Novel Insights into the Role of Turbulent Fluxes and Diabatic Heating in Shaping the Spatio-Temporal Structure of the Hadley Cell

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***Abstract*—Due to global warming, the Hadley cell is expanding, significantly impacting environmental patterns worldwide. The effects of the eddy momentum flux, turbulent heat flux, and diabatic heating on the Hadley cell is an understudied issue, and could provide insights into the structure and behavior of global air circulation patterns. However, the limitations of current methods do not fully capture the time-dependent structure of the Hadley cell because they use ideal distributions of the eddy fluxes and diabatic heating in simulations. We propose functional distributions for eddy momentum and heat flux parameterizations using a combination of symmetric and asymmetric components to more effectively model the non-linearities. For this, we developed a novel computational framework using the mixed finite difference-spectral representation to model the spatiotemporal structure of the Hadley cell. Our results demonstrate that a) increased diabatic heating led to pronounced long-term Hadley cell expansion, b) asymmetric components of the eddy momentum flux significantly influences the strength of the Hadley cell, and c) a larger asymmetric component of the turbulent heat flux changes the meridional position. These findings reveal the complex interplay between atmospheric heat transport and climate forcing mechanisms, which we believe could provide critical insights for improving climate model accuracy and long-term weather prediction capabilities.**

***Index Terms*—climate modeling, diabatic heating, eddy momentum flux, General Circulation Models (GCMs), Hadley cell, turbulent heat flux**

I. Introduction

The Hadley Cell is a fundamental component of Earth's atmospheric circulation, playing a pivotal role in organizing tropical weather patterns and facilitating the poleward transport of energy and angular momentum [1]. This large-scale circulation pattern significantly influences the distribution of precipitation and aridity across tropics and subtropics, with its rising branch associated with the Intertropical Convergence Zone (ITCZ) and its descending branch linked to subtropical deserts [2]. The dynamic nature of the Hadley cell is evident in its seasonal and annual migrations, which have profound implications for regional climates and ecosystems [3]. In the context of anthropogenic climate change, understanding the behavior and potential alterations of the Hadley circulation becomes increasingly critical. The expansion of Hadley cell circulation plays a significant role in large-scale weather events, including hurricanes, monsoons, droughts, heat waves, and crop failure [4]. Thus, long-term predictions into the scale and expansion of the Hadley cell may provide insight into weather and climate migrations that affect millions worldwide. Paleoclimatic records have revealed substantial shifts in the Hadley cell and ITCZ positions throughout Earth's history, illustrating the system's sensitivity to climate forcings [5]. In this paper, eddy fluxes refer to the eddy momentum flux and turbulent heat flux. Eddy momentum flux is the transport of momentum through turbulent air motions, driving the movement of air within the Hadley cell. It helps transfer momentum from the Earth's surface to the upper atmosphere, influencing tropical atmospheric circulation. These eddies also affect the width and strength of the Hadley cell [6]. Turbulent heat flux is the vertical transfer of heat due to turbulent air motions, critical for convection and atmospheric stability. It drives warm air upward in the tropics, influencing cloud formation and weather patterns [7]. Diabatic heating rate refers to the heating or cooling of the atmosphere from processes like radiation and latent heat release. It influences the strength and position of the Hadley cell by affecting the temperature structure and convection in the tropics [8]. Traditional atmospheric models use diffusive parameterizations for eddy fluxes and diabatic heating:

$$\overline{u'v'} = D_e \frac{du}{dy} \tag{1}$$

where D_e is the drag coefficient, u is the zonal flow, and $\overline{u'v'}$ represents the turbulent heat flux [9]. However, this method often fails to capture complex eddy-driven processes [6], [10]. In our approach, we propose a novel composition for the distribution functions that accounts for the minute differences due to noise.

II. Methodology

A. Modeling of the General Circulation

Hadley cells can be represented by the general circulation of air, where warmer air at the equator (due to insolation) rises, moves towards the poles, and descends while cooling. The relationship between the eddy fluxes (eddy momentum and turbulent heat fluxes) with atmospheric variables can be described by Holton's formulation, as detailed in Appendix A.

