Homework 1

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1

1.1 A14, B25, B28, C32, C36, D44, D50

A14

$5x : |2x| \leq 8$ and $x \in \mathbb{Z}$
$x = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} \Rightarrow \boxed{\{-20, -15, -10, -5, 0, 5, 10, 15, 20\}}$

B25

$\left\{\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8\right\}$
The pattern is $x = 2^n$ where n is an integer and the set is $\boxed{S = \{x \in \mathbb{R} : x = 2^n, n \in \mathbb{Z}\}}$

B28

$\left\{\frac{-3}{2}, \frac{-3}{4}, 0, \frac{3}{4}, \frac{3}{2}, \frac{9}{4}, \frac{12}{4}, \frac{15}{4}, \frac{9}{2}\right\}$
The pattern is $S = \left\{\frac{3}{4}x, x \in \mathbb{Z}\right\}$

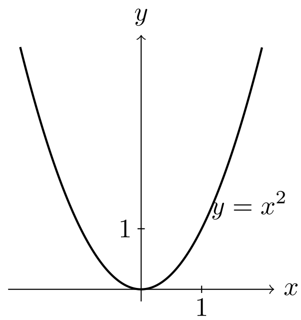
C32

$|\,\{\{\{1,4\}, a, b, \{\{3,4\}\}, \{\varnothing\}\}\}\,|$
Cardinality: 1. Contains 1 subset

C36

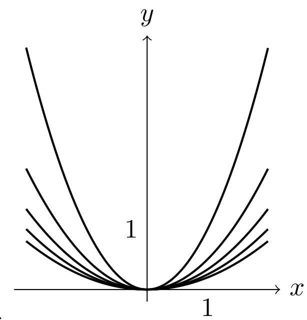
$\{x \in \mathbb{N} : x^2 < 10\}$
Elements: $\{1, 2, 3\}$ where $\mathbb{N} \equiv \mathbb{Z}^+$
Cardinality: 3

D44



$\{(x, x^2) : x \in \mathbb{R}\}$

D50 for y=1,2,3,4,5



$\left\{(x, \frac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\right\}$

1.2 A2, A3, B14, B20

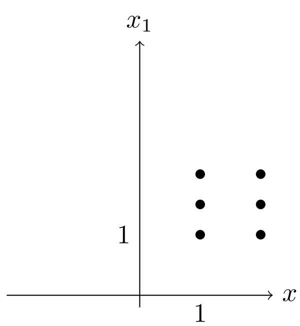
A2

$A = \{\pi, e, 0\}, B = \{0, 1\}$
(a): $A \times B = \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$
(b): $B \times A = \{(0, \pi), (1, \pi), (0, e), (1, e), (0, 0), (1, 0)\}$
(c): $A \times A = \{(\pi, \pi), (e, \pi), (0, \pi), (e, e), (\pi, e), (0, e), (e, 0), (\pi, 0), (0, 0)\}$
(d): $B \times B = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
(e): $A \times \varnothing = \varnothing$, no possible pairs
(f): $(A \times B) \times B = \{((\pi, 0), 0), ((\pi, 1), 0), ((e, 0), 0), ((e, 1), 0), ((0, 0), 0), ((0, 1), 0),$
$((\pi, 0), 1), ((\pi, 1), 1), ((e, 0), 1), ((e, 1), 1), ((0, 0), 1), ((0, 1), 1)\}$
(g): $A \times (B \times B) = \{(\pi, (0, 0)), (\pi, (0, 1)), (\pi, (1, 0)), (\pi, (1, 1)), (e, (0, 0)),$
$(e, (0, 1)), (e, (1, 0)), (e, (1, 1)), (0, (0, 0)), (0, (0, 1)), (0, (1, 0)), (0, (1, 1))\}$
(h): $A \times B \times B = (\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1), (e, 0, 0), (e, 0, 1), (e, 1, 0), (e, 1, 1), (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)$

A3

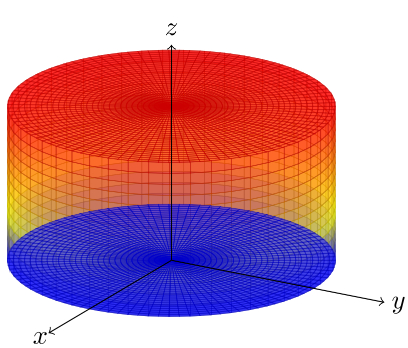
$\left\{x \in \mathbb{R} : x^2 = 2\right\} \times \{a, c, e\}$
$= \left\{-\sqrt{2}, \sqrt{2}\right\} \times \{a, c, e\}$
$= \left\{(-\sqrt{2}, a), (-\sqrt{2}, c), (-\sqrt{2}, e), (\sqrt{2}, a), (\sqrt{2}, c), (\sqrt{2}, e)\right\}$

