University of Waterloo
MATH 213, Spring 2015
Assignment 6

Question 1

Use the Laplace transform to find the particular solution of the following functions

$$y' + y = \sin(t-3)u(t-3) + tu(t-3) - 3u(t-3), y(0) = 0$$

Where

$$u(t) = \begin{cases} 0 & \text{when } t < 0 \\ 1 & \text{when } t \geq 0 \end{cases}$$

Take the Laplace of both sides:

$$\begin{aligned}
&y' + y = \sin(t-3)u(t-3) + tu(t-3) - 3u(t-3) \\
&y' + y = \sin(t-3)u(t-3) + (t-3)u(t-3) \\
&sY(s) + Y(s) = \frac{e^{-3s}}{s^2+1} + \frac{e^{-3s}}{s^2} \\
&(s+1)Y(s) = e^{-3s}(\frac{1}{s^2+1} + \frac{1}{s^2}) \\
&Y(s) = e^{-3s}(\frac{1}{(s^2+1)(s+1)} + \frac{1}{s^2(s+1)})
\end{aligned}$$

Use partial fraction decomposition:

$$\begin{aligned}
&\frac{1}{(s^2+1)(s+1)} = \frac{As}{s^2+1} + \frac{B}{s^2+1} + \frac{C}{s+1} \\
&1 = As^2 + As + Bs + B + Cs^2 + C \\
&A + C = 0, A + B = 0, B + C = 1 \\
&\therefore A = -0.5, C = 0.5, B = 0.5
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} \\
&1 = As^2 + As + Bs + B + Cs^2 \\
&A + C = 0, A + B = 0, B = 1 \\
&\therefore A = -1, B = 1, C = 1
\end{aligned}$$

Take the inverse Laplace:

$$Y(s) = e^{-3s}(\frac{1}{(s^2+1)(s+1)} + \frac{1}{s^2(s+1)})$$
$$Y(s) = e^{-3s}(-\frac{1}{2}\frac{s}{s^2+1} + \frac{1}{2}\frac{1}{s^2+1} + \frac{1}{2}\frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1})$$
$$y(t) = (-\frac{1}{2}cos(t-3) + \frac{1}{2}sin(t-3) + \frac{3e^{-(t-3)}}{2} - 1 + (t-3))u(t-3)$$

Question 2