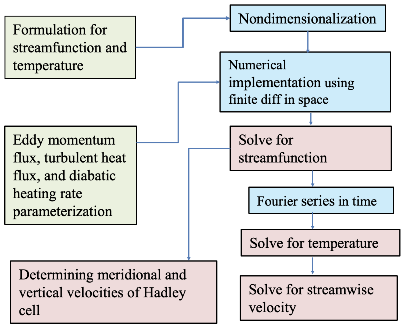


Fig. 1: Computational workflow for Hadley cell structure determination. Green represents data inputs, blue represents computational steps, and pink represents data outputs.

When considering the response of climate forcings, we focus on changes in the stream function and temperature.

B. Formulation for Stream Function Distribution

The coupled system can be written in matrix operator form:

$$\begin{bmatrix} -i\omega I & \frac{N^2HR^{-1}}{\rho_o}\frac{\partial}{\partial y} \\ 0 & L \end{bmatrix} \begin{bmatrix} \overline{T} \\ \overline{\chi} \end{bmatrix} = \begin{bmatrix} b_T \\ b_\chi \end{bmatrix} \tag{2}$$

where L is the operator from Holton's equation and b_T, b_χ are the corresponding right-hand sides as derived in Appendix A [11]. To parameterize our model, we consider the constants for dry air as described in a moist general circulation model by [12]. Since our model focuses on the Hadley cell at a smaller scale, we assume the coriolis parameter to be constant. To additionally uncover the role of the zonal component of drag and meridional velocity, we also consider the following equation:

$$\frac{\partial u}{\partial t} - f_o v + \frac{\partial \phi}{\partial x} = \overline{X} \tag{3}$$

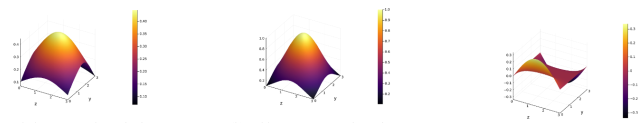
If the unresolved small-scale eddies do not introduce a damping effect on larger structures, the stream function equation can be solved independently. In principle, the presence of the $\overline{X}$ damping term would necessitate coupling between the stream function and the streamwise velocity $\overline{u}$. However, due to the thermal wind balance, this would consequently require coupling between the stream function and temperature. The stream function equation indicates that, in the absence of coupling with the temperature equation, the stream function structure is independent of frequency. This implies that the structure of the cell itself does not depend on the time scale.

C. Formulation for Temperature Distribution

Unlike streamfunction, mean temperature $\overline{T}$ does have a dependence on frequency ω. And, assuming thermal wind balance still holds, $\overline{u}$ will also have to directly depend on frequency ω due to its thermal wind balance with temperature. On the other hand, thermal wind balance being maintained throughout the time allows streamfunction $\overline{\chi}$ to not depend on frequency ω. Using the parameterizations of eddy momentum flux, eddy heat flux, and the diabatic heating rate, $\dfrac{\partial \overline{T}}{\partial t}$ can be determined. We devise a frequency operator $-i\omega I$ where I is the identity matrix given the meridional and vertical dimensions of our plot and perform matrix division with $\dfrac{\partial \overline{T}}{\partial t}$ to obtain the T as a function of ω. Once both temperature $\overline{T}$ and streamfunction $\overline{\chi}$ have been found, streamwise velocity $\overline{u}$ can be found through thermal wind balance relation:

$$f_o\frac{\partial \overline{u}}{\partial z} + RH^{-1}\frac{\partial \overline{T}}{\partial y} = 0 \tag{4}$$

D. Eddy Flux and Diabatic Heating Parameterization



(a) Turbulent Heat Flux Distribution (b) Eddy Momentum Flux Distribution (c) Diabatic Heating Distribution

Fig. 2: Parameterizations of the turbulent heat flux, eddy momentum flux, and diabatic heating

Our approach to modeling the spatial distribution of eddy heat and momentum fluxes involves a combination of even and non-even functions to capture both symmetric and asymmetric transport processes:

$$a \cdot f_{\text{even}}(y,z) + b \cdot f_{\text{non-even}}(y,z) \tag{5}$$

where f_{even} is an even symmetric distribution function and $f_{\text{non-even}}$ is an asymmetric function. The even function represents the symmetric component of the flux distribution, while the non-even function accounts for asymmetric transport processes [13]. Figure 2 shows the symmetric distributions for the eddy fluxes and diabatic heating. We apply a similar approach to the zonally averaged diabatic heating rate utilizing a Gaussian distribution. The diabatic heating rate distribution was modeled using two components: a symmetric (even) heating function and an asymmetric (odd) heating function. The symmetric component is represented by a Gaussian distribution, capturing the zonally averaged heating structure. The asymmetric component introduces nonlinear variations that represent the influence of localized phenomena such as convective or advective processes [14].

