RH 1.1

MATH 5, Jones

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23: Find h where system is consistent

$\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right] \rightarrow$ Scale Row 1 by 2 $\rightarrow \left[\begin{array}{cc|c} 3 & 3h & 12 \\ 3 & 6 & 8 \end{array}\right]$

$\cancel{3x_1} + 3hx_2 - 12 = \cancel{3x_1} + 6x_2 - 8$

$3hx_2 = 6x_2 + 4$

$(3h - 6)x_2 = 4$

$3h - 6 \neq 0$ to avoid $0y = 4$

$\boxed{h \neq 2}$

26: Find h where system is consistent

$\left[\begin{array}{cc|c} 2 & -3 & h \\ -6 & 9 & 5 \end{array}\right] \rightarrow$ Scale Row 1 by -3 $\rightarrow \left[\begin{array}{cc|c} -6 & 9 & -3h \\ -6 & 9 & 5 \end{array}\right]$

$-3h = 5 \rightarrow \boxed{h = -\frac{5}{3}}$

33: T/F The two fundamental questions involve existence and uniqueness

True

The questions are:

Question 1: Does a solution exist? (Existence)

Question 2: If a solution does exist, is that solution the only one? (Uniqueness)

Computer Homework Problems

1: Solve the system

$x_1 + 2x_2 = -1 \qquad 4x_1 + 5x_2 = -10$

$4x_1 + 8x_2 = -4 \qquad 4x_1 + 5x_2 = -10$

$4x_1 = -4 - 8x_2 \qquad 4x_1 = -10 - 5x_2$

$-4 - 8x_2 = -10 - 5x_2$
$6 = 3x_2$

$x_2 = 2$
$x_1 + 2(2) = -1$
$x_1 = -5$
$\boxed{(-5, 2)}$

2: Solve the system

$x_1 + 2x_2 = 8 \qquad x_1 - x_2 = -1$

$x_1 = 8 - 2x_2 \qquad x_1 = x_2 - 1$

$8 - 2x_2 = x_2 - 1$
$9 = 3x_2$
$x_2 = 3$
$x_1 - 3 = -1$
$x_1 = 2$
$\boxed{(2, 3)}$

3: Consider the accompanying matrix as the augmented matrix of a linear system. State in words the next two elementary row operations that should be performed in the process of solving the system.

$$\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -2 \\ 0 & 2 & -6 & 0 & 5 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 4 & 13 & 4 \end{array}\right]$$

First elementary row operation:

Replace row 4 by its sum with -4 times row 3.
Resultant array:

$$\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -2 \\ 0 & 2 & -6 & 0 & 5 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & -3 & 12 \end{array}\right]$$

Second elementary row operation:

Scale row 4 by $-\frac{1}{3}$
Resultant array:

$$\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -2 \\ 0 & 2 & -6 & 0 & 5 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 1 & -4 \end{array}\right]$$

4: Describe the solution set of the system.

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 2 \\ 1 & 6 & 3 & -5 \end{array}\right]$$

$\boxed{\text{Solution set is empty. In the first row } 0 \neq 2}$

5: Solve the system

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 13 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 3 & 1 \end{array}\right]$$

Add row 3 to row 1 $\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 14 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 3 & 1 \end{array}\right]$

Scale row 1 by 2 $\rightarrow \left[\begin{array}{ccc|c} 2 & 4 & 0 & 28 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 3 & 1 \end{array}\right]$

Subtract row 2 from row 1 $\rightarrow \left[\begin{array}{ccc|c} 0 & 0 & -7 & 21 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 3 & 1 \end{array}\right]$

Scale row 1 by $-\frac{1}{7} \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 1 & -3 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 3 & 1 \end{array}\right]$

Subtract 3 times row 1 from row 3 $\rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 1 & -3 \\ 2 & 4 & 7 & 7 \\ 0 & 2 & 0 & 10 \end{array}\right]$

Scale row 3 by $\frac{1}{2} \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 1 & -3 \\ 2 & 4 & 7 & 7 \\ 0 & 1 & 0 & 5 \end{array}\right]$

Subtract 7 times row 1 from row 2 and 4 times row 3 from row 2 $\rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 1 & -3 \\ 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \end{array}\right]$

Solution: $\boxed{(4, 5, -3)}$

6: Determine if the system is consistent

$$\left[\begin{array}{cccc|c} 2 & 0 & 6 & 0 & 10 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & -2 & 6 & 2 & 3 \\ 6 & 0 & 0 & 7 & -4 \end{array}\right]$$

$\boxed{\text{The system is consistent as there is a relation between every variable without any duplicate relations}}$

7: Determine the h values for which the system is consistent

$\left[\begin{array}{cc|c} 1 & h & 3 \\ 4 & 20 & 9 \end{array}\right] \rightarrow$ Scale row 1 by 4 $\rightarrow \left[\begin{array}{cc|c} 4 & 4h & 12 \\ 4 & 20 & 9 \end{array}\right]$
$(4h-20)y=3$
to avoid $0y=3 \downarrow$
$4h-20 \neq 0$
$4h \neq 20$
$h \neq 5$

8: Suppose the system is consistent for all f and g. What can you say about the coefficients c and d?

$x_1+4x_2=f$
$cx_1+dx_2=g$
$\left[\begin{array}{cc|c} 1 & 4 & f \\ c & d & g \end{array}\right]$ to $\left[\begin{array}{cc|c} 1 & 4 & f \\ 0 & d-4c & g-cf \end{array}\right]$ shows that $d-4c \neq 0$ since f and g are arbitrary, otherwise the second row would imply $0=b$ where b is nonzero. Therefore, $d \neq 4c$

9: Find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 3 & -2 & 2 & -5 \\ 0 & 3 & -2 & 8 \end{array}\right], \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 7 & -4 & -5 \\ 0 & 3 & -2 & 8 \end{array}\right]$
First matrix to second: Replace row 2 by its sum with -3 times row 1.
Second matrix to first: Replace row 2 by its sum with 3 times row 1.