Assignment 1

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May 15, 2015

1 Problem Set 1

1.1 Problem 1

a) $y' - 2y = 0$

$$y' - 2y = 0$$
$$Ne^{Nx} - 2e^{Nx} = 0$$
$$Ne^{Nx} = 2e^{Nx}$$
$$N = 2$$

Therefore $N = 2$.

b) $y'' + 4y = 0$

$$y'' + 4y = 0$$
$$N^2 e^{Nx} + 4e^{Nx} = 0$$
$$N^2 e^{Nx} = -4e^{Nx}$$
$$N^2 = -4$$

Since $N^2 >= 0$, there are no such N's.

1.2 Problem 2

a) First we shall find the derivatives of y.

$$y = N\cos(2x) + x$$
$$y' = -2N\sin(2x) + 1$$
$$y'' = -4N\cos(2x)$$

Next we shall check if y is a solution.

$$y'' + 4y$$
$$= -4N\cos(2x) + 4(N\cos(2x) + x)$$
$$= -4N\cos(2x) + 4N\cos(2x) + 4x$$
$$= 4x$$
$$= RHS$$

Therefore, y is a solution.

b)

$$y'(x) = -2N\sin(2x) + 1$$
$$y'(0) = -2N\sin(0) + 1$$
$$2 = -2N\sin(0) + 1$$
$$2 = -2N0 + 1$$
$$2 = 1$$

Therefore, there are no possible solutions.

c) Linear, as it can be written in the form $a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + ... + a_0(x)y(t) = g(x)$. Where $n = 2$, $a_2 = 1$, $a_1 = 4$, $a_0 = 0$, and $g(x) = 4x$.

2 Problem Set 2

2.1 Problem 1

Given $y'' = 2y + y'$, verify the following solutions:

$$y_1(x) = sinh(2x) + cosh(2x)$$
$$y_2(x) = sin(2x + 3)$$

For y_1,

$$y_1(x) = sinh(2x) + cosh(2x)$$
$$y_1'(x) = 2cosh(2x) + 2sinh(2x)$$
$$y_1''(x) = 4sinh(2x) + 4cosh(2x)$$

Substituting into the differential equation,

$$4sinh(2x) + 4cosh(2x) = 2(sinh(2x) + cosh(2x)) + 2cosh(2x) + 2sinh(2x)$$

This statement holds so y_1 is a solution. For y_2,

$$y_2(x) = sin(2x + 3)$$
$$y_2'(x) = 2cos(2x + 3)$$
$$y_2''(x) = 4cos(2x + 3)$$

Substituting into the differential equation,

$$4cos(2x + 3) = 2sin(2x + 3) + 2cos(2x + 3)$$

This statement holds so y_2 is a solution.

2.2 Problem 2

Given the following differential equations:

$$y_1(x) = e^x + y' + y'' = 3$$

$$y_1(x) = e^y + y' + y'' = 3$$

Determine which equation is linear. For the linear equation, verify that

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

is a solution, for all constants A and B. Then, determine one set of possible values for A and B given that $y_3(-1) = 2e - \frac{1}{2e}$.

We know y_1 is the linear equation as y_2 contains a term e^y which is non-linear. To verify that y_3 is a solution,

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

$$y_3'(x) = -Ae^{-x} + 3 - \frac{e^x}{2}$$

$$y_3''(x) = Ae^{-x} - \frac{e^x}{2}$$

Substituting into the differential equation,

$$e^x + y' + y'' = 3$$

$$e^x + (-Ae^{-x} + 3 - \frac{e^x}{2}) + (Ae^{-x} - \frac{e^x}{2}) = 3$$

$$3 = 3$$

This holds for all values of A and B. Using the given parameters,

$$y_3(-1) = 2e - \frac{1}{2e}$$

$$2e - \frac{1}{2e} = Ae^{-(-1)} + B + 3(-1) - \frac{e^{-1}}{2}$$

$$2e - \frac{1}{2e} = Ae + B - 3 - \frac{1}{2e}$$

Thus we get $A = 2, B = 3$.