Assignment 8

Jerry Jiang
TianQi Shi

July 21, 2015

Problem Set 1

Question 1

$$f(x) = x^2 \text{ from } -2 \text{ to } 2$$

First, we find a_0.

$$\begin{aligned}
a_0 &= \frac{1}{4}\int_{-2}^{2} x^2 dx \\
&= \frac{1}{2}\int_{0}^{2} x^2 dx \\
&= \frac{1}{2}\frac{8}{3} \\
&= \frac{4}{3}
\end{aligned}$$

Next, we find a_n.

$$\begin{aligned}
a_n &= \frac{1}{2}\int_{-2}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx \\
&= \int_{0}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx \\
&= \frac{x^2 \sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\bigg|_0^2 - \int_0^2 \frac{2x\sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}} dx \\
&= \frac{2x^2 \sin(\frac{n\pi x}{2})}{n\pi}\bigg|_0^2 - \frac{4}{n\pi}\int_0^2 x\sin\left(\frac{n\pi x}{2}\right) dx \\
&= -\frac{4}{n\pi}\left(-\frac{x\cos(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\bigg|_0^2 + \frac{2}{n\pi}\int_0^2 \cos\left(\frac{n\pi x}{2}\right) dx \right) \\
&= -\frac{4}{n\pi}\left(-\frac{4(-1)^n}{n\pi} + \frac{2}{n\pi}\left(\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\bigg|_0^2\right)\right) \\
&= -\frac{4}{n\pi}\left(-\frac{4(-1)^n}{n\pi}\right) \\
&= \frac{16(-1)^n}{(n\pi)^2}
\end{aligned}$$

Next, we find b_n.

$$\begin{aligned}
b_n &= \frac{1}{2}\int_{-2}^{2} x^2 \sin\left(\frac{n\pi x}{2}\right) dx \\
&= 0
\end{aligned}$$

Therefore,

$$f(x) = \frac{4}{3} + \sum_{1}^{\infty} \frac{16(-1)^n}{(n\pi)^2}\cos(\frac{n\pi x}{2})$$

This converges for all x.

Question 2

Assuming $f(x)$ is periodic.

$$f(x) = e^{2x}$$

$$\begin{aligned}
c_n &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{2x} e^{inx} dx \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{(2+in)x} dx \\
&= \frac{1}{2\pi} \frac{e^{(2+in)x}}{2+in} \bigg|_{-\pi}^{\pi} \\
&= \frac{1}{2\pi(2+in)} \left(e^{(2+in)\pi} - e^{-(2+in)\pi} \right)
\end{aligned}$$

Therefore,

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi(2+in)} \left(e^{(2+in)\pi} - e^{-(2+in)\pi} \right) e^{inx}$$

Problem Set 2

Question 1

Question 2