The University of Alabama in Huntsville
Electrical and Computer Engineering
Homework #2 Solution
CPE 633 01
Spring 2008

Chapter 2: Problems 5(20 points), 7(20 points), 15(20 points), 16(20 points), and 17(20 points)

5. The lifetime of each of the seven blocks in Figure 2.3 is exponentially distributed with parameter λ. Derive an expression for the reliability function of the system, Rsystem(t), and plot it over the range t = [0; 100] for _ = 0:02.

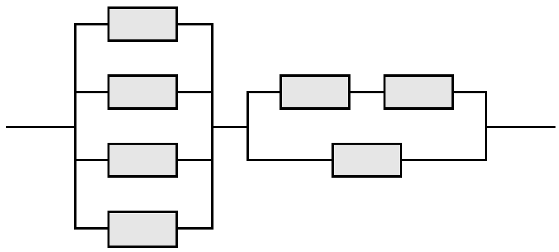
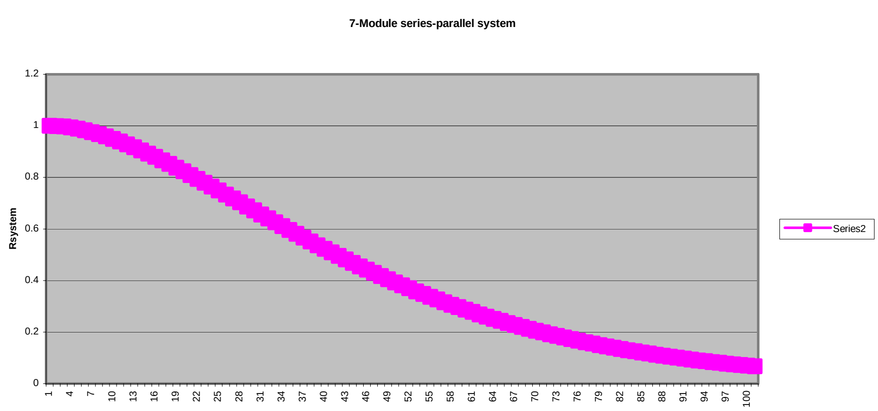


Figure 2.3: A 7-module series-parallel system.

As before, we decompose this structure into two substructures connected in series. The left substructure is four blocks in parallel; the right substructure consists of a series arrangement of two blocks in parallel with one block. The reliability of this system is thus given by:

$$R_{system} = \left[1-(1-R(t))^4\right]\left[1-(1-R^2(t))(1-R(t)\right]$$
$$= \left[1-(1-2R(t)+R^2(t))(1-2R(t)+R^2(t))\right]\left[1-(1-R(t)-R^2(t)+R^3(t)\right]$$

$$= \left[1-(1-2R(t)+R^2(t)-2R(t)+4R^2(t)-2R^3(t)+R^2(t)-2R^3(t)+R^4(t))\right]\left[R(t)+R^2(t)-R^3(t)\right]$$
$$= \left[1-(1-4R(t)+6R^2(t)-4R^3(t)+R^4(t))\right]\left[R(t)+R^2(t)-R^3(t)\right]$$
$$= \left[4R(t)-6R^2(t)+4R^3(t)-R^4(t))\right]\left[R(t)+R^2(t)-R^3(t)\right]$$
$$= 4R^2(t)+4R^3(t)-4R^4(t)-6R^3(t)-6R^4(t)+6R^5(t)+4R^4(t)+4R^5(t)-4R^6(t)-R^5(t)-R^6(t)+R^7(t)$$
$$= 4R^2(t)-2R^3(t)-6R^4(t)+9R^5(t)-5R^6(t)+R^7(t)$$



7. Write the expression for the reliability of a 5MR system and calculate its MTTF. Assumethat failures occur as a Poisson process with rate λ per node, that failures are independent and permanent, and that the voter is failure-free.

Denote by R(t) = $e^{-\lambda t}$ the reliability of an individual node. The reliability of the 5MR

$$R_{5MR}(t) = \sum_{i=3}^{i=5} \binom{5}{i} R^i(t)(1-R(t))^{5-i} = \frac{5!}{3!2!} R^3(t)(1-R(t))^2 + \frac{5!}{4!1!} R^4(t)(1-R(t))^1 + \frac{5!}{5!0!} R^5(t)(1-R(t))^0$$

$$= 10R^3(t)(1-2R(t)+R^2(t)) + 5R^4(t)(1-R(t)) \quad + R^5(t)$$
$$= 10R^3(t) - 20R^4(t) + 10R^5(t) + 5R^4(t) - 5R^5(t) \quad + R^5(t) = 6R^5(t) - 15R^4(t) + 10R^3(t)$$

