Assignment 8

Jerry Jiang
TianQi Shi

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Problem Set 1

Question 1

$$f(x) = x^2 \text{ from } -2 \text{ to } 2$$

First, we find a_0.

$$a_0 = \frac{1}{4} \int_{-2}^{2} x^2 dx$$
$$= \frac{1}{2} \int_{0}^{2} x^2 dx$$
$$= \frac{1}{2} \frac{8}{3}$$
$$= \frac{4}{3}$$

Next, we find a_n.

$$a_n = \frac{1}{2} \int_{-2}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx$$
$$= \int_{0}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx$$
$$= \frac{x^2 \sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\Big|_{0}^{2} - \int_{0}^{2} \frac{2x \sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}} dx$$
$$= \frac{2x^2 \sin(\frac{n\pi x}{2})}{n\pi}\Big|_{0}^{2} - \frac{4}{n\pi} \int_{0}^{2} x \sin\left(\frac{n\pi x}{2}\right) dx$$
$$= -\frac{4}{n\pi}\left(-\frac{x \cos(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\Big|_{0}^{2} + \frac{2}{n\pi} \int_{0}^{2} \cos\left(\frac{n\pi x}{2}\right) dx\right)$$
$$= -\frac{4}{n\pi}\left(-\frac{4(-1)^n}{n\pi} + \frac{2}{n\pi}\left(\frac{2}{n\pi} \sin\left(\frac{n\pi x}{2}\right)\Big|_{0}^{2}\right)\right)$$
$$= -\frac{4}{n\pi}\left(-\frac{4(-1)^n}{n\pi}\right)$$
$$= \frac{16(-1)^n}{(n\pi)^2}$$

Next, we find b_n.

$$b_n = \frac{1}{2} \int_{-2}^{2} x^2 \sin\left(\frac{n\pi x}{2}\right) dx$$
$$= 0$$

Therefore,

$$f(x) = \frac{4}{3} + \sum_{1}^{\infty} \frac{16(-1)^n}{(n\pi)^2} \cos(\frac{n\pi x}{2})$$

This converges for all x.

Question 2

Assuming $f(x)$ is periodic.

$$f(x) = e^{2x}$$

$$
\begin{aligned}
c_n &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{2x} e^{-inx} dx \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{(2-in)x} dx \\
&= \frac{1}{2\pi} \frac{e^{(2-in)x}}{2-in} \bigg|_{-\pi}^{\pi} \\
&= \frac{1}{2\pi(2-in)} \left(e^{(2-in)\pi} - e^{-(2-in)\pi} \right)
\end{aligned}
$$

Therefore,

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi(2-in)} \left(e^{(2-in)\pi} - e^{-(2-in)\pi} \right) e^{inx}$$

Problem Set 2

Question 1

Question 2

Simulation

Simulation code:

```
[y, fs] = audioread('recording.wav');
[y1, fs1] = audioread('azim90elev0.wav');

out1 = conv(y, y1(:,1));
out2 = conv(y, y1(:,2));

out = horzcat(out1, out2);

sound(out, fs);
```

The code loads the wav file that we recorded, as well as one of the wav files that was given. It then convolutes each side of the given stereo wav file with the recorded wav file. Finally, it combines the two audio streams together, and plays the resultant sound.

We observed that the resultant sound preserves the content of the recording, but the perceived direction that the sound is coming from has been changed. We could change the direction that the sound was projected from by convoluting it with a different impulse file.

See `sounds.zip` for the collection of resultant wav files that