SOS 2020

FINANCIAL MATHEMATICS

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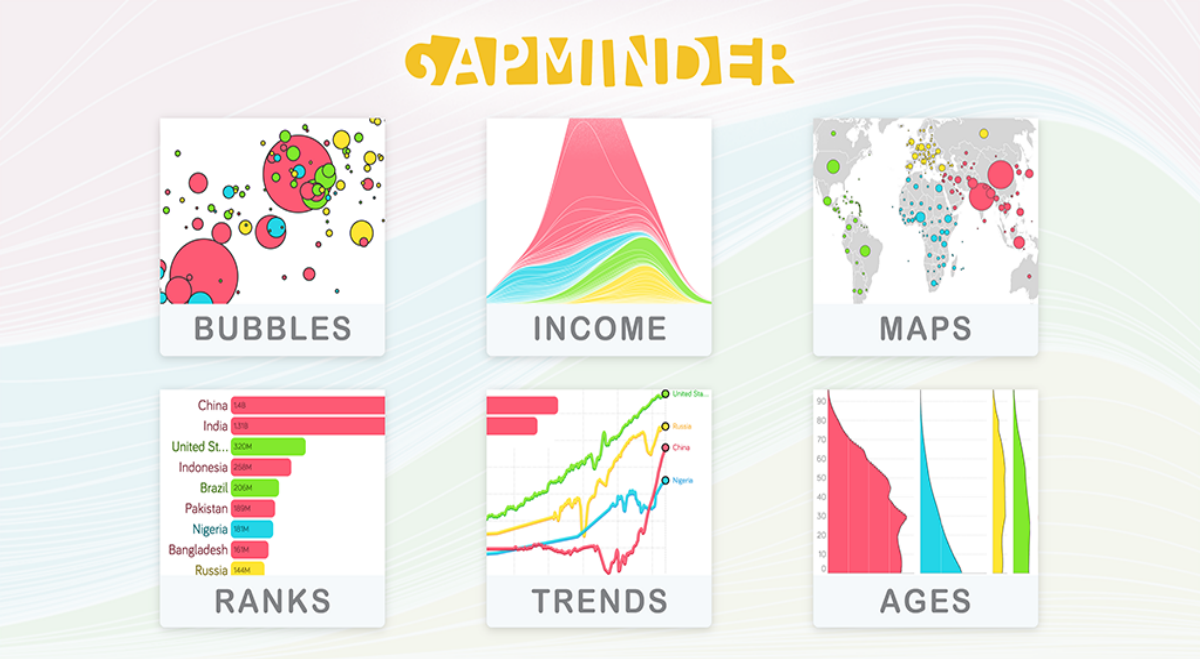
Quantitative modelling



(source:https://www.gapminder.org/tools/)

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FUNDAMENTALS OF MATLAB:

Matlab is one of the softwares and a programming language too that helps business analysts and engineers a lot. It can be used in various ways starting from a calculator to every task that a programming language does. But unlike other programming languages it's syntax is easy to get used to and it has many more in-built functions available which help engineers in solving the problems directly instead of writing down huge complicated functions.

Two online courses were completed to get acquainted with the software and its environment.one of the courses was provided by the official website of mathworks which every individual is supposed to complete upon installing the software "MATLAB". The other course was provided by coursera namely "Introduction to Programming with MATLAB" by Vanderbilt university . This was a nine week course but was completed very early as another fundamental course by mathworks had already been done.

During these courses the following book was suggested "COMPUTER PROGRAMMINGwith MATLAB" by J. MICHAEL FITZPATRICK AND ÁKOS LÉDECZI. There are three sections in the book

So far study till chapter 3 has been done. Some of the concepts in the book were too complicated to be done at that moment so they are supposed to be done later once completing the entire text.

