Homework 6

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12.1

6

$f = \{(x, 4x+5) : x \in \mathbb{Z}\}$
The domain is $\mathbb{Z}$
Codomain is $\mathbb{Z}$, since all outputs are integers
Range is $4\mathbb{Z} + 1$

10

Yes (x^3, x) is a function from $\mathbb{R}$ to $\mathbb{R}$
Every input has exactly one output and all inputs and outputs are real numbers

12.2

Injective is 1-1, Surjective is onto

6

$(3n - 4m)$
The function is surjective, with every value of $\mathbb{Z}$ being mapped to a linear combination of 3 and -4
It is not injective as the counterexamples are $(4, 3)$ and $(8, 6)$

9

NOT COMPLETE

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$f(x) = \left(\dfrac{x+1}{x-1}\right)^3$ is defined for all $x \neq 1$

Solving for x we get $x = \dfrac{\sqrt[3]{f(x)}}{\sqrt[3]{f(x)} - 1}$, which is also defined for all $f(x), f(x) \neq -1$, meaning the function is surjective.
Take $\dfrac{x_1+1}{x_1-1} = \dfrac{x_2+1}{x_2-1}$
$(x_1+1)(x_2-1) = (x_2+1)(x_1-1)$
$x_1x_2 - x_1 + x - 2 - 1 = x_1x_2 - x_2 + x_1 - 1$
$x_2 - x_1 = x_1 - x_2$ which is only true when $x_1 = x_2$

18

We have $f(n) = \dfrac{(-1)^n(2n-1)+1}{4}$.
For n even, $(-1)^n = 1$ so $f(n) = \dfrac{n}{2}$, giving the nonnegative integers.
For n odd, $(-1)^n = -1$ so $f(n) = \dfrac{1-n}{2}$, giving the nonpositive integers. Both parts are strictly monotone and do not overlap, so f is injective.
For surjectivity, let $k \in \mathbb{Z}$. If $k \geq 0$, set $n = 2k$. Then $f(2k) = k$. If $k \leq 0$, set $n = 1 - 2k$. Then $f(1-2k) = k$. Every integer has a preimage, so f is surjective.
Therefore f is bijective.

12.4

3

$g \cdot f = \{(1,1),(2,2),(3,2)\}$
$f \cdot g = \{(1,1),(2,1),(3,3)\}$

4

$g \cdot f = \{(a,a),(b,a),(c,a)\}$
$f \cdot g = \{(a,c),(b,c),(c,c)\}$

5

$g \cdot f = x + 1$
$f \cdot g = \sqrt[3]{x^3+1}$

6

$g \cdot f = \dfrac{3}{x^2+1} + 2$
$f \cdot g = \dfrac{1}{(3x+2)^2+1}$

10

$f(x,y) = (xy, x^3)$
$(f \cdot f)(x,y) = f(f(x,y))$
$= f(x \cdot y, x^3)$
$= (x \cdot y \cdot x^3, (x \cdot y)^3)$
$= (x^4 y, x^3 y^3)$

12.5

1

Let $y = f(n)$ and $x = n$
$y = 6 - x$.
Injective: If $f(a) = f(b)$, then $6 - a = 6 - b$, so $a = b$. Injective.
Surjective: For any $y \in \mathbb{R}$, solve $y = 6 - x$ to get $x = 6 - y$. Always real. Surjective.
So f is bijective.
Inverse: $y = 6 - x$ rearranges to $x = 6 - y$, so $f^{-1}(x) = 6 - x$.

2

$y = \dfrac{5x-1}{x-2}$.

Injective: Assume $f(a) = f(b)$: $\dfrac{5a-1}{a-2} = \dfrac{5b-1}{b-2}$. Cross-multiplying and simplifying yields $a = b$. Injective.

Surjective: Solve $y = \dfrac{5x-1}{x-2}$: $xy - 2y = 5x - 1$, $xy - 5x = 2y - 1$, $x(y-5) = 2y - 1$, so $x = \dfrac{2y-1}{y-5}$. This works for all $y \neq 5$, so f is surjective onto $\mathbb{R} \setminus \{5\}$.

Thus $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{5\}$ is bijective.

Inverse: $f^{-1}(x) = \dfrac{2x-1}{x-5}$.

