

Assignment 2

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1 Problem Set 1

1.1 Problem 1

a) $y' + 3x^2y^2 = 0, y(0) = 1$

$$y' + 3x^2y^2 = 0, y(0) = 1$$

$$y' = -3x^2y^2$$

$$\frac{dy}{dx} = -3x^2y^2$$

$$\frac{dy}{y^2} = -3x^2 dx$$

$$\int \frac{dy}{y^2} = \int -3x^2 dx$$

$$\frac{-1}{y} = -x^3 + C$$

$$-1 = (-x^3 + C)y$$

$$y = \frac{-1}{-x^3 + C}$$

Particular solution:

$$y = \frac{-1}{-x^3 + C}$$

$$1 = \frac{-1}{0 + C}$$

$$1 = \frac{-1}{C}$$

$$C = -1$$

$$\therefore y = \frac{-1}{-x^3 - 1}$$

b) $x^2y' = 1 + y^2$

$$x^2y' = 1 + y^2$$

$$\frac{dyx^2}{dx} = 1 + y^2$$

$$\frac{dy}{1 + y^2} = \frac{1}{x^2} dx$$

$$\int \frac{dy}{1 + y^2} = \int \frac{1}{x^2} dx$$

$$\arctan y = \frac{-1}{x} + C$$

$$y = \tan\left(\frac{-1}{x} + C\right)$$

1.2 Problem 2

$$\begin{aligned}(x-1)y' + \frac{2(x-1)y}{x} &= (x-1)(x+1) \\ y' + \frac{2y}{x} &= x+1 \quad (x \neq 1) \\ \sigma y' + \sigma \frac{2y}{x} &= \sigma(x+1)\end{aligned}$$

Use integrating factor method:

$$\begin{aligned}(\sigma y)' &= \sigma y' + \sigma' y = \sigma y' + \sigma \frac{2y}{x} \\ \sigma' y &= \sigma \frac{2y}{x} \\ \sigma' &= \sigma \frac{2}{x} \\ \frac{d\sigma}{dx} &= \sigma \frac{2}{x} \\ \frac{d\sigma}{\sigma} &= \frac{2}{x} dx \\ \int \frac{d\sigma}{\sigma} &= \int \frac{2}{x} dx \\ \ln |\sigma| &= 2 \ln |x| + C \\ |\sigma| &= e^C |x|^2 \\ \sigma &= Dx^2\end{aligned} \quad (x \neq 0)$$

Calculate y :

$$\begin{aligned}\frac{dDx^2y}{dx} &= Dx^2(x+1) \\ \frac{dx^2y}{dx} &= x^2(x+1) \\ dx^2y &= x^2(x+1)dx \\ dx^2y &= x^3 + x^2dx \\ \int dx^2y &= \int x^3 + x^2dx \\ x^2y &= \frac{x^4}{4} + \frac{x^3}{3} + C \\ y &= \frac{x^2}{4} + \frac{x}{3} + \frac{C}{x^2}\end{aligned} \quad (x \neq 0)$$

In addition, $x \neq 1$, since we divided the original equation by $(x-1)$.
Find particular solution:

$$y = \frac{x^2}{4} + \frac{x}{3} + \frac{C}{x^2}$$

$$\frac{7}{3} = \frac{2^2}{4} + \frac{2}{3} + \frac{C}{2^2}$$

$$\frac{7}{3} = 1 + \frac{2}{3} + \frac{C}{4}$$

$$\frac{7}{3} = \frac{5}{3} + \frac{C}{4}$$

$$\frac{2}{3} = \frac{C}{4}$$

$$\frac{8}{3} = C$$

$$\therefore y = \frac{x^2}{4} + \frac{x}{3} + \frac{8}{3x^2}$$

2 Problem Set 2

2.1 Problem 1

2.2 Problem 2