1 Problems Set 1

1.1 Problem 1

a) $y' - 2y = 0$

$$\begin{aligned} y' - 2y &= 0 \\ Ne^{Nx} - 2e^{Nx} &= 0 \\ Ne^{Nx} &= 2e^{Nx} \\ N &= 2 \end{aligned}$$

Therefore $N = 2$.

b) $y'' + 4y = 0$

$$\begin{aligned} y'' + 4y &= 0 \\ N^2e^{Nx} + 4e^{Nx} &= 0 \\ N^2e^{Nx} &= -4e^{Nx} \\ N^2 &= -4 \end{aligned}$$

Since $N^2 >= 0$, there are no such N's.

1.2 Problem 2

a) First we shall find the derivatives of y.

$$\begin{aligned} y &= Ncos(2x) + x \\ y' &= -2Nsin(2x) + 1 \\ y'' &= -4Ncos(2x) \end{aligned}$$

Next we shall check if y is a solution.

$$\begin{aligned} &y'' + 4y \\ &= -4Ncos(2x) + 4(Ncos(2x) + x) \\ &= -4Ncos(2x) + 4Ncos(2x) + 4x \\ &= 4x \\ &= RHS \end{aligned}$$

Therefore, y is a solution.

b)

$$\begin{aligned} y'(x) &= -2Nsin(2x) + 1 \\ y'(0) &= -2Nsin(0) + 1 \\ 2 &= -2Nsin(0) + 1 \\ 2 &= -2N0 + 1 \\ 2 &= 1 \end{aligned}$$

Therefore, there are no possible solutions.

c) Linear, as it can be written in the form $a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + ... + a_0(x)y(t) = g(x)$. Where $n = 2$, $a_2 = 1$, $a_1 = 4$, $a_0 = 0$, and $g(x) = 4x$.

2 Problem Set 2

2.1 Problem 1

Given $y'' = 2y + y'$, verify the following solutions:

$$y_1(x) = sinh(2x) + cosh(2x)$$
$$y_2(x) = sin(2x + 3)$$

For y_1,

$$y_1(x) = sinh(2x) + cosh(2x)$$
$$y_1'(x) = 2cosh(2x) + 2sinh(2x)$$
$$y_1''(x) = 4sinh(2x) + 4cosh(2x)$$

Substituting into the differential equation,

$$4sinh(2x) + 4cosh(2x) = 2(sinh(2x) + cosh(2x)) + 2cosh(2x) + 2sinh(2x)$$

This statement holds so y_1 is a solution. For y_2,

$$y_2(x) = sin(2x + 3)$$
$$y_2'(x) = 2cos(2x + 3)$$
$$y_2''(x) = 4cos(2x + 3)$$

Substituting into the differential equation,

$$4cos(2x + 3) = 2sin(2x + 3) + 2cos(2x + 3)$$

This statement holds so y_2 is a solution.

2.2 Problem 2

Given the following differential equations:

$$y_1(x) = e^x + y' + y'' = 3$$

$$y_1(x) = e^y + y' + y'' = 3$$

Determine which equation is linear. For the linear equation, verify that

$$y_3(x) = Ae^{-x} + B + 3x - \frac{e^x}{2}$$

is a solution, for all constants A and B. Then, determine one set of possible values for A and B given that $y_3(-1) = 2e - \frac{1}{2e}$.

We know y_1 is the linear equation as y_2 contains a term e^y which is non-linear. To verify that y_3 is a solution,

$$\begin{aligned}
y_3(x) &= Ae^{-x} + B + 3x - \frac{e^x}{2} \\
y_3'(x) &= -Ae^{-x} + 3 - \frac{e^x}{2} \\
y_3''(x) &= Ae^{-x} - \frac{e^x}{2}
\end{aligned}$$

Substituting into the differential equation,

$$\begin{aligned}
e^x + y' + y'' &= 3 \\
e^x + (-Ae^{-x} + 3 - \frac{e^x}{2}) + (Ae^{-x} - \frac{e^x}{2}) &= 3 \\
3 &= 3
\end{aligned}$$

This holds for all values of A and B. Using the given parameters,

$$\begin{aligned}
y_3(-1) &= 2e - \frac{1}{2e} \\
2e - \frac{1}{2e} &= Ae^{-(-1)} + B + 3(-1) - \frac{e^{-1}}{2} \\
2e - \frac{1}{2e} &= Ae + B - 3 - \frac{1}{2e}
\end{aligned}$$

Thus we get $A = 2, B = 3$.