Skills gained :

1) Matlab syntax and semantics
2) Plotting
3) accessing , combining and transforming matrices
4) Matrix arithmetic
5) Variable function arguments and outputs-robust functions
6) Logical indexing
7) Data types: character arrays, structs, cells
8) Input and output file- excel and text files
9) Linear algebra

Programming assignments:

1) Simple matlab calculation
2) Matrix indexing
3) Matrix arithmetic
4) Using functions “nargin” and “nargout”
5) Using cell arrays
6) Saddle points
7) Image blur
8) Echo generator
9) Some practice problems from matlab website

All the informations and detail given above can be found from the following websites

Matlab course : https://matlabacademy.mathworks.com/

Coursera course: https://www.coursera.org/learn/matlab

Book referred: http://cs103.net/buy

Credentials gained:

Coursera completion certificate:
https://www.coursera.org/account/accomplishments/certificate/W8TFZET2DJXE

Mathworks completion certificate:
https://matlabacademy.mathworks.com/progress/share/certificate.html?id=0cfd95b5-bfe8-4d6d-9827-24efff8abcbb

FUNDAMENTALS OF QUANTITATIVE MODELLING

1) Understanding the language of modeling

_a model is a formal description of a business process involving mathematical equations sought as a simplification of more complex business problems relying upon a set of assumptions.

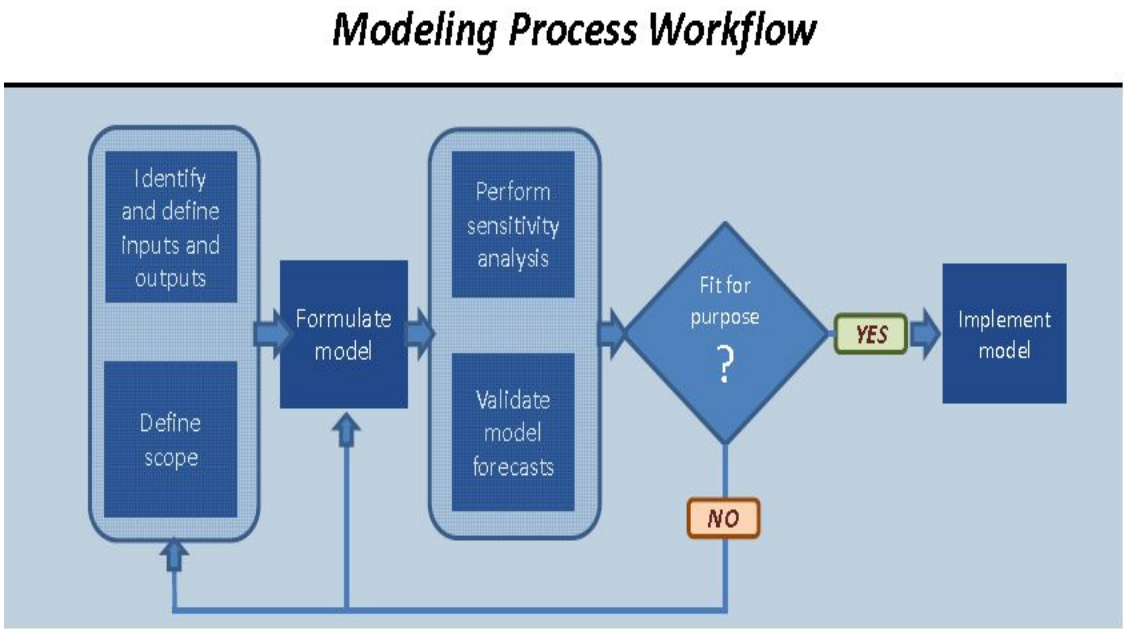
2) Uses of models

Models are used in prediction, forecasting,optimisation, ranking and targeting as well as exploring whatif scenarios.

Sometimes interpretation of coefficients and sensitivity of the model to key assumptions are also very important.

Modelling makes assumptions explicit and can be used as a decision support tool. It also helps to realise gaps in current understanding of a business problem

3) Key steps in modelling process:



(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

4) The Modelling Lexicon

Theory driven model Derives logical consequences using a set of assumptions and relations	**Data driven model** Given a set of observations, tries to approximate the process that generated these observations
Deterministic model Given a fixed set of inputs the model always give same output	**Probabilistic model** Even with identical inputs the model output can vary from instance to instance
Discrete variable Distinct values are used in models	**Continuous variable** Smooth process with possibly infinite number of values in an interval
Static model Only deals with a single snapshot of the business process	**Dynamic model** Deals with the process itself along with the outcome of the business process.

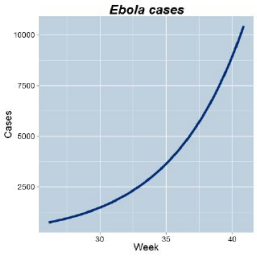
5)Basic Models

Following models were dealt with during the study of fundamentals of quantitative modelling

- The price of a diamond as a function of its weight

  This model is the best example of a linear model.

