RH 1.4

MATH 5, Jones

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Refrigerator Homework

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$\begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ becomes the system $\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix}$ and can be solved using row reduction

$R_2+ = 2R_3 \rightarrow \begin{bmatrix} 3 & -5 & 0 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_1- = 3R_3 \rightarrow \begin{bmatrix} 0 & -8 & -12 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_1+ = R_2 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_2/ = 8 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1.5 \\ 1 & 1 & 4 \end{bmatrix}$ From here, $X_2 = 1.5, X_1 + 1.5 = 4$, so $X_1 = 2.5$

Answer: $\boxed{\text{Yes, and the solution is } X_1 = 2.5, \quad X_2 = 1.5}$

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Part a: **Counterexample:** $b_1 = 0, \quad b_2 = 1$

Part b: $\begin{bmatrix} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{bmatrix} \rightarrow R_1 * = -3 \rightarrow \begin{bmatrix} -6 & 3 & -3b_1 \\ -6 & 3 & b_2 \end{bmatrix}$

this shows the system is consistent for $\boxed{\text{all possibilities where } b_2 = -3b_1}$

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$$\begin{bmatrix}1 & 3 & -2 & 2\\0 & 1 & 1 & -5\\1 & 2 & -3 & 7\\-2 & -8 & 2 & -1\end{bmatrix} \to R_1- = R_3 \;\&\; R_4+ = 2R_3 \to \begin{bmatrix}0 & 1 & 1 & -5\\0 & 1 & 1 & -5\\1 & 2 & -3 & 7\\0 & -4 & -4 & 13\end{bmatrix} \to R_2- = R_1 \to \begin{bmatrix}0 & 1 & 1 & -5\\0 & 0 & 0 & 0\\1 & 2 & -3 & 7\\0 & -4 & -4 & 13\end{bmatrix}$$

By Theorem 1.4, since there is no pivot in all 4 rows, it it not possible for matrix B to span $\mathbb{R}^4$

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Part a: No, also by Theorem 1.4, since B does not span $\mathbb{R}^4$, not all vectors in $\mathbb{R}^4$ can be written as a linear combination of the colum of B

Part b: $\begin{bmatrix}0 & 1 & 1 & -5\\0 & 0 & 0 & 0\\1 & 2 & -3 & 7\\0 & -4 & -4 & 13\end{bmatrix} \to R_4+ = 4R_1 \to \begin{bmatrix}0 & 1 & 1 & -5\\0 & 0 & 0 & 0\\1 & 2 & -3 & 7\\0 & 0 & 0 & -7\end{bmatrix} \to R_4/ = -7 \to \begin{bmatrix}0 & 1 & 1 & -5\\0 & 0 & 0 & 0\\1 & 2 & -3 & 7\\0 & 0 & 0 & 1\end{bmatrix}$

$R_3- = 2R_1 \to \begin{bmatrix}0 & 1 & 1 & -5\\0 & 0 & 0 & 0\\1 & 0 & -5 & 3\\0 & 0 & 0 & 1\end{bmatrix} \to R_3- = 3R_4 \;\&\; R_1+ = 5R_4 \to \begin{bmatrix}0 & 1 & 1 & 0\\0 & 0 & 0 & 0\\1 & 0 & -5 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

Rearrange the rows: $\begin{bmatrix}1 & 0 & -5 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}$

No, it does not span all of $\mathbb{R}^3$ and the counterexample is $\{5, -1, 1, 0\}$

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False. It's a Matrix Equation. That's the title of this section.

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True. Distributing the $\mathbf{x}$ out and tacking on the b into the end of the new matrix, you will end up with a system that is in the form of an augmented matrix

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True. By Theorem 1.4, if it is inconsistent for any b then it is not true that there is a pivot in every row.

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Computer Homework