E. Numerical Implementation for Stream Function

We solve for χ employing finite difference formulas:

$$D_0u(\overline{x}) = \frac{u(\overline{x}+h) - u(\overline{x}-h)}{2h} \tag{6}$$

$$Du^2(\overline{x}) = \frac{u(\overline{x}-h) - 2u(\overline{x}) + u(\overline{x}+h)}{h^2} \tag{7}$$

Using the double-derivative finite difference approximation given in (7), the corresponding difference matrices are generated with the appropriate periodic boundary conditions.

F. *Nondimensionalization*

The non-dimensionalization reveals the underlying relationships between atmospheric variables. By nondimensionalizing the equations, we obtain:

$$\frac{\partial \overline{u}}{\partial t} - \frac{\overline{v}}{Ro} = -\frac{\partial \overline{u'v'}}{\partial y} + \alpha_1 \overline{X} \tag{8}$$

$$\frac{\partial \overline{T}}{\partial t} + r_T \overline{w} = -\frac{\partial \overline{v'T'}}{\partial y} + \alpha_2 \overline{J} \tag{9}$$

where $Ro = \frac{U}{f_o L}$ is the Rossby number, $r_T = \frac{N^2 H}{f_o U}$, and α_1 and α_2 are dimensionless scaling factors. α_1 controls the contribution of unresolved small-scale eddies. α_2 controls the weight of the zonally-averaged diabatic heating $\overline{J}$, and is what we further explore the effects of through controlled experimentation.

III. Results

TABLE I: Physical Constants Used in Hadley Cell Modeling

Physical Constant	Symbol	Value
Coriolis Parameter	f_0	1×10^{-4} s^{-1}
Specific Heat Capacity	C_p	1004.64 J/(kg·K)
Brunt-Väisälä Frequency	N	0.02 s^{-1}
Reference Density	ρ_0	1.225 kg/m^3
Von Kármán Constant	κ	0.40
Gas Constant	R	287.04 J/(kg·K)
Scale Height	H	8500 m

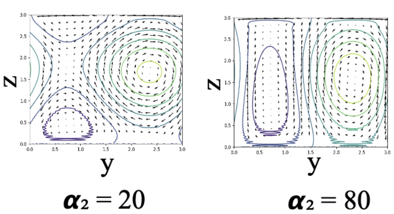


Fig. 3: Effects of diabatic heating parameter variations on Hadley cell circulation patterns.

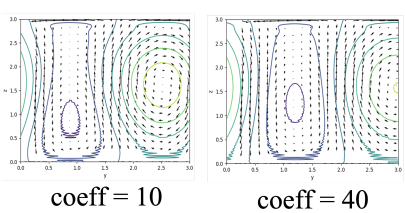


Fig. 4: Effects of turbulent heat flux odd parameter variations on Hadley cell circulation patterns.

Computational analysis was performed using Julia programming language to solve the stream function, temperature, and velocity vector fields. Fixed parameterization values were set with $r_T = 0.15$ and $Ro = 20000$. The physical constants implemented for our model shown in Table I are derived from

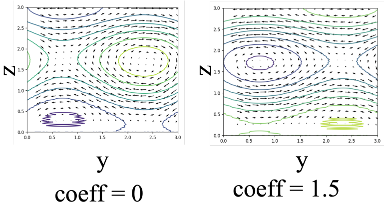


Fig. 5: Effects of eddy momentum flux odd parameter variations on Hadley cell circulation patterns.

relevant literature used to model other GCMs. In three separate experiments, we determine the changes in Hadley cell structure due to increase in diabatic heating by raising the α_2 and to increase in noise from the non-even asymmetric component in the eddy fluxes. Computation of the stream function allowed us to devise the individual streamwise velocities for meridional and vertical velocities. We visualize results as vector fields with streamfunction contours and analyze structural changes from independent manipulation of each eddy variable and diabatic heating.

IV. Discussion

A. *Interpretative Validation of Circulation Patterns*

The validity of results is confirmed by the presence of physically meaningful circulation patterns in the stream function for both Hadley cells. The stream function effectively captures the expected meridional overturning circulation shown in Figures 3, 4, and 5, with positive values corresponding to clockwise circulation and negative values indicating counterclockwise circulation, consistent with theoretical expectations shown in other GCMs [15].

