RH 2.3

MATH 5, Jones

Tejas Patel

Refrigerator Homework

15

Counterexample: $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ where $ad - bc = 0$

24

Suppose A is a 5x5 matrix whose columns do not span $\mathbb{R}^5$. This means that the columns of A are linearly dependent, so at least one of column can be written as a linear combination of the others. If the columns are linearly dependent, then the equation $Ax = 0$ has a nontrivial solution, meaning there exists a nonzero vector x such that $Ax = 0$. However, if A were invertible, the only solution to $Ax = 0$ would be $x = 0$, which contradicts the existence of a nontrivial solution. Thus, A cannot be invertible if its columns do not span $\mathbb{R}^5$.

35

Since the columns of A are linearly independent, one of the statements of the Invertible Matrix Theorem tells us that A is invertible (i.e., A has an inverse). This means that A creates a one-to-one and onto transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. Now, consider A^2, which is just A multiplied by itself. Because A is invertible, the product A^2 is also invertible. Applying the Invertible Matrix Theorem again, since A^2 is invertible, its columns must also be linearly independent and, importantly, they span $\mathbb{R}^n$.

42

$$T(x_1, x_2) = \begin{bmatrix} 6x_1 - 8x_2 \\ -5x_1 + 7x_2 \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{7}{2} & 4 \\ \frac{5}{2} & 3 \end{bmatrix}$$

$$T^{-1} = \left(\frac{7}{2}x_1 + 4x_2, \frac{5}{2}x_1 + 3x_2 \right)$$

Computer Homework: Next 10 Pages

Question list

Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 4 & 5 \\ -2 & -5 \end{bmatrix}$$

Choose the correct answer below.

- A. The matrix is not invertible because its determinant is zero.
- B. The matrix is invertible because its determinant is not zero.
- C. The matrix is not invertible because the matrix has 2 pivot positions.
- D. The matrix is invertible because its columns are multiples of each other. The columns of the matrix form a linearly dependent set.

Help me solve this View an example Get more help ▲ Similar question Next

Question list
Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 2 & 0 & 0 \\ -5 & -4 & 0 \\ 5 & 4 & -5 \end{bmatrix}$$

Choose the correct answer below.

A. The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has no solution for some **b** in $\mathbb{R}^3$.

B. The matrix is not invertible. The given matrix has two pivot positions.

C. The matrix is invertible. The given matrix has three pivot positions.

D. The matrix is invertible. If the given matrix is A, there is a 3×3 matrix C such that $CI = A$.

Help me solve this View an example Get more help ▴ Similar q

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:33 PM

Review Homework: Section 2.3 Homework

Question 3, 2.3.4

HW Score: 100%, 17 of 17 points

Points: 1 of 1

Close

Question list

Question 1

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8

Question 9

Question 10

Question 11

Question 12

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 0 & -3 \\ 2 & 0 & 4 \\ -4 & 0 & 5 \end{bmatrix}$$

Choose the correct answer below.

A. The matrix is invertible. The columns of the given matrix span $\mathbb{R}^3$.

B. The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

C. The matrix is invertible. The given matrix has 2 pivot positions.

D. The matrix is not invertible. If the given matrix is A, the columns of A do not form a linearly independent set.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:33 PM

Review Homework: Section 2.3 Homework

Question 4, 2.3.5

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 0 & 4 & -6 \\ 1 & 0 & 3 \\ -4 & -12 & 6 \end{bmatrix}$$

Choose the correct answer below.

- A. The matrix is invertible. The given matrix has 3 pivot positions.
- B. The matrix is invertible. The columns of the given matrix span are linearly dependent.
- C. The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has a solution for at least one **b** in $\mathbb{R}^3$.
- D. The matrix is not invertible. If the given matrix is A, A is not row equivalent to the n × n identity matrix.

Help me solve this View an example Get more help ▴

Similar q

Question list

Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 0 & 4 & -6 \\ 1 & 0 & 3 \\ -4 & -12 & 6 \end{bmatrix}$$

Choose the correct answer below.

