Homework 1

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2 October, 2025

2.28

Applicants defined:
Applicants 1-3 are part of the plurality group, and applicant 4 is in the minority group The set of applicants is defined as $S = \{1, 2, 3, 4\}$

a

Possible outcomes: $(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)$

b

Assuming each outcome has a random probability of occuring, the probability of each one happening is $\frac{1}{6}$
There is a discrete set of outcomes, quantity N so the probability of each one occuring is $\frac{1}{N}$

c

Of the 6 outcomes, 3 include applicant 4, making their chance of hire $\frac{3}{6}$ or $\frac{1}{2}$

2.60

a

$\frac{n_a}{N}$ where $n_a = \frac{k!}{(k-a)!}$ and $N = k^a$ making $\frac{k!}{(k-a)! \cdot k^a}$ where k is 365 and a is the number of people
For this problem we can use $k = 365$ to simplify the expression to $\frac{365!}{(365-a)! \cdot 365^a}$
Then, to get the opposite, where 2 people share a birthday, we subtrace that from 1, getting $1 - \frac{365!}{(365-a)! \cdot 365^a}$

b

Plugging in

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\frac{365!}{(365-x)! \cdot 365^x} = 0.5
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into Wolfram, the output is $x = 22.7677$, meaning when 23 people are present the probability of a shared birthday is above 50%

2.64

This is a classic selection with replacement problem. We are looking for all unique numbers in 6 rolls. For the first roll, the probability is $\frac{6}{6}$, since all numbers are on the board. The 2nd is $\frac{5}{6}$, the third has 4 unchosen numbers so it is $\frac{4}{6}$ so on and so forth. $\frac{k!}{k^n}$ where $k = 6, n = 6$ is $\frac{5}{324} \approx 0.0154\%$

2.82

Suppose that $A \subset B$ and that $P(A) > 0$ and $P(B) > 0$. Show that $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$

1: If outcome set A is a subset of outcome set B, then every outcome part of A must also be part of outcome set B too, giving the proposition $P(B|A)$ a 100% chance of being true. $\boxed{P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1}$

2: If we know the outcome was from the set of outcomes B and A is known to be a subset of B, $\frac{|A|}{|B|}$ will be from A $\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}}$

2.88

a

Yes. It is possible the overlap between A and B is 0.1. As long as it is under 0.3 (the smallest individual probability), it can be valid

b

0. Anything below 0.3 is valid, as that is the lowest probability of an individual outcome set

c

Not possible. 0.7 is greater the maximum of 0.3

d

The largest value of $A \cap B$ is 0.3 as that is the lowest probability of an individual outcome set and occurs when $A \subset B$

2.96

$P(A \cap B) = 0.1$
$P(A) = 0.5$
$P(B) = 0.2$
$P(\bar{A}) = 0.5$
$P(\bar{B}) = 0.8$

a

Given $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = 0.5 + 0.2 - 0.1 = 0.6$

b

$P(\bar{A} \cap \bar{B}) = 0.5 \times 0.8 = 0.4$

c

$P(\bar{A} \cup \bar{B}) = P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B}) = 0.5 + 0.8 - 0.4 = 0.9$

2.124

Of the population, $R = 0.4, D = 0.6$
30% of Republicans makes 0.12 of the population and 70% of the democrats makes 0.42 of the population.
The probability of the person chosen being a democrat is $\frac{0.42}{0.42 + 0.12}$ or $\frac{7}{9}$

2.134

The probability of the worker getting trained by method A is 0.7, B is 0.3
Of the A's 0.14 of the population fails training and 0.56 passes
Of the B's 0.03 of the population fails training and 0.27 passes
0.17 of the population fails training, of which 0.14 took method A, making the probability $\frac{0.14}{0.17}$ or 82.353%