- The spread of an epidemic over time



$$\text{Model: Cases} = 6.69\, e^{0.18\,\text{Weeks}}$$

(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

This model is an example of an exponential growth model. This model is also a very good example of a continuous model.

- The relationship between demand for, and price of, a product

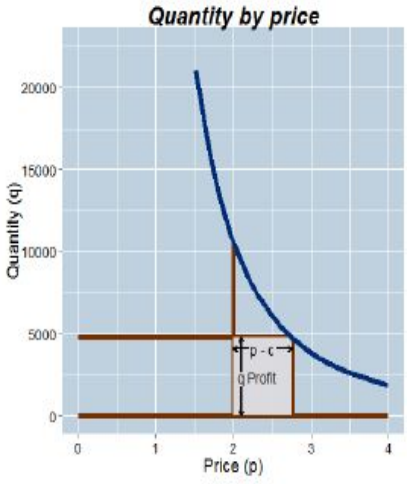
This scenario gives the best application of a model for optimisation. The objective of the optimization is to find the value of an input that maximizes/minimizes an output.

This is called demand model:

Quantity = $60{,}000\ \text{Price}^{-2.5}$

If the price of production is constant at c = 2 for each unit, then at what price is profit maximized?(this is called optimisation)

The price can be calculated using a calculus approach and the coefficient (-2.5) is called the price elasticity of demand model.



Price	Profit
1.75	-3702.509
2.0	0.000
2.5	3035.787
3.0	3849.002
3.5	3927.104
4.0	3750.000
4.5	3491.885
5.0	3219.938

Visualisation of calculus approach Brute force approach

(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

- The uptake of a new product in a market

6) STUDYING ABOUT PROBABILISTIC MODELS:

These models include random variable and probability distributions. Probabilistic approach is required because it introduces uncertainty in outcome variables.

Following probabilistic models were studied :

- Regression model

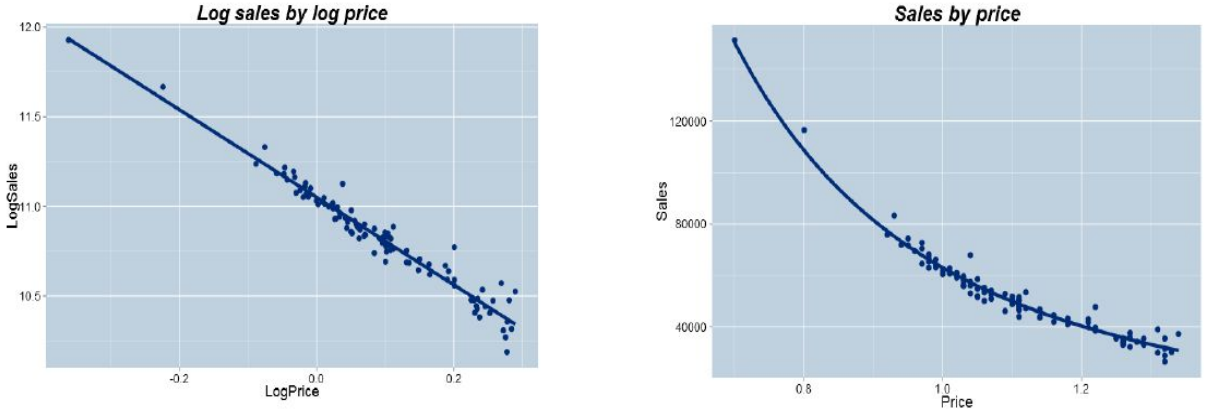
  When a set of data is given, the process is modelled by finding out the averaged outcome variable using one or more predictor variables. If multiple predictor variables are used then it is called multiple regression. If the relationship is linear then it is called linear regression model. Example is the weight of diamonds. Most regression models are fit using least squares methods.

$$E(Price \mid Weight) = -260 + 3721 \times Weight$$

Diamond model-using regression

(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

Often relations are nonlinear too. So on observing the curvature transform accordingly. Example is the log log model.