B. *Diabatic Heating Effects*

The diabatic heating parameter α_2 functions as a multiplier for the meridional gradient of heating $\frac{\partial \overline{J}}{\partial y}$. Increased α_2 values amplify thermal forcing strength, intensifying overturning circulation and modifying Hadley cell structure. Enhanced heating produces a more extensive and well-defined left Hadley cell with greater vertical reach, indicating stronger atmospheric response to imposed heating gradients shown in Figure 3. This mechanism creates more vigorous meridional flow that redistributes energy more effectively across latitudes. Previous research demonstrates that climate change-induced diabatic heating increases result in mid-latitude shifts of the Hadley cell [16]. The present model suggests that these heating increases also drive Hadley cell expansion, potentially affecting large-scale climate phenomena.

C. *Eddy Momentum Flux Impacts*

Asymmetric eddy momentum flux components significantly influence right Hadley cell circulation characteristics. Strengthened asymmetry enhances momentum redistribution, extending circulation over larger latitudinal ranges shown in Figure 5. Interestingly, these changes seemingly invert the sizes and configuration of the Hadley cells as well. This expansion affects momentum and heat distribution patterns, potentially altering wind systems, precipitation zone locations, and climate event frequency and intensity across different latitudes.

D. Turbulent Heat Flux Effects

Asymmetric turbulent heat flux components redistribute heat between hemispheres, causing Hadley cell positional shifts as illustrated in Figure 4. Modified interhemispheric temperature gradients force rising branch migration toward warmer regions. Resulting circulation changes affect subtropical jet streams, precipitation patterns, and climate systems including monsoons and storm tracks.

E. Temperature Distributions

Temperature analysis revealed purely imaginary values in the Fourier transform domain, indicating sinusoidal spatial variation in the temperature field. This result suggests that the temperature structure exhibits harmonic behavior with no real component in the frequency-domain solution. Increasing the α_2 scaling factor for diabatic heating did not change the structure of the temperature distribution, but did proportionally increase the magnitude of the temperature according to Appendix C. However, increasing the contribution of the asymmetric component of the turbulent heat flux resulted in an asymmetric change in temperature on both sides of the Hadley cells, suggesting the presence of directional forcings that further impacts heat distribution across the equator.

F. Model Limitations

The current framework excludes small-scale eddy forcings due to the structure of the upper-triangular matrix operator, allowing independent stream function determination without temperature coupling. Horizontal domain periodicity introduces artificial boundary effects that may require larger computational domains to minimize. However, domain expansion increases computational requirements, necessitating careful balance between accuracy and feasibility. Additionally, frequency-independent stream function solutions require temperature calculations through division by $-i\omega$, resulting in decreased temperature amplitude at higher frequencies and emphasizing lower-frequency temperature variations.

V. Conclusion

This research presents a novel computational approach for analyzing the time-dependent structure of the Hadley cell, providing valuable insights into long-term tropical circulation dynamics under varying climate conditions. The mixed finite difference-spectral representation successfully captures the spatiotemporal evolution of Hadley cell structure, which we believe offers a more comprehensive understanding than traditional snapshot-based analyses. Key findings demonstrate that both eddy momentum and heat fluxes contribute significantly to Hadley cell expansion through weighted symmetric and asymmetric components. Increased diabatic heating rates produce pronounced long-term expansion effects, while asymmetric components of eddy momentum flux and turbulent heat flux substantially influence right Hadley cell positioning and strength. These results highlight the complex interactions between atmospheric heat transport, air movement dynamics, and climate forcing mechanisms. By elucidating Hadley cell expansion mechanisms, we believe these findings enhance our capability to predict and address climate change impacts on precipitation patterns. The time-dependent modeling approach provides a more dynamic perspective on Hadley cell behavior, potentially leading to improved climate model development and enhanced long-term forecasting accuracy. This work advances understanding of Hadley cell responses to global warming and emphasizes the importance of time-dependent structural analysis in climate research applications.

VI. Acknowledgement

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References

[1] K. M. Grise and S. M. Davis, "Hadley cell expansion in cmip6 models," *Atmospheric Chemistry and Physics*, vol. 20, no. 9, pp. 5249–5268, 2020.

