RH 1.6

MATH 5, Jones

Tejas Patel

Refrigerator Homework

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$\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$ RREF: $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

The set of column vectors is linearly dependent, because there are more vectors (4) than the dimension of the space they occupy (at most 3). This means at least one of the columns can be written as a linear combination of the others.

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$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -3 \\ 10 \\ -6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

$c_1 \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 10 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$ a: $c_1(1) + c_2(-3) = 2$

$c_1(-3) + c_2(10) = -7$

$c_1(2) + c_2(-6) = h$

$c_1 - 3c_2 = 2$

$-3c_1 + 10c_2 = -7$

$3c_1 - 9c_2 = 6$

$(3c_1 - 9c_2) + (-3c_1 + 10c_2) = 6 - 7 \qquad c_2 = -1$

$c_1 + 3 = 2$

Thus, v_3 is in Span$\{v_1, v_2\}$ when $h = 4$

b: Solving using the determinant $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 10 & -7 \\ 2 & -6 & h \end{bmatrix}$

$\det(A) = 1\begin{vmatrix} 10 & -7 \\ -6 & h \end{vmatrix}(-3)\begin{vmatrix} -3 & -7 \\ 2 & h \end{vmatrix}2\begin{vmatrix} -3 & 10 \\ 2 & -6 \end{vmatrix}$

$\begin{vmatrix} 10 & -7 \\ -6 & h \end{vmatrix} = 10h + 42 = 10h + 42$

$\begin{vmatrix} -3 & -7 \\ 2 & h \end{vmatrix} = (-3h + 14)$

$\det(A) = 1(10h + 42) + 3(-3h + 14) + 2(-2)$

$h + 80 = 0 \qquad h = -80$

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Since one of the vectors v_3 is the zero vector, the set of vectors is linearly dependent

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False The columns of a matrix A are linearly independent if the only solution to $Ax = 0$ is the trivial solution $x = 0$.

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False: If S is a linearly **dependent** set, then at least one vector in S can be written as a linear combination of the others

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True: The set x, y being linearly independent means that neither x nor y can be written as a linear combination of the other.
The set x, y, z being linearly dependent means that there exists a nontrivial linear combination such that: $c_1x + c_2y + c_3z = 0$

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Since A is a 3×3 matrix with linearly independent columns, this means:

1. The columns form a basis for $\mathbb{R}^3$, so the matrix is invertible.
2. The rank of A is 3, meaning all three rows contain leading 1s in its echelon form.
3. The reduced row echelon form (RREF) of A must be the identity matrix, because an invertible matrix always reduces to the identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

Since A has linearly independent columns, it has full rank (3). This guarantees that its echelon form will always have three pivot positions, leading to either:

1. The identity matrix (in RREF).
2. An upper triangular form (in REF).

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a: n b: A matrix A with n columns has linearly independent columns if the only solution to $A\mathbf{x} = 0$ is the trivial solution ($\mathbf{x} = 0$).

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$\{-1, -1, 1\}$

42

False
The set $\{v_1, v_2, v_3, v_4\}$ is linearly independent if no vector in the set is a linear combination of the others. The given statement only ensures v_3 is independent of v_1, v_2, v_4 , but it does not rule out dependencies involving all four vectors.

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True

• A set of vectors is linearly independent if the only solution to:
$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$
is the trivial solution $c_1 = c_2 = c_3 = c_4 = 0$.

• Since v_1, v_2, v_3, v_4 are linearly independent, no vector in this set can be written as a linear combination of the others.

• Now consider the subset $\{v_1, v_2, v_3\}$. If we assume:
$c_1v_1 + c_2v_2 + c_3v_3 = 0$

then we can rewrite it as $c_1v_1 + c_2v_2 + c_3v_3 + 0 \cdot v_4 = 0$

Since v_1, v_2, v_3, v_4 were already independent, the only solution to this equation is:

$c_1 = c_2 = c_3 = 0$

This confirms that $\{v_1, v_2, v_3\}$ is also linearly independent

Computer Homework

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Determine if the columns of the matrix form a linearly independent set.
Justify your answer.

$$\begin{bmatrix} 0 & -8 & 16 \\ 3 & 1 & -14 \\ -1 & 5 & -5 \\ 1 & -5 & -2 \end{bmatrix}$$

Select the correct choice below and fill in the answer box within your choice.
(Type an integer or simplified fraction for each matrix element.)

A. If A is the given matrix, then the augmented matrix $\begin{bmatrix} 0 & -8 & 16 & 0 \\ 3 & 1 & -14 & 0 \\ -1 & 5 & -5 & 0 \\ 1 & -5 & -2 & 0 \end{bmatrix}$ represents the equation $A\mathbf{x} = \mathbf{0}$. The reduced echelon form of this matrix indicates that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Therefore, the columns of A form a linearly independent set.

B. If A is the given matrix, then the augmented matrix ☐ represents the equation $A\mathbf{x} = \mathbf{0}$. The reduced echelon form of this matrix indicates that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Therefore, the columns of A do not form a linearly independent set.

C. If A is the given matrix, then the augmented matrix ☐ represents the equation $A\mathbf{x} = \mathbf{0}$. The reduced echelon form of this matrix indicates that $A\mathbf{x} = \mathbf{0}$ has more than one solution. Therefore, the columns of A do not form a linearly independent set.

