RH 1.2

MATH 5, Jones

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3: Convert matrix to RREF

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right]$ Subtract $4R_1$ from R_2 and $6R_1$ from $R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array}\right]$

Scale R_2 by $-\frac{1}{3}$ and R_3 by $-\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]$

Subtract R_2 from $R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Subtract $2R_2$ from $R_1 \rightarrow \boxed{\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]}$ is the resultant matrix in RREF

7: Find the general solution to the system

$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right] \rightarrow$ Subtract $3R_1$ from $R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right]$

Scale R_2 by $-\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Subtract $4R_2$ from $R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Free variable: $X_2 = t$
$X_1 = -5 - 3t$
$X_2 = t$
$X_3 = 3$
$\boxed{(-5 - 3t, t, 3)}$

14: Find the general solution to the system

$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -4 & 0 & -5 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$ Subtract $2R_2$ from $R_1 \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 4 & 0 & -9 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Free Variables $X_3 = s,\ X_4 = t$
Solution: $X_1 = -9 - 7s - 4t,\ X_2 = 2 + 6s + 4t,\ X_3 = s,\ X_4 = t,\ X_5 = 0$
$\boxed{-9 - 7s - 4t,\ 2 + 6s + 4t,\ s,\ t,\ 0}$

21: Find h where the system is consistent

$\left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array}\right] \rightarrow$ Scale R_1 by $2 \rightarrow \left[\begin{array}{cc|c} 4 & 6 & 2h \\ 4 & 6 & 7 \end{array}\right] \rightarrow 2h = 7 \rightarrow \boxed{h = 3.5}$

24: Choose h & k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right]$$

a

At $h = 9$ and $k \neq 6$, you'd need to scale R_1 by 3 and show $6 \neq 6$ meaning there would be no solution to the system

b

At $h = 4$ and $k = 3$ there is one solution, as long as $h \neq 9$ and $k \neq 6$ there will be one solution. In this case the solution is $X_1 = \frac{1}{5} \qquad X_2 = \frac{3}{5}$

c

At $h = 9$ and $k = 6$ the two equations will be scalar multiples of each other, meaning multiple solutions exist.

35: Suppose a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Why or why not?

3 Rows $\times$ 5 Columns means there will be 5 variables and only 3 equations to relate the variables to each other. Since there are 5 variables but 3 equations and 3 pivot points, we know there are 2 free variables, which means $\boxed{\text{the system could have an infinite solution set, meaning it is consistent}}$

37: Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

If there is a pivot in every row, that means every row is defined to equal the constant it shares a row with. Since every variable is mapped to a constant, there $\boxed{\text{is a unique solution and the system is consistent}}$