RH 1.9

MATH 5, Jones

Tejas Patel

Refrigerator Homework

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$T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then rotates points $\dfrac{3\pi}{2}$ radians.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to$ Reflection through the x_2 axis from textbook table $\to \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\dfrac{3\pi}{2}$ rad rotation (90 degree clockwise)$\to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Multiplying them together: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}$

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Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$

T = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Since all values inside the matrix are real and make the transformation linear and nothing other than linear, the transformation can be considered linear

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True. The transformation matrix is calculated using the identity matrix, meaning

Identitiy Matrix $\to$ Transformation Matrix

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Yes. Rotations are linear transformations as they scale by 1 and dont have any nonlinear effect on the original point

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$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ only has 3 pivots so it only maps into $\mathbb{R}^3$, not $\mathbb{R}^4$ so it is neither onto and also means its not one-to-one since there's a row of zeores

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$T\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ makes the transformation matrix $A = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}$. It is onto since there are 2 pivots, one for each row, but is not one-to-one since its not true that the only element of each row is the pivot

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For $\mathbb{R}^n \to \mathbb{R}^m$ A linear transformation T is onto if its image spans the entire codomain $\mathbb{R}^m$, which means the rank of the transformation matrix A must be equal to m (i.e., the number of linearly independent rows must be m). That wasy $n \geq m$
A linear transformation T is one-to-one if the null space of A is trivial, meaning the only solution to Ax $=0$ is the zero vector. This occurs when A has full column rank, meaning: $n \leq m$
For T to be both onto ($n \geq m$) and one-to-one ($n \leq m$), we require: $n = m$

Computer Homework: Next 10 Pages

Question list

Assume that T is a linear transformation. Find the standard matrix of T.

T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$, rotates points (about the origin) through $\frac{\pi}{6}$ radians.

$$A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element. Type exact answers, using radicals as needed.)

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Homework: **Section 1.9 Homework**

Question 2, 1.9.5

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Assume that T is a linear transformation. Find the standard matrix of T.

T: $\mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps $\mathbf{e}_1$ into $\mathbf{e}_1 - 18\mathbf{e}_2$ but leaves the vector $\mathbf{e}_2$ unchanged.

$$A = \begin{bmatrix} 1 & 0 \\ -18 & 1 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

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Homework: **Section 1.9 Homework** Question 3, 1.9.7

HW Score: 100%, 10 of 10 points
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Assume that T is a linear transformation. Find the standard matrix of T.

T: $\mathbb{R}^2 \to \mathbb{R}^2$ first rotates points through $-\frac{\pi}{3}$ radians and then reflects points through the horizontal x_1-axis.

$$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element. Type exact answers, using radicals as needed.)

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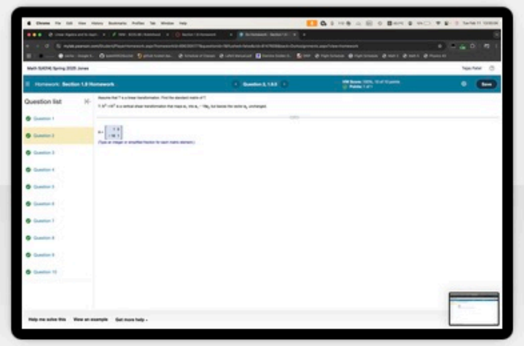

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Homework: **Section 1.9 Homework**

Question 4, 1.9.13

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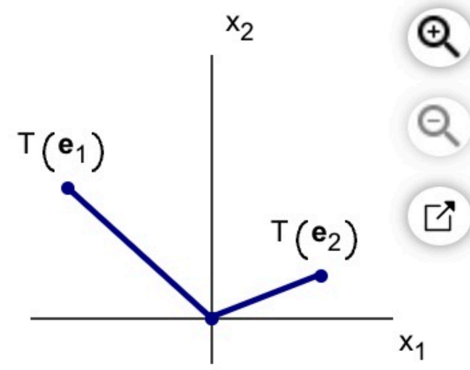
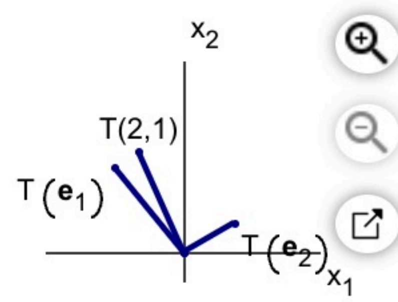
Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $T(\mathbf{e}_1)$ and $T(\mathbf{e}_2)$ are the vectors shown in the figure. Using the figure, sketch the vector T(2,1).



Choose the correct graph below.

A.



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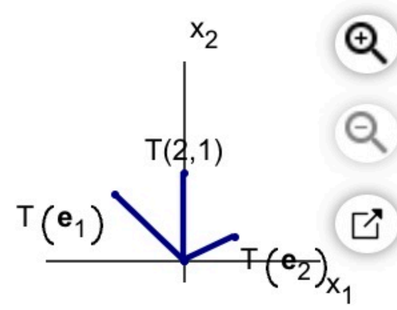
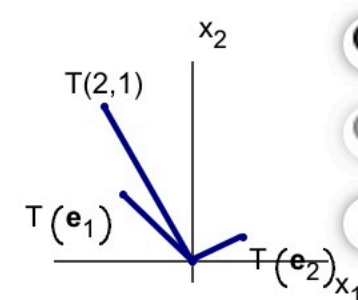
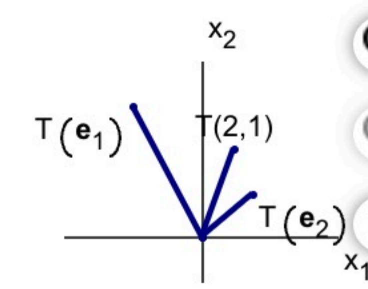


C.



