RH 1.4

MATH 5, Jones

Tejas Patel

Refrigerator Homework

13

$\begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ becomes the system $\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix}$ and can be solved using row reduction

$R_2+ = 2R_3 \rightarrow \begin{bmatrix} 3 & -5 & 0 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_1- = 3R_3 \rightarrow \begin{bmatrix} 0 & -8 & -12 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_1+ = R_2 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & 12 \\ 1 & 1 & 4 \end{bmatrix}$

$R_2/ = 8 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1.5 \\ 1 & 1 & 4 \end{bmatrix}$ From here, $X_2 = 1.5, X_1 + 1.5 = 4$, so $X_1 = 2.5$

Answer: $\boxed{\text{Yes, and the solution is } X_1 = 2.5, \quad X_2 = 1.5}$

15

Part a: **Counterexample:** $b_1 = 0, \quad b_2 = 1$

Part b: $\begin{bmatrix} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{bmatrix} \rightarrow R_1* = -3 \rightarrow \begin{bmatrix} -6 & 3 & -3b_1 \\ -6 & 3 & b_2 \end{bmatrix}$

this shows the system is consistent for $\boxed{\text{all possibilities where } b_2 = -3b_1}$

18

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \to R_1 -= R_3 \ \& \ R_4 += 2R_3 \to \begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ 0 & -4 & -4 & 13 \end{bmatrix} \to R_2 -= R_1 \to \begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & -3 & 7 \\ 0 & -4 & -4 & 13 \end{bmatrix}$$

By Theorem 1.4, since there is no pivot in all 4 rows, it it not possible for matrix B to span $\mathbb{R}^4$

20

Part a: No, also by Theorem 1.4, since B does not span $\mathbb{R}^4$, not all vectors in $\mathbb{R}^4$ can be written as a linear combination of the colum of B

Part b: $\begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & -3 & 7 \\ 0 & -4 & -4 & 13 \end{bmatrix} \to R_4 += 4R_1 \to \begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & -3 & 7 \\ 0 & 0 & 0 & -7 \end{bmatrix} \to R_4 /= -7 \to \begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & -3 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_3 -= 2R_1 \to \begin{bmatrix} 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to R_3 -= 3R_4 \ \& \ R_1 += 5R_4 \to \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rearrange the rows: $\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

No, it does not span all of $\mathbb{R}^3$ and the counterexample is $\{5, -1, 1, 0\}$

23

False. It's a Matrix Equation. That's the title of this section.

32

True. Distributing the $\mathbf{x}$ out and tacking on the b into the end of the new matrix, you will end up with a system that is in the form of an augmented matrix

33

True. By Theorem 1.4, if it is inconsistent for any b then it is not true that there is a pivot in every row.

43

For a 4 row $\times$ 3 column matrix to have a unique solution for all b, there will be 4 equations and only 3 variables for them to link, meaning if all the variables are solved then there will be one row of zeroes, and there will have been more equations than necessary. This is because only one variable can take up a row when a unique solution is solved and there are rows that won't contain a solved variable. Since all 3 variables are solved for, this means all of the variables can be scaled to different amounts and the solutions to the matrix span all of $\mathbb{R}^3$

Computer Homework