Response to the Handling Editor and Reviewers

Submission No.: **DSP-D-16-00375**
Title: **Low Complexity Performance Assessment of a Sensor Array via Unscented Transformation**

Dear handling editor and reviewers,

The authors would like to thank the Associate Editor and the Reviewers for volunteering their time in reviewing our manuscript and providing us with valuable comments which allowed us to significantly improve the readability and presentation of the paper. We have carefully revised the manuscript based on the reviewer's comments and suggestions. All changes in the paper have been marked with a red color. The following is our point-by-point responses to the raised comments.

Kind regards,
The authors.

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1. Reply to the Handling Editor:

Four review reports are received. In summary, the low-complexity of the proposed method can be demonstrated in a clearer manner and the use of unscented transform should be better motivated. The readability and English usage should also be enhanced.

REPLY: *Thank you very much for considering our paper for publication. We thoroughly went through the reviwers comments in order to improve the papers quality. Please find the reponse to the reviewers bellow.*

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2. Reply to Reviewer #1:

The author propose a beamforming performance evaluation technique in the presence of direction of arrival (DOA) and positioning errors by using the unscented transform (UT). The signal to interference plus noise ratio (SINR) is used for the performance measure metric and only uniform linear array is considered. The UT based SINR computation is compared with the Manto-Carlo (MC) based SINR computation. The motivation of the work is only in reducing the computational burden of the MC based SINR computation while maintaining reliability.

1. Section 3 Page 11: The author first assumes that the random variable SINR has a fixed mean. First of all, what kind of random variable the SINR would be is not established in the paper.

REPLY: *Thank you for your comment. We consider that the analyseble* $\overline{\text{SINR}}$ *is actually the average or expected value of SINR which is a non-linear function of a random variable. First, a general distribution is considered, although the extension for other types of random variables is straightforward. In the proposed multivariate case and in the simulations, a Gaussian random variable is considered. We made improvements throughout the text to make it clearer.*

2. In Section 4.2 : The author assumes the positioning errors D_x, D_y, D_z are Gaussian random variables without giving any justification.

REPLY: *Thank you for pointing out this aspect. We added the following text in order to elaborate more on the justification:*

Even though Gaussian distributions have non-zero tails, the actual generation of very large or very small numbers is unlikely. This work, as in common practice [21, 22], uses Gaussian distributions to model the array elements positioning errors in order to simplify the UT validation while making very little to no comprimise to the validation process. For a theoretically more precise distribution, the reader is referred to the circular normal distribution, also known as the Von Mises distribution [23].

3. Figure 4 has not been discussed in the paper.

REPLY: *Thank you for your observation. Actually Figure 4 has been discussed in Section 4, but it has been shown up in Section 5 for the sake of formatting. In order to avoid confusion, we decided to move the Figure to Section 4.*

4. Nothing has been said about the accuracy of the UT based SINR evaluation over MC based evaluation.

REPLY: *Figures 7 and 11 show the UT error in comparison to* 10^6 *MC simulations by assuming that* 10^6 *MC simulations is a close-to-true value. We inserted the following text to make it clearer.*

For our simulations we assume that the mean over 10^6 MC realizations, represented by $\overline{\mathrm{SINR}}_{\mathrm{MC}10^6}$, is very close to the ground truth mean. Therefore, our proposed methods error is defined as

$$\mathrm{Error} = |\overline{\mathrm{SINR}} - \overline{\mathrm{SINR}}_{\mathrm{MC}10^6}|. \tag{32}$$

Equation (32) is used in sections 5.1 and 5.2 for the validation of the proposed UT-based methods.

5. Overall all the claims about reduced computational burden is based on simulations. This reviewer does not get any intuition about how reduction in computational burden can necessarily be assured over the MC based evaluation. In addition, the author fails to give why they choose UT while it is inferior to the Cubature Transform (CT). Based on these, I believe this paper cannot be published in its present form.

REPLY: *Thank you very much for your suggestion. To the best of our knowledge no previous work has been done on the UT as an evaluation method for beamforming-related problems. In our survey, we have not found ~~on~~ the literature applications of the Cubature Transform (CT) for low complexity assessment. Therefore, we consider the CT as future work. We added a text to make it explicit containing a reference to the CT:*

Also, other integration methods such as the quadrature and cubature transforms [15] showed better results than the UT for filtering purposes. Even though the quadrature and cubature transforms might also be considered for sensor array performance assessment, we regard them as future work and focus on the simplicity and ease of implementation of the UT.

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3. Reply to Reviewer #2:

In this work, the authors present a novel performance assessment method which has very low computational complexity. The description is clear and the method is technically sound. I have the following comments:

REPLY: *Thank you for you time reviewing our paper.*

1. In general, Monte Carlo trials are easy to implement and it is based on the randomization of the s, e.g., DOA or positions of the elements. So, there may be many randomizations on the errors that are very close and thus the corresponding computations are meaningless. In contrast, in this paper, the authors give determinist of the errors by sampling them based on their probability distribution function. It is sure to avoid redundant computation in Monte Carlo simulations.

