

# Assignment 1

Jerry Jiang  
Dinah Shi

May 15, 2015

# 1 Problem Set 1

## 1.1 Problem 1

a)  $y' - 2y = 0$

$$\begin{aligned}y' - 2y &= 0 \\Ne^{Nx} - 2e^{Nx} &= 0 \\Ne^{Nx} &= 2e^{Nx} \\N &= 2\end{aligned}$$

Therefore  $N = 2$ .

b)  $y'' + 4y = 0$

$$\begin{aligned}y'' + 4y &= 0 \\N^2e^{Nx} + 4e^{Nx} &= 0 \\N^2e^{Nx} &= -4e^{Nx} \\N^2 &= -4\end{aligned}$$

Since  $N^2 \geq 0$ , there are no such  $N$ 's.

## 1.2 Problem 2

a) First we shall find the derivatives of  $y$ .

$$\begin{aligned}y &= N\cos(2x) + x \\y' &= -2N\sin(2x) + 1 \\y'' &= -4N\cos(2x)\end{aligned}$$

Next we shall check if  $y$  is a solution.

$$\begin{aligned}y'' + 4y &= -4N\cos(2x) + 4(N\cos(2x) + x) \\&= -4N\cos(2x) + 4N\cos(2x) + 4x \\&= 4x \\&= RHS\end{aligned}$$

Therefore,  $y$  is a solution.

b)

$$\begin{aligned}y'(x) &= -2N\sin(2x) + 1 \\y'(0) &= -2N\sin(0) + 1 \\2 &= -2N\sin(0) + 1 \\2 &= -2N \cdot 0 + 1 \\2 &= 1\end{aligned}$$

Therefore, there are no possible solutions.

c) Linear, as it can be written in the form  $a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \dots + a_0(x)y(t) = g(x)$ . Where  $n = 2$ ,  $a_2 = 1$ ,  $a_1 = 4$ ,  $a_0 = 0$ , and  $g(x) = 4x$ .

## 2 Problem Set 2

### 2.1 Problem 1

Given  $y'' = 2y + y'$ , verify the following solutions:

$$y_1(x) = \sinh(2x) + \cosh(2x)$$

$$y_2(x) = \sin(2x + 3)$$

For  $y_1$ ,

$$y_1(x) = \sinh(2x) + \cosh(2x)$$

$$y_1'(x) = 2\cosh(2x) + 2\sinh(2x)$$

$$y_1''(x) = 4\sinh(2x) + 4\cosh(2x)$$

Substituting into the differential equation,

$$4\sinh(2x) + 4\cosh(2x) = 2(\sinh(2x) + \cosh(2x)) + 2\cosh(2x) + 2\sinh(2x)$$

This statement holds so  $y_1$  is a solution. For  $y_2$ ,

$$y_2(x) = \sin(2x + 3)$$

$$y_2'(x) = 2\cos(2x + 3)$$

$$y_2''(x) = -4\sin(2x + 3)$$

Substituting into the differential equation,

$$-4\sin(2x + 3) = 2\sin(2x + 3) + 2\cos(2x + 3)$$

This statement holds so  $y_2$  is a solution.

### 2.2 Problem 2

Given the following differential equations:

$$y_1(x) = e^x + y' + y'' = 3$$

$$y_1(x) = e^y + y' + y'' = 3$$

Determine which equation is linear. For the linear equation, verify that

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

is a solution, for all constants  $A$  and  $B$ . Then, determine one set of possible values for  $A$  and  $B$  given that  $y_3(-1) = 2e - \frac{1}{2e}$ .

We know  $y_1$  is the linear equation as  $y_2$  contains a term  $e^y$  which is non-linear. To verify that  $y_3$  is a solution,

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

$$y'_3(x) = -Ae^{-x} + 3 - \frac{e^x}{2}$$

$$y''_3(x) = Ae^{-x} - \frac{e^x}{2}$$

Substituting into the differential equation,

$$\begin{aligned} e^x + y' + y'' &= 3 \\ e^x + (-Ae^{-x} + 3 - \frac{e^x}{2}) + (Ae^{-x} - \frac{e^x}{2}) &= 3 \\ 3 &= 3 \end{aligned}$$

This holds for all values of  $A$  and  $B$ . Using the given parameters,

$$\begin{aligned} y_3(-1) &= 2e - \frac{1}{2e} \\ 2e - \frac{1}{2e} &= Ae^{-(-1)} + B + 3(-1) - \frac{e^{-1}}{2} \\ 2e - \frac{1}{2e} &= Ae + B - 3 - \frac{1}{2e} \end{aligned}$$

Thus we get  $A = 2, B = 3$ .