-I:

3.10. At the end of the year of service, the reliability of a component is 0.96.
1. What is the failure rate of the component?
2. If two components are connected in parallel, what will be the reliability of the resulting system during the first year of operation?

Solution

To solve the first part we can consider the following expression for the exponential failure law of components:
$R(t) = e^{-\lambda t}$
where $R(t)$ is the reliability and λ is the failure rate constant.
Inverting the previous expression we find that the failure rate is expressed as:
$\lambda = -\frac{lnR(t)}{t} = -\frac{ln0.96}{8760} = 4.66E-06$
where $t = 8760$ because time is expressed as the number of hours in one year and λ is given in reference to time in hours. To solve the second part we can consider the fact that reliability of a parallel of components is given as:
$R_{parallel} = 1 - Q_{parallel} = 1 - \prod_{1=i}^{n}(1 - R_i(t)) = 1 - (1 - 0.96)(1 - 0.96) = 0.9984 = 99.84\%$

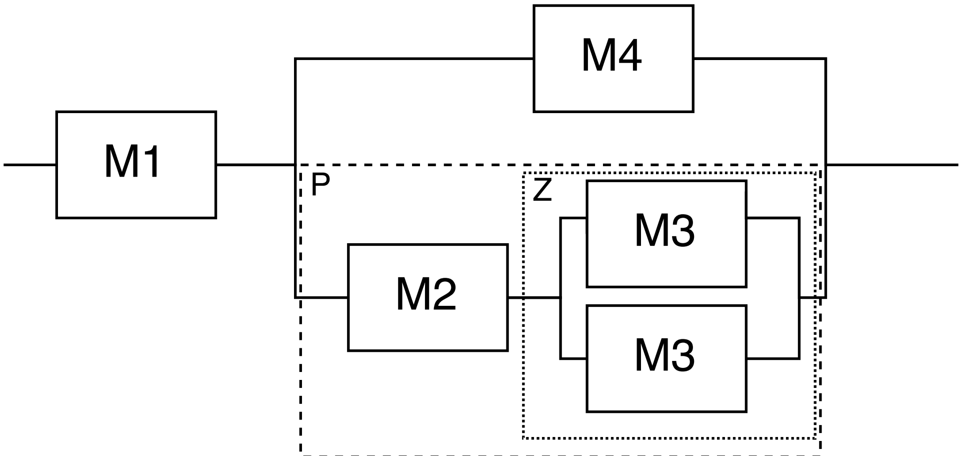
3.14. A system has MTTF = 12E+3 h. An engineer is to set the design life time,so that the end-of-life reliability is 95%.
1. Determine the design life time.
2. If two systems are placed in parallel, to what value may the design life time be increased without decreasing the end-of-life reliability?

Solution

To solve the first point we can consider that:
$MTTF = \frac{1}{\lambda}$
From the expression for the reliability we can obtain that the design life time is given by:
$t = -MTTF \cdot \ln R(t) = 615.51h = 25.64d = 25d7h40m$ For the second point, we can work the following way.
$1 - (1 - R(t))^2 = 0.95$
If we write the Reliability $R(t)$ referred to $12000h$ we find:
$1 - (1 - e^{-t/12000})^2 = 0.95$
Which leads to a life time of $t = 3037.15h$

3.21. Write an expression for the reliability of the system shown by the RBD in Fig. 3.18. Assume that Ri (t) is the reliability of the module Mi, for i 1, 2, 3, 4.

Solution



$$
\begin{aligned}
R_Z &= 1-(1-R_{M_3})^2 = 2R_{M_3} - R_{M_3}^2\\
R_P &= R_Z \cdot R_{M_2} = 2R_{M_3}R_{M_2} - R_{M_3}^2 R_{M_2}\\
R_{System} &= (1-(1-R_P)(1-R_{M_4}))R_{M_4}\\
&= [1-(1-2R_{M_3}R_{M_2} + R_{M_2}R_{M_3}^2)(1-R_{M_4})]R_{M_1}\\
&= [2R_{M_2}R_{M_3} - R_{M_2}R_{M_3}^2 + R_{M_4} - R_{M_2}R_{M_3}R_{M_4} + R_{M_2}R_{M_3}^2R_{M_4}^2]R_{M_1}\\
&= 2R_{M_1}R_{M_2}R_{M_3} - R_{M_1}R_{M_2}R_{M_3}^2 + R_{M_1}R_{M_4} - 2R_{M_1}R_{M_2}R_{M_3}R_{M_4} + R_{M_1}R_{M_2}R_{M_3}^2R_{M_4}
\end{aligned} \tag{1}
$$

