RH 2.1

MATH 5, Jones

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Refrigerator Homework

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$T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then rotates points $\frac{3\pi}{2}$ radians.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to$ Reflection through the x_2 axis from textbook table $\to \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{3\pi}{2}$ rad rotation (90 degree clockwise)$\to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Multiplying them together: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}$

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Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.
$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$

T = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Since all values inside the matrix are real and make the transformation linear and nothing other than linear, the transformation can be considered linear

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True. The transformation matrix is calculated using the identity matrix, meaning
Identitiy Matrix $\to$ Transformation Matrix

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Yes. Rotations are linear transformations as they scale by 1 and dont have any nonlinear effect on the original point

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$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ only has 3 pivots so it only maps into $\mathbb{R}^3$, not $\mathbb{R}^4$ so it is neither onto and also means its not one-to-one since there's a row of zeores

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$T\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ makes the transformation matrix $A = \begin{bmatrix}1 & -5 & 4\\0 & 1 & -6\end{bmatrix}$. It is onto since there are 2 pivots, one for each row, but is not one-to-one since its not true that the only element of each row is the pivot

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For $\mathbb{R}^n \to \mathbb{R}^m$ A linear transformation T is onto if its image spans the entire codomain $\mathbb{R}^m$, which means the rank of the transformation matrix A must be equal to m (i.e., the number of linearly independent rows must be m). That wasy $n \geq m$
A linear transformation T is one-to-one if the null space of A is trivial, meaning the only solution to Ax $= 0$ is the zero vector. This occurs when A has full column rank, meaning: $n \leq m$
For T to be both onto $(n \geq m)$ and one-to-one $(n \leq m)$, we require: $n = m$

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