RH 2.3

MATH 5, Jones

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Refrigerator Homework

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Counterexample: $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ where $ad - bc = 0$

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Suppose A is a 5x5 matrix whose columns do not span $\mathbb{R}^5$. This means that the columns of A are linearly dependent, so at least one of column can be written as a linear combination of the others. If the columns are linearly dependent, then the equation $Ax = 0$ has a nontrivial solution, meaning there exists a nonzero vector x such that $Ax = 0$. However, if A were invertible, the only solution to $Ax = 0$ would be $x = 0$, which contradicts the existence of a nontrivial solution. Thus, A cannot be invertible if its columns do not span $\mathbb{R}^5$.

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Since the columns of A are linearly independent, one of the statements of the Invertible Matrix Theorem tells us that A is invertible (i.e., A has an inverse). This means that A creates a one-to-one and onto transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. Now, consider A^2, which is just A multiplied by itself. Because A is invertible, the product A^2 is also invertible. Applying the Invertible Matrix Theorem again, since A^2 is invertible, its columns must also be linearly independent and, importantly, they span $\mathbb{R}^n$.

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$$T(x_1, x_2) = \begin{bmatrix} 6x_1 - 8x_2 \\ -5x_1 + 7x_2 \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{7}{2} & 4 \\ \frac{5}{2} & 3 \end{bmatrix}$$

$$T^{-1} = \left(\frac{7}{2}x_1 + 4x_2, \frac{5}{2}x_1 + 3x_2 \right)$$

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