# Assignment 3

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# 1 Problem Set 1

## 1.1 Problem 1

a) $y'' + 3y' - 10y = 0$

$$\lambda^2 + 3\lambda - 10 = 0$$
$$(\lambda + 5)(\lambda - 2) = 0$$
$$\therefore \lambda = 2, -5$$

So $y = c_1 e^{2x} + c_2 e^{-5x}$.
b) $y'' + 2y' + 3y = 0$

$$\lambda^2 + 2\lambda + 3 = 0$$
$$\lambda = \frac{-2 \pm \sqrt{2^2 - 4 * 1 * 3}}{2}$$
$$\lambda = \frac{-2 \pm \sqrt{4 - 12}}{2}$$
$$\lambda = \frac{-2 \pm i\sqrt{8}}{2}$$
$$\lambda = -1 \pm i\sqrt{2}$$

So $y = e^{-x}(c_1 \sin(\sqrt{2}x) + c_1 \cos(\sqrt{2}x))$.

## 1.2 Problem 2

$$y'' - 3y' = e^x + x$$

First find homogeneous solution.

$$\lambda^2 - 3\lambda = 0$$
$$\lambda(\lambda - 3) = 0$$
$$\lambda = 0, 3$$

So $y_h = c_1 e^{0x} + c_2 e^{3x}$.
Second find particular solution. Guess $y_p = Ae^x + Bx + C$.
$C$ is part of homogeneous solution. Guess $y_p = Ae^x + Bx^2 + Cx$ instead.

$$y_p' = Ae^x + 2Bx + C$$
$$y_p'' = Ae^x + 2B$$

$$y'' - 3y' = e^x + x$$
$$Ae^x + 2B - 3(Ae^x + 2Bx + C) = e^x + x$$
$$(A - 3A)e^x - 6Bx + (2B - 3C) = e^x + x$$

Find $A$.

$$
\begin{aligned}
A - 3A &= 1 \\
-2A &= 1 \\
A &= -\frac{1}{2}
\end{aligned}
$$

Find $B$.

$$
\begin{aligned}
-6B &= 1 \\
B &= -\frac{1}{6}
\end{aligned}
$$

Find $C$.

$$
\begin{aligned}
2B - 3C &= 0 \\
-\frac{2}{6} &= 3C \\
C &= -\frac{1}{9}
\end{aligned}
$$

So $y_p = -\frac{1}{2}e^x - \frac{1}{6}x^2 - \frac{1}{9}x$.
So $y = -\frac{1}{2}e^x - \frac{1}{6}x^2 - \frac{1}{9}x + c_1 + c_2e^{3x}$.

# 2 Problem Set 2

## 2.1 Problem 1

## 2.2 Problem 2