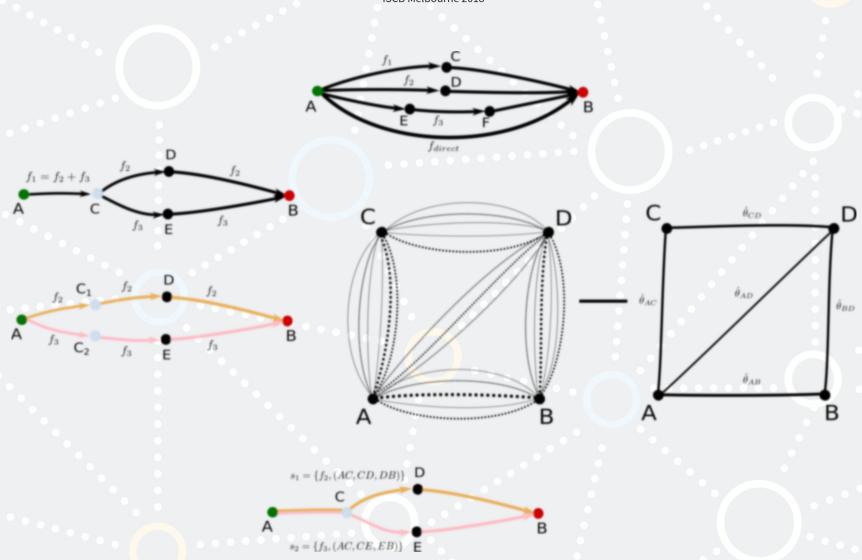
USING FLOW TO ESTIMATE THE PERCENTAGE CONTRIBUTION OF STUDIES IN NETWORK META-ANALYSIS

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CONTRIBUTION OF STUDIES IN PAIRWISE META-ANALYSIS

Pairwise meta-analysis

- Multiple studies
- Single comparison A:B

 c_i : contribution of study i:

•
$$c_i = \frac{w_i}{\sum_{1}^{n} w_i}$$
, $w_i = \frac{1}{v_i}$
• $\hat{\theta}_{AB} = \sum_{i=1}^{n} c_i y_i$

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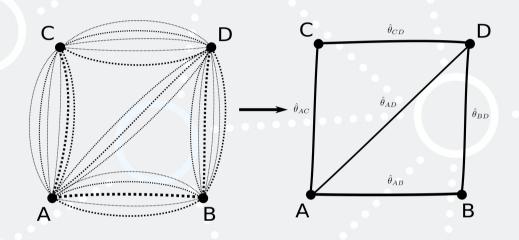


Contribution depends on the variance of studies not the effect size

$$\bullet \quad \sum_{i=1}^n c_i = 1$$

FROM PAIRWISE TO NETWORK META-ANALYSIS

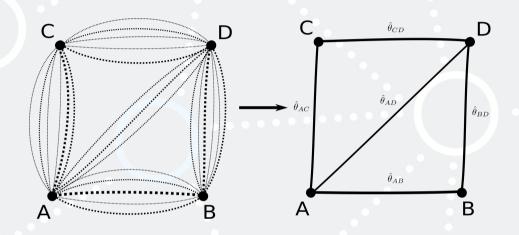
- Multiple studies
- Multiple comparisons:
 - Direct: A:B, A:C, C:D, B:D
 - Indirect: B:C



Two-Stage network meta-analysis

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 - Direct: A:B, A:C, C:D, B:D
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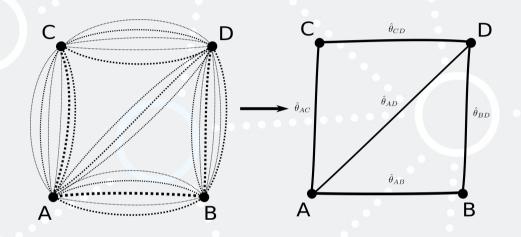


Two-Stage network meta-analysis

• Estimate pairwise summary effects: $\hat{\boldsymbol{\theta}}^{\mathbf{D}} = \{\hat{\theta}_{AB}, \hat{\theta}_{AC}, \hat{\theta}_{AD}, \hat{\theta}_{BD}, \hat{\theta}_{CD}\}$

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Two-Stage network meta-analysis

