

# MATH 2 Lecture Notes

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# 1 Chapter 1

## 1.1 Terminology

**Definition** A differential equation is an equation containing the derivatives or differentials of one or more dependent variables, with respect to one or more independent variables.

· An Ordinary Differential Equation (ODE) involves only ordinary derivatives

· A Partial Differential Equation (PDE) involves partial derivatives.

**Definition** The order of a DE is the order of the highest-order derivative that appears in the DE

**Notation**  $F(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2})$

**Definition** A linear DE is any DE that can be written in form:

$$a_0(x)y + a_1(x)y' + a_2(x)y'' \cdots + a_n(x)y^{(n)} = b(x)$$

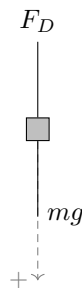
For a DE to be linear:

1. Y and all of its derivatives must be of the 1st degree
2. Any term that does not include y or any of its derivatives must be a function of x

## 1.2 Some Mathematical Models

### I. Free-falling body

Goal: Find  $s(t)$ .



Set up a differential equation in S, model it, then solve

$$ma = mg$$

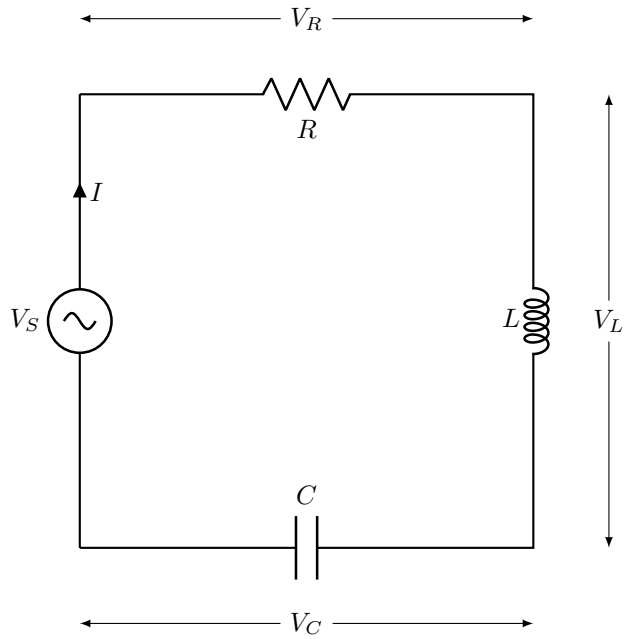
$$\frac{d^2s}{dt^2} = g$$

$$v = \frac{ds}{dt}, g = \frac{dv}{dt}$$

What if there is air resistance. Assume force scales linear with velocity

$$\frac{dv}{dt} = g - \frac{kv}{m} \rightarrow \frac{dv}{dt} = g - \frac{k}{m} \cdot \frac{ds}{dt}$$

## II: Series Circuit



Voltage drops:

$$V = L \frac{dI}{dt}, V = L \frac{d^2 q}{dt^2}$$

$$V = IR, V = R \frac{dq}{dt}$$

$$V = \frac{q}{C}$$

$$E(t) = L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C}$$

## III: Population Growth

$P = P(t)$  = population at time  $t$  — use exponential model

$$\frac{dp}{dt} \propto P \rightarrow \frac{dp}{dt} = kP \rightarrow C e^{kt} \text{ where } C \text{ is the initial population}$$

## IV: Population Growth with Finite Capacity

”Carrying Capacity”

## 2 Example Problems with Solutions

### 2.1

$$\frac{dy}{dx} = y^2 + 2xy$$