REPLY: *Thank you for your comments. We appreciate your positive feedback.*

2. The noise covariance matrix $\mathbf{R_n}$ is also very important in evaluating the SINR performance. There are many works trying to show the SINR performance in small number of snapshots. Could the author comment on this? How the proposed method can be used to evaluate the SINR performance with respect to the number of snapshots?

REPLY: *Thank you for mentioning the covariace matrix importance. The covariance is estimated via a quadratic, thus non-linear, function and, therefore, can be estimated via the UT. The covariance matrix estimation usually needs more samples than atennas to have good characteristics such as good condition number. This means that besides the number of combinations of sigma points needed for each antennas element, this number of combinations will also be needed for each sample causing the UT to have high multivariate complexity, more specifically, an average over* $N_{\mathrm{ut}}^{M \times N}$ *sigma points are needed, where* N_{ut} *is the number of UT sigma points,* N *is number of snapshots and* M *is number of antennas. Due to the high complexity in this case, the UT is not recommended for covariance matrix estimation.*

3. Maybe the performance of the proposed method in evaluating the SINR depends on the accuracy of the discretized probability distribution function. If we can reconstruct (in some degree) the continuous probability distribution function from the discretized one, the performance is good. If the reconstruction error increases, the performance degrades. Please comment

on this.

REPLY: *Thank you for your note. That is right. The more points are added to the UT, the more it approximates the true distribution such that when,* $N_{\text{ut}} \to \infty$, $w(p_i) = p(x)$. *We show this by considering the 3-point UT and the 5-point UT and veryfying that the 5-point UT has a better precision than the 3-point UT. However, our goal is not to have a good approximation of the distributions, but use as few points as possible while maintaing relevant statistical information. In that regard, we have added the 5-point UT to Figure 2 to give this sense to the reader. Also the following text was added:*

Note that the increase in UT points translates to better PDF approximation such that the UT with infinite sigma points is equivalent the continuous PDF.

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4. Reply to Reviewer #3:

Please see the attached page on EES. The manuscript addresses performance assessment of the average SINR of a beamformer under the DOA estimation error and imperfections of the array elements positions. The proposed method reduces the computational burden of the evaluation via unscented transformation, while maintain the reliability of the simulations. There are some minor issues needed to be modified by the authors.

REPLY: *Thank you for your time reviewing our paper. We appreciate the positive comments.*

1. The steering vector contains the phase delays $a_{m,i} = e^{j2\pi\mu_i}$, where $\mu_i = (m-1)d\cos\theta_i$. (56 line)

REPLY: *The inter-element spacing d is given in wavelengths, therefore* λ *was dropped. We wrote it explicitly to avoid confusion as follows:*

Since we are dealing with a ULA, the steering vector contains the phase delays $a_{m,i} = e^{j2\pi\mu_i}$, where $\mu_i = (m-1)\frac{d}{\lambda}\cos\theta_i$, m is the antenna element index and d is the inter-element spacing in wavelengths. If d is given in wavelengths, the wavelength can be dropped from the phase delay μ_i.

2. The equation (3) occurs θ_i and ϕ_i In order to make the express more clear, I suggest θ_i and ϕ_i can be marked in Figure 1.

REPLY: *Thank for your suggestion. We have marked θ_i and ϕ_i in Figure 1.*

3. The UT for a single random variable is review in Subsection 3.2 and its extension for multiple i.i.d random variables is reviewed in Subsection 3.1. This sentence is opposite and the author need to switch the 3.1 and 3.2. (85 line)

REPLY: *Thank you for pointing that out. The section numbers were corrected.*

4. The result of inserting perturbations, as modeled in (28) and (4), implies the replacement of $\mathbf{a}(\theta_i)$ for $\mathbf{a}(\theta_i+\Theta)$ in (20) and (21) for the case where DOA estimation error is considered. When perturbations on the antenna element positions are considered, we replace $\mathbf{a}(\theta_i)$ for $\mathbf{a}^{\Theta}(\theta_i)$. The (28) should be replaced by (2) and the $\mathbf{a}^{\Theta}(\theta_i)$ a should be replaced by $\mathbf{a}^{D}(\theta_i)$. (under 110 line above 115 line)

REPLY: *Thank you for noticing. The equation numbers and steering vector notation were corrected accordingly.*

In a similar manner as in (23) and (24), the SOI and interference correlation matrices in the presence of element positioning perturbations are written as:

$$\mathbf{R}_{\mathrm{ss}}^{(\boldsymbol{\mathcal{D}})}(\boldsymbol{\mathcal{D}}[n]) = \mathbf{a}^{(\boldsymbol{\mathcal{D}}[n])}(\theta_0,\phi_0)[\mathbf{a}^{(\boldsymbol{\mathcal{D}}[n])}(\theta_0,\phi_0)]^{\mathrm{H}}, \tag{27}$$

$$\mathbf{R}_{\mathrm{int}}^{(\boldsymbol{\mathcal{D}})}(\boldsymbol{\mathcal{D}}[n]) = \sum_{i\neq 0}\mathbf{a}^{(\boldsymbol{\mathcal{D}}[n])}(\theta_i,\phi_i)[\mathbf{a}^{(\boldsymbol{\mathcal{D}}[n])}(\theta_i,\phi_i)]^{\mathrm{H}}. \tag{28}$$

5. For Figure 10, the legend and explain under the figure exist problems. MC5 should be revised by UT5. The explain sentence has two “to”.