- Estimate pairwise summary effects: $\hat{\boldsymbol{\theta}}^{\mathbf{D}} = \{\hat{\theta}_{AB}, \hat{\theta}_{AC}, \hat{\theta}_{AD}, \hat{\theta}_{BD}, \hat{\theta}_{CD}\}$ Calculate relative network effect sizes: $\hat{\boldsymbol{\theta}}^{N} = \mathbf{H}\hat{\boldsymbol{\theta}}^{D} = \{\hat{\theta}_{AB}^{N}, \hat{\theta}_{AC}^{N}, \hat{\theta}_{AD}^{N}, \hat{\theta}_{BC}^{N}, \hat{\theta}_{BD}^{N}, \hat{\theta}_{CD}^{N}\}$

PROJECTION H MATRIX

- Resembles the **hat matrix** in a linear regression model
- Each row h_{AB} refers to a single comparison

$$\hat{\boldsymbol{\theta}}^N = \mathbf{H}\hat{\boldsymbol{\theta}}^D$$

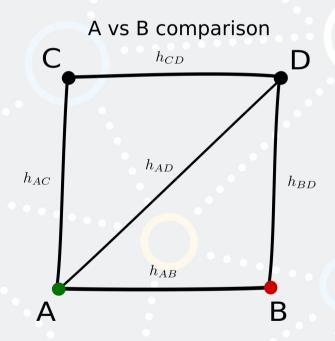
$$\begin{pmatrix} \hat{\theta}_{AB}^{N} \\ \hat{\theta}_{AC}^{N} \\ \hat{\theta}_{AC}^{N} \\ \hat{\theta}_{AD}^{N} \\ \hat{\theta}_{BC}^{N} \\ \hat{\theta}_{BD}^{N} \\ \hat{\theta}_{CD}^{N} \end{pmatrix} = \begin{pmatrix} h_{AB}^{AB} & h_{AC}^{AB} & h_{AD}^{AB} & h_{BD}^{AB} & h_{CD}^{AB} \\ h_{AB}^{AC} & h_{AC}^{AC} & h_{AD}^{AC} & h_{BD}^{AC} & h_{CD}^{AC} \\ h_{AB}^{AD} & h_{AC}^{AD} & h_{AD}^{AD} & h_{BD}^{AD} & h_{CD}^{AD} \\ h_{AB}^{BC} & h_{AC}^{BC} & h_{AD}^{BC} & h_{BD}^{BC} & h_{CD}^{BC} \\ h_{AB}^{D} & h_{AC}^{CD} & h_{AD}^{CD} & h_{BD}^{CD} & h_{CD}^{CD} \\ h_{AB}^{CD} & h_{AC}^{CD} & h_{AD}^{CD} & h_{BD}^{CD} & h_{CD}^{CD} \end{pmatrix} \times \begin{pmatrix} \hat{\theta}_{AB} \\ \hat{\theta}_{AC} \\ \hat{\theta}_{AD} \\ \hat{\theta}_{BD} \\ \hat{\theta}_{CD} \end{pmatrix}$$

PROJECTION H MATRIX

- Resembles the hat matrix in a linear regression model
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$$\hat{\theta}_{AB}^{N} = h_{AB}^{AB} \hat{\theta}_{AB} + h_{AC}^{AB} \hat{\theta}_{AC} + h_{AD}^{AB} \hat{\theta}_{AD} + h_{BD}^{AB} \hat{\theta}_{BD} + h_{CD}^{AB} \hat{\theta}_{CD}$$

The elements of hatmatrix can be seen as generalization of weights in pairwise meta-analysis **but** they do not add up to $1 \sum h_{XY}^{AB} \neq 1$ and are not strictly positive since $h_{XY}^{AB} = -h_{YX}^{AB}$.



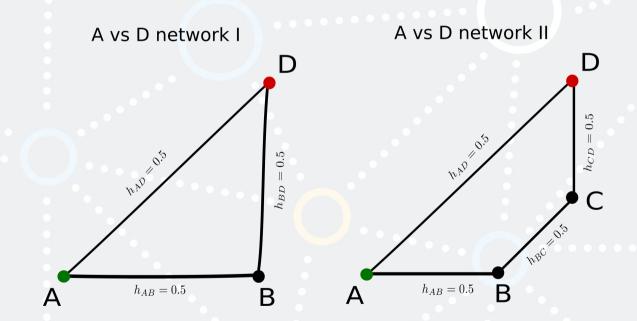
superscript AB denoting the comparison is omitted

NO EASY FIX

Naive normalization:
$$c_{XY} = \frac{|h_{XY}|}{\sum_i |h_i|}$$

Contribution should be **independent** for parallel comparisons.