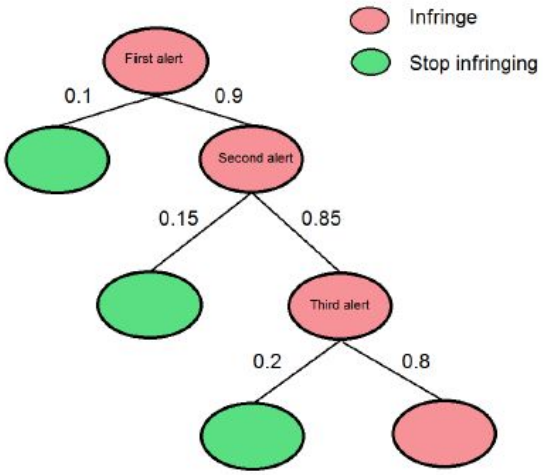


(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

- Probability trees

  Probability trees are used to propagate probabilities through a sequence of events.

  Given figure is a model which shows the probability tree.



(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

- Monte carlo simulation

In the demand model introduced above if instead of a definite coefficient if a range was given then a range of outcomes was possible.i.e. A range of optimal prices. This is called monte carlo simulations

- Markov models
  These are dynamic models for discrete time state space transitions. For example modelling a process of employment status of an individual.

7) Building blocks of probability models

Random variables can be discrete or continuous. But the key summaries of any probability distribution are the mean and spread around the mean. This spread around the mean can be estimated by using the standard deviation or variance.

Types of probability distributions dealt with during the study:

1) Bernoulli distribution: it can be seen as an experiment which has only two outcomes- true or false.
2) Binomial distribution: a binomial variable is the number of successes in n independent Bernoulli trials.
3) Normal distributions: it is famously known as the bell curve. There is universality to normal distribution. I.e. most of the processes when modelled turn out to have bell shaped data. It is symmetric about it's mean.

   EMPIRICAL RULE:

   It is a rule for calculating probabilities when the underlying distribution is approximately normal.

   "There is an approximate 68% chance that an observation falls within one standard deviation from the mean . There is an approximate 95% chance that an observation falls within two standard deviations from the mean . There is an approximate 99.7% chance that an observation falls within three standard deviations from the mean"

INTRODUCTION TO SPREADSHEETS AND MODELS:

- ## SPREADSHEET AS A TOOL FOR THINKING WITH NUMBERS:

  The revolution in computer technology was not in fact due to the personal computers but because of the “killer app” called spreadsheets which allowed common individuals and firms to replace them with their ledgers.

  This was what apple became famous for.

The spreadsheet as the original “killer app”

Dan Bricklin & Bob Frankston, circa 1979

(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

Following are some of the basic requirements for modelling in spreadsheets:

- Gain familiarity and comfort in navigating a spreadsheet,
- Identity the different types of data used in a spreadsheet and options for displaying them
- Use spreadsheet notation for mathematical operations on cells and arrays
- Understand and control the order of processing in formulas
- Use shortcuts for copying data and formulas

Following functions were studied in detail and were practiced too:

1) Different uses of sum and sumproduct functions
2) Basic statistical functions like max, min , average and standard deviation
3) Using conditional expressions and their applications
4) Understanding relative and absolute references:

   Relative reference deals with numbers that change when going from one cell to another so we need a series instead of a constant value to do any operation.

   Absolute reference is required when a value in a particular cell is constant in all it's operations

5) Identifying and correcting circular references:

   Circular references come into play when using a value in a cell which has not been yet calculated.

6) Audit formulas. Special addins must be installed for statistical functions.

- ## From spreadsheets to models:

Recognising assumptions and decision variables is an important practice in modelling businesses. Different types of metrics for evaluating outcome of business processes being modeled were identified. Spreadsheet functions can be incorporated into the models for this purpose.

PRACTICING SPREADSHEETS AND MODELLING:

A basic cash flow model was created. Some critical decisions were tested by using simple statistical functions.

What-if analysis was conducted and key variables were identified using sensitivity analysis.

Some basic characteristics of linear programming models and deterministic models were revealed like not so many real life scenarios are as deterministic as the results provided by the spreadsheets.

- ### Addressing uncertainty and probability in models:

  Though many programming languages like python and matlab have standard libraries for analysis tasks, spreadsheets also provide support for such tasks. The difference lies in the user interface. Though matlab also has a graphical user interface it is a bit complicated to use.

