Homework 2

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3 October, 2025

2.1

6

Some sets are finite
Yes, it is a statement and its truth value is $\boxed{\text{True}}$. The example needed is $\{1, 2\}$

8

$\mathbb{N} \notin \mathscr{P}(\mathbb{N})$ Yes it is a statement, and it is $\boxed{\text{False}}$. Every set is always an element in its own Power Set

10

$(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$
Yes it is a statement, and it is $\boxed{\text{True}}$. Since $\mathbb{N} \subset \mathbb{R}$, the statement holds

2.2

6

There is a quiz scheduled for Wednesday or Friday.
$W \vee F$ means the quiz will be delivered on W (Wednesday) or F (Friday)

8

At least one of the numbers x and y equals 0.
$\exists n \in \{x, y\} : n = 0$
or $(x = 0) \vee (y = 0)$

10

$x \in A \cup B$
$x \in A \vee x \in B$

2.3

2

For a function to be continuous, it is sufficient that it is differentiable.
If differentiable, then continuous

8

A geometric series with ratio r converges if $|r| < 1$
If ratio $|r| < 1$ then the geometric series converges.

10

The discriminant is negative only if the quadratic equation has no real solutions.
If no real solutions for quadratic equation, then discriminant is negative.

2.4

2

If a function has a constant derivative then it is linear, and conversely
A function is linear if and only if it has a constant derivative

4

If $a \in \mathbb{Q}$ then $5a \in \mathbb{Q},$ and if $5a \in \mathbb{Q}$ then $a \in \mathbb{Q}$
$\boxed{a \in \mathbb{Q} \Leftrightarrow 5a \in \mathbb{Q}}$

2.5

4

P	Q	¬ P	¬ (P ∨ Q)	¬ (P ∨ Q) ∨ (¬ P)
T	T	F	F	F
F	T	T	F	T
T	F	F	F	F
F	F	T	T	T

6

P	Q	P ∧¬ P	$(P \wedge \neg P) \wedge Q$
T	T	F	F
F	T	F	F
T	F	F	F
F	F	F	F

8

P	Q	R	Q $\wedge\neg$ R	P $\vee$ Q $\wedge\neg$ R
T	T	T	F	T
F	T	T	F	F
T	F	T	F	T
F	F	T	F	F
T	T	F	T	T
F	T	F	T	T
T	F	F	F	T
F	F	F	F	F

2.6

A2

P	Q	R	Q $\wedge$ R	P $\vee$ Q	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	F	T	F	F	T	F	F
T	T	F	F	T	T	T	T
F	T	F	F	T	F	F	F
T	F	F	F	T	T	T	T
F	F	F	F	F	F	F	F

B10

$(P \rightarrow Q) \vee R \equiv \neg((P \wedge \neg Q) \wedge \neg R)$
$(\neg P \vee Q) \vee R \equiv \neg(P \wedge \neg Q) \vee R$
$(\neg P \vee Q) \vee R \equiv \neg P \vee Q \vee R$
$\neg P \vee Q \vee R \equiv \neg P \vee Q \vee R$
The statements are logically equivalent

2.7

2

$\forall x \in \mathbb{R}, \exists n \in \mathbb{N},\, x^n \geq 0$
True
For all real numbers x, there exists an integer n where $x^n \geq 0$

4

$\forall X \in \mathscr{P}(\mathbb{N}),\, X \subseteq \mathbb{R}$
All subsets X of the power set of $\mathbb{N}$ are subsets of the set of all real numbers $\mathbb{R}$
True, $\mathbb{N} \subset \mathbb{R}$ so the relation will apply to all subsets of $\mathbb{N}$ too

6

$\exists n \in \mathbb{N}, \forall X \in \mathscr{P}(\mathbb{N}), |X| < n$ False
There exists a natural number n where all subsets X of the power set of $\mathbb{N}$ have a cardinality less than n
False because $\mathbb{N}$ is an infinite set, meaning no matter what value of n you choose, there will always be a bigger subset of size $n+1$ and n can not be of infinite size

2.10

2

If x is prime, then $\sqrt{x}$ is not a rational number.
x is prime and $\sqrt{x}$ is a rational number.

4

For every positive number ε, there is a positive number δ such that $|x-a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.
There exists a positive number ϵ for which all positive numbers δ satisfy $|x-a| < \delta$ and $|f(x) - f(a) \geq \epsilon$
$\exists \epsilon > 0, \forall \delta > 0 : |x-a| < \delta \wedge |f(x) - f(a)| \geq \epsilon$

6

There exists a real number a for which $a + x = x$ for every real number x. For all real numbers a there is a number x where $a + x \neq x$