[2] C. Deser and J. M. Wallace, "Large-scale atmospheric circulation features of warm and cold episodes in the tropical pacific," *Journal of Climate*, vol. 3, no. 11, pp. 1254–1281, 1990.

[3] I. M. Dima and J. M. Wallace, "On the seasonality of the hadley cell," *Journal of the atmospheric sciences*, vol. 60, no. 12, pp. 1522–1527, 2003.

[4] D. Choudhury, D. Nath, and W. Chen, "Impact of indian ocean warming on increasing trend in pre-monsoon rainfall and hadley circulation over bay of bengal," *Theoretical and Applied Climatology*, vol. 137, pp. 2595–2606, 2019.

[5] C. M. Flynn and T. Mauritsen, "Hadley cell size and strength responses depend on turbulent drag," *Journal of the Atmospheric Sciences*, vol. 80, no. 4, pp. 1047–1064, 2023.

[6] I. M. Held and A. Y. Hou, "Nonlinear axially symmetric circulations in a nearly inviscid atmosphere," *Journal of the Atmospheric Sciences*, vol. 37, no. 3, pp. 515–533, 1980.

[7] A. Sheshadri, M. Borrus, M. Yoder, and T. Robinson, "Midlatitude error growth in atmospheric gcms: The role of eddy growth rate," *Geophysical Research Letters*, vol. 48, no. 23, p. e2021GL096126, 2021.

[8] R. Chemke, L. Polvani, and C. Deser, "The effect of arctic sea ice loss on the hadley circulation," *Geophysical Research Letters*, vol. 46, no. 2, pp. 963–972, 2019.

[9] C.-Z. Zou and T. Gal-Chen, "Parameterization of the meridional eddy heat and momentum fluxes," *Journal of the atmospheric sciences*, vol. 56, no. 12, pp. 1830–1842, 1999.

[10] B. F. Farrell, "Equable climate dynamics," *Journal of Atmospheric Sciences*, vol. 47, no. 24, pp. 2986–2995, 1990.

[11] J. R. Holton, *An introduction to dynamic meteorology. volume 23*. Academic Press, 1979.

[12] D. M. Frierson, I. M. Held, and P. Zurita-Gotor, "A gray-radiation aquaplanet moist gcm. part i: Static stability and eddy scale," *Journal of the atmospheric sciences*, vol. 63, no. 10, pp. 2548–2566, 2006.

[13] H. Wang, B. Qiu, H. Liu, and Z. Zhang, "Doubling of surface oceanic meridional heat transport by non-symmetry of mesoscale eddies," *Nature Communications*, vol. 14, no. 1, p. 5460, 2023.

[14] J. Ling and C. Zhang, "Diabatic heating profiles in recent global reanalyses," *Journal of Climate*, vol. 26, no. 10, pp. 3307–3325, 2013.

[15] M. Pikovnik, v. Zaplotnik, L. Boljka, and N. Žagar, "Metrics of the hadley circulation strength and associated circulation trends," *Weather and Climate Dynamics*, vol. 3, no. 2, pp. 625–644, 2022.

[16] S. Ghosh, O. Lachmy, and Y. Kaspi, "The role of diabatic heating in the midlatitude atmospheric circulation response to climate change," *Journal of Climate*, vol. 37, no. 10, pp. 2987–3009, 2024.

Appendix A
Derivation of Matrix Equation for General Circulation

$$\frac{\partial^2\overline{\chi}}{\partial y^2} + \frac{f_o^2}{N^2}\rho_o\frac{\partial}{\partial z}\left(\frac{1}{\rho_o}\frac{\partial\overline{\chi}}{\partial z}\right) + \frac{\rho_o f_o}{N^2}\frac{\partial\chi}{\partial z} = \frac{\rho_o}{N^2}\left[\frac{\partial}{\partial y}\left(\frac{\kappa\overline{J}}{H} - \frac{R}{H}\frac{\partial\overline{v'T'}}{\partial y}\right) - f_o\frac{\partial^2}{\partial z\partial y}(\overline{u'v'})\right] \tag{10}$$

