

RH 1.3

MATH 5, Jones

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1 Refrigerator Homework

Practice Problem 3

Let w_1, w_2, w_3, u, v be vectors in \mathbb{R}^n . Suppose the vectors \mathbf{u} and \mathbf{v} are in $\text{Span } w_1, w_2, w_3$. Show that $u + v$ is also in $\text{Span } w_1, w_2, w_3$.

Since u and v exist in $\text{Span } w_1, w_2, w_3$, there must be constants x_n for u and y_n for v where $u = \sum_{m=1}^n x_m w_m$ and

$$v = \sum_{m=1}^n y_m w_m. \text{ With this, } u+v = x_1 w_1 + x_2 w_2 + x_3 w_3 + y_1 w_1 + y_2 w_2 + y_3 w_3 = (y_1 + x_1)w_1 + (y_2 + x_2)w_2 + (y_3 + x_3)w_3$$

Since all three of those coefficients are scalar quantities, $\boxed{u + v \text{ is in } \text{Span } w_1, w_2, w_3}$

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$$a_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} \quad a_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix} \text{ for what value(s) of } h \text{ is } b \text{ in the plane spanned by } a_1 \text{ and } a_2?$$

$$a_1 - 2a_2 = 4 \quad 4a_1 - 3a_2 = 1$$

$$5a_2 = -15 \rightarrow a_2 = -3$$

$$a_1 + 6 = 4 \rightarrow a_1 = -2$$

$$-2(-2) + -3(7) = h \rightarrow \boxed{h = -17}$$

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$$-2c_1 + 2c_2 = h$$

$$c_1 + c_2 = k$$

Second equation:

$$c_1 = k - c_2$$

Substituting into the first equation:

$$-2(k - c_2) + 2c_2 = h \rightarrow -2k + 2c_2 + 2c_2 = h$$

$$-2k + 4c_2 = h \rightarrow c_2 = \frac{h + 2k}{4}$$

Substituting back:

$$c_1 = k - \frac{h + 2k}{4} = \frac{4k - h - 2k}{4}$$

$$c_1 = \frac{2k - h}{4}$$

Since division by 0 never occurs and the system always has solutions for any h and k , it follows that the span of \mathbf{u} and \mathbf{v} covers the entire \mathbb{R}^2 , meaning they form a basis for \mathbb{R}^2 .

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T/F $\begin{bmatrix} -4 \\ 3 \end{bmatrix} \equiv [-4 \ 3]$ is **false**. Row vectors and column vectors have different meanings in the way they are read and are only equivalent when marked as transposed

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T/F $(-2,5)$ and $(-5,2)$ lie on a line passing through the origin. This statement is **false** as the line equation is $y = x + 7$ and $(0,0)$ is not a solution

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a: No, 3

b: $\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix}$

$$R_3+ = 2R_1 \rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix}$$

$$R_3- = 2R_2 \rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$R_1+ = 4R_3 \text{ and } R_2+ = 2R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$R_2/ = 3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \text{ Yes } \mathbf{b} \text{ is in } W \text{ and there are an infinite number of vectors in } W$$

c: $a_1 = 1 \cdot a_1 + 0a_2 + 0a_3$ ✓

Computer Homework

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