Homework 2

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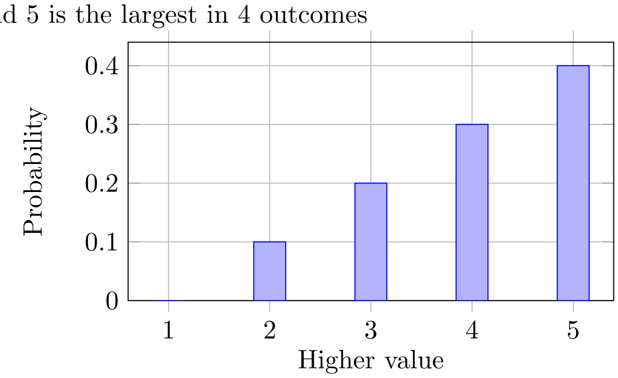
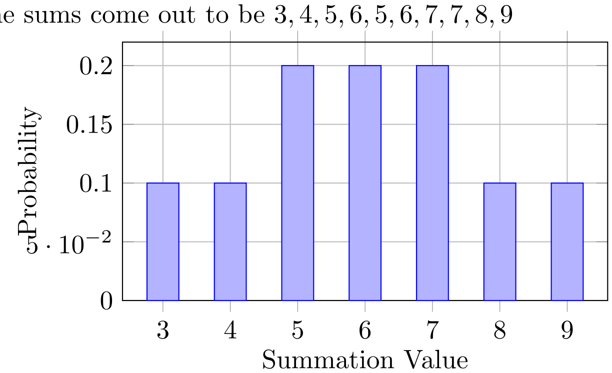
3.6

Using brute force calculations, all the possible outcomes are:
$(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)$
2 is the largest in 1 outcome
3 is the larger in 2 outcomes
4 is the larger in 3 outcomes
and 5 is the largest in 4 outcomes



The sums come out to be $3,4,5,6,5,6,7,7,8,9$



3.8

Generation 0: 1 chance of 1 cell
Generation 1: 0.9 chance of 2 cells, 0.1 chance of 0 cells
Generation 2: 0.18 chance of 2 cells, 0.01 chance of 0 cells, 0.81 chance of 4 cells

3.12

$E(Y) = 0.4 * 1 + 0.3 * 2 + 0.2 * 3 + 0.1 * 4 = 2$
$E(Y^2 - 1) = 0.4 * 0 + 0.3 * 3 + 0.2 * 8 + 0.1 * 15 = 4$

$E(1/Y) = 0.4 * 1 + 0.3 * 0.5 + 0.2/3 + 0.1 * 0.25 = \frac{77}{120}$

$V(Y) = (1 - 2.5)^2 * 0.4 + (2 - 2.5)^2 * 0.3 + (3 - 2.5)^2 * 0.2 + (4 - 2.5)^2 * 0.1 = 1.25$

3.30

a

Larger than, by 1. Since the whole set of possibilities was shifted up by 1, the mean would also be shifted up by 1

b

$E(X)$. Using linearity of expectation and $E(1) = 1$, $E(X) = E(Y + 1) = E(Y) + E(1) = \mu + 1$, which matches the answer in part (a).

c

Equal to. Since Y is uniformly larger than X by 1, the mean median and mode change, but the variance and standard deviation do not since the data moves uniformly.

d

From part (b), $E(X) = \mu + 1 \quad \Rightarrow \quad X - E(X) = (Y + 1) - (\mu + 1) = Y - \mu$.
Therefore, $V(X) = E\big[(X - E(X))^2\big] = E\big[(Y - \mu)^2\big] = \sigma^2$.

3.38

a

No, it is not a binomial distribution because the set of possible values is above two. There are 31 possibilities

b

No, it is also not a binomial experiment because the number of possible outcomes is infinite, not two.

3.48

Binom (4,3) = 4. $4p^4q + 4pq^4 = 4 * 0.5^5 * 2 = 0.25$

3.60

$\mathbb{E}(Y) = 0.1 * 4 = 0.4$ defective on average
Take variance as $np(1 - p) = 0.1 * 0.9 * 4 = 0.36$
$E(\mathbb{Y}^2) = \text{Var}(Y) + \mathbb{E}(Y)^2) = 0.36 + 0.4^2 = 0.52$
$\mathbb{E}[C] = 3 * 0.52 + 0.4 + 2 = 3.96$

3.70

$0.8^2 - 0.8^3 = 12.8\%$
After 10 drillings, there is a $0.8^{10} = 10.737\%$ chance of not striking oil

3.82

The prospector would assume that the amount of time he would need to drill would be the value at which a random sample of holes would contain a successful oil strike. For success rate of 0.2, that means on average he would expect to dig $\frac{1}{0.2} = 5$ holes.