

RH 1.2

MATH 5, Jones

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1 Refrigerator Homework

3: Convert matrix to RREF

$$\left[\begin{array}{ccc|c} \underline{1} & 2 & 3 & 4 \\ 4 & \underline{5} & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array} \right] \text{ Subtract } 4R_1 \text{ from } R_2 \text{ and } 6R_1 \text{ from } R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array} \right]$$

$$\text{Scale } R_2 \text{ by } -\frac{1}{3} \text{ and } R_3 \text{ by } -\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array} \right]$$

$$\text{Subtract } R_2 \text{ from } R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{Subtract } 2R_2 \text{ from } R_1 \rightarrow \boxed{\left[\begin{array}{ccc|c} \underline{1} & 0 & -1 & -2 \\ 0 & \underline{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]}$$

is the resultant matrix in RREF, the pivot points are underlined in the solution and original matrices

7: Find the general solution to the system

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array} \right] \rightarrow \text{Subtract } 3R_1 \text{ from } R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array} \right]$$

$$\text{Scale } R_2 \text{ by } -\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\text{Subtract } 4R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Free variable: $X_2 = t$

$$X_1 = -5 - 3t$$

$$X_2 = t$$

$$X_3 = 3$$

$$\boxed{(-5 - 3t, t, 3)}$$

14: Find the general solution to the system

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -4 & 0 & -5 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \text{Subtract } 2R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 4 & 0 & -9 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Free Variables $X_3 = s$, $X_4 = t$

Solution: $X_1 = -9 - 7s - 4t$, $X_2 = 2 + 6s + 4t$, $X_3 = s$, $X_4 = t$, $X_5 = 0$

$$\boxed{-9 - 7s - 4t, 2 + 6s + 4t, s, t, 0}$$

21: Find h where the system is consistent

$$\left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array} \right] \rightarrow \text{Scale } R_1 \text{ by } 2 \rightarrow \left[\begin{array}{cc|c} 4 & 6 & 2h \\ 4 & 6 & 7 \end{array} \right] \rightarrow 2h = 7 \rightarrow \boxed{h = 3.5}$$

24: Choose h & k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array} \right]$$

a

At $h = 9$ and $k \neq 6$, you'd need to scale R_1 by 3 and show $6 \neq 6$ meaning there would be no solution to the system

b

At $h = 4$ and $k = 3$ there is one solution, as long as $h \neq 9$ and $k \neq 6$ there will be one solution. In this case the solution is $X_1 = \frac{1}{5}$ $X_2 = \frac{3}{5}$

c

At $h = 9$ and $k = 6$ the two equations will be scalar multiples of each other, meaning multiple solutions exist.

35: Suppose a 3×5 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

3 Rows \times 5 Columns means there will be 5 variables and only 3 equations to relate the variables to each other. Since there are 5 variables but 3 equations and 3 pivot points, we know there are 2 free variables, which means the system could have an infinite solution set, meaning it is consistent

37: Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

If there is a pivot in every row, that means every row is defined to equal the constant it shares a row with. Since every variable is mapped to a constant, there is a unique solution and the system is consistent

Computer Homework

1: Convert matrix to RREF

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 6 & 7 & 8 & 9 \end{array} \right] \rightarrow \text{Subtract } 6R_1 \text{ from } R_3 \text{ and } 3R_1 \text{ from } R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -4 & -6 \\ 0 & -5 & -10 & -15 \end{array} \right]$$

$$\text{Subtract } 2.5R_2 \text{ from } R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -4 & -6 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{Scale } R_2 \text{ by } -\frac{1}{2} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{Subtract } R_2 \text{ from } R_1 \rightarrow \boxed{\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]}$$
 The matrix is now in RREF with pivot columns 1 and 2

2: Select the example matrices that can be echelon form

$$\text{The answer was A, D, and E } \begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix} \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$$

3: Find the general solution to the matrix

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 2 & 8 & 3 & 7 \end{array} \right] \rightarrow R_2 - 2R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & -5 & -25 \end{array} \right] \rightarrow R_2 / -5 \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & 1 & 5 \end{array} \right] \rightarrow R_1 - 4R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 0 & -4 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$X_2 = t \text{ is a free variable, meaning the solution set is: } \boxed{(-4 - 4t, t, 5)}$$

4: Find the general solution to the matrix

$$\left[\begin{array}{ccc|c} 0 & 1 & -5 & 6 \\ 1 & -3 & 13 & -12 \end{array} \right] \rightarrow R_2 - 3R_1 \rightarrow \left[\begin{array}{ccc|c} 0 & 1 & -5 & 6 \\ 1 & 0 & -2 & 6 \end{array} \right] \rightarrow \text{Swap rows} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 6 \\ 0 & 1 & -5 & 6 \end{array} \right]$$

$$\text{Making } X_3 = t \text{ a free variable and the solution set } \boxed{(6 + 2t, 6 + 5t, t)}$$

5: Find the general solution to the matrix

$$\left[\begin{array}{ccc|c} 5 & -3 & 7 & 0 \\ 10 & -6 & 14 & 0 \\ 15 & -9 & 21 & 0 \end{array} \right] \rightarrow \text{Divide } R_2 \text{ by 2 and } R_3 \text{ by 3} \rightarrow \left[\begin{array}{ccc|c} 5 & -3 & 7 & 0 \\ 5 & -3 & 7 & 0 \\ 5 & -3 & 7 & 0 \end{array} \right] \text{ making } X_2 = s \text{ and } X_3 = t \text{ free}$$

$$\text{variables and the solution to the system } \boxed{\left(\frac{3}{5}s - \frac{7}{5}t, s, t \right)}$$

6: Find the general solution to the matrix

$$\begin{bmatrix} 1 & -4 & 0 & -1 & 0 & -9 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow R_1 += 4R_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & -20 & 3 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 += R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -14 & 10 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Variables $X_3 = s$, $X_5 = t$ is are free variables, and the solution to the system is $\boxed{10 + 14t, 3 + 5t, s, 7 - 6t, t}$

7: Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions.

System:
$$\begin{cases} x_1 + hx_2 = 5 \\ 5x_1 + 15x_2 = k \end{cases}$$

a: No solutions when $h = 3$ and $k \neq 25$

b: Unique solution when $h \neq 3$

c: Many solutions when $h = 3$ and $k = 25$

8: Suppose a 3×6 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

The augmented matrix will have seven columns and will not have a row of the form $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]$, so the system is consistent. This is because every row has a pivot, so there will be no blank coefficient rows

9: Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is not a pivot column. Is the system consistent? Why or why not?

A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column. That is, if and only if an echelon form of the augmented matrix has no row of the form $[0 \ \dots \ 0 \ b]$ with b nonzero.

In the augmented matrix described above, is the rightmost column a pivot column? No

In the echelon form of the augmented matrix, is there a row of the form $[0 \ 0 \ 0 \ 0 \ b]$ with b nonzero? No

Therefore, by the Existence and Uniqueness Theorem, the linear system is consistent.

10: Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

The system is consistent because the rightmost column of the augmented matrix is not a pivot column.