REPLY: *Thank you for taking your time to report the typos. The figure’s legend and caption were corrected.*

6. From the equation (29), we can see that the weight w of the DS filter is relate to the perturbation of the antenna elements, but the matrix $\mathbf{R}_{ss}$ and $\mathbf{R}_{\text{int}}$ hasn't any changes with respect to equation (19). This point makes me confused and the author need to take it into account.

REPLY: *Thank you for checking this problem. We agree that the change in notation becomes confusing. We unified the notation in equations (27)-(28) and (29) to include the perturbation in the correlation matrix in Section 4.1 as follows:*

$$\overline{\text{SINR}}_{\text{UT}}^{(\mathcal{D})} = \sum_{n_{x,1}=1}^{N_{\text{UT}}} \sum_{n_{y,1}=1}^{N_{\text{UT}}} \sum_{n_{z,1}=1}^{N_{\text{UT}}} \sum_{n_{x,2}=1}^{N_{\text{UT}}} \cdots \sum_{n_{z,M}=1}^{N_{\text{UT}}} \omega_{n_{x,1}} \omega_{n_{y,1}} \omega_{n_{z,1}} \omega_{n_{x,2}} \cdots \omega_{n_{z,M}} \cdot \cdot \frac{\mathbf{w}^{\text{H}} \mathbf{R}_{\text{ss}}^{(\mathcal{D})}(\mathbf{p}_{n_{x,1},\ldots,n_{z,M}}) \mathbf{w}}{\mathbf{w}^{\text{H}} \left(\mathbf{R}_{\text{int}}^{(\mathcal{D})}(\mathbf{p}_{n_{x,1},\ldots,n_{z,M}}) + \sigma^2 \mathbf{I} \right) \mathbf{w}}, \tag{29}$$

7. In this paper, the authors make sufficient simulations to illustrate that the average SINRs using the UT method follow the MC curve. In order to express the low complexity of the proposed method, I suggest the authors can give a figure or table to compare the cost time of the proposed method and MC method.

REPLY: *Thank you for your suggestion. We added figures 9 and 13 to compare the computational time of the proposed techniques with MC simulations. The results are in accordance with the number of points over which the mean is computed and shown in Section 4.*

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5. Reply to Reviewer #4:

In this paper, a low complexity performance assessment of the average SINR via unscented transformation is proposed,instead of traditional Monte Carlo (MC) simulations. The work is interesting, however, the only example in the paper seems too simple.

REPLY: *Thank you for your positive feedback.*

For DS beamformer, the average SINR versus DOA error can be computed without UT. As to positioning error, the advantage of UT is not so remarkable, for other types antenna errors such as mutual coupling, amplitude and phase error, multivariate scenario arises just like positioning error. Therefore, more work should be done to prove the effectiveness of proposed method.

REPLY: *Thank you very much for your suggestion. We agree that the UT can also be applied for other purposes and some of them give raise to a multivariate analysis and we understood that most perturbations should have a similar analysis. Therefore we considered that the chosen scenarios suffice at this stage and suggested other scenarios for future work. Thus, the following was added to the introduction:*

In this work we consider two perturbations, a DOA estimation error and array element positioning error. The evaluation of these perturbations give raise to a univariate and multivariate UT, respectively. For the sake of demonstration, one type of perturbation is considered for each case. In future work, the UT may also be applied for other types of perturbations such as frequency shift, mutual coupling, amplitude error and phase error.

Some detailed comments are as follows:

1. The details of deducing Equation (25) should be given, from which expression can Equation (25) be obtained?

REPLY: *Thank you for checking this problem. The reference to Equation (9) was missing in the explanation, therefore, we added the reference as follows:*

The UT average of the SINR is obtained by replacing $g(\cdot)$ in Equation (9) by the SINR and Θ_n by $p_n^{(\Theta)}$:

2. The structure of section 5 should be improved. It would be better to add subheadings to classify all figures into different categories, i.e., the performance, the error, ... Otherwise, the purpose of explanations and figures is not so clear.

REPLY: *Thank you for your comment. We now divided Section 5 in* *accordance to the previous sections of the paper, i.e. univariate and multivariate case. In sucsections 5.1 and 5.2 we evaluate the the performance, the error and computational time.*

3. There are several grammar mistakes, i.e.,
line 36-line 37, ”more precisely” appears twice;
line 37, the use of ”computational” is wrong;
the first line in page 9, ”we can achieve an easy to” is not appropriate;
line 116, ”This mean can be used as a” is confusing.

REPLY: *Thank you for pointing them out. The text was improved along the entirety of the paper.*