RH 1.8

MATH 5, Jones

Tejas Patel

Refrigerator Homework

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Since T is linear $T(a\,\mathbf{x}) \;=\; a\,T(\mathbf{x})$ and $T(\mathbf{x}+\mathbf{y}) \;=\; T(\mathbf{x})+T(\mathbf{y})$.
Given $T(\mathbf{u}) \;=\; \begin{bmatrix}2\\1\end{bmatrix}$ and $T(\mathbf{v}) \;=\; \begin{bmatrix}-1\\3\end{bmatrix}$.

$$\boxed{3u = 3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}6\\3\end{bmatrix} \qquad 2v = 2\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix} \qquad 3u+2v = \begin{bmatrix}6-2\\3+6\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}}$$

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$\mathbf{e}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix}0\\1\end{bmatrix}.$
The first column of A is $T(\mathbf{e}_1)$, and the second column is $T(\mathbf{e}_2)$.
$T\big(x_1, x_2\big) = x_1\,v_1 + x_2\,v_2$
Given that$T(x_1, x_2) \;=\; x_1\,v_1 \;+\; x_2\,v_2$, then for the standard basis vectors $\mathbf{e}_1 = (1,0)$ and $\mathbf{e}_2 = (0,1)$ we have$T(\mathbf{e}1) \;=\; v_1, \quad T(\mathbf{e}2) \;=\; v_2$.
$A \;=\; \begin{bmatrix}v_1 & v_2\end{bmatrix} \;=\; \boxed{\begin{bmatrix}-2 & 7\\5 & -3\end{bmatrix}}$

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True. A linear transformation is indeed a function (one that takes vectors as inputs and returns vectors as outputs) but it satisfies two special properties:
1. Additivity: $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$.
2. Homogeneity: $T(c\,\mathbf{u}) = c\,T(\mathbf{u})$ for any scalar c
Any function that meets both of these properties is called a linear transformation, so it's a specialized type of function.

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False. The set of all linear combinations of the columns of A is the range (or image) of the transformation function, a subset of its codomain. When we write $T\colon \mathbb{R}^n \to \mathbb{R}^m$ via $T(\mathbf{x}) = A\mathbf{x}$, the usual codomain is $\mathbb{R}^m$. The range—the set of all possible outputs that T actually attains—coincides with the column space of A. So the column space is a subspace of the codomain, not the entire codomain itself

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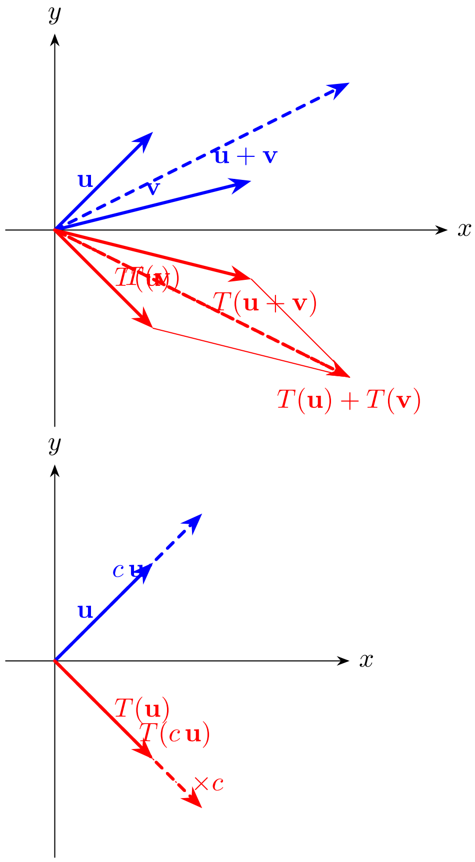

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1. Every $\mathbf{x} \in \mathbb{R}^n$ can be written as a linear combination of the spanning vectors $v_1, \ldots, v_p$. In symbols,
$\mathbf{x} \;=\; c_1\,v_1 \;+\; c_2\,v_2 \;+\; \cdots \;+\; c_p\,v_p.$
2. Apply T and use linearity:
$T(\mathbf{x}) \;=\; T\big(c_1\,v_1 + c_2\,v_2 + \cdots + c_p\,v_p\big) \;=\; c_1\,T(v_1) \;+\; c_2\,T(v_2) \;+\; \cdots \;+\; c_p\,T(v_p).$
3. But each $T(v_i) = \mathbf{0}$, by assumption. Hence every term on the right is zero, so
$T(\mathbf{x}) = \mathbf{0}$.
Because $\mathbf{x}$ is an arbitrary vector that can span $\mathbb{R}^n$, it follows that $T(\cdot)$ sends every vector to $\mathbf{0}$.