B14



$[1, 2] \times \{1, 1.5, 2\}$

B20



1.3 A3, A6, B12, C14, C15

A3

$\{\{\mathbb{R}\}\}$
$\{\varnothing, \{\mathbb{R}\}\}$

A6

$\{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}$
$\{\varnothing\}, \{\mathbb{R}, \mathbb{Q}\}, \{\mathbb{R}, \mathbb{N}\}, \{\mathbb{Q}, \mathbb{N}\}, \{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{N}\}$

B12

$X : X \subseteq \{3, 2, a\}$ and $|X| = 4$
$\{3, 2, a\}$

C14

$\mathbb{R}^2 \subseteq \mathbb{R}^3$
No, its a 2 dimensional space in $\mathbb{R}^3$, not a subset. For it to be a subset it would need to have 3 axes

C15

$\{(x, y) \in \mathbb{R}^2 : x - 1 = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\}$
True, $\{1\}$ is a subset of $\{1, 0\}$

1.4 A5, A12, B17, B18

A5

$\mathscr{P}(\mathscr{P}(\{2\}))$
$\mathscr{P}(\{\{2\}\}) = \{\varnothing, 2\}$
$\mathscr{P}(\mathscr{P}(2)) = \{\varnothing, \{\varnothing\}, \{2\}, \{\varnothing, 2\}\}$

A12

$\{X \in \mathscr{P}(\{1, 2, 3\}) : 2 \in X\}$
One of each length, following pascals triangle up to the length of the original set. $\{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

B17

$|\{X \in \mathscr{P}(A) : |X| \leq 1\}|$
$m + 1$

B18

$|\mathscr{P}(A \times \mathscr{P}(B))|$
$|\mathscr{P}(A \times \mathscr{P}(B))| = 2^{(A \times \mathscr{P}(B))} = 2^{(m \cdot 2^n)}$

1.5 4,7

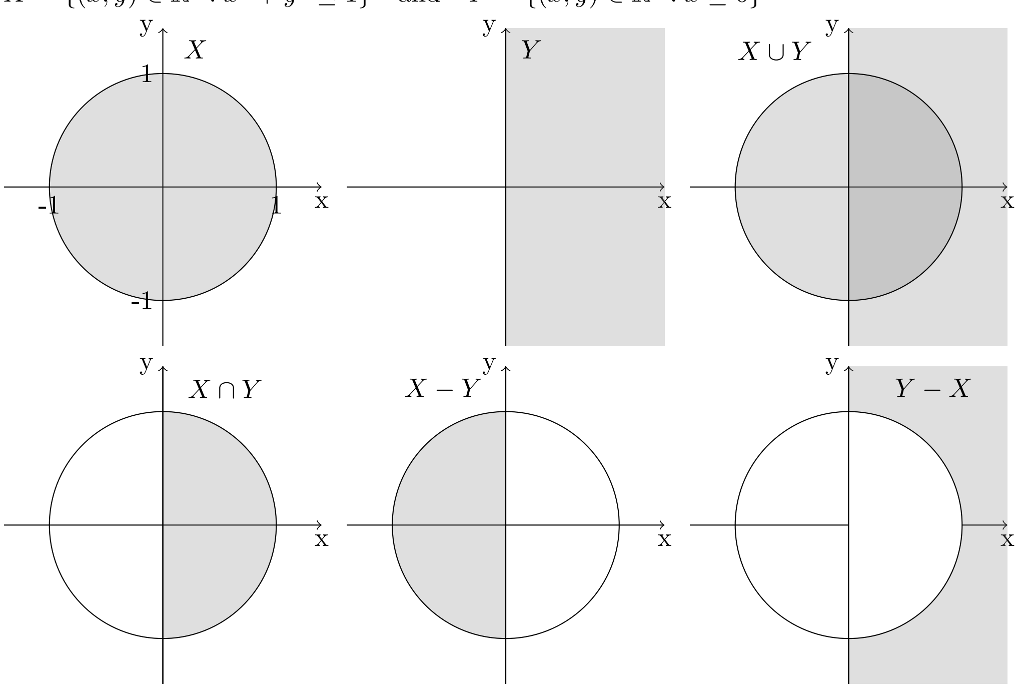
4

Suppose $A = \{b, c, d\}$ and $B = \{a, b\}$. Find:

