MATH 1C Test 2 Notecard

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15.1 Double Integrals over Rectangles

The single variable definite integral: $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$

Volume as a Double Riemann Sum: $V \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$

Volume as a Double Integral: $\int \int_R f(x,y)dA = \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$

Fubini's Theorem

If f is continuous on the rectangle $R = \{(x,y) \mid a \le x \le b, c \le y \le d\}$

then it is known $\iint\limits_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\ dy\ dx = \int_c^d \int_a^b f(x,y)\ dx\ dy$

Special Case

In the special case that $f(x,y)$ can be factored as a product of a function of x only and a function of y only then it is known the following is true:

$$\iint\limits_R g(x)\,h(y)\,dA = \int_a^b g(x)\ dx \int_c^d h(y)\ dy$$

Average Value

The average value of a function f of two variables defined on a rectangle R is:

$$f_{avg} = \frac{1}{A(R)} \iint\limits_R f(x,y)dA$$

where A(R) is the area of R.

15.2 Double Integrals over General Regions

$$F(x,y) = \begin{cases} f(x,y) & \text{if } (x,y) \text{ s in } D \\ 0 & \text{if } (x,y) \text{ is in } R \text{ but not in } D \end{cases}$$

$$\iint\limits_D f(x,y)\, dA = \iint\limits_D F(x,y)\, dA$$

If f is continuous on a Type I region D such that the fixed bounds are vertical,
$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$, then

$$\iint\limits_D f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

If f is continuous on a Type II region D such that the fixed bounds are horizontal,
$D = \{(x,y) \mid c \le x \le d,\ h_1(x) \le y \le h_2(x)\}$, then

$$\iint\limits_D f(x,y)\, dA = \int_a^b \int_{h_1(x)}^{h_2(x)} f(x,y)\, dx\, dy$$

15.3 Double Integrals in Polar Coordinates

$$\iint\limits_R f(x,y)\, dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Put all half angle and double angle identities on notecard If f is continuous on a polar region of the form

$$D \ = \ \{(r,\theta) | \alpha \le \theta \le \beta,\ h_1(\theta) \le r \le h_2(\theta)\}$$

15.4 Applications of Double Integrals

Electric Charge

$$Q = \iint\limits_D \sigma(x,y)\, dA$$

Center of Mass of a Lamina

[0.1in] $\bar{x} = \dfrac{M_y}{m} = \dfrac{1}{m} \iint\limits_D x\rho(x,y)\, dA$ and $\bar{y} = \dfrac{M_x}{m} = \dfrac{1}{m} \iint\limits_D y\rho(x,y)\, dA$ where

$$m = \iint\limits_D \rho(x,y)\, dA$$

15.5 Surface Area

The surface area with equation $z = f(x, y), (x, y)\epsilon D$ where f_x and f_y are continuous is

$$A(S) = \iint\limits_D \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\; dA$$

$$A(S) = \iint\limits_D \sqrt{\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial y}{\partial z}\right]^2 + 1}\; dA$$

For the noteacrd:

$$A(S) = \iint\limits_D \sqrt{[f_x]^2 + [f_y]^2 + 1}\; dA$$

15.6 Triple Integrals

15.7 Triple Integrals in Cylndrical Coordinates

15.8 Triple Integrals in Spherical Coordinates

15.9 Change of Variables in Multiple Integrals