In both networks I and II contribution should be equal but normalisation in network I gives: $c_{AD}=\frac{1}{3}$ and in network II gives: $c_{AD}=\frac{1}{4}$



contribution of direct should be: $c_{direct} = h_{direct} = 1 - h_{indirect}$

h^{AB} ROW AS A **COMPARISON GRAPH** G_{AB}

- Each h matrix row can be transformed into a directed graph $G_{AB} = (V, E, F)$
 - *V*: Set of interventions
 - *E*: Comparisons with direct evidence (studies)
 - F: Flow, property of edges, equals the elements of h row.

Köning J. Krahn U. Binder H. Statistics in Medicine 2013

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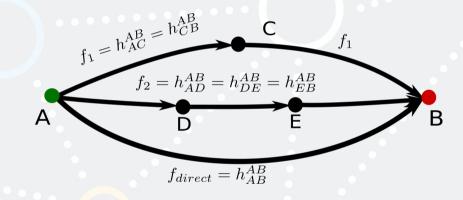
Assumptions for contribution in a contribution graph

Contribution of *parallel* paths (sequence of edgescomparisons) is **equal to the their flow**.

$$c_{ACB} = c_{AC} + c_{CB} = f_1$$

$$c_{ADEB} = c_{AD} + c_{DE} + c_{EB} = f_2$$

$$c_{AB} = f_{direct}$$



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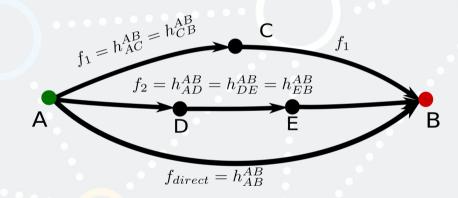
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Contribution of individual edge inside a path Each comparison **contributes equally** in a path so the contribution of each comparison is its **flow** divided by its **length**:

$$c_{AC} = c_{CD} = c_{DB} = \frac{f_{ACDB}}{3}$$
 $\mathbf{c_{comparison}} = \frac{\mathbf{f_{path}}}{\mathbf{length of path}}$

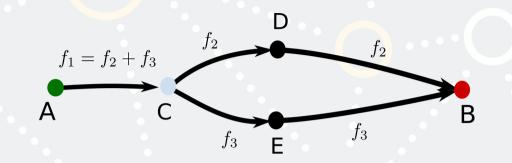




STREAMS

Mixed paths

A comparison (edge) is shared between two parallel paths.



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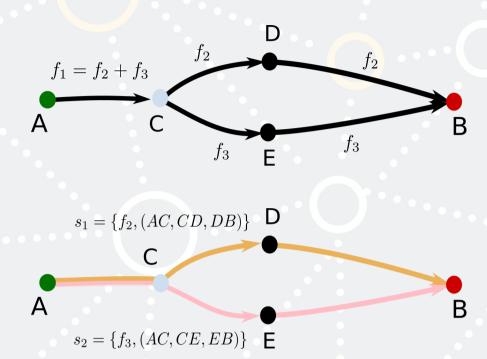
Decompose flow into streams

A stream is defined by its flow ϕ and the path π

$$\mathbf{s_i} = \{\phi_i, \pi_i\}$$

Total contribution of a comparison is the sum over the streams that contain

$$c_{AC} = \frac{f_2}{3} + \frac{f_3}{3}$$



Comparison A:B The first step is to locate all streams by decomposing flow

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Repeat until all flow is depleted and all streams are found: $S = \{s_1, s_2, \dots s_k\}$

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Calculate contribution of comparison X vs Y to the network estimate of A vs B: $c_{XY}^{AB} = \sum_{i=|\pi_i|}^k \frac{\phi_i}{|\pi_i|}$, $(XY) \in \pi_i$ where $|\pi_i|$ is the length of path i of stream s_i and k the number of streams.