3.22. Your company produces a system which consists of a memory and a processor.To increase the systems reliability, the memory is duplicated and the processor is triplicated (see RBD in Fig. 3.19).The reliabilities of the memory and the processor for the first year of operation are RM(1year) = 0.95 and RP(1year) = 0.9, and their costs are 50 and 30$,respectively. Your boss decides that the system is too expensive. She would like you to reduce its cost by using processors with a higher reliability. There are three manufacturers producing processors which are suitable for your system. They offer processors with the following reliabilities and costs:
1. RP1 (1year) = 0.9683 for 40$
2. RP2 (1year) = 0.9812 for 45$
3. RP3 (1year) = 0.9989 for 50$
Your task is to estimate which processor minimizes the cost of the system without decreasing its reliability compared to the original system. You are allowed to combine processors from different manufacturers. Draw an RBD of the new system.
Solution
We first consider the reliability of the original system:

$$
\begin{aligned}
R_{system} &= [1-(1-R_P)^3][1-(1-R_M)^2]\\
&= (3R_P - 3R_P^2 + R_P^3)(2R_M - R_M^2)\\
&= (3\cdot 0.9 - 3\cdot 0.9^2 + 0.9^3)(2\cdot 0.95 - 0.95^2) = 0.9965
\end{aligned} \tag{2}
$$

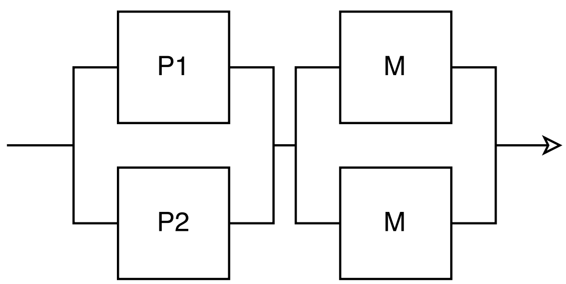
To reduce the cost and maintain the same reliability we reduce the number of processors and we use processors with an higher reliability. We will then minimize the cost. For a system with two processors in parallel:

$$\begin{aligned} R_{system} &= [1-(1-R_{P_A})(1-R_{P_B})][1-(1-R_M)^2] \\ &= [1-(1-R_{P_A})(1-R_{P_B})](2R_M - R_M^2) \\ &= [1-(1-R_{P_A})(1-R_{P_B})] \cdot 0.9975 \end{aligned} \tag{3}$$

We consider each processor separately:

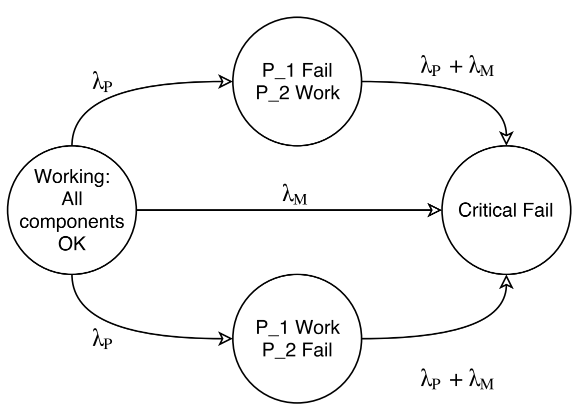
P	R	$
P1	0.9964	140$
P2	0.9971	150$
P3	0.9974	160$
P1-P2	0.9969	135$

The solution that minimizes the cost without loosing on reliability is the one using P2 and P1. The RBD of the final system is:



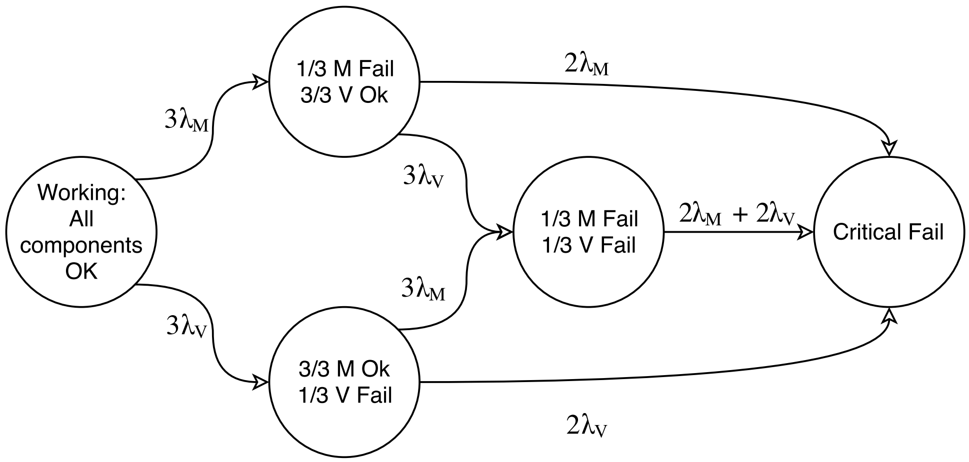
3.25. Draw a Markov chain for reliability evaluation of the system shown in Fig. 3.5. The failure rate of the processors 1 and 2 is λ_p. The failure rates of the memory is λ_m. No repairs are allowed.

Solution



4.3. Draw a Markov chain for reliability evaluation of the TMR with three voters shown in Fig. 4.6. Assume that the failure rate of each module is λ_m, and the failure rate of each voter is λ_v. No repairs are allowed.

Solution



Part-II:

Suppose that the reliability of a system consisting of 4 blocks, two of which are identical, is given by the following equation:

$R_{system} = R_1 R_2 R_3 + R_1^2 - R_1^2 R_2 R_3$ Draw the reliability block diagram representing the system

Solution