RH 1.9

MATH 5, Jones

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Refrigerator Homework

10

$T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then rotates points $\dfrac{3\pi}{2}$ radians.
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to$ Reflection through the x_2 axis from textbook table $\to \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\dfrac{3\pi}{2}$ rad rotation (90 degree clockwise)$\to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Multiplying them together: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}$

17

Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.
$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$
T = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Since all values inside the matrix are real and make the transformation linear and nothing other than linear, the transformation can be considered linear

23

True. The transformation matrix is calculated using the identity matrix, meaning
Identitiy Matrix $\to$ Transformation Matrix

25

Yes. Rotations are linear transformations as they scale by 1 and dont have any nonlinear effect on the original point

33

$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ only has 3 pivots so it only maps into $\mathbb{R}^3$, not $\mathbb{R}^4$ so it is neither onto and also means its not one-to-one since there's a row of zeores

35

$T\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ makes the transformation matrix $A = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}$. It is onto since there are 2 pivots, one for each row, but is not one-to-one since its not true that the only element of each row is the pivot

43

For $\mathbb{R}^n \to \mathbb{R}^m$ A linear transformation T is onto if its image spans the entire codomain $\mathbb{R}^m$, which means the rank of the transformation matrix A must be equal to m (i.e., the number of linearly independent rows must be m). That wasy $n \geq m$
A linear transformation T is one-to-one if the null space of A is trivial, meaning the only solution to Ax $= 0$ is the zero vector. This occurs when A has full column rank, meaning: $n \leq m$
For T to be both onto ($n \geq m$) and one-to-one ($n \leq m$), we require: $n = m$

Computer Homework: Next 10 Pages