  Using power, exponential and logarithmic functions. For random numbers using rand() and randBetween() and correlation function. It is not a deniable fact that spreadsheet is as efficient as any other programming language for analysis if used properly.

- ### Using simulations to model uncertainty and risk in spreadsheets:

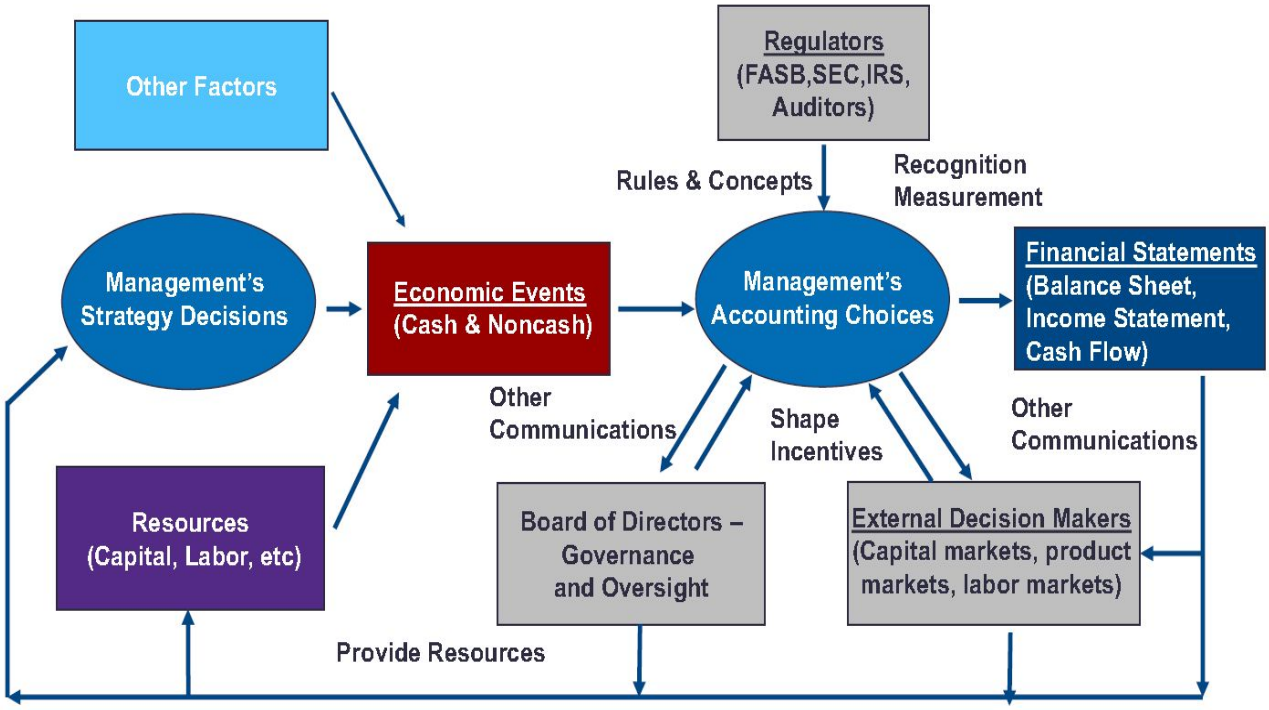
  As an example of linear programs, using Excel's solver to identify optimal allocations of resources to reach a desired outcome.

  Following link is provided to be considered as a resource for practicing spreadsheets:

  https://drive.google.com/drive/folders/1SgQBe8I_wiE3kBAFsL6VcIEI8GFPxbNH?usp=sharing

FINANCIAL ACUMEN :

This is one of the most important parts of financial modelling. As an analyst one should be aware of the terms of finance and economy. Financial statements give the food for thought for an analyst. These statements can be found in annual or semi-annual reports of any firm. Following is a chart that gives an idea of how the financial report is affected.



(Source: https://www.coursera.org/learn/wharton-quantitative-modeling)

There are three primary financial statements:

1) Balance sheet

Balance sheets describe the resources of a firm(assets) and also the claim on the resources(liabilities) using numerals and a single and most important equation called the *" balance equation "*.

ASSETS = LIABILITIES + SHAREHOLDER'S EQUITY

Recognition and classification of financial objects as assets, liabilities and shareholders equity is important. Following shows if these are assets or not:

Cash ✓

Accounts receivable ✓

Customers' promises to buy products in the future ✗

Prepaid insurance ✓

Inventory ✓

Brand name ?