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Because v_1, v_2, v_3 is dependent, there exist scalars c_1, c_2, c_3, not all zero, such that
$c_1\,v_1 \;+\; c_2\,v_2 \;+\; c_3\,v_3 \;=\; \mathbf{0}.$
Apply the linear map T to both sides:
$T\big(c_1v_1 + c_2v_2 + c_3v_3\big) \;=\; c_1\,T(v_1) \;+\; c_2\,T(v_2) \;+\; c_3\,T(v_3) \;=\; T(\mathbf{0}) \;=\; \mathbf{0}.$
This shows that
$c_1\,T(v_1) \;+\; c_2\,T(v_2) \;+\; c_3\,T(v_3) \;=\; \mathbf{0},$
with $(c_1, c_2, c_3) \neq (0,0,0)$, meaning $\;T(v_1), T(v_2), T(v_3)$ is also linearly dependent.

Computer Homework: Next 10 Pages

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Homework: Section 1.8 Homework

Question 1, 1.8.4 Part 2 of 2

HW Score: 100%, 10 of 10 points
Points: 1 of 1

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If T is defined by T(**x**) = A**x**, find a vector **x** whose image under T is **b**, and determine whether **x** is unique. Let $A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -3 \\ 3 & -13 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5 \\ -9 \\ -3 \end{bmatrix}$.

Find a single vector **x** whose image under T is **b**.

$\mathbf{x} = \begin{bmatrix} -50 \\ -12 \\ -1 \end{bmatrix}$

Is the vector **x** found in the previous step unique?

- A. No, because there is a free variable in the system of equations.
- B. Yes, because there are no free variables in the system of equations.
- C. No, because there are no free variables in the system of equations.
- D. Yes, because there is a free variable in the system of equations.

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Homework: **Section 1.8 Homework**

Question 2, 1.8.6
Part 2 of 2

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Find a vector **x** whose image under T, defined by T(**x**) = A**x**, is **b**, and determine whether **x** is unique. Let $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 12 & 18 \\ 0 & 1 & 1 \\ -2 & -9 & -15 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 14 \\ 36 \\ 2 \\ -30 \end{bmatrix}$.

Find a single vector **x** whose image under T is **b**.

$\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

Is the vector **x** found in the previous step unique?

- A. Yes, because there are no free variables in the system of equations.
- B. Yes, because there is a free variable in the system of equations.
- C. No, because there is a free variable in the system of equations.
- D. No, because there are no free variables in the system of equations.

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Homework: **Section 1.8 Homework**

Question 3, 1.8.9

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Find all **x** in $\mathbb{R}^4$ that are mapped into the zero vector by the transformation $\mathbf{x} \mapsto A\mathbf{x}$ for the given matrix A.

$$A = \begin{bmatrix} 1 & -5 & 16 & -9 \\ 0 & 1 & -5 & 3 \\ 3 & -12 & 33 & -18 \end{bmatrix}$$

Select the correct choice below and fill in the answer box(es) to complete your choice.

A. There is only one vector, which is $\mathbf{x} =$ ☐.

B. $x_3 \begin{bmatrix} 9 \\ 5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -6 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

C. x_3 ☐

D. x_1 ☐ $+ x_2$ ☐ $+ x_4$ ☐

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Let $\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, and let A be the matrix $\begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & -4 \end{bmatrix}$. Is **b** in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

Is **b** in the range of the linear transformation? Why or why not?

- A. No, **b** is not in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is consistent.
- B. Yes, **b** is in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is consistent.
- C. Yes, **b** is in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is inconsistent.
- D. No, **b** is not in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is inconsistent.

Let $\mathbf{b} = \begin{bmatrix} -11 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, and let A be the matrix $\begin{bmatrix} 1 & 4 & 11 & 5 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 1 & -4 & 1 \end{bmatrix}$. Is **b** in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

Is **b** in the range of the linear transformation? Why or why not?

A. No, **b** is not in the range of the linear transformation because the system represented by the appropriate augmented matrix is inconsistent.