$$MTTF = \int_{t=0}^{\infty} 6R^5(t) - 15R^4(t) + 10R^3(t) = \int_{t=0}^{\infty} 6e^{-5\lambda t} - 15e^{-4\lambda t} + 10e^{-3\lambda t}$$
$$= \left[\frac{6}{-5\lambda} e^{-5\lambda\infty} - \frac{15}{-4\lambda} e^{-4\lambda\infty} + \frac{10}{-3\lambda} e^{-3\lambda\infty}\right] - \left[\frac{6}{-5\lambda} e^{-5\lambda 0} - \frac{15}{-4\lambda} e^{-4\lambda 0} + \frac{10}{-3\lambda} e^{-3\lambda 0}\right]$$
$$= [0 - 0 + 0] - \left[\frac{6}{-5\lambda} - \frac{15}{-4\lambda} + \frac{10}{-3\lambda}\right] = \frac{-72 + 225 - 200}{60\lambda} = \frac{47}{60\lambda}$$

15. The system shown in Figure 2.7 consists of a TMR core with a single spare a which can serve as a spare only for module 1. Assume that modules 1 and a are active. When either of the two modules 1 or a fails, the failure is detected by the perfect comparator C, and the single operational module is used to provide an input to the voter.

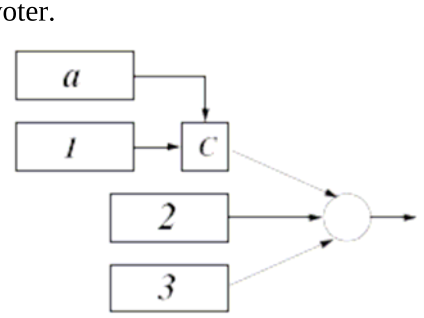


Figure 2.7: A TMR with a spare.

(a) Assuming that the voter is perfect as well, which one of the following expressions for the system reliability is correct (where each module has a reliability R and the modules are independent).

(1) $R_{system} = R^4 + 4R^3(1 - R) + 3R^2(1 - R)^2$
(2) $R_{system} = R^4 + 4R^3(1 - R) + 4R^2(1 - R)^2$
(3) $R_{system} = R^4 + 4R^3(1 - R) + 5R^2(1 - R)^2$
(4) $R_{system} = R^4 + 4R^3(1 - R) + 6R^2(1 - R)^2$

(b) Write an expression for the reliability of the system if instead of a perfect comparator for modules 1 and a there is a coverage factor c (c is the probability that a failure in one module is detected, the faulty module is correctly identifed and the operational module is successfully connected to the voter which is still perfect).

$R_{duplex} = R^2(t) + 2cR(t)\,[1 - R(t)]$, c = 1 for active spare
$R_{duplex} = R^2(t) + 2R(t)\,[1 - R(t)] = R^2(t) + 2R(t) - 2R^2(t) = 2R(t) - R^2(t)$

$R_{system} = R(_{\text{duplex good plus other two good}}) + R(_{\text{duplex good and one other good}}) + R(_{\text{duplex bad and both others good}})$

$= (2R(t) - R^2(t))R^2(t) + (2R(t) - R^2(t))(2!/(1!1!))R(t)(1 - R(t)) + (1 - (2R(t) - R^2(t))R^2(t)$

$= 2R^3(t) - R^4(t) + 2R(t)(2R(t) - 2R^2(t) - R^2(t) + R^3(t)) + R^2(t) - 2\,R^3(t) + R^4(t)$

$= 4R^2(t) - 4R^3(t) - 2R^3(t) + 2R^4(t) + R^2(t) = 2R^4(t) - 6R^3(t) + 5R^2(t)$

This is the same as number (3) above

$R_{duplex} = R^2(t) + 2cR(t)\,[1 - R(t)]$

$R_{system} = R(_{\text{duplex good plus other two good}}) + R(_{\text{duplex good and one other good}}) + R(_{\text{duplex bad and both others good}})$

$= (R^2(t) + 2cR(t)\,[1 - R(t)])R^2(t) + (R^2(t) + 2cR(t)\,[1 - R(t)])(2!/(1!1!))R(t)(1 - R(t)) + (1 - (R^2(t) + 2cR(t)\,[1 - R(t)])R^2(t)$

$= (R^2(t) + 2cR(t) - 2cR^2(t))\,R^2(t) + (R^2(t) + 2cR(t) - 2cR^2(t))(2R(t) - 2R^2(t))$
$+ (1 - (R^2(t) + 2cR(t) - 2cR^2(t))R^2(t)$