- A. The matrix is invertible. The given matrix has 3 pivot positions.
- B. The matrix is invertible. The columns of the given matrix span are linearly dependent.
- C. The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{b}$ has a solution for at least one $\mathbf{b}$ in $\mathbb{R}^3$.
- D. The matrix is not invertible. If the given matrix is A, A is not row equivalent to the $n \times n$ identity matrix.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:33 PM

Review Homework: Section 2.3 Homework

Question 5, 2.3.21

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

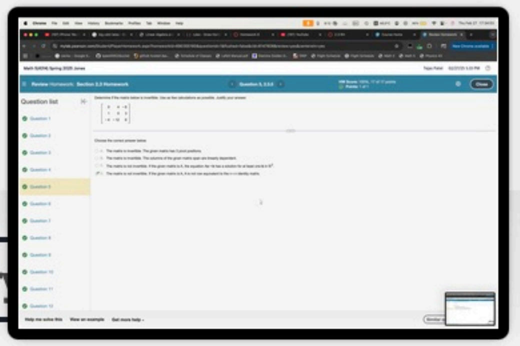
- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

An $m \times n$ upper triangular matrix is one whose entries below the main diagonal are zeros, as is shown in the matrix to the right. When is a square upper triangular matrix invertible? Justify your answer.

$$\begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose the correct answer below.

- A. A square upper triangular matrix is invertible when the matrix is equal to its own transpose. For such a matrix A, $A = A^T$ means that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
- B. A square upper triangular matrix is invertible when all entries on its main diagonal are nonzero. If all of the entries on its main diagonal are nonzero, then the $n \times n$ matrix has n pivot positions.
- C. A square upper triangular matrix is invertible when all entries on the main diagonal are ones. If any entry on the main diagonal is not one, then the equation $A\mathbf{x} = \mathbf{b}$, where A is an $n \times n$ square upper triangular matrix, has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^n$.
- D. A square upper triangular matrix is invertible when all entries above the main diagonal are zeros as well. This means that the matrix is row equivalent to the $n \times n$ identity matrix.



Chrome File Edit View History Bookmarks Profiles Tab Window Help 8:12 48.4°C 95% Thu Feb 27 17:34:07

(197) iPhone 16e R | big cdot latex - Go | Linear Algebra and | rules - Draw Horiz | Homework 6 | (197) YouTube | 2.3 RH | Course Home | Review Homework

mylab.pearson.com/Student/PlayerHomework.aspx?homeworkId=690355180&questionId=1&flushed=false&cId=8147608&review=yes&centerwin=yes

New Chrome available

.... | cache: - Google S... | tpate0562/Quizlet | github hosted das... | Schedule of Classes | LaTeX Manual.pdf | Diamine Golden O... | DRIP | Flight Schedule | Flight Schedule | Math 2 | Math 5 | Physics 43

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 6, 2.3.23

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

Can a square matrix with two identical columns be invertible? Why or why not?

Select the correct choice below.

A. It depends on the values in the matrix. According to the Invertible Matrix Theorem, if the two columns are larger than any other columns the matrix will be invertible, otherwise it will not.

B. The matrix is not invertible. If a matrix has two identical columns then its columns are linearly dependent. According to the Invertible Matrix Theorem this makes the matrix not invertible.

C. The matrix is invertible. If a matrix has two identical columns then its columns are linearly independent. According to the Invertible Matrix Theorem this makes the matrix invertible.

D. The matrix is not invertible. According to the Invertible Matrix Theorem a square matrix can never be invertible.

Textbook Ask my instructor

https://tdx.acs.pearson.com/Player/Player.aspx?cultureId=&theme=math&style=highered&disableStan...

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 7, 2.3.24

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12

Is it possible for a 5×5 matrix to be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?

Select the correct choice below.

- A. It will depend on the values in the matrix. According to the Invertible Matrix Theorem, a square matrix is only invertible if it is row equivalent to the identity.
- B. It is possible; according to the Invertible Matrix Theorem an $n \times n$ matrix can be invertible when its columns do not span $\mathbb{R}^n$.
- C. It is not possible; according to the Invertible Matrix Theorem an $n \times n$ matrix cannot be invertible when its columns do not span $\mathbb{R}^n$.
- D. It is possible; according to the Invertible Matrix Theorem all square matrices are always invertible.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 8, 2.3.26

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

If C is 6×6 and the equation $C\mathbf{x} = \mathbf{v}$ is consistent for every $\mathbf{v}$ in $\mathbb{R}^6$, is it possible that for some $\mathbf{v}$, the equation $C\mathbf{x} = \mathbf{v}$ has more than one solution? Why or why not?

