RH 1.8

MATH 5, Jones

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Refrigerator Homework

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Since T is linear $T(a\,\mathbf{x}) \;=\; a\,T(\mathbf{x})$ and $T(\mathbf{x}+\mathbf{y}) \;=\; T(\mathbf{x})+T(\mathbf{y})$.
Given $T(\mathbf{u}) \;=\; \begin{bmatrix}2\\1\end{bmatrix}$ and $T(\mathbf{v}) \;=\; \begin{bmatrix}-1\\3\end{bmatrix}$.

$$\boxed{3u = 3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}6\\3\end{bmatrix} \qquad 2v = 2\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix} \qquad 3u+2v = \begin{bmatrix}6-2\\3+6\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}}$$

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$\mathbf{e}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix}0\\1\end{bmatrix}.$
The first column of A is $T(\mathbf{e}_1)$, and the second column is $T(\mathbf{e}_2)$.
$T\big(x_1, x_2\big) = x_1\,v_1 + x_2\,v_2$
Given that$T(x_1, x_2) \;=\; x_1\,v_1 \;+\; x_2\,v_2$, then for the standard basis vectors $\mathbf{e}_1 = (1,0)$ and $\mathbf{e}_2 = (0,1)$ we have$T(\mathbf{e}1) \;=\; v_1, \quad T(\mathbf{e}2) \;=\; v_2$.
$A \;=\; \begin{bmatrix}v_1 & v_2\end{bmatrix} \;=\; \boxed{\begin{bmatrix}-2 & 7\\5 & -3\end{bmatrix}}$

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True. A linear transformation is indeed a function (one that takes vectors as inputs and returns vectors as outputs) but it satisfies two special properties:
1. Additivity: $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$.
2. Homogeneity: $T(c\,\mathbf{u}) = c\,T(\mathbf{u})$ for any scalar c
Any function that meets both of these properties is called a linear transformation, so it's a specialized type of function.

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False. The set of all linear combinations of the columns of A is the range (or image) of the transformation function, a subset of its codomain. When we write $T\colon \mathbb{R}^n \to \mathbb{R}^m$ via $T(\mathbf{x}) = A\mathbf{x}$, the usual codomain is $\mathbb{R}^m$. The range—the set of all possible outputs that T actually attains—coincides with the column space of A. So the column space is a subspace of the codomain, not the entire codomain itself

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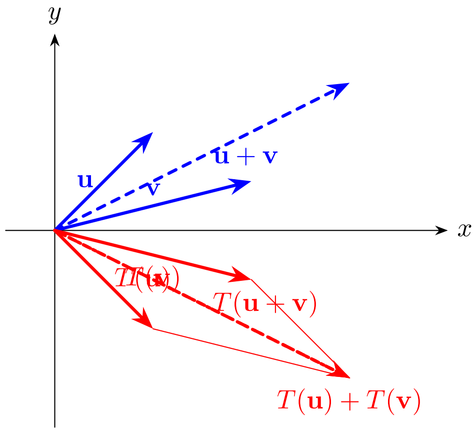

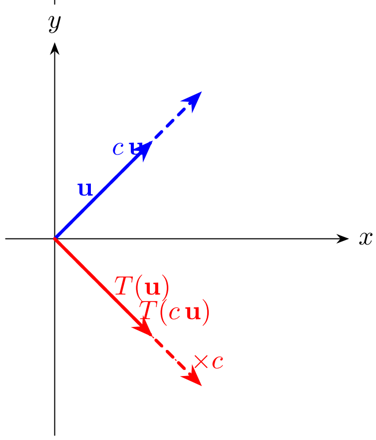

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1. Every $\mathbf{x} \in \mathbb{R}^n$ can be written as a linear combination of the spanning vectors $v_1, \ldots, v_p$. In symbols,
   $\mathbf{x} \;=\; c_1\, v_1 \;+\; c_2\, v_2 \;+\; \cdots \;+\; c_p\, v_p.$
   2. Apply T and use linearity:
   $T(\mathbf{x}) \;=\; T\big(c_1\, v_1 + c_2\, v_2 + \cdots + c_p\, v_p\big) \;=\; c_1\, T(v_1) \;+\; c_2\, T(v_2) \;+\; \cdots \;+\; c_p\, T(v_p).$
   3. But each $T(v_i) = \mathbf{0}$, by assumption. Hence every term on the right is zero, so
   $T(\mathbf{x}) = \mathbf{0}.$
   Because $\mathbf{x}$ is an arbitrary vector that can span $\mathbb{R}^n$, it follows that $T(\cdot)$ sends every vector to $\mathbf{0}$.

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Because v_1, v_2, v_3 is dependent, there exist scalars c_1, c_2, c_3, not all zero, such that
$c_1\, v_1 \;+\; c_2\, v_2 \;+\; c_3\, v_3 \;=\; \mathbf{0}.$
Apply the linear map T to both sides:
$T\big(c_1 v_1 + c_2 v_2 + c_3 v_3\big) \;=\; c_1\, T(v_1) \;+\; c_2\, T(v_2) \;+\; c_3\, T(v_3) \;=\; T(\mathbf{0}) \;=\; \mathbf{0}.$
This shows that
$c_1\, T(v_1) \;+\; c_2\, T(v_2) \;+\; c_3\, T(v_3) \;=\; \mathbf{0},$
with $(c_1, c_2, c_3) \neq (0, 0, 0)$, meaning $T(v_1), T(v_2), T(v_3)$ is also linearly dependent.

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