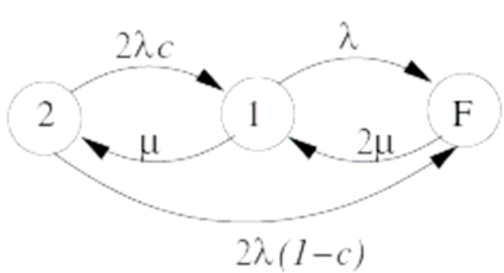
$= R^4(t) + 2cR^3(t) - 2cR^4(t) + 2R^3(t) - 2R^4(t) + 4cR^2(t) - 4cR^3(t) - 4cR^3(t) + 4cR^4(t)$
$+ (1 - R^2(t) - 2cR(t) + 2cR^2(t))R^2(t)$

$= R^4(t)(1 - 2c - 2 + 4c) + R^3(t)(2c + 2 - 4c - 4c) + 4cR^2(t) + R^2(t) - R^4(t) - 2cR^3(t) + 2cR^4(t)$

$= R^4(t)(2c - 1 + 2c - 1) + R^3(t)(2 - 6c - 2c) + R^2(t)(4c + 1)$

$= R^4(t)(4c - 2) + R^3(t)(2 - 8c) + R^2(t)(4c + 1)$

16. A duplex system consists of two active units and a comparator. Assume that each unit has a failure rate of _ and a repair rate of _. The outputs of the two active units are compared and when a mismatch is detected, a procedure to locate the faulty unit is performed. The probability that upon a failure, the faulty unit is correctly identi_ed and the fault-free unit (and consequently, the system) continues to run properly, is the coverage factor c. Note that when a coverage failure occurs, the entire system fails and both units have to be repaired (at a rate _ each). When the repair of one unit is complete, the system becomes operational and the repair of the second unit continues, allowing the system to return to its original state.
(a) Show the Markov model for this duplex system.
(b) Derive an expression for the long-term availability of the system assuming that _ = 2_.



For the long-term availability, we write the steady-state balance equations.

$$2\lambda c P_2 = \mu P_1$$
$$2\lambda(1 - c)P_2 + \lambda P_1 = 2\mu P_F$$

$P_2 + P_1 + P_F = 1$

For convenience, denote $\rho = \lambda/\mu$. The solution to the above equations is

$$P_2 = \frac{1}{1+\rho(3-c)+\rho^2}$$

$$P_1 = \frac{2\rho}{1+\rho(3-c)+\rho^2}$$

$$P_1 + P_2 = \frac{1+2\rho}{1+\rho(3-c)+\rho^2}$$

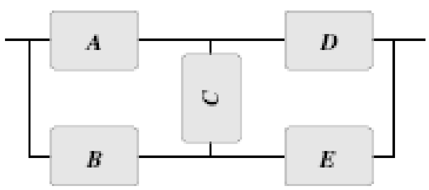
If $\lambda = 2\mu$, $\rho = 0.5$ and the availability is $P_1 + P_2 = \frac{1+2\rho}{1+\rho(3-c)+\rho^2} = \frac{2}{2.75-0.5c} = \frac{8}{11-2c}$

17. (a) Your manager in the Reliability and Quality Department asked you to verify her calculation of the reliability of a certain system. The equation that she derived is

$$R_{system} = R_C\,[1 - (1 - R_A)(1 - R_B)]\,[1 - (1 - R_D)(1 - R_E)] + (1 - R_C)\,[1 - (1 - R_AR_D)(1 - R_BR_E)]$$

However, she lost the system diagram. Can you draw the diagram based on the expression above?

(b) Write expressions for the upper and lower bounds on the reliability of the system and calculate these values and the exact reliability for the case $R_A = R_B = R_C = R_D = R_E = R = 0.9$.



The paths are: AD, BE, ACE and BCD. The upper bound is therefore:

$$R_{system} \le 1 - (1 - R_AR_D)(1 - R_BR_E)(1 - R_AR_CR_E)(1 - R_BR_CR_D)$$

This upper bound is equal to 0.99735 for R = 0.9.

The cut sets are: AB, DE, ACE and BCD. The lower bound is:

$$R_{system} \ge [1-(1- R_A)(1- R_B)][1- (1- R_D)(1- R_E)][1- (1 -R_A)(1- R_C)(1- R_E)][1- (1 - R_B)(1 - R_C)(1- R_D)]$$

$$= [1 - (1 - R)^2]^2[1 - (1 - R)^3]^2$$

This lower bound is equal to 0.9781 for R = 0.9.

The value of the exact reliability for the case of $R_A = R_B = R_C = R_D = R_E = 0.9$

$$R_{system} = R_C[(1 - (1 - R_A)(1 - R_B))(1 - (1 - R_D)(1 - R_E))] + (1 - R_C)(1 - R_AR_D)(1-R_BR_E)$$

$$R_{system} = R(1 - (1 - R)^2)^2 + (1 - R)[1 - (1 - R^2)^2] = 0.97848.$$