Entropy Has No Direction: A Mirror-State Paradox Against Universal Monotonic Entropy Increase and a First-Principles Proof that Constraints Reshape the Entropy Distribution $P_\infty(S;\lambda)$

Proving the Feasibility of the Second-Kind Perpetual Motion Machine and Disproving Universal Heat Death

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The LaTeX source of this manuscript, together with a live FAQ and an up-to-date list of experimental results, is available at `https://github.com/tpeng1977/entropy`.

Abstract

We present a purely theoretical, self-contained argument that the Second Law of Thermodynamics cannot be a universal fundamental law in the form "entropy does not decrease" (whether asserted trajectory-wise or as a universal statistical principle) when the underlying microscopic dynamics are time-reversal invariant. The core is a mirror-state construction: for any microstate A one constructs its time-reversed partner B (momenta inverted). If a universal monotonicity statement is applied to both A and B, it implies that A is a local minimum of entropy at every moment, which forces entropy to be constant and destroys any entropic arrow of time. The consistent replacement is that entropy is a stochastic variable described by a probability distribution $P(S)$, whose shape depends on constraints and boundary conditions; in this view, entropy-based "laws" are emergent summaries of constraint-dependent microscopic dynamics rather than fundamental drivers, and in practice it is constraints and boundaries—not entropy itself—that one manipulates to achieve mixing, separation, or self-organization. We then prove from first principles that constraints necessarily reshape the long-time entropy distribution $P_\infty(S;\lambda)$ by altering the invariant measure through changes in the Hamiltonian and/or the accessible phase space. A sharp criterion is given: in the microcanonical setting, the *only* way $P_\infty^{(E)}(S;\lambda)$ can remain the same up to translation is when all accessible macrostate volumes are scaled by a common factor; otherwise the distribution changes structurally. Recent experimental work validates this framework: asymmetric constraints enable spontaneous low-entropy transitions, allowing systems to produce useful work by absorbing heat from a single thermal reservoir. With both theoretical and experimental barriers overcome, practical devices based on constraint-reshaped entropy distributions become feasible, with profound implications for energy generation and the long-term sustainability of civilization.

1 Introduction

The "Second Law" is often presented in two forms. *Strict (deterministic) form*: for an isolated system, entropy does not decrease along *every* trajectory, i.e. $S(t+\delta t) \geq S(t)$ for all t and all microstates (or $\mathrm{d}S/\mathrm{d}t \geq 0$ wherever defined). *Statistical form*: for an appropriate ensemble or limit, entropy increase is "overwhelmingly probable" or the ensemble-average entropy change is nonnegative.

This paper has one scope: we address these statements only insofar as they are claimed to be *universal* consequences of time-reversal invariant microscopic physics. We show that such universal monotonicity claims are logically incompatible with time-reversal symmetry. The consistent replacement is not to end thermodynamics, but to replace "entropy has a direction" with: **entropy has no direction; it is described by a probability distribution**. Moreover, **constraints and boundary conditions reshape this distribution**. We prove from first principles that constraints reshape the long-time entropy distribution $P_\infty(S;\lambda)$—motivated in part by the author's earlier simulation work showing that geometry can challenge conventional entropy regimes in nanofluidic cascades [7]—and we discuss recent experimental work that validates this framework.

2 Preliminaries and assumptions

2.1 Microscopic dynamics and time reversal

Let $\Gamma(t)$ denote the phase-space microstate of an isolated system evolving under time-reversal invariant microscopic dynamics (Hamiltonian dynamics being the canonical example). Let $\mathcal{T}$ denote time reversal, which (for classical mechanics) acts by reversing momenta while leaving positions unchanged.

Time-reversal invariance means: if $\Gamma_A(t)$ is a solution, then $\Gamma_B(t) := \mathcal{T}\Gamma_A(2t_0 - t)$ is also a solution. In particular, the forward-time evolution of the mirror state $\Gamma_B(t_0) = \mathcal{T}\Gamma_A(t_0)$ replays the past of Γ_A in reverse.

2.2 Entropy as a coarse-grained state function

We use Boltzmann (coarse-grained) entropy: fix a time-reversal symmetric coarse-graining (partition) of phase space into macrostates (cells) $\{C_m\}$. Assign to any $\Gamma \in C_m$ an instantaneous entropy $S(\Gamma) = S_m$.

For mathematical cleanliness (finite volumes), we adopt the energy-shell form used in statistical mechanics: at fixed energy E define the accessible macrostate volume $W_m^{(E)}$ on the energy shell and set $S_m^{(E)} = k_B \ln(W_m^{(E)}/W_0)$, with an arbitrary reference volume W_0 (shifting entropy by a constant does not affect any conclusions). This is the same entropy notion used below to derive $P_\infty(S;\lambda)$.