Here, $\overline{\chi}$ is the zonally averaged streamfunction, N the buoyancy frequency, $\overline{v'T'}$ the turbulent heat flux, $\overline{u'v'}$ the eddy momentum flux, f_o the Coriolis parameter, and $\overline{J}$ the diabatic heating rate. To examine climate forcing responses, we consider the zonally averaged temperature equation:

$$\frac{\partial\overline{T}}{\partial t} + \frac{N^2H}{R}\overline{w} = -\frac{\partial\overline{v'T'}}{\partial y} + \frac{\overline{J}}{c_p} \tag{11}$$

where $\overline{T}$ is temperature, H is scale height, $\overline{w}$ is vertical velocity, R the gas constant, and c_p the specific heat at constant pressure. To avoid explicitly solving for $\overline{w}$, we use the streamfunction relationships:

$$\rho_o\overline{v} = -\frac{\partial\overline{\chi}}{\partial z}, \quad \rho_o\overline{w} = \frac{\partial\overline{\chi}}{\partial y} \tag{12}$$

Substituting (12) into (11), we obtain:

$$\frac{\partial\overline{T}}{\partial t} + \frac{N^2HR^{-1}}{\rho_o}\frac{\partial\overline{\chi}}{\partial y} = -\frac{\partial\overline{v'T'}}{\partial y} + \frac{\overline{J}}{c_p} \tag{13}$$

Applying a Fourier transform in time to express the system in frequency space, we write the coupled equations in operator form as shown in (2), where L represents the operator on the left-hand side of Equation (10), and b_T, b_χ the respective right-hand sides of Equations (13) and (10).

Appendix B
Nondimensionalized Form of Holton's Formulation

This nondimensionalized form of Holton's formulation [11] for the general circulation equation was used in our numerical model:

$$r_T\frac{\partial^2\overline{\chi}}{\partial y^2} + \frac{\rho_o}{Ro}\frac{\partial}{\partial z}[\frac{1}{\rho_o}\frac{\partial\overline{\chi}}{\partial z}] = \rho_o[\alpha_1\frac{\partial\overline{X}}{\partial z} + \alpha_2\frac{\partial\overline{J}}{\partial y} - \frac{\partial^2\overline{u'v'}}{\partial y\partial z} - \frac{\partial^2\overline{v'T'}}{\partial y^2}] \tag{14}$$

Appendix C
Dimensionless Temperature Distributions

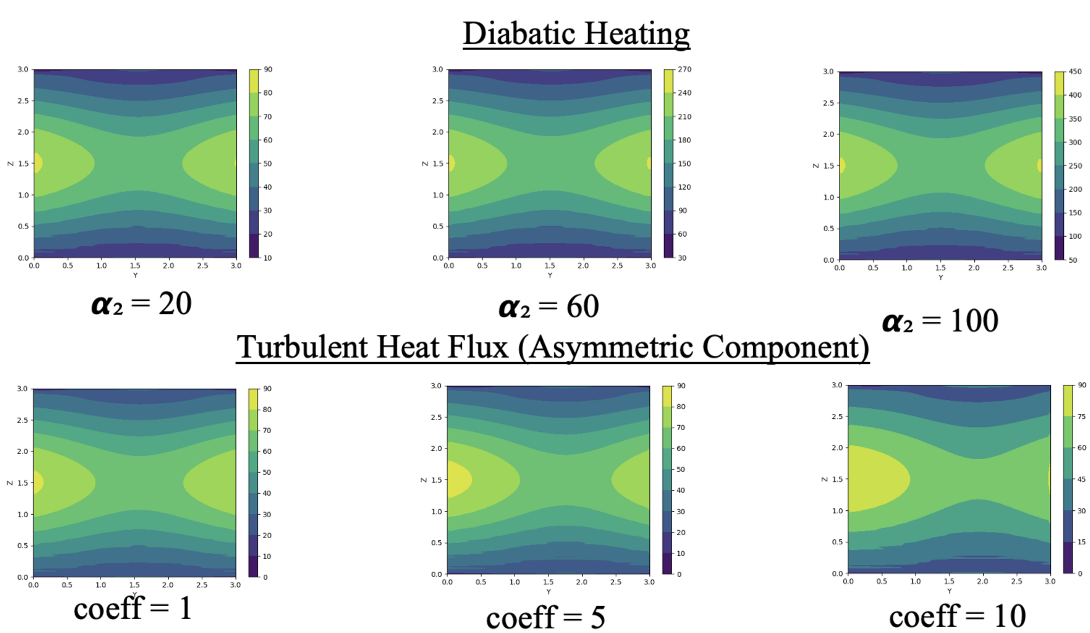

Fig. 6: Temperature distributions by manipulating α_2 scaling factor for diabatic heating and asymmetric component in turbulent heat flux.