Select the correct choice below.

- A. It will depend on the values in the 6×6 matrix. According to the Invertible Matrix Theorem if any of the values are zero this makes $C\mathbf{x} = \mathbf{v}$ have more than one solution. If none of the values are zero, then $C\mathbf{x} = \mathbf{v}$ has a unique solution.
- B. It is not possible. Since $C\mathbf{x} = \mathbf{v}$ is consistent for every $\mathbf{v}$ in $\mathbb{R}^6$, according to the Invertible Matrix Theorem that makes the 6×6 matrix invertible. Since it is invertible, $C\mathbf{x} = \mathbf{v}$ has a unique solution.
- C. It is possible. Since $C\mathbf{x} = \mathbf{v}$ is consistent for every $\mathbf{v}$ in $\mathbb{R}^6$, according to the Invertible Matrix Theorem that makes the 6×6 matrix not invertible. Since it is not invertible, $C\mathbf{x} = \mathbf{v}$ does not have a unique solution.
- D. It is possible because 6×6 is a square matrix, and according to the Invertible Matrix Theorem all square matrices are not invertible. Since it is not invertible, $C\mathbf{x} = \mathbf{v}$ does not have a unique solution.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 9, 2.3.27

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

If the columns of a 7×7 matrix D are linearly independent, what can you say about the solutions of $D\mathbf{x} = \mathbf{b}$? Why?

Select the correct choice below.

A. Equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, a matrix is not invertible if the columns of the matrix form a linearly independent set, and the equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^n$.

B. Equation $D\mathbf{x} = \mathbf{b}$ has no solutions for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, the equation $D\mathbf{x} = \mathbf{0}$ has only the trivial solution.

C. Equation $D\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, a matrix is invertible if the columns of the matrix form a linearly independent set; this would mean that the equation $D\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.

D. It will depend on the values in the matrix. If the diagonal of the matrix is zero, $D\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^7$. However, if the diagonal is all non-zero, equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^7$.

Textbook Ask my instructor

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: **Section 2.3 Homework**

Question 10, 2.3.30

HW Score: 100%, 17 of 17 points

Points: 1 of 1

Close

Question list

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12

Suppose H is an $n \times n$ matrix. If the equation $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $H\mathbf{x} = \mathbf{0}$? Why?

Select the correct choice below.

- A. The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has no solution for some $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are false, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution. Thus, $H\mathbf{x} = \mathbf{0}$ has no solution.
- B. The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has no solution for some $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are false, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution. Thus, $H\mathbf{x} = \mathbf{0}$ has a nontrivial solution.
- C. The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has a solution for every $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are true, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- D. The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has a solution for every $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are true, including the statement that the columns of H form a linearly independent set. Thus, $H\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.

Textbook Ask my instructor

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 11, 2.3.31

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12

If an $n \times n$ matrix K cannot be row reduced to I_n, what can you say about the columns of K? Why?

Select the correct choice below.

- A. The columns of K are linearly dependent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
- B. The columns of K are linearly independent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n, the equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$.
- C. The columns of K are linearly independent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n that matrix is invertible.
- D. The columns of K are linearly dependent and the columns do not span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n that matrix is non invertible.

Question list

- Question 5
- Question 6
- Question 7
- Question 8
- Question 9
- Question 10
- Question 11
- Question 12
- Question 13
- Question 14
- Question 15
- Question 16

Let A and B be square matrices. Show that if $AB = I$ then A and B are both invertible, with $B = A^{-1}$ and $A = B^{-1}$.

It is given that $AB = I$. The Invertible Matrix Theorem states that if there is an $n \times n$ matrix B such that $AB = I$, then it is true that matrix A is invertible.

It is given that $AB = I$. The Invertible Matrix Theorem states that if there is an $n \times n$ matrix A such that $AB = I$, then it is true that matrix B is invertible.

It is given that $AB = I$. Left-multiply each side of the equation by A^{-1}.

$$A^{-1}AB = A^{-1}I \qquad \text{Left-multiply by } A^{-1}.$$
$$B = A^{-1} \qquad \text{Simplify.}$$

It is given that $AB = I$. Right-multiply each side of the equation by B^{-1}.

$$ABB^{-1} = IB^{-1} \qquad \text{Right-multiply by } B^{-1}.$$
$$A = B^{-1} \qquad \text{Simplify.}$$

In conclusion, it can be stated that A and B are both invertible, with $B = A^{-1}$ and $A = B^{-1}$.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 13, 2.3.34

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12
Question 13
Question 14
Question 15
Question 16

Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of an $n \times n$ matrix A are linearly independent.