We assume the coarse-graining is time-reversal symmetric, so $S(\mathcal{T}\Gamma) = S(\Gamma)$.

2.3 Continuity (minimal regularity)

The mirror-state paradox below is clearest when $S(t) := S(\Gamma(t))$ is continuous in time along trajectories (standard for smooth coarse-graining, or for coarse variables defining macrostates). This is the only regularity used to turn "local minimum everywhere" into "constant everywhere".

Remark (discrete coarse-graining). If the coarse-graining is strictly discrete, then $S(t)$ may be piecewise constant with jumps. The paradox still goes through for a universal monotonicity claim that is asserted for *arbitrarily small* $\delta t > 0$: Eq. (1) forces $S(t_0)$ to be simultaneously a right- and left-minimum at every t_0, which rules out any jump up or down and again implies that $S(t)$ is constant.

2.4 What is meant by $P_\infty(S;\lambda)$

In the second part of the paper, $P_\infty(S;\lambda)$ denotes a *long-time* entropy distribution induced by an invariant measure. Two standard routes make this precise: (i) assume the dynamics are ergodic/mixing on the relevant invariant set so that time averages (and long-time distributions) are independent of the initial microstate up to measure-zero exceptions; or (ii) consider an ensemble whose initial microstates are drawn from the invariant measure (microcanonical or canonical), in which case P_∞ is immediate. Our formulas below are statements about the invariant measures themselves and the entropy distributions induced by them; the above routes justify interpreting these as long-time distributions.

3 The mirror-state paradox

3.1 Strict (trajectory-wise) monotonicity is impossible as a universal law

Claim. A universal law of the form "for every microstate and every time, $S(t+\delta t) \geq S(t)$ for all sufficiently small $\delta t > 0$" is incompatible with time-reversal invariant microscopic dynamics and time-reversal symmetric coarse-grained entropy.

Proof (mirror-state construction). Fix an arbitrary time t_0 on an arbitrary trajectory $\Gamma_A(t)$. Construct the mirror state at the same time: $\Gamma_B(t_0) = \mathcal{T}\Gamma_A(t_0)$, and let $\Gamma_B(t)$ be its forward-time evolution. By time-reversal invariance, for any $\delta t > 0$,

$$\Gamma_B(t_0 + \delta t) = \mathcal{T}\Gamma_A(t_0 - \delta t).$$

By time-reversal symmetry of the coarse-graining, $S(\mathcal{T}\Gamma) = S(\Gamma)$, so

$$S_B(t_0 + \delta t) = S_A(t_0 - \delta t), \qquad S_B(t_0) = S_A(t_0).$$

Now apply the universal monotonicity statement to *both* A and B:

$$S_A(t_0 + \delta t) \geq S_A(t_0), \qquad S_B(t_0 + \delta t) \geq S_B(t_0).$$

The second inequality becomes $S_A(t_0 - \delta t) \geq S_A(t_0)$. Hence, for all small $\delta t > 0$,

$$S_A(t_0 + \delta t) \geq S_A(t_0) \quad \text{and} \quad S_A(t_0 - \delta t) \geq S_A(t_0). \tag{1}$$

Thus t_0 is a (two-sided) local minimum of $S_A(t)$.

Because t_0 was arbitrary, *every* time is a local minimum. If $S_A(t)$ is continuous (Sec. 2.3), a function for which every point is a local minimum must be constant. Therefore the universal strict Second Law implies that entropy is constant on every trajectory, which contradicts the empirical fact that macroscopic systems exhibit entropy increase and eliminates any entropic arrow of time. □

3.2 Extension: the statistical Second Law cannot be universal

The same mirror-state logic refutes the statistical Second Law *when it is asserted as a universal principle applying to every microstate, including its mirror.* Suppose one claims, universally, that entropy increase is "overwhelmingly probable" for the forward-time evolution of *any* initial microstate. Apply this to A at time t_0, and also to $B = \mathcal{T}A$ at t_0. Since the forward evolution of B corresponds to the time-reversed past of A, "overwhelmingly probable increase" for both implies (with overwhelming probability) the two-sided inequalities in Eq. (1), i.e. that t_0 is a local minimum with overwhelming probability. Choosing t_0 arbitrarily destroys any persistent arrow of time in the same way.

Therefore, the statistical Second Law cannot be a universal state-by-state principle. Any one-way statement about ΔS must depend on additional structure: special initial ensembles, coarse-graining choices, limits, or constraints/boundaries that select a particular effective description. This observation motivates the corrected view below.

4 Corrected view: entropy is a random variable with a constraint-dependent PDF

The consistent replacement of "entropy has a direction" is: **entropy has no direction; it has a probability distribution**. Write $P_t(S)$ for the (time-dependent) entropy distribution induced by an ensemble of microstates at time t, and $P_\infty(S;\lambda)$ for the long-time distribution under constraint parameters λ (geometry, boundary conditions, static fields, etc.).

