Assignment 2

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1 Problem Set 1

1.1 Problem 1

a)

$$y'' + 3y' - 10y = 0$$

$$\lambda^2 + 3\lambda - 10 = 0$$
$$(\lambda + 5)(\lambda - 2) = 0$$
$$\therefore \lambda = 2, -5$$

So

$$y = c_1 e^{2x} + c_2 e^{-5x}$$

b)

$$y'' + 2y' + 3y = 0$$

$$\lambda^2 + 2\lambda + 3 = 0$$
$$\lambda = \frac{-2 \pm \sqrt{2^2 - 4 * 1 * 3}}{2}$$
$$\lambda = \frac{-2 \pm \sqrt{4 - 12}}{2}$$
$$\lambda = \frac{-2 \pm i\sqrt{8}}{2}$$
$$\lambda = -1 \pm i\sqrt{2}$$

So

$$y = e^{-x}(c_1 \sin(\sqrt{2}x) + c_1 \cos(\sqrt{2}x))$$

1.2 Problem 2

$$y'' - 3y' = e^x + x$$

First find homogeneous solution.

$$\lambda^2 - 3\lambda = 0$$
$$\lambda(\lambda - 3) = 0$$
$$\lambda = 0, 3$$

So

$$y_h = c_1 e^{0x} + c_2 e^{3x}$$

Second find particular solution. Guess $y_p = Ae^x + Bx + C$.
C is part of homogenous solution. Guess $y_p = Ae^x + Bx^2 + Cx$ instead.

$$y_p' = Ae^x + 2Bx + C$$
$$y_p'' = Ae^x + 2B$$

$$y'' - 3y' = e^x + x$$
$$Ae^x + 2B - 3(Ae^x + 2Bx + C) = e^x + x$$

$$(A - 3A)e^x - 6Bx + (2B - 3C) = e^x + x$$
$$A - 3A = 1$$
$$-2A = 1$$
$$A = -\frac{1}{2}$$
$$-6B = 1$$
$$B = -\frac{1}{6}$$
$$2B - 3C = 0$$
$$-\frac{2}{6} = 3C$$
$$C = -\frac{1}{9}$$

So $y_p = -\frac{1}{2}e^x - \frac{1}{6}x^2 - \frac{1}{9}x$.
So $y = -\frac{1}{2}e^x - \frac{1}{6}x^2 - \frac{1}{9}x + c_1 + c_2e^{3x}$.

2 Problem Set 2

2.1 Problem 1

2.2 Problem 2