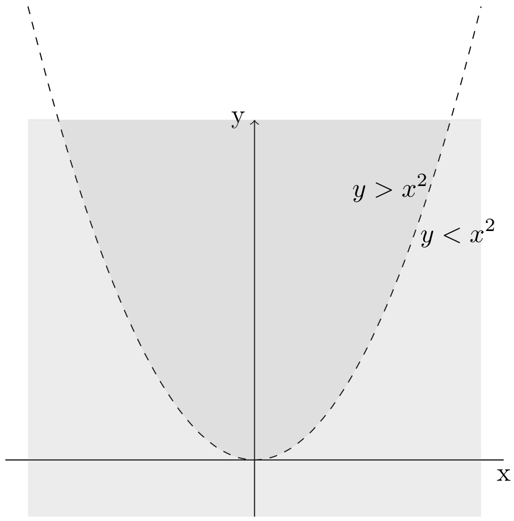
(a)$(A \times B) \cap (B \times B)$ (b)$(A \times B) \cup (B \times B)$
(c)$(A \times B) - (B \times B)$ (d)$(A \cap B) \times A$
(e)$(A \times B) \cap B$ (f)$\mathscr{P}(A) \cap \mathscr{P}(B)$
(g)$\mathscr{P}(A) - \mathscr{P}(B)$ (h)$\mathscr{P}(A \cap B)$
(i)$\mathscr{P}(A) \times \mathscr{P}(B)$
(a): $\{\{b, b\}, \{b, a\}\}$
(b): $\{\{b, a\}, \{b, b\}, \{c, a\}, \{c, b\}, \{d, a\}, \{d, b\}, \{a, b\}, \{a, a\},\}$
(c): $\{\{c, a\}, \{c, b\}, \{d, a\}, \{d, b\}\}$
(d): $\{\{b, b\}, \{b, c\}, \{b, d\}\}$
(e): $\varnothing$
(f): $\{\{\varnothing\}, \{b\}\}$
(g): $\{\varnothing, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\} - \{\varnothing, \{a\}, \{b\}, \{a, b\}\} = \boxed{\{\{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}}$
(h): $\{\varnothing\}, \{\varnothing, b\}$
(i): $\mathscr{P}(A) = \{\varnothing, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$
$\mathscr{P}(B) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$
$\mathscr{P}(A) \times \mathscr{P}(B) = \{(\varnothing, \varnothing), (\varnothing, \{a\}), (\varnothing, \{b\}), (\varnothing, \{a, b\})(\{b\}, \varnothing), (\{b\}, \{a\}), (\{b\}, \{b\}), (\{b\}, \{a, b\})$
$(\{c\}, \varnothing), (\{c\}, \{a\}), (\{c\}, \{b\}), (\{c\}, \{a, b\})\ (\{d\}, \varnothing), (\{d\}, \{a\}), (\{d\}, \{b\}), (\{d\}, \{a, b\})$
$(\{b, c\}, \varnothing), (\{b, c\}, \{a\}), (\{b, c\}, \{b\}), (\{b, c\}, \{a, b\})\ (\{b, d\}, \varnothing), (\{b, d\}, \{a\}), (\{b, d\}, \{b\}), (\{b, d\}, \{a, b\})$
$(\{c, d\}, \varnothing), (\{c, d\}, \{a\}), (\{c, d\}, \{b\}), (\{c, d\}, \{a, b\})\ (\{b, c, d\}, \varnothing), (\{b, c, d\}, \{a\}), (\{b, c, d\}, \{b\}), (\{b, c, d\}, \{a, b\})\}$

7

$X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $Y = \{(x, y) \in \mathbb{R}^2 : x \geq 0\}$

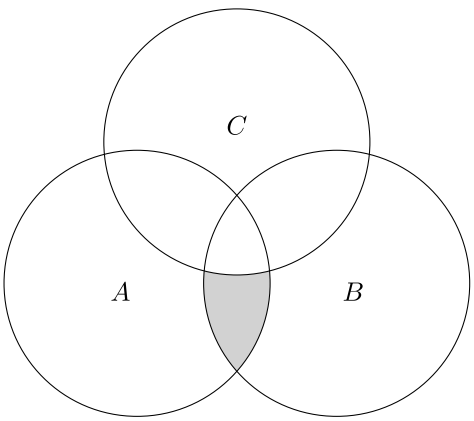


1.6 6



$X = \{(x, y) \in \mathbb{R}^2 : y < x^2\}$

1.7 9



$(A \cap B) - C$

1.8 9,10,12

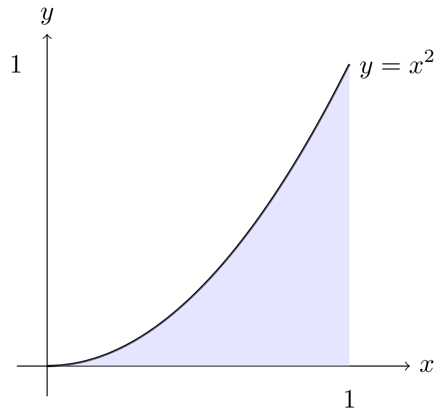
9

$\bigcup_{X \in \mathscr{P}(\mathbb{N})} X =$ and $\bigcap_{X \in \mathscr{P}(\mathbb{N})} X =$

(a): The collection of all elements from the power sets of all naturals results in the set of all naturals. $\boxed{\mathbb{N}}$
(b): The intersection of elements from the power sets of all naturals results in only the empty set, which is present in all power sets. $\boxed{\varnothing}$

10

$\bigcup_{x\in[0,1]} [x,1]\times[0,x^2] =$ and $\bigcap_{x\in[0,1]} [x,1]\times[0,x^2] =$



(b): Expanding out the cross set for both values of x, we get an intersection of $\{1,0\}$ for both $x=1$ and $x=0$

12

$\bigcap_{\alpha\in I} A_\alpha = \bigcup_{\alpha\in I} A_\alpha$

We can state that the sets are identical, as if they were not, then the intersection would be a subset of the union, not equal to it