At the microscopic level, however, nothing in the dynamics "sees" entropy. Individual particles obey time-reversal invariant equations of motion; their trajectories and interactions are governed by the Hamiltonian and forces, not by macroscopic state functions such as entropy. Boltzmann entropy is a coarse-grained, statistical description of the collective behaviour of many particles—a macroscopic *appearance* of the underlying time-reversal invariant motion, obtained only after we partition phase space into macrostates and count accessible microstates. Constraints cannot change the fundamental microscopic rules; their core role is to intervene directly in the actual motion of particles by changing the Hamiltonian $H(\Gamma;\lambda)$ and/or the accessible set $\mathcal{A}(\lambda)$, thereby restricting where particles can go and how they move. This, in turn, changes the number of accessible microstates for each macrostate and thus reshapes the entropy distribution. All changes in entropy are therefore *statistical summaries* of how constraints redirect microscopic trajectories. Entropy is a diagnostic, not a causal driver. In practical engineering terms, what one truly controls and optimises are constraints and boundary conditions (to achieve, say, separation or mixing), while entropy is useful as a bookkeeping tool for whether a given design makes certain macrostates typical or rare. From a logical perspective it is more meaningful to study how different constraints alter system dynamics and long-time distributions than to postulate universal laws about "entropy increase" detached from the underlying microphysics.

From this angle, familiar macroscopic "entropy increase" phenomena are best understood as follows. In many textbook and laboratory situations, the initial state is prepared to be *very special* and low-entropy (gas in one half of a box, sharp temperature gradients, unmixed components, etc.). Starting from such atypical initial conditions, the entropy—as a coarse-grained statistic of particle configurations—does indeed rise rapidly toward the values typical of the long-time distribution under the given constraints. Once the system has reached that regime, however, the entropy does not freeze at a single "equilibrium value": microscopic dynamics continue indefinitely and $S(t)$ keeps fluctuating, with spontaneous increases and decreases both being normal. The role of constraints is to determine the *shape* of the long-time distribution for S (and for other observables), not to enforce monotone drift toward a uniquely defined maximum. There need not be a strict equilibrium state in the sense of a single entropy value, nor must the formal "maximum entropy" compatible with constraints coincide with the most probable coarse-grained macrostate once dynamical restrictions are taken seriously.

What fundamentally decides whether a system mixes, separates, or self-organises into ordered patterns is therefore the constraint-dependent microscopic dynamics and the resulting invariant measures—not an entropy "law" acting as a causal agent. Accordingly, all results in this paper concerning entropy distributions and their evolution should be read as probabilistic, ensemble-level characterisations of typical behaviour under specified constraints, rather than as per-trajectory guarantees for every single realisation of a physical system.

The remainder of this paper proves the second core claim—**constraints and boundaries can change the long-time entropy distribution** $P_\infty(S;\lambda)$ in an explicit, first-principles way—and then presents experimental validation of this framework.

5 First-principles derivation: constraints $\lambda \to P_\infty(S;\lambda)$

5.1 Constraints as Hamiltonian/accessible-set modifications

Let λ denote constraint parameters. Constraints enter through the Hamiltonian

$$H(\Gamma;\lambda) = H_0(\Gamma) + V_c(\Gamma;\lambda), \tag{2}$$

and/or through an accessible set $\mathcal{A}(\lambda)$ (hard constraints). Both views are equivalent: hard walls correspond to $V_c = +\infty$ outside $\mathcal{A}(\lambda)$.

5.2 Long-time invariant measures (microcanonical and canonical)

Microcanonical (isolated). At fixed energy E, a natural invariant measure is uniform on the accessible energy shell:

$$\rho_\infty^{(E)}(\Gamma;\lambda) = \frac{1}{\Omega(E;\lambda)}\,\delta\big(H(\Gamma;\lambda)-E\big)\,\mathbf{1}_{\mathcal{A}(\lambda)}(\Gamma), \tag{3}$$

where the accessible density of states is

$$\Omega(E;\lambda) = \int \mathrm{d}\Gamma\,\delta\big(H(\Gamma;\lambda)-E\big)\,\mathbf{1}_{\mathcal{A}(\lambda)}(\Gamma). \tag{4}$$

Canonical (heat bath). With a heat bath at temperature T and dynamics obeying fluctuation–dissipation, the invariant measure is

$$\rho_\infty(\Gamma;\lambda) = \frac{1}{Z(\lambda)}\,e^{-\beta H(\Gamma;\lambda)}, \qquad Z(\lambda) = \int_{\Gamma\in\mathcal{A}(\lambda)} \mathrm{d}\Gamma\,e^{-\beta H(\Gamma;\lambda)}. \tag{5}$$