D.



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Show that T is a linear transformation by finding a matrix that implements the mapping. Note that x_1, x_2, ... are not vectors but are entries in vectors.

$$T(x_1,x_2,x_3,x_4) = (x_1 + 9x_2,\ 0,\ 2x_2 + x_4,\ x_2 - x_4)$$

$$A = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

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Homework: Section 1.9 Homework

Question 6, 1.9.33
Part 2 of 2

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Determine if the specified linear transformation is (a) one-to-one and (b) onto. Justify your answer.

$$T(x_1,x_2,x_3,x_4) = (x_2 + x_3, x_1 + x_2, x_2 + x_3, 0)$$

a. Is the linear transformation one-to-one?

- A. T is not one-to-one because the columns of the standard matrix A are linearly independent.
- B. T is not one-to-one because the standard matrix A has a free variable.
- C. T is one-to-one because $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.
- D. T is one-to-one because the column vectors are not scalar multiples of each other.

b. Is the linear transformation onto?

- A. T is not onto because the columns of the standard matrix A span $\mathbb{R}^4$.
- B. T is not onto because the fourth row of the standard matrix A is all zeros.
- C. T is onto because the columns of the standard matrix A span $\mathbb{R}^4$.
- D. T is onto because the standard matrix A does not have a pivot position for every row.

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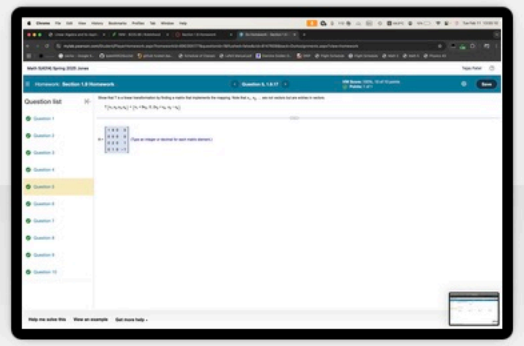

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Determine if the specified linear transformation is (**a**) one-to-one and (**b**) onto. Justify each answer.

$$T(x_1,x_2,x_3) = (x_1 - 5x_2 + 3x_3,\ x_2 - 6x_3)$$

(**a**) Is the linear transformation one-to-one?

- A. T is one-to-one because $T(x) = 0$ has only the trivial solution.
- B. T is not one-to-one because the columns of the standard matrix A are linearly independent.
- C. T is not one-to-one because the columns of the standard matrix A are linearly dependent.
- D. T is one-to-one because the column vectors are not scalar multiples of each other.

(**b**) Is the linear transformation onto?

- A. T is onto because the columns of the standard matrix A span $\mathbb{R}^2$.
- B. T is onto because the standard matrix A does not have a pivot position for every row.
- C. T is not onto because the standard matrix A does not have a pivot position for every row.
- D. T is not onto because the columns of the standard matrix A span $\mathbb{R}^2$.

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Homework: **Section 1.9 Homework**

Question 8, 1.9.37

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Describe the possible echelon forms of the standard matrix for a linear transformation T where T: $\mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one.

Give some examples of the echelon forms. The leading entries, denoted ■, may have any nonzero value. The starred entries, denoted *, may have any value (including zero). Select all that apply.

A. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

B. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

C. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & * \\ 0 & 0 & \blacksquare \end{bmatrix}$

D. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

F. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

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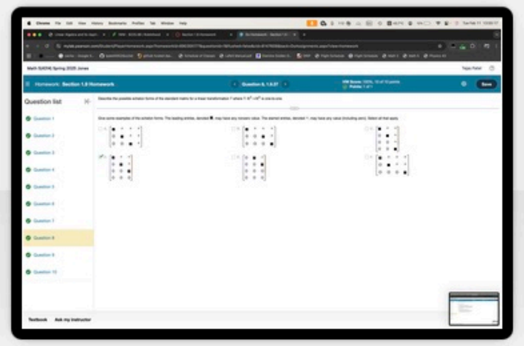
Describe the possible echelon forms of the standard matrix for a linear transformation T where T: $\mathbb{R}^4 \to \mathbb{R}^3$ is onto.

Give some examples of the echelon forms. The leading entries, denoted ■, may have any nonzero value; the starred entries, denoted *, may have any value (including zero). Select all that apply.

- [ ] A. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- [x] B. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$
- [x] C. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$
- [x] D. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$
- [ ] E. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- [ ] F. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- [x] G. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$
- [ ] H. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & \blacksquare & * & * \\ 0 & \blacksquare & * & * \end{bmatrix}$
- [ ] I. $\begin{bmatrix} 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \\ 0 & 0 & 0 & 0 \end{bmatrix}$



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Let T be the linear transformation whose standard matrix is given. Decide if T is a one-to-one mapping. Justify your answer.

$$\begin{bmatrix} -5 & 7 & -6 & -15 \\ 7 & 4 & -8 & 2 \\ 6 & 9 & 3 & 27 \\ -3 & 2 & 8 & 12 \end{bmatrix}$$

Choose the correct answer below.

- A. The transformation T is not one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.
- B. The transformation T is not one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has a nontrivial solution.
- C. The transformation T is one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.
- D. The transformation T is one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has a nontrivial solution.