Macfadyen CA, Acuin JM, Gamble C: Cochrane Database Syst Rev. 2005; (4): CD004618.

x: no treatment, y: quinolone antibiotic, u: non-quinolone antibiotic, v: antiseptic

h matrix

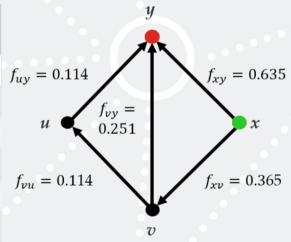
	xy	χv	yu	yv	uv
хy	0.635	0.365	-0.114	-0.251	-0.114
хu	0.603	0.397	0.632	-0.029	-0.368
χv	0.545	0.455	0.170	0.375	0.170
yu	-0.032	0.032	0.745	0.223	-0.255
yv	-0.090	0.090	0.284	0.627	0.284
uv	-0.058	0.058	-0.462	0.404	0.538

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comparison x : y

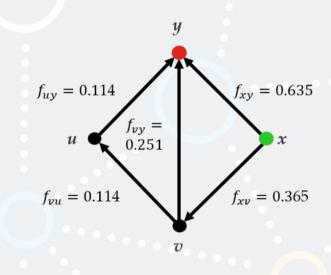
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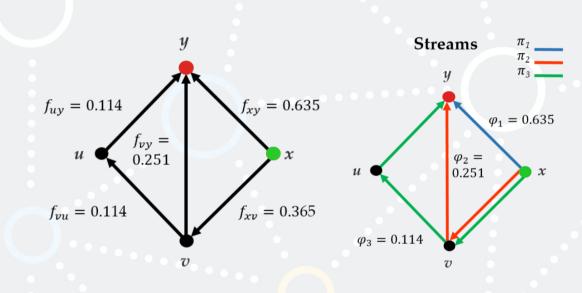
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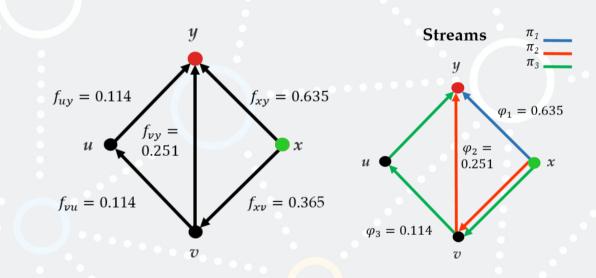
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comparison x : y



•
$$c_{xy} = \phi_1 = 0.635$$

•
$$c_{vy} = \frac{\phi_2}{2} = \frac{0.251}{2} = 0.126$$

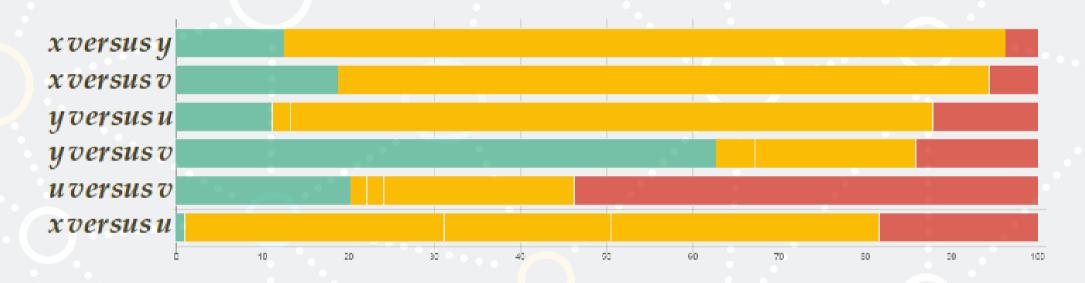
•
$$c_{xy} = \frac{\overline{\phi_2}}{2} + \frac{\phi_3}{3} = \frac{0.251}{2} + \frac{0.114}{3} = 0.164$$

•
$$c_{vu} = c_{uy} = \frac{\phi_3}{3} = \frac{0.114}{3} = 0.038$$

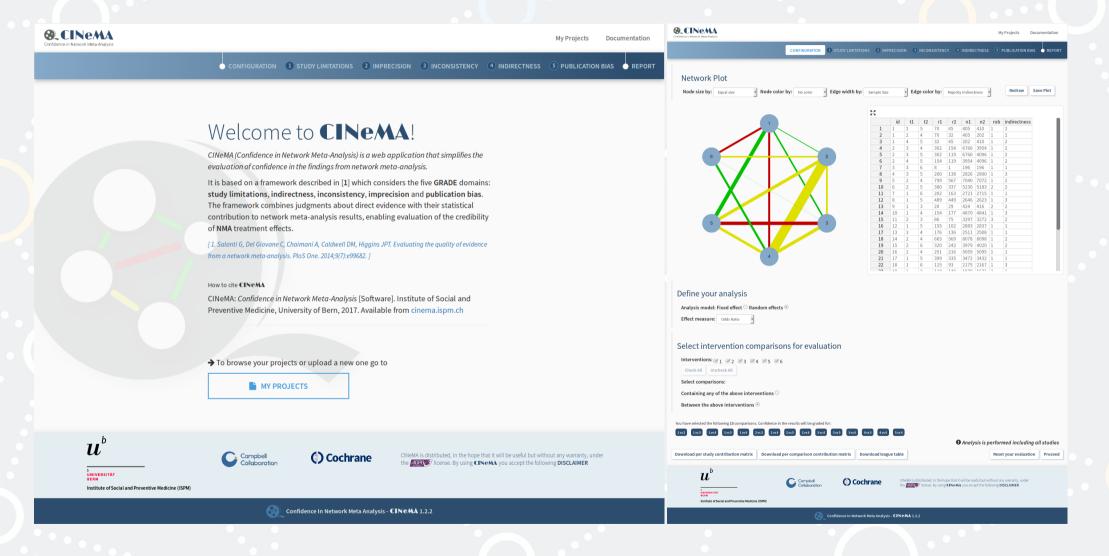
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Bar chart for Risk of Bias for all comparisons

