Project 2, Matrix Algebra, Markov Chains
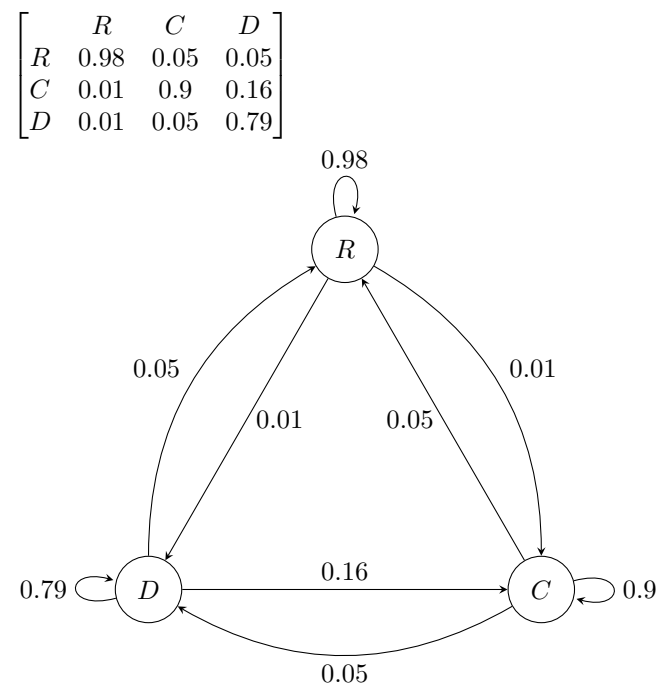
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Matrices Defined: $x_0 = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.6 \end{bmatrix}$, $A = \begin{bmatrix} 0.98 & 0.05 & 0.05 \\ 0.01 & 0.9 & 0.16 \\ 0.01 & 0.05 & 0.79 \end{bmatrix}$

a: State Diagram

$$\begin{bmatrix} & R & C & D \\ R & 0.98 & 0.05 & 0.05 \\ C & 0.01 & 0.9 & 0.16 \\ D & 0.01 & 0.05 & 0.79 \end{bmatrix}$$

b:

The weight of $D \to C$ is 0.16, meaning 16% of disposable cup users will switch to compostable cups

c:

$$\vec{x_1} = \begin{bmatrix} 0.98 & 0.05 & 0.05 \\ 0.01 & 0.9 & 0.16 \\ 0.01 & 0.05 & 0.79 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.2 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.2*0.98 + 0.2*0.05 + 0.6*0.05 \\ 0.2*0.01 + 0.2*0.9 + 0.6*0.16 \\ 0.2*0.01 + 0.2*0.05 + 0.6*0.79 \end{bmatrix} = \begin{bmatrix} 0.236 \\ 0.278 \\ 0.486 \end{bmatrix}$$

This means after the first year passes, 23.6% of people use Recyclable cups, 27.8% use compostable, and 48.6% of people use disposable cups

d: Computed using SageMath:

$\vec{x_1}$	$\vec{x_2}$	$\vec{x_3}$	$\vec{x_4}$	$\vec{x_5}$	$\vec{x_6}$	$\vec{x_7}$	$\vec{x_8}$	$\vec{x_9}$
$\begin{bmatrix} 0.236 \\ 0.278 \\ 0.486 \end{bmatrix}$	$\begin{bmatrix} 0.2695 \\ 0.3303 \\ 0.4002 \end{bmatrix}$	$\begin{bmatrix} 0.3006 \\ 0.3640 \\ 0.3354 \end{bmatrix}$	$\begin{bmatrix} 0.3296 \\ 0.3843 \\ 0.2861 \end{bmatrix}$	$\begin{bmatrix} 0.3565 \\ 0.3949 \\ 0.2486 \end{bmatrix}$	$\begin{bmatrix} 0.3815 \\ 0.3988 \\ 0.2197 \end{bmatrix}$	$\begin{bmatrix} 0.4048 \\ 0.3979 \\ 0.1973 \end{bmatrix}$	$\begin{bmatrix} 0.4265 \\ 0.3937 \\ 0.1798 \end{bmatrix}$	$\begin{bmatrix} 0.4466 \\ 0.3873 \\ 0.1660 \end{bmatrix}$
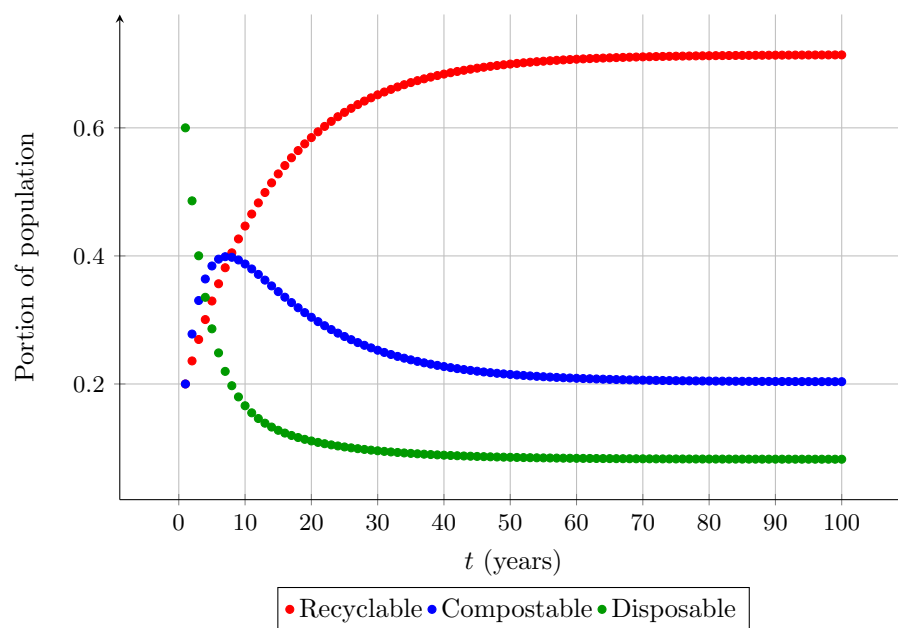
Observations

The percentage of people using Recyclable cups increases at a decreasing rate, compostable cups go up then start dropping, and disposable decreases at a decreasing rate

e:

After infinite time, the portion of the population using each type of cup came to the following:
Recyclable: $\frac{130}{182}$ of the population, or about 71.43%
Compostable: $\frac{37}{182}$ of the population, or about 20.33%
Disposable: $\frac{15}{182}$ of the population, or about 8.24%