Choose the correct answer below. Note that the invertible matrix theorem is abbreviated IMT.

- A. If the columns of A are linearly independent and A is square, then A is invertible, by the IMT. Thus, A^2, which is the product of invertible matrices, is not invertible. So, the columns of A^2 span $\mathbb{R}^n$.
- B. If the columns of A are linearly independent and A is square, then A is invertible, by the IMT. Thus, A^2, which is the product of invertible matrices, is also invertible. So, by the IMT, the columns of A^2 span $\mathbb{R}^n$.
- C. If the columns of A are linearly independent, then it directly follows that the columns of A^2 span $\mathbb{R}^n$.
- D. If the columns of A are linearly independent and A is square, then A is not invertible. Thus, A^2, which is the product of non invertible matrices, is also not invertible. So, the columns of A^2 span $\mathbb{R}^n$.

Math 5(4314) Spring 2025 Jones

Tejas Patel 02/27/25 5:34 PM

Review Homework: Section 2.3 Homework

Question 14, 2.3.36

HW Score: 100%, 17 of 17 points
Points: 1 of 1

Close

Question list

Question 5
Question 6
Question 7
Question 8
Question 9
Question 10
Question 11
Question 12
Question 13
Question 14
Question 15
Question 16

Let A and B be $n \times n$ matrices. Show that if AB is invertible so is B.

Choose the correct answer below. Note that the invertible matrix theorem is abbreviated IMT.

- A. Since AB is invertible then by the IMT AB^T is an invertible matrix. Therefore, matrix B is invertible by part (l) of the IMT.
- B. Since AB is invertible, then it directly follows that $A = B^{-1}$ and $B = A^{-1}$ by the IMT. Therefore, matrix B is invertible.
- C. Let W be the inverse of AB. Then $WAB = I$ and $(WA)B = I$. Therefore, matrix B is invertible by part (j) of the IMT.
- D. Let W be the inverse of AB. Then $WAB = B$. Therefore, since B is the product of two invertible matrices, W and AB, matrix B is invertible.

The given T is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$. Show that T is invertible and find a formula for T^{-1}.

$$T(x_1,x_2) = (5x_1 - 8x_2, -5x_1 + 9x_2)$$

To show that T is invertible, calculate the determinant of the standard matrix for T. The determinant of the standard matrix is 5.
(Simplify your answer.)

$$T^{-1}(x_1,x_2) = \left(\frac{9}{5}x_1 + \frac{8}{5}x_2, x_1 + x_2\right)$$

(Type an ordered pair. Type an expression using x_1 and x_2 as the variables.)

Let T be a linear transformation that maps $\mathbb{R}^n$ onto $\mathbb{R}^n$. Show that T^{-1} exists and maps $\mathbb{R}^n$ onto $\mathbb{R}^n$. Is T^{-1} also one-to-one?

If T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$, then according to the Invertible Matrix Theorem, the columns of its standard matrix A span $\mathbb{R}^n$ and so A is invertible.

It is a known theorem that if $T: \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation and A is the standard matrix for T, then T is invertible if and only if A is invertible. Given the information determined in the previous step, it can be stated that matrix T is invertible.

The rest of the theorem states that in the case that T and A are both invertible, the linear transformation S given by $S(\mathbf{x}) = A^{-1}\mathbf{x}$ is the unique function satisfying the equations $S(T(\mathbf{x})) = \mathbf{x}$ and $T(S(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$. Therefore, it can be stated that A^{-1} is the standard matrix of T^{-1}.

Since A^{-1} is invertible, according to the Invertible Matrix Theorem, its columns are linearly independent and span $\mathbb{R}^n$.

It is a known theorem that if $T: \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation and A is the standard matrix for T, then T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ if and only if the columns of A span $\mathbb{R}^n$, and T is one-to-one if and only if the columns of A are linearly independent. Applying this theorem to the transformation T^{-1}, it can be concluded that T^{-1} is a one-to-one mapping of $\mathbb{R}^n$ onto $\mathbb{R}^n$.