

Assignment 1

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1 Problems Set 1

1.1 Problem 1

a) $y' - 2y = 0$

$$\begin{aligned}y' - 2y &= 0 \\Ne^{Nx} - 2e^{Nx} &= 0 \\Ne^{Nx} &= 2e^{Nx} \\N &= 2\end{aligned}$$

Therefore $N = 2$.

b) $y'' + 4y = 0$

$$\begin{aligned}y'' + 4y &= 0 \\N^2e^{Nx} + 4e^{Nx} &= 0 \\N^2e^{Nx} &= -4e^{Nx} \\N^2 &= -4\end{aligned}$$

Since $N^2 \geq 0$, there are no such N 's.

1.2 Problem 2

a) First we shall find the derivatives of y .

$$\begin{aligned}y &= N\cos(2x) + x \\y' &= -2N\sin(2x) + 1 \\y'' &= -4N\cos(2x)\end{aligned}$$

Next we shall check if y is a solution.

$$\begin{aligned}y'' + 4y &= -4N\cos(2x) + 4(N\cos(2x) + x) \\&= -4N\cos(2x) + 4N\cos(2x) + 4x \\&= 4x \\&= RHS\end{aligned}$$

Therefore, y is a solution.

b)

$$\begin{aligned}y'(x) &= -2N\sin(2x) + 1 \\y'(0) &= -2N\sin(0) + 1 \\2 &= -2N\sin(0) + 1 \\2 &= -2N \cdot 0 + 1 \\2 &= 1\end{aligned}$$

Therefore, there are no possible solutions.

c) Linear, as it can be written in the form $a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \dots + a_0(x)y(t) = g(x)$. Where $n = 2$, $a_2 = 1$, $a_1 = 4$, $a_0 = 0$, and $g(x) = 4x$.

2 Problem Set 2

2.1 Problem 1

Given $y'' = 2y + y'$, verify the following solutions:

$$y_1(x) = \sinh(2x) + \cosh(2x)$$

$$y_2(x) = \sin(2x + 3)$$

For y_1 ,

$$y_1(x) = \sinh(2x) + \cosh(2x)$$

$$y_1'(x) = 2\cosh(2x) + 2\sinh(2x)$$

$$y_1''(x) = 4\sinh(2x) + 4\cosh(2x)$$

Substituting into the differential equation,

$$4\sinh(2x) + 4\cosh(2x) = 2(\sinh(2x) + \cosh(2x)) + 2\cosh(2x) + 2\sinh(2x)$$

This statement holds so y_1 is a solution. For y_2 ,

$$y_2(x) = \sin(2x + 3)$$

$$y_2'(x) = 2\cos(2x + 3)$$

$$y_2''(x) = 4\cos(2x + 3)$$

Substituting into the differential equation,

$$4\cos(2x + 3) = 2\sin(2x + 3) + 2\cos(2x + 3)$$

This statement holds so y_2 is a solution.

2.2 Problem 2

Given the following differential equations:

$$y_1(x) = e^x + y' + y'' = 3$$

$$y_1(x) = e^y + y' + y'' = 3$$

Determine which equation is linear. For the linear equation, verify that

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

is a solution, for all constants A and B . Then, determine one set of possible values for A and B given that $y_3(-1) = 2e - \frac{1}{2e}$.

We know y_1 is the linear equation as y_2 contains a term e^y which is non-linear. To verify that y_3 is a solution,

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

$$y'_3(x) = -Ae^{-x} + 3 - \frac{e^x}{2}$$

$$y''_3(x) = Ae^{-x} - \frac{e^x}{2}$$

Substituting into the differential equation,

$$\begin{aligned} e^x + y' + y'' &= 3 \\ e^x + (-Ae^{-x} + 3 - \frac{e^x}{2}) + (Ae^{-x} - \frac{e^x}{2}) &= 3 \\ 3 &= 3 \end{aligned}$$

This holds for all values of A and B . Using the given parameters,

$$\begin{aligned} y_3(-1) &= 2e - \frac{1}{2e} \\ 2e - \frac{1}{2e} &= Ae^{-(-1)} + B + 3(-1) - \frac{e^{-1}}{2} \\ 2e - \frac{1}{2e} &= Ae + B - 3 - \frac{1}{2e} \end{aligned}$$

Thus we get $A = 2, B = 3$.