D. If A is the given matrix, then the augmented matrix ☐ represents the equation $A\mathbf{x} = \mathbf{0}$. The reduced echelon form of this matrix indicates that $A\mathbf{x} = \mathbf{0}$ has more than one solution. Therefore, the columns of A form a linearly independent set.

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Determine if the columns of the matrix form a linearly independent set.

$$\begin{bmatrix} 1 & 3 & -3 & 5 \\ 2 & 7 & -3 & 1 \\ 2 & 8 & 3 & -17 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box(es) within your choice.

A. The columns of the matrix do not form a linearly independent set because there are more entries in each vector, ☐, than there are vectors in the set, ☐.
(Type whole numbers.)

B. The columns of the matrix do not form a linearly independent set because the set contains more vectors, 4, than there are entries in each vector, 3.
(Type whole numbers.)

C. Let A be the given matrix. Then the columns of the matrix form a linearly independent set since the vector equation, $A\mathbf{x} = \mathbf{0}$, has only the trivial solution.

D. The columns of the matrix form a linearly independent set because at least one vector in the set is a constant multiple of another.

Homework: **Section 1.7 Homework**

Question 3, 1.7.17

HW Score: 100%, 10 of 10 points
Points: 1 of 1

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Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 4 \end{bmatrix}$$

Choose the correct answer below.

A. The set of vectors is linearly independent because ☐ times the first vector is equal to the second vector.
(Type an integer or a simplified fraction.)

B. The set of vectors is linearly dependent because ☐ times the first vector is equal to the third vector.
(Type an integer or a simplified fraction.)

C. The set of vectors is linearly dependent because one of the vectors is the zero vector.

D. The set of vectors is linearly independent because none of the vectors are multiples of the other vectors.

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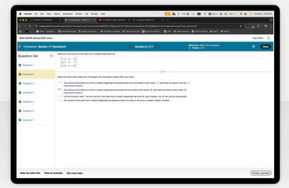

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Homework: **Section 1.7 Homework**

Question 4, 1.7.33

HW Score: 100%, 10 of 10 points
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Suppose A is a 7×5 matrix. How many pivot columns must A have if its columns are linearly independent? Why?

Select the correct answer below.

A. The matrix must have 5 pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of A would be linearly dependent.

B. The matrix must have ___ pivot columns. If A had fewer pivot columns, then the equation $A\mathbf{x} = \mathbf{0}$ would have only the trivial solution.

C. The matrix must have ___ pivot columns. The statements "A has a pivot position in every row" and "the columns of A are linearly independent" are logically equivalent.

D. None of the columns of A are pivot columns. Any column of A that is a pivot column is linearly dependent with the other pivot columns.

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Question list

Suppose A is a 5×7 matrix. How many pivot columns must A have if its columns span $\mathbb{R}^5$? Why?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The matrix must have ___ pivot columns. If A had fewer pivot columns, then the equation $A\mathbf{x} = \mathbf{0}$ would have only the trivial solution.

B. The matrix must have ___ pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of A would not span $\mathbb{R}^5$.

C. The matrix must have 5 pivot columns. The statements "A has a pivot position in every row" and "the columns of A span $\mathbb{R}^5$" are logically equivalent.

D. The columns of a 5×7 matrix cannot span $\mathbb{R}^5$ because having more columns than rows makes the columns of the matrix dependent.

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Homework: **Section 1.7 Homework**

Question 6, 1.7.37

HW Score: 100%, 10 of 10 points
Points: 2 of 2

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Given $A = \begin{bmatrix} 8 & 3 & 5 \\ -3 & 3 & -6 \\ -8 & -1 & -7 \\ 1 & 0 & 1 \end{bmatrix}$, observe that the first column is the sum of the second and third columns. Find a nontrivial solution of $A\mathbf{x} = \mathbf{0}$ without performing row operations. [Hint: Write $A\mathbf{x} = \mathbf{0}$ as a vector equation.]

$\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

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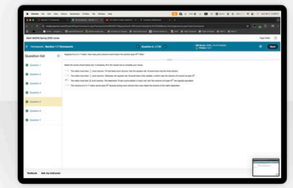

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Determine if **v** is in the set spanned by the columns of B.

$$B = \begin{bmatrix} 3 & -3 & 2 \\ 9 & -6 & 8 \\ 6 & 3 & 13 \\ 9 & -3 & 16 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 0 \\ 15 \\ 45 \\ 30 \end{bmatrix}$$

Choose the correct answer below and, if necessary, fill in the answer box(es) to complete your choice.

A. Vector **v** is not in the set spanned by the columns of B because the reduced echelon form of the matrix formed by writing B with a fourth column equal to **v** is ☐.

B. Vector **v** is not in the set spanned by the columns of B because the columns of B, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ are linearly independent.

C. Vector **v** is in the set spanned by the columns of B because $5\,\mathbf{b}_1 + 5\,\mathbf{b}_2 + 0\,\mathbf{b}_3 = \mathbf{v}$.

D. Vector **v** is in the set spanned by the columns of B because the columns of B span $\mathbb{R}^4$.