MATH 2 Lecture Notes

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1 Chapter 1

1.1 Terminology

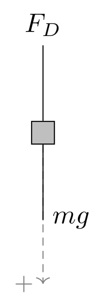
Definition A differential equation is an equation containing the derivatives or differentials of one or more dependent variables, with respect to one or more independent variables.
· An Ordinary Differential Equation (ODE) involves only ordinary derivatives
· A Partial Differential Equation (PDE) involves partial derivatives.
Definition The order of a DE is the order of the highest-order derivative that appears in the DE
Notation $F(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2})$
Definition A linear DE is any DE that can be written in form:
$a_0(x)y + a_1(x)y' + a_2(x)y'' \cdots + a_n(x)y^{(n)} = b(x)$
For a DE to be linear:

1. Y and all of its derivatives much be of the 1st degree
2. Any term that does not include y or any of its derivatives must be a function of x

1.2 Some Mathematical Models

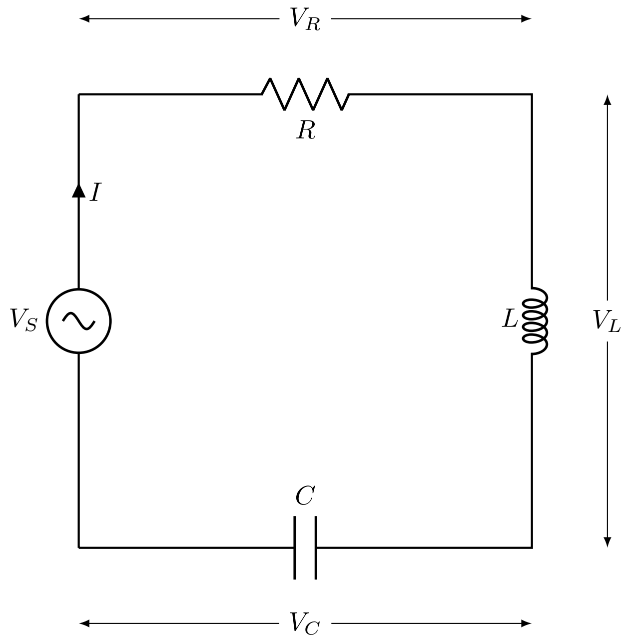
I. Free-falling body

Goal: Find s(t).



Set up a differential equation in S, model it, then solve
$ma = mg$
$\frac{d^2s}{dt^2} = g$
$v = \frac{ds}{dt}, g = \frac{dv}{dt}$
What if there is air resistance. Assume force scales linear with velocity
$\frac{dv}{dt} = g - \frac{kv}{m} \rightarrow \frac{dv}{dt} = g - \frac{k}{m} \cdot \frac{ds}{dt}$

II: Series Circuit



Voltage drops:

$V = L\dfrac{dI}{dt}, V = L\dfrac{d^2q}{dt^2}$

$V = IR, V = R\dfrac{dq}{dt}$

$V = \dfrac{q}{C}$

$E(t) = L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C}$

III: Population Growth

$P = P(t) =$ population at time t — use exponential model

$\dfrac{dp}{dt} \propto P \rightarrow \dfrac{dp}{dt} = kP \rightarrow= Ce^{kt}$ where C is the initial population

IV: Population Growth with Finite Capacity

"Carrying Capacity" = N — uses the logistic growth model

$\dfrac{dp}{dt} \propto$ both P and amount to carrying capacity (N-P)

$\dfrac{dp}{dt} = kP(N - P)$

V: Chemical Reaction

$A + B \rightarrow C$ Concentrations of A and B decreases by amount of C formed

Can we write DE governing the concentration of C x(t)?

The rate at which the reaaction takes place $\propto$ Product of the remaining concentrations of A and B

α initial concentration of A

β initial concentration of B

$\dfrac{dx}{dt} = k(\alpha - x)(\beta - x)$

2 Example Problems with Solutions

2.1

$\frac{dy}{dx} = y^2 + 2xy$