Project 1, Transformations

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Matrices defined: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 10 \end{bmatrix}$, $V = \begin{bmatrix} 1 & 4 & 0 \end{bmatrix}$

When `show(A), show(B), show(V)` was input into the python terminal, the output from SageMath was $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 10 \end{bmatrix}$, $\begin{bmatrix} 1 & 4 & 0 \end{bmatrix}$ respectively. The matrices were repeated back to me as I originially entered them

$$\text{For } QQ \to RR \Rightarrow \begin{bmatrix} 1.00000000000000 & 3.00000000000000 \\ 2.00000000000000 & 4.00000000000000 \end{bmatrix}$$

$$\text{For } QQ \to RDF \Rightarrow \begin{bmatrix} 1.0 & 2.0 \\ 3.0 & 4.0 \end{bmatrix}$$

RR and RDF seem to be data types for the matrices. RR appears to be an aribtrary precision floating point, outputting the maximum number of decimal places allowed. It won't be as precise as QQ, which is symbolic math, nor will it be as fast as RDF, which seems to be reduced decimal form, but it will get you a precise decimal answer when youre looking for one.

$$\text{RREF}(B) = \begin{bmatrix} 1 & 0 & \frac{21}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

The command `I4 = identity matrix(n)` creates an identity matrix of the degree of the input `n`.

For the original command: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ For input $= 2$: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ For input $= 5$: $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

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New matrices defined: $C = \begin{bmatrix} 3 & 7 \\ 1 & 6 \end{bmatrix}$

a: In SageMath, to add: `A+C` results in $\begin{bmatrix} 4 & 9 \\ 4 & 10 \end{bmatrix}$, to subtract `A-C` results in $\begin{bmatrix} -2 & -5 \\ 2 & -2 \end{bmatrix}$

Lastly, to multiply: `A*C` results in $\begin{bmatrix} 5 & 19 \\ 13 & 45 \end{bmatrix}$, bonus: `det(A*C)` results in -22

b:

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ -1 & 10 \end{bmatrix} \text{ shows that } \forall x \in B\,\{x_{ij} \to x_{ji}\}$$

`A*C==C*A` returned `false`, meaning $AC \neq CA$

`(A*C).transpose()==A.transpose()*C.transpose()` returned `false`, meaning $(AC)^T \neq A^T C^T$

`(A+C).transpose()==A.transpose()+C.transpose()` returned `true`, meaning $(A+C)^T = A^T + C^T$

`(c*A).transpose()==c*A.transpose()` returned `true` for $c = 6$ and $c = 10$, meaning $(cA)^T = c * A^T$

The left number of columns in the left matrix must equal the number of rows in the right matrix. If m is the number of columns in A and p is the number of rows in B, then $m = p$ must be true. The output of matrix multiplication is going to be of dimension $n \times q$

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a: The output from the code is 4 graphs, each one containing 2 vectors. The blue arrows represent $P1, P2, P3, P4$ and the purple arrows represent $Q1, Q2, Q3, Q4$. See the output tikzpictures here

b: Reflects across the y axis (or x_2 aixs). Vector $(2, -4)$ becomes $(-2, -4)$

c: Reflects across $y = x$, $(3, 7)$ becomes $(7, 3)$ and $(1, 0)$ becomes $(0, 1)$

d: Reflects across $y = -x$, $(3, 7)$ becomes $(-7, -3)$ and $(1, 0)$ becomes $(0, -1)$

e: Maintains x value, sets y value to 0. $(3, 7)$ becomes $(3, 0)$ and $(0, 1)$ becomes $(0, 0)$

f: Pretty much using the reciprocal of the columns of M, the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ resolves the reflection across the x_1 axis, where $(3, 7)$ becomes $(-3, 7)$

g: The transformation $\begin{bmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ rotates the vector it is multiplied by by angle θ counterclockwise. $(3, 7)$ becomes approx. $(-2.828, 7.071)$, creating a $45°$ dihedral angle

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4.1 Derivation

If $\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = a + b$, then $x_1 a + x_2 b = a + b$, meaning $x_1 = 1,\ x_2 = 1$

If $\begin{bmatrix} x_3 & x_4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = a - b$, then $x_3 a + x_4 b = a - b$, meaning $x_3 = 1,\ x_4 = -1$

Combining these together, we get transformation matrix $\boxed{A = T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$

4.2 Check/Proof

Checking $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ with the Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The expected answer is $\begin{bmatrix} 1+0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 0 + -1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

We can observe the answer matches the expected, showing the transformation matrix A is the correct transformation. The answer was checked with SageMath

5 Determining the Transformation Matrix J

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ x_{41} & x_{42} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 5x_2 \\ 0 \\ x_1 + 4x_2 \\ x_2 \end{bmatrix}$$

5.1 Determining individual rows

Row 1: $x_1 x_{11} + x_2 x_{12} = 2x_1 - 5x_2 \Rightarrow x_{11} = 2$ and $x_{12} = -5$

Row 2: $x_1 x_{21} + x_2 x_{22} = 0 \Rightarrow x_{21} = 0$ and $x_{22} = -0$

Row 3: $x_1 x_{31} + x_2 x_{32} = x_1 + 4x_2 \Rightarrow x_{31} = 1$ and $x_{32} = 4$

Row 4: $x_1 x_{41} + x_2 x_{42} = x_2 \Rightarrow x_{41} = 0$ and $x_{42} = 1$

5.2 Combining the results

Combining the results back into the original matrix form: $J = \begin{bmatrix} 2 & -5 \\ 0 & 0 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

6 Extras

6.1 Section 3 tikzpictures