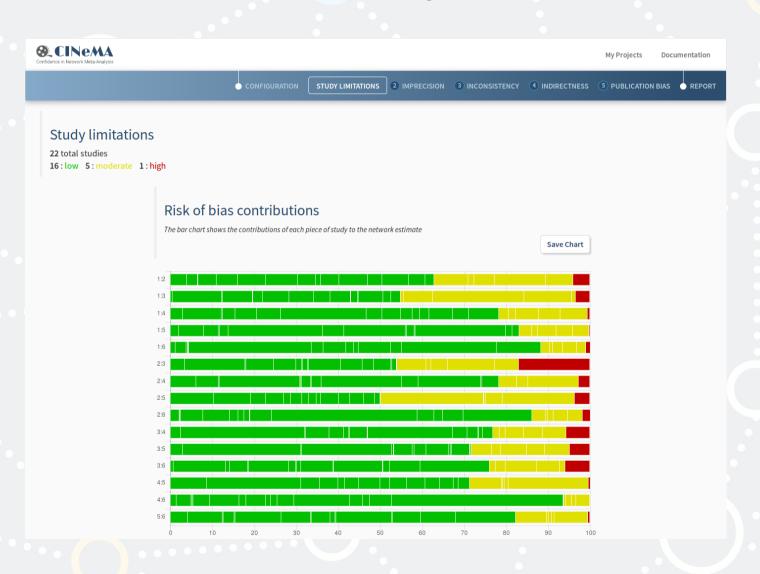


http://cinema.ispm.ch



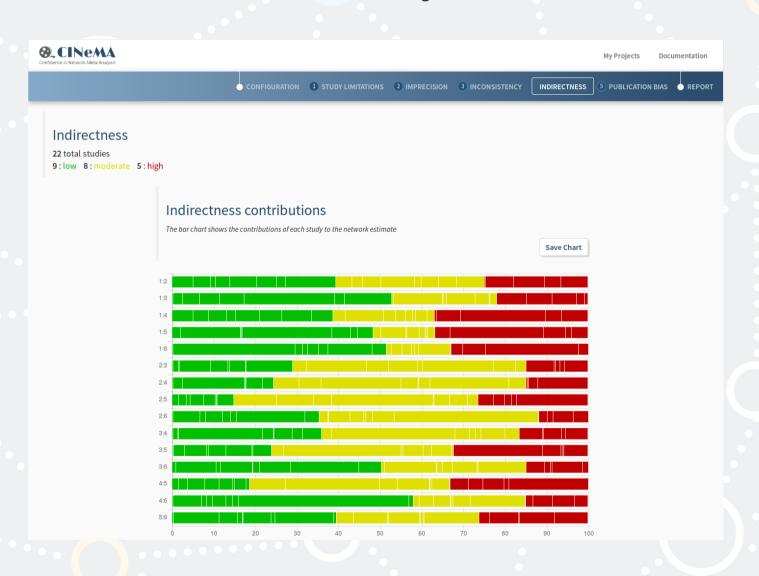
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contributions for assessing Risk of Bias



http://cinema.ispm.ch

contributions for assessing Indirectness



http://cinema.ispm.ch

final report



Thank you

paper: https://f1000research.com/articles/7-610/v1

Estimating the contribution of studies in network meta-analysis: paths, flows and streams



Adriani Nikolakopoulou Gerta Rücker Anna Chaimani Guido Schwarzer Matthias Egger Georgia Salanti