

MATH 1C Test 2 Notecard

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15.1 Double Integrals over Rectangles

The single variable definite integral: $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*)\Delta x$

Volume as a Double Riemann Sum: $V \approx \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*)\Delta A$

Volume as a Double Integral: $\int \int_R f(x, y) dA = \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*)\Delta A$

Fubini's Theorem

If f is continuous on the rectangle $R = \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}$

then it is known $\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx = \int_c^d \int_a^b f(x, y) dx dy$

Special Case

In the special case that $f(x, y)$ can be factored as a product of a function of x only and a function of y only then it is known the following is true:

$$\iint_R g(x) h(y) dA = \int_a^b g(x) dx \int_c^d h(y) dy$$

Average Value

The average value of a function f of two variables defined on a rectangle R is:

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) dA$$

where $A(R)$ is the area of R .

15.2 Double Integrals over General Regions

$$F(x, y) = \begin{cases} f(x, y) & \text{if } (x, y) \text{ is in } D \\ 0 & \text{if } (x, y) \text{ is in } R \text{ but not in } D \end{cases}$$

$$\iint_D f(x, y) \, dA = \iint_D F(x, y) \, dA$$

If f is continuous on a Type I region D such that the fixed bounds are vertical, $D = \{(x, y) \mid a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$, then

$$\iint_D f(x, y) \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx$$

If f is continuous on a Type II region D such that the fixed bounds are horizontal, $D = \{(x, y) \mid c \leq x \leq d, h_1(x) \leq y \leq h_2(x)\}$, then

$$\iint_D f(x, y) \, dA = \int_a^b \int_{h_1(x)}^{h_2(x)} f(x, y) \, dx \, dy$$

15.3 Double Integrals in Polar Coordinates

$$\iint_R f(x, y) \, dA = \int_\alpha^\beta \int_a^b f(r \cos \theta, r \sin \theta) r \, dr \, d\theta$$

Put all half angle and double angle identities on notecard If f is continuous on a polar region of the form

$$D = \{(r, \theta) \mid \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\}$$

Identities: $x^2 + y^2 = r^2$ $x = r \cos \theta$ $y = r \sin \theta$

15.4 Applications of Double Integrals

Electric Charge

$$Q = \iint_D \sigma(x, y) \, dA$$

Center of Mass of a Lamina

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m} \iint_D x \rho(x, y) \, dA \quad \text{and} \quad \bar{y} = \frac{M_x}{m} = \frac{1}{m} \iint_D y \rho(x, y) \, dA \quad \text{where}$$

$$m = \iint_D \rho(x, y) \, dA$$

15.5 Surface Area

The surface area with equation $z = f(x, y)$, $(x, y) \in D$ where f_x and f_y are continuous is

$$A(S) = \iint_D \sqrt{[f_x(x, y)]^2 + [f_y(x, y)]^2 + 1} \, dA$$

$$A(S) = \iint_D \sqrt{\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2 + 1} \, dA$$

For the notecard:

$$A(S) = \iint_D \sqrt{[f_x]^2 + [f_y]^2 + 1} \, dA$$

15.6 Triple Integrals

$$\iiint_B f(x, y, z) \, dV = \lim_{l, m, n \rightarrow \infty} \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*) \Delta V \quad \text{Wont need on notecard}$$

$$\iiint_B f(x, y, z) \, dV = \int_r^s \int_c^d \int_a^b f(x, y, z) \, dx \, dy \, dz$$

Solutions for general regions:

$$\text{For a shadow on the } x, y \text{ plane: } \iiint_B f(x, y, z) \, dV = \iint_D \left[\int_{u_1(x, y)}^{u_2(x, y)} f(x, y, z) \, dz \right] \, dA$$

$$\text{For a shadow on the } y, z \text{ plane: } \iiint_B f(x, y, z) \, dV = \iint_D \left[\int_{u_1(y, z)}^{u_2(y, z)} f(x, y, z) \, dx \right] \, dA$$

$$\text{For a shadow on the } x, z \text{ plane: } \iiint_B f(x, y, z) \, dV = \iint_D \left[\int_{u_1(x, z)}^{u_2(x, z)} f(x, y, z) \, dy \right] \, dA$$

15.7 Triple Integrals in Cylindrical Coordinates

15.8 Triple Integrals in Spherical Coordinates

15.9 Change of Variables in Multiple Integrals