B. Yes, **b** is in the range of the linear transformation because the system represented by the appropriate augmented matrix is consistent.

C. Yes, **b** is in the range of the linear transformation because the system represented by the appropriate augmented matrix is inconsistent.

D. No, **b** is not in the range of the linear transformation because the system represented by the appropriate augmented matrix is consistent.

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Homework: **Section 1.8 Homework**

Question 6, 1.8.13
Part 3 of 3

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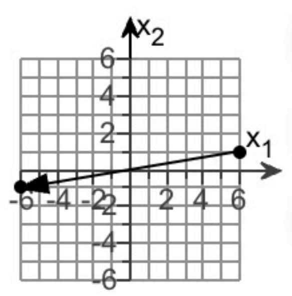
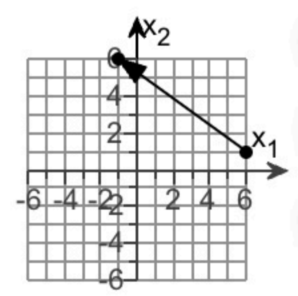
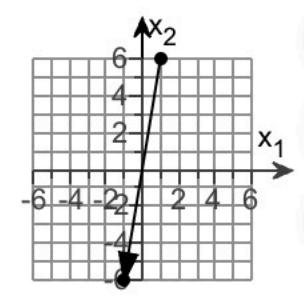
Use a rectangular coordinate system to plot $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

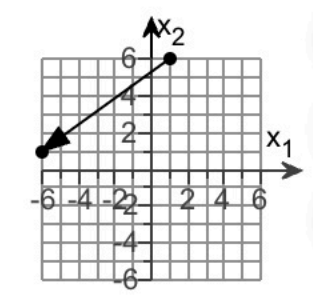
$$T(\mathbf{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Which graph below shows **u** and its image under the given transformation?

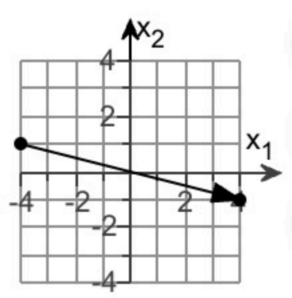
A.


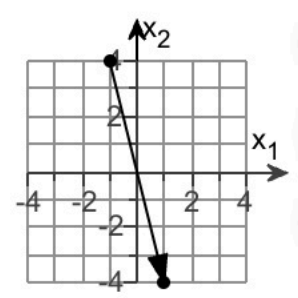
B.


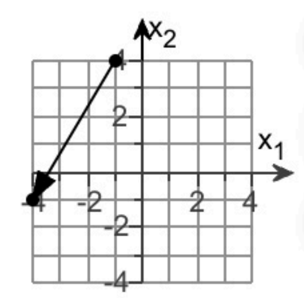
C.


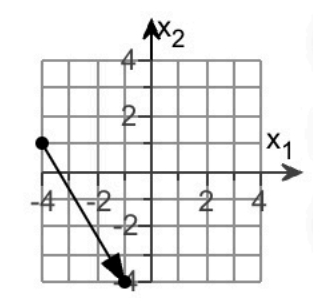
D.


Which graph below shows **v** and its image under the given transformation?

A.


B.


C.


D.


What does T do geometrically to each vector $\mathbf{x}$ in $\mathbb{R}^2$?

- A. A shear transformation
- B. A dilation transformation over the x_1-axis
- C. A projection onto the x_1-axis
- D. A reflection through the origin

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Homework: **Section 1.8 Homework**

Question 7, 1.8.17
Part 3 of 3

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Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Use the fact that T is linear to find the images under T of $4\mathbf{u}$, $3\mathbf{v}$, and $4\mathbf{u} + 3\mathbf{v}$.

The image of $4\mathbf{u}$ is $\begin{bmatrix} 24 \\ 4 \end{bmatrix}$.

The image of $3\mathbf{v}$ is $\begin{bmatrix} -3 \\ 6 \end{bmatrix}$.

The image of $4\mathbf{u} + 3\mathbf{v}$ is $\begin{bmatrix} 21 \\ 10 \end{bmatrix}$.

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Show that the transformation T defined by $T(x_1, x_2) = (3x_1 - 4x_2, x_1 + 4, 5x_2)$ is not linear.

If T is a linear transformation, then $T(\mathbf{0}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$ for all vectors **u**, **v** in the domain of T and all scalars c, d.

