University of Waterloo
MATH 213, Spring 2015
Assignment 5

Question 1

Find the inverse of the given transform.

a)

$$\frac{3s^2 - 12s^2 - 2s - 56}{(s-7)s(s^2+4)}$$

b)

$$\frac{s^4 + 3s^3 + 2s^2 + 27s + 18}{s^3(s^2+9)}$$

Question 2

Use the Laplace transform to find the particular solution.

$$y''' - 2y'' + 4y' - 8y = 0$$
$$y(0) = 0, y'(0) = 0, y''(0) = 4$$

First, take the Laplace transform of the equation:

$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2(s^2Y(s) - sy(0) - y'(0)) + 4(sY(s) - y(0)) - 8Y(s) = 0$
$s^3Y(s) - 4 - 2s^2Y(s) + 4sY(s) - 8Y(s) = 0$
$(s^3 - 2s^2 + 4s - 8)Y(s) = 4$
$Y(s) = \dfrac{4}{s^3 - 2s^2 + 4s - 8}$
$Y(s) = \dfrac{4}{(s-2)(x^2+4)}$

Using partial fraction decomposition:

$$\frac{4}{(s-2)(x^2+4)} = \frac{A}{s-2} + \frac{Bs}{s^2+4} + \frac{2C}{s^2+4}$$
$$4 = As^2 + 4A + Bs^2 - 2Bs + 2Cs - 4C$$
$$A + B = 0$$
$$-2B + 2C = 0$$
$$4A - 4C = 4$$

Solving for these equations gives $A = \frac{1}{2}$, $B = -\frac{1}{2}$, $C = -\frac{1}{2}$.
Therefore,

$$Y(s) = \frac{1}{2} * \frac{1}{s-2} - \frac{1}{2} * \frac{s}{s^2+4} - \frac{1}{2} * \frac{2C}{s^2+4}$$

Taking the inverse Laplace:

$$y(t) = \frac{1}{2}e^{2t} - \frac{1}{2}\cos(2t) - \frac{1}{2}\sin(2t)$$