MATH 1C Test 2 Notecard

Tejas Patel

10/3/2024

15.1 Double Integrals over Rectangles

The single variable definite integral: $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$

Volume as a Double Riemann Sum: $V \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$

Volume as a Double Integral: $\int \int_R f(x,y)dA = \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$

Fubini's Theorem

If f is continuous on the rectangle $R = \{(x,y) \mid a \le x \le b, c \le y \le d\}$

then it is known $\iint\limits_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\; dy\; dx = \int_c^d \int_a^b f(x,y)\; dx\; dy$

Special Case

In the special case that $f(x,y)$ can be factored as a product of a function of x only and a function of y only then it is known the following is true:

$$\iint\limits_R g(x)\, h(y)\, dA = \int_a^b g(x)\; dx \int_c^d h(y)\; dy$$

Average Value

The average value of a function f of two variables defined on a rectangle R is:

$$f_{avg} = \frac{1}{A(R)} \iint\limits_R f(x,y)dA$$

where A(R) is the area of R.

15.2 Double Integrals over General Regions

15.3 Double Integrals in Polar Coordinates

15.4 Applications of Double Integrals

15.5 Surface Area

15.6 Triple Integrals

15.7 Triple Integrals in Cylndrical Coordinates

15.8 Triple Integrals in Spherical Coordinates

15.9 Change of Variables in Multiple Integrals