# Assignment 8

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July 21, 2015

# Problem Set 1

## Question 1

$$f(x) = x^2 \text{ from } -2 \text{ to } 2$$

First, we find $a_0$.

$$\begin{aligned}
a_0 &= \frac{1}{4}\int_{-2}^{2} x^2 dx \\
&= \frac{1}{2}\int_{0}^{2} x^2 dx \\
&= \frac{1}{2}\frac{8}{3} \\
&= \frac{4}{3}
\end{aligned}$$

Next, we find $a_n$.

$$\begin{aligned}
a_n &= \frac{1}{2}\int_{-2}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx \\
&= \int_{0}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right) dx \\
&= \frac{x^2 \sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\Bigg|_0^2 - \int_0^2 \frac{2x \sin(\frac{n\pi x}{2})}{\frac{n\pi}{2}} dx \\
&= \frac{2x^2 \sin(\frac{n\pi x}{2})}{n\pi}\Bigg|_0^2 - \frac{4}{n\pi}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx \\
&= -\frac{4}{n\pi}\left( - \frac{x \cos(\frac{n\pi x}{2})}{\frac{n\pi}{2}}\Bigg|_0^2 + \frac{2}{n\pi}\int_0^2 \cos\left(\frac{n\pi x}{2}\right) dx\right) \\
&= -\frac{4}{n\pi}\left( - \frac{4(-1)^n}{n\pi} + \frac{2}{n\pi}\left(\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Bigg|_0^2\right)\right) \\
&= -\frac{4}{n\pi}\left( - \frac{4(-1)^n}{n\pi}\right) \\
&= \frac{16(-1)^n}{(n\pi)^2}
\end{aligned}$$

Next, we find $b_n$.

$$\begin{aligned}
b_n &= \frac{1}{2}\int_{-2}^{2} x^2 \sin\left(\frac{n\pi x}{2}\right) dx \\
&= 0
\end{aligned}$$

Therefore,

$$f(x) = \frac{4}{3} + \sum_{1}^{\infty} \frac{16(-1)^n}{(n\pi)^2} \cos(\frac{n\pi x}{2})$$

This converges for all x.

### Question 2

Assuming $f(x)$ is periodic.

$$f(x) = e^{2x}$$

$$\begin{aligned} c_n &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{2x}e^{-inx}dx \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(2-in)x}dx \\ &= \frac{1}{2\pi}\frac{e^{(2-in)x}}{2-in}\Bigg|_{-\pi}^{\pi} \\ &= \frac{1}{2\pi(2-in)}\left(e^{(2-in)\pi} - e^{-(2-in)\pi}\right) \end{aligned}$$

Therefore,

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi(2-in)}\left(e^{(2-in)\pi} - e^{-(2-in)\pi}\right)e^{inx}$$

## Problem Set 2

### Question 1

### Question 2