(Type a column vector.)

Check if $T(\mathbf{0})$ follows the correct property to be linear.

$T(0,0) = (3(0) - 4(0), (0) + 4, 5(0))$ Substitute.

$= (0, 4, 0)$ Simplify.

What is true about $T(\mathbf{0})$?

- A. $T(\mathbf{0}) = \mathbf{0}$
- B. $T(\mathbf{0}) \neq \mathbf{0}$
- C. $T(\mathbf{0}) = (1,1,1)$
- D. $T(\mathbf{0}) = 4$

Therefore, T is not linear.

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The given matrix determines a linear transformation T. Find all **x** such that T(**x**) = **0**.

$$\begin{bmatrix} 4 & -2 & 6 & -5 \\ -1 & 3 & -8 & 0 \\ -5 & 7 & 6 & 4 \\ 7 & -5 & 8 & -8 \end{bmatrix}$$

Select the correct choice below and fill in the answer box within your choice.

A. $\mathbf{x} = x_2 \square + x_3 \square + x_4 \square$

B. There is only one vector, which is $\mathbf{x} = \square$.

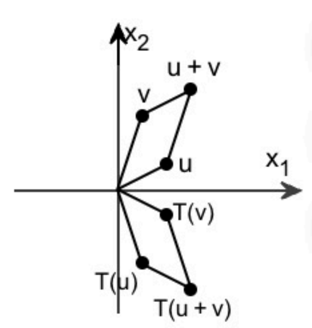
C. $\mathbf{x} = x_4 \begin{bmatrix} 1.5 \\ 0.5 \\ 0 \\ 1 \end{bmatrix}$

D. $\mathbf{x} = x_1 \square + x_4 \square$

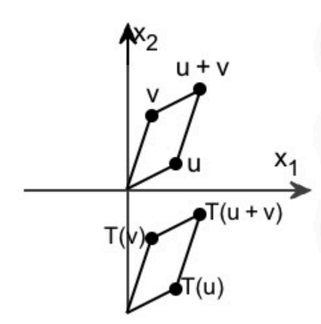
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects each point through the x_1-axis, such that $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ where $\mathbf{x} \mapsto A\mathbf{x}$. Make two sketches that illustrate the two properties of a linear transformation.

Choose the correct graph below that shows the property $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$.

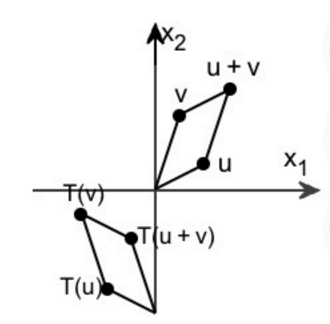
A.



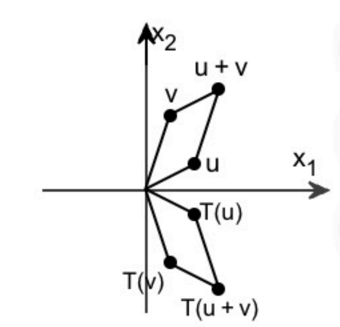
B.



C.

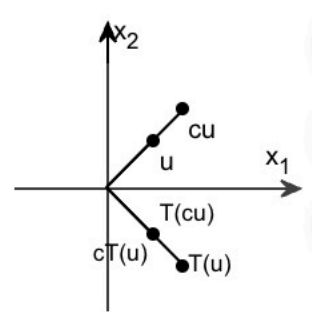


D.

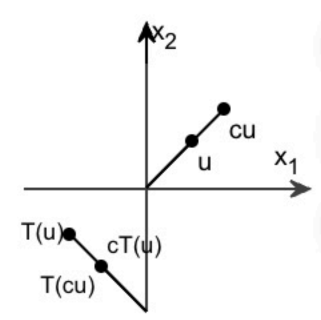


Choose the correct graph below that shows the property $T(c\mathbf{u}) = cT(\mathbf{u})$.

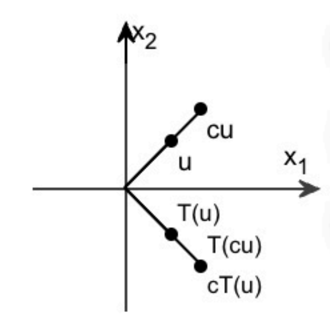
A.



B.



C.



D.