MATH 5 Lecture Notes

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1 Chapter 1

1.1 Systems of Equations

$$\left[\begin{array}{cc|c} a & b & c \\ c & d & e \end{array}\right]$$

Linear Equation Example:
$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$
a_{ij} = are coefficients
x_i = variables = unknowns
b_i = constants
The set $S_1, ...S_n$ is a solution if $a_iS_1 + ... + a_{in}S_n = b_i$ is True = consistent for all i
3 possible outcomes
One Solution
No Solutions
Infinitely many Solutions
Row Operations
Interchange Rows
Row Multiplication
Row Addition: Add a constant multiple of one row to another row

$$\left[\begin{array}{cc|c} 1 & 0 & S_1 \\ 0 & 1 & S_2 \end{array}\right]$$

1.2 Row Reductions and Echelon Form
1.3 Vector Equations
1.4 The Matrix Equations Ax=b
1.5 Solution Sets for Linear Systems
1.6 Linear Independence
1.7 Linear Transformations
1.8 The Matrix of a Linear Transformation

2 Chapter 2

2.1 Matrix Operations
2.2 The Inverse of a Matrix
2.3 Characterizations of Invertible Matrices

3 Chapter 3

3.1 Introduction to Determinants
3.2 Properties of Determinants
3.3 Cramer's Rule and Linear Transformations

4 Chapter 4

4.1 Vector Spaces and Subspaces
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5 Chapter 5

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5.3 Diagonalization
5.4 Eigenvectors and Linear Transformations
5.5 Complex Eigenvalues
5.6 Discrete Dynamical Systems
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6 Chapter 6

6.1 Inner Product Spaces
6.2 Orthogonal Sets

7 Example Problems with Solutions