RH 1.2

MATH 5, Jones

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1 Refrigerator Homework

3: Convert matrix to RREF

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right]$ Subtract $4R_1$ from R_2 and $6R_1$ from R_3 $\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array}\right]$

Scale R_2 by $-\dfrac{1}{3}$ and R_3 by $-\dfrac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]$

Subtract R_2 from $R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Subtract $2R_2$ from $R_1 \rightarrow \boxed{\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]}$ is the resultant matrix in RREF

7: Find the general solution to the system

$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right] \rightarrow$ Subtract $3R_1$ from $R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right]$

Scale R_2 by $-\dfrac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Subtract $4R_2$ from $R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Free variable: $X_2 = t$
$X_1 = -5 - 3t$
$X_2 = t$
$X_3 = 3$
$\boxed{(-5 - 3t, t, 3)}$

14: Find the general solution to the system

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -4 & 0 & -5 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \text{Subtract } 2R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 4 & 0 & -9 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Free Variables $X_3 = s,\ X_4 = t$
Solution: $X_1 = -9 - 7s - 4t,\ X_2 = 2 + 6s + 4t,\ X_3 = s,\ X_4 = t,\ X_5 = 0$

$\boxed{-9 - 7s - 4t,\ 2 + 6s + 4t,\ s,\ t,\ 0}$

21: Find h where the system is consistent

$$\left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array}\right] \rightarrow \text{Scale } R_1 \text{ by } 2 \rightarrow \left[\begin{array}{cc|c} 4 & 6 & 2h \\ 4 & 6 & 7 \end{array}\right] \rightarrow 2h = 7 \rightarrow \boxed{h = 3.5}$$

24: Choose h & k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right]$$

a

At $h = 9$ and $k \neq 6$, you'd need to scale R_1 by 3 and show $6 \neq 6$ meaning there would be no solution to the system

b

At $h = 4$ and $k = 3$ there is one solution, as long as $h \neq 9$ and $k \neq 6$ there will be one solution. In this case the solution is $X_1 = \frac{1}{5} \qquad X_2 = \frac{3}{5}$

c

At $h = 9$ and $k = 6$ the two equations will be scalar multiples of each other, meaning multiple solutions exist.

35: Suppose a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Why or why not?

3 Rows $\times$ 5 Columns means there will be 5 variables and only 3 equations to relate the variables to each other. Since there are 5 variables but 3 equations and 3 pivot points, we know there are 2 free variables, which means $\boxed{\text{the system could have an infinite solution set, meaning it is consistent}}$

37: Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

If there is a pivot in every row, that means every row is defined to equal the constant it shares a row with. Since every variable is mapped to a constant, there $\boxed{\text{is a unique solution and the system is consistent}}$

Computer Homework

1: Convert matrix to RREF

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 6 & 7 & 8 & 9 \end{array}\right] \to$ Subtract $6R_1$ from R_3 and $3R_1$ from R_2 $\to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -4 & -6 \\ 0 & -5 & -10 & -15 \end{array}\right]$

Subtract $2.5R_2$ from $R_3 \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -4 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Scale R_2 by $-\dfrac{1}{2} \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Subtract R_2 form $R_1 \to \boxed{\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]}$ The matrix is now in RREF with pivot columns 1 and 2

2: Select the example matrices that can be echelon form

The answer was A, D, and E $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix} \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

3: Find the general solution to the matrix

$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 2 & 8 & 3 & 7 \end{array}\right] \to R_2 -= 2R_1 \to \left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & -5 & -25 \end{array}\right] \to R_2 /= -5 \to \left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & 1 & 5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 16 \\ 0 & 0 & 1 & 5 \end{array}\right] \to R_1 -= 4R_2 \to \left[\begin{array}{ccc|c} 1 & 4 & 0 & -4 \\ 0 & 0 & 1 & 5 \end{array}\right]$

$X_2 = t$ is a free variable, meaning the solution set is: $\boxed{(-4-4t, t, 5)}$

4: Find the general solution to the matrix

$\left[\begin{array}{ccc|c} 0 & 1 & -5 & 6 \\ 1 & -3 & 13 & -12 \end{array}\right] \to R_2 -= 3R_1 \to \left[\begin{array}{ccc|c} 0 & 1 & -5 & 6 \\ 1 & 0 & -2 & 6 \end{array}\right] \to$ Swap rows $\to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 6 \\ 0 & 1 & -5 & 6 \end{array}\right]$

Making $X_3 = t$ a free variable and the solution set $\boxed{(6+2t, 6+5t, t)}$

5: Find the general solution to the matrix

$\left[\begin{array}{ccc|c} 5 & -3 & 7 & 0 \\ 10 & -6 & 14 & 0 \\ 15 & -9 & 21 & 0 \end{array}\right] \to$ Divide R_2 by 2 and R_3 by 3 $\to \left[\begin{array}{ccc|c} 5 & -3 & 7 & 0 \\ 5 & -3 & 7 & 0 \\ 5 & -3 & 7 & 0 \end{array}\right]$ making $X_2 = s$ and $X_3 = t$ free variables and the solution to the system $\boxed{\left(\dfrac{3}{5}s - \dfrac{7}{5}t, s, t\right)}$

6: Find the general solution to the matrix

$$\begin{bmatrix} 1 & -4 & 0 & -1 & 0 & -9 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to R_1 \mathrel{+}= 4R_2 \to \begin{bmatrix} 1 & 0 & 0 & -1 & -20 & 3 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \mathrel{+}= R_3 \to \begin{bmatrix} 1 & 0 & 0 & 0 & -14 & 10 \\ 0 & 1 & 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Variables $X_3 = s,\ X_5 = t$ is are free variables, and the solution to the system is $\boxed{10 + 14t, 3 + 5t, s, 7 - 6t, t}$

7: Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions.

System: $\begin{cases} x_1 + hx_2 = 5 \\ 5x_1 + 15x_2 = k \end{cases}$

a: No solutions when $h = 3$ and $k \neq 25$

b: Unique solution when $h \neq 3$

c: Many solutions when $h = 3$ and $k = 25$

8: Suppose a 3×6 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

The augmented matrix will have seven columns and will not have a row of the form $\left[\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$, so the system is consistent. This is because every row has a pivot, so there will be no blank coefficient rows

9: Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is not a pivot column. Is the system consistent? Why or why not?

A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column. That is, if and only if an echelon form of the augmented matrix has no row of the form [0 ... 0 b] with b nonzero.

In the augmented matrix described above, is the rightmost column a pivot column? No

In the echelon form of the augmented matrix, is there a row of the form [0 0 0 0 b] with b nonzero? No

$\boxed{\text{Therefore, by the Existence and Uniqueness Theorem, the linear system is consistent.}}$

10: Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

$\boxed{\text{The system is consistent because the rightmost column of the augmented matrix is not a pivot column.}}$