Discovery of a new medicine ✗

Competitor goes bankrupt ✗

Hire a new CEO ✗

Following shows if these come under liabilities or not:

Accounts payable ✓

New customer signs a contract to buy product in the future ✗

Receipt of payment in advance of providing service ✓

Long-term debt ✓

Product warranties ✓

Employee pensions ✓

Lawsuit is filed against the company ✗

But if all the firms would have stuck to just these measures in accounting then there was no way the firms would have covered up the losses or exaggerated the profits. So some adjusting entries are needed. A typical balance sheet looks like this:

Consolidated Statement of Financial Position

at 31 December

	Notes	2xx4 $m	2xx3 $m	2xx2 $m
Assets				
Non-current assets				
Property, plant and equipment	7	**6,010**	5,818	6,089
Goodwill	8	**11,550**	9,981	9,898
Intangible assets	9	**20,981**	16,047	16,448
Investments in joint ventures	10	**59**	–	–
Other investments	11	**502**	281	199
Derivative financial instruments	12	**465**	365	389
Other receivables	13	**1,112**	1,867	352
Deferred tax assets	4	**1,219**	1,205	1,111
		41,898	35,564	34,486
Current assets				
Inventories	14	**1,960**	1,909	2,061
Trade and other receivables	15	**7,232**	7,879	7,629
Other investments	11	**795**	796	823
Derivative financial instruments	12	**21**	40	31
Income tax receivable		**329**	494	803
Cash and cash equivalents	16	**6,360**	9,217	7,701
		16,697	20,335	19,048
Total assets		**58,595**	55,899	53,534
Liabilities				
Current liabilities				
Interest-bearing loans and borrowings	17	**(2,446)**	(1,788)	(901)
Trade and other payables	18	**(11,886)**	(10,362)	(9,221)
Derivative financial instruments	12	**(21)**	(2)	(3)
Provisions	19	**(623)**	(823)	(916)
Income tax payable		**(2,354)**	(3,076)	(2,862)
		(17,330)	(16,051)	(13,903)
Non-current liabilities				
Interest-bearing loans and borrowings	17	**(8,397)**	(8,588)	(9,409)
Derivative financial instruments	12	**–**	(1)	–
Deferred tax liabilities	4	**(1,796)**	(2,827)	(2,576)
Retirement benefit obligations	20	**(2,951)**	(2,261)	(2,271)
Provisions	19	**(484)**	(566)	(428)
Other payables	18	**(7,991)**	(2,352)	(1,001)
		(21,619)	(16,595)	(15,685)
Total liabilities		**(38,949)**	(32,646)	(29,588)
Net assets		**19,646**	23,253	23,946

Equity				
Capital and reserves attributable to equity holders of the Company				
Share capital	22	**316**	315	312
Share premium account		**4,261**	3,983	3,504
Capital redemption reserve		**153**	153	153
Merger reserve		**448**	433	433
Other reserves	21	**1,420**	1,380	1,374
Retained earnings	21	**13,029**	16,960	17,955
		19,627	23,224	23,731
Non-controlling interests		**19**	29	215
Total equity		**19,646**	23,253	23,946

(source: Eta-Zee Inc financial statements)

2) Income statement

Income statement tells us the profitability in a year. Unlike a balance sheet, it is prepared for a period of time. It helps to link changes in the balance sheet. Income statement generally has the following format:

Revenue (or Sales)

- Cost of Goods Sold

Gross Profit

- Operating (SG&A) Expense

Operating Income

- Interest, Other Gains, and Losses

Pre-tax Income

- Income Tax Expense

Net Income

Revenue is an increase in shareholder's equity.Expenses are the decrease in the shareholder's equity.

