University of Waterloo
MATH 213, Spring 2015
Assignment 7

Question 1

Convolution Theorem

We separate $F(s)$ into the following equation:

$$7 * \frac{1}{s^3} * \frac{1}{s-3}$$

Now we do the convolution:

$$\begin{aligned}
f(t) &= 7\int_0^t \frac{1}{2}\tau^2 e^{3(t-\tau)}d\tau \\
&= \frac{7e^{3t}}{2}\int_0^t \tau^2 e^{-3\tau}d\tau \\
&= \frac{7e^{3t}}{2}\left(\frac{\tau^2 e^{-3\tau}}{-3}\bigg|_0^t + \frac{2}{3}\int_0^t \tau e^{-3\tau}d\tau\right) \\
&= \frac{7e^{3t}}{2}\left(-\frac{t^2e^{-3t}}{3} + \frac{2}{3}\left(-\frac{\tau e^{-3\tau}}{3}\bigg|_0^t + \frac{1}{3}\int_0^t e^{-3\tau}d\tau\right)\right) \\
&= \frac{7e^{3t}}{2}\left(-\frac{t^2e^{-3t}}{3} + \frac{2}{3}\left(-\frac{te^{-3t}}{3} - \frac{1}{9}e^{-3\tau}\bigg|_0^t\right)\right) \\
&= \frac{7e^{3t}}{2}\left(-\frac{t^2e^{-3t}}{3} + \frac{2}{3}\left(-\frac{te^{-3t}}{3} - \frac{1}{9}e^{-3t} + \frac{1}{9}\right)\right) \\
&= -\frac{7t^2}{6} + \frac{7e^{3t}}{3}\left(-\frac{te^{-3t}}{3} - \frac{1}{9}e^{-3t} + \frac{1}{9}\right) \\
&= -\frac{7t^2}{6} - \frac{7t}{9} - \frac{7}{27} + \frac{7e^{3t}}{27}
\end{aligned}$$

Partial Fraction Expansion

Decompose the rational equation.

$$\frac{7}{s^3(s-3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-3}$$

$$7 = A(s^3 - 3s^2) + B(s^2 - 3s) + C(s-3) + Ds^3$$

$$A + D = 0, -3A + B = 0, -3B + C = 0, -3C = 7$$

$$\therefore C = \frac{-7}{3}, B = \frac{-7}{9}, A = \frac{-7}{27}, D = \frac{7}{27}$$

$$\therefore \frac{7}{s^3(s-3)} = \frac{-7}{27}\frac{1}{s} + \frac{-7}{9}\frac{1}{s^2} + \frac{-7}{3}\frac{1}{s^3} + \frac{7}{27}\frac{1}{s-3}$$

$$L\{\frac{7}{s^3(s-3)}\} = \frac{-7}{27} + \frac{-7}{9}t + \frac{-7}{6}t^2 + \frac{7}{27}e^{3t}$$

Question 2