

RH 1.2

MATH 5, Jones

Tejas Patel

3: Convert matrix to RREF

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array} \right] \text{ Subtract } 4R_1 \text{ from } R_2 \text{ and } 6R_1 \text{ from } R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array} \right]$$

$$\text{Scale } R_2 \text{ by } -\frac{1}{3} \text{ and } R_3 \text{ by } -\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array} \right]$$

$$\text{Subtract } R_2 \text{ from } R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{Subtract } 2R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ is the resultant matrix in RREF}$$

7: Find the general solution to the system

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array} \right] \rightarrow \text{Subtract } 3R_1 \text{ from } R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array} \right]$$

$$\text{Scale } R_2 \text{ by } -\frac{1}{5} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\text{Subtract } 4R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Free variable: $X_2 = t$

$$X_1 = -5 - 3t$$

$$X_2 = t$$

$$X_3 = 3$$

$$\boxed{(-5 - 3t, t, 3)}$$

14: Find the general solution to the system

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -4 & 0 & -5 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \text{Subtract } 2R_2 \text{ from } R_1 \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 4 & 0 & -9 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Free Variables $X_3 = s$, $X_4 = t$

Solution: $X_1 = -9 - 7s - 4t$, $X_2 = 2 + 6s + 4t$, $X_3 = s$, $X_4 = t$, $X_5 = 0$

$$\boxed{-9 - 7s - 4t, 2 + 6s + 4t, s, t, 0}$$

21: Find h where the system is consistent

$$\left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array} \right] \rightarrow \text{Scale } R_1 \text{ by } 2 \rightarrow \left[\begin{array}{cc|c} 4 & 6 & 2h \\ 4 & 6 & 7 \end{array} \right] \rightarrow 2h = 7 \rightarrow \boxed{h = 3.5}$$

24: Choose h & k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array} \right]$$

a

At $h = 9$ and $k \neq 6$, you'd need to scale R_1 by 3 and show $6 \neq 6$ meaning there would be no solution to the system

b

At $h = 4$ and $k = 3$ there is one solution, as long as $h \neq 9$ and $k \neq 6$ there will be one solution. In this case the solution is $X_1 = \frac{1}{5}$ $X_2 = \frac{3}{5}$

c

At $h = 9$ and $k = 6$ the two equations will be scalar multiples of each other, meaning multiple solutions exist.

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