Following is how a typical income statement look like:

Consolidated Statement of Comprehensive Income

for the year ended 31 December

	Notes	2xx4 $m	2xx3 $m	2xx2 $m
Revenue	1	**26,095**	25,711	27,973
Cost of sales		**(5,842)**	(5,261)	(5,393)
Gross profit		**20,253**	20,450	22,580
Distribution costs		**(324)**	(306)	(320)
Research and development expense	2	**(5,579)**	(4,821)	(5,243)
Selling, general and administrative costs	2	**(13,000)**	(12,206)	(9,839)
Other operating income and expense	2	**787**	595	970
Operating profit	2	**2,137**	3,712	8,148
Finance income	3	**78**	50	42
Finance expense	3	**(963)**	(495)	(544)
Share of after tax losses in joint ventures	10	**(6)**	–	–
Profit before tax		**1,246**	3,267	7,646
Taxation	4	**(11)**	(696)	(1,376)
Profit for the period		**1,235**	2,571	6,270

3) Cash flow statement

Cash flow is about liquidity whereas net income is about profitability. It allows classification of business activities into different categories:

- OPERATING ACTIVITIES

  All the transactions related to providing goods and services to customers and to paying expenses related to revenue generating activities. Operating section of a typical cash flow statement:

Amounts in millions; Years ended June 30	2xx3	2xx2	2xx1
CASH AND CASH EQUIVALENTS, BEGINNING OF YEAR	**$ 4,436**	$ 2,768	$ 2,879
OPERATING ACTIVITIES			
Net earnings	**11,402**	10,904	11,927
Depreciation and amortization	**2,982**	3,204	2,838
Share-based compensation expense	**346**	377	414
Deferred income taxes	**(307)**	(65)	128
Gain on sale and purchase of businesses	**(916)**	(2,106)	(203)
Goodwill and indefinite-lived intangible asset impairment charges	**308**	1,576	—
Change in accounts receivable	**(415)**	(427)	(426)
Change in inventories	**(225)**	77	(501)
Change in accounts payable, accrued and other liabilities	**1,253**	(22)	358
Change in other operating assets and liabilities	**68**	(444)	(1,221)
Other	**377**	210	16
TOTAL OPERATING ACTIVITIES	**14,873**	13,284	13,330

- ## INVESTING ACTIVITIES

  Transactions related to acquisitions and disposal of long term assets. Investing section of a typical cash flow statement.

INVESTING ACTIVITIES			
Capital expenditures	**(4,008)**	(3,964)	(3,306)
Proceeds from asset sales	**584**	2,893	225
Acquisitions, net of cash acquired	**(1,145)**	(134)	(474)
Purchases of available-for-sale investment securities	**(1,605)**	—	—
Change in other investments	**(121)**	112	73
TOTAL INVESTING ACTIVITIES	**(6,295)**	(1,093)	(3,482)

- ## FINANCING ACTIVITIES

  Transactions related to owners and creditors(except payment of interest). Financing activities of a typical cash flow statement:

FINANCING ACTIVITIES			
Dividends to shareholders	**(6,519)**	(6,139)	(5,767)
Change in short-term debt	**3,406**	(3,412)	151
Additions to long-term debt	**2,331**	3,985	1,536
Reductions of long-term debt	**(3,752)**	(2,549)	(206)
Treasury stock purchases	**(5,986)**	(4,024)	(7,039)
Impact of stock options and other	**3,449**	1,729	1,203
TOTAL FINANCING ACTIVITIES	**(7,071)**	(10,410)	(10,122)

CONCLUSION:

The subject of study was financial mathematics. It was dealt not in the conventional way. Instead the focus was only on modelling businesses . Since basic knowledge of finance is important, and also some technical knowledge about modelling any process, the study was done merging these two subfields of financial mathematics. This effort by me and the direction of study concentrated more on diversifying one into analysis of data using matlab and then linking it to finance. Technicalities regarding excel were not summarised in the report as it was considered basic and not the motto of learning financial mathematics, especially financial modelling. Modelling is all about analysing data and understanding the trend and coming up with a mathematical base. My study concentrated more on the analysis of data and understanding trends. Coming up with a mathematical base needs rigorous statistical mathematics. One interested in this field of statistical mathematics will surely be able to complete the third step of financial modelling.

REFERENCES:

MATLAB:

Matlab course : https://matlabacademy.mathworks.com/

Coursera course: https://www.coursera.org/learn/matlab

Book referred: http://cs103.net/buy

Learn MS-Excel for modelling:

“Hector Guerrero-Excel Data Analysis(Modeling and Simulation)Second Edition”

Source: https://link.springer.com/book/10.1007%2F978-3-030-01279-3

Coursera course for quantitative modelling:

https://www.coursera.org/specializations/finance-quantitative-modeling-analysts?