3

Let $y = f(n)$ and $x = n$

$y = 2^x$.

Injective: $2^a = 2^b$ implies $a = b$ since the exponential is strictly increasing. Injective.

Surjective: Codomain is $(0, \infty)$. For any $y > 0$, solving $y = 2^x$ gives $x = \log_2(y)$. Surjective.

So $f : \mathbb{R} \to (0, \infty)$ is bijective.

Inverse: $f^{-1}(x) = \log_2(x)$.

4

$y = e^(x^3 + 1)$

$\ln(y) = x^3 + 1$

$\ln(y) - 1 = x^3$

$x = \sqrt[3]{\ln(y) - 1} \Rightarrow$

$y = \sqrt[3]{\ln(x) - 1}$

6

Given $(x, y) = f(m, n)$, we solve $5m + 4n = x$ and $4m + 3n = y$. Subtracting $3(5m + 4n) = 3x$ from $4(4m + 3n) = 4y$ gives $m = 4y - 3x$. Substituting into $4m + 3n = y$ yields $n = 4x - 5y$. Thus $f^{-1}(x, y) = (4y - 3x,\ 4x - 5y)$

12.6

1

$f(x) = x^2 + 3 \quad f'(x) = 2x$

$2x = 0$ at $x = 0$, $f(x) = 3$

$f(-3) = 12, \quad f(5) = 28$

So the image is $[3, 28]$

$f^-1([12, 19])$

$12 \leq x^2 + 3 \leq 19$ is true when $3 \leq |x| \leq 4$

So, $f^-1([12, 19]) = [-4, -3] \cup [3, 4]$

2

$f = \{(1, 3), (2, 8), (3, 3), (4, 1), (5, 2), (6, 4), (7, 6)\}$

$f(\{1, 2, 3\}) = \{3, 8\}$

$f(\{4, 5, 6, 7\}) = \{1, 2, 4, 6\}$

$f(\varnothing) = \varnothing$

No element maps to any of those outputs so $f^{-1}(\{0, 5, 9\}) = \varnothing$

$f^{-1}(\{0, 3, 5, 9\}) = \{1, 3\}$

10

First we prove $f^{-1}(Y \cap Z) \subseteq f^{-1}(Y) \cap f^{-1}(Z)$.
Let $a \in f^{-1}(Y \cap Z)$. By definition of preimage, this means $f(a) \in Y \cap Z$. Therefore $f(a) \in Y$ and $f(a) \in Z$.
Again using the definition of preimage, we get $a \in f^{-1}(Y)$ and $a \in f^{-1}(Z)$.
Hence $a \in f^{-1}(Y) \cap f^{-1}(Z)$, so $f^{-1}(Y \cap Z) \subseteq f^{-1}(Y) \cap f^{-1}(Z)$
Next, the reverse inclusion $f^{-1}(Y) \cap f^{-1}(Z) \subseteq f^{-1}(Y \cap Z)$.
Let $a \in f^{-1}(Y) \cap f^{-1}(Z)$. Then $a \in f^{-1}(Y)$ and $a \in f^{-1}(Z)$, so $f(a) \in Y$ and $f(a) \in Z$.
Hence $f(a) \in Y \cap Z$, which means $a \in f^{-1}(Y \cap Z)$. Therefore $f^{-1}(Y) \cap f^{-1}(Z) \subseteq f^{-1}(Y \cap Z)$
Since we have both inclusions, the two sets are equal: $f^{-1}(Y \cap Z) = f^{-1}(Y) \cap f^{-1}(Z)$

1 Newtons Method

Derivative of $y = x^2 - x^3 + 2 \Rightarrow y' = 2x - 3x^2$
Start off with guess $n = 1.5$
$1.5 - \dfrac{0.875}{-3.75} = 1.7\bar{3}$
Now with $1.7\bar{3}$, rounded to 1.7333
$1.7333 - \dfrac{-0.203074}{-5.54639} \approx 1.696686$
With 1.696686
$1.696686 - \dfrac{-0.00558021}{-5.24286} \approx 1.6956216$
One more iteration for good measure, the intended number is $\equiv 1.69562076955986$
$1.6956216 - \dfrac{-0.000004346649275}{-5.2341546312} \equiv 1.6956207695604$ yeah thats close enough