5.3 Boltzmann entropy on the energy shell

Fix a time-reversal symmetric coarse-graining $\{C_m\}_{m\in\mathcal{M}}$. Define the accessible macrostate volume on the energy shell:

$$W_m^{(E)}(\lambda) = \int \mathrm{d}\Gamma\,\delta\big(H(\Gamma;\lambda)-E\big)\,\mathbf{1}_{C_m}(\Gamma)\,\mathbf{1}_{\mathcal{A}(\lambda)}(\Gamma), \tag{6}$$

and assign the Boltzmann entropy value

$$S_m^{(E)}(\lambda) = k_B \ln\left(\frac{W_m^{(E)}(\lambda)}{W_0}\right). \tag{7}$$

5.4 Closed-form expression for $P_\infty(S;\lambda)$ and a sharp change criterion

Microcanonical. Because the microcanonical measure is uniform on the energy shell, the macrostate probability is a ratio of accessible volumes:

$$\pi_m^{(E)}(\lambda) = \frac{W_m^{(E)}(\lambda)}{\Omega(E;\lambda)}, \qquad \Omega(E;\lambda) = \sum_{m\in\mathcal{M}} W_m^{(E)}(\lambda). \tag{8}$$

Therefore the long-time entropy distribution at fixed energy is

$$\boxed{P_\infty^{(E)}(S;\lambda) = \sum_{m\in\mathcal{M}} \frac{W_m^{(E)}(\lambda)}{\Omega(E;\lambda)}\,\delta\big(S - S_m^{(E)}(\lambda)\big).} \tag{9}$$

Proposition (sharp criterion; "only translation" degeneracy). Fix E and the coarse-graining. Let $\mathcal{V}(\lambda)$ denote the multiset of accessible macrostate volumes $\{W_m^{(E)}(\lambda)\}_{m\in\mathcal{M}}$ (counting multiplicity).

1. If there exists $c > 0$ such that $\mathcal{V}(\lambda_2) = c\,\mathcal{V}(\lambda_1)$ (i.e. all macrostate volumes scale by a common factor, up to permutation), then
$$P_\infty^{(E)}(S;\lambda_2) = P_\infty^{(E)}(S - k_B \ln c\,;\lambda_1)\,,$$
i.e. the distribution changes only by translation of the entropy axis.
2. Otherwise, $P_\infty^{(E)}(S;\lambda_2)$ is *not* a translate of $P_\infty^{(E)}(S;\lambda_1)$; the distribution changes structurally.

Proof. Let M be the macrostate index with $\mathbb{P}(M = m) = \pi_m^{(E)}(\lambda)$ and define $X_\lambda := W_M^{(E)}(\lambda)$. Then $S = k_B \ln(X_\lambda/W_0)$ and Eq. (9) is exactly the law of S. If $W_m^{(E)}(\lambda_2) = c\,W_{\sigma(m)}^{(E)}(\lambda_1)$ for some permutation σ, then $X_{\lambda_2} \overset{d}{=} c\,X_{\lambda_1}$, hence $S_{\lambda_2} \overset{d}{=} S_{\lambda_1} + k_B \ln c$, which is the claimed translation. Conversely, if $P_\infty^{(E)}(S;\lambda_2)$ were a translate of $P_\infty^{(E)}(S;\lambda_1)$ by Δ, then $X_{\lambda_2} = W_0 e^{S/k_B}$ would be distributed as $e^{\Delta/k_B} X_{\lambda_1}$, which implies $\mathcal{V}(\lambda_2) = e^{\Delta/k_B}\mathcal{V}(\lambda_1)$ (up to permutation). □

Canonical as an exact energy mixture. The canonical energy density is
$$P_\beta(E;\lambda) = \frac{\Omega(E;\lambda)\,e^{-\beta E}}{Z(\lambda)}, \qquad Z(\lambda) = \int \mathrm{d}E\;\Omega(E;\lambda)\,e^{-\beta E}. \tag{10}$$
Since $P_\infty^{(E)}(S;\lambda)$ is the entropy distribution conditioned on energy E, the canonical long-time entropy distribution is the exact mixture
$$\boxed{P_{\infty,\beta}(S;\lambda) = \int \mathrm{d}E\;P_\beta(E;\lambda)\,P_\infty^{(E)}(S;\lambda).} \tag{11}$$

5.5 Bulletproof qualifier: changing rates is not changing the invariant measure

The above results concern changes that alter the invariant measure by changing $\mathcal{A}(\lambda)$ and/or $H(\Gamma;\lambda)$. By contrast, a purely kinetic modification that only rescales crossing rates or mixing times *while leaving the invariant measure unchanged* can change the finite-time distribution $P_t(S)$ but does not change the long-time distribution $P_\infty(S)$.

6 Experimental validation: asymmetric constraints enable spontaneous low-entropy transitions

Recent experimental work by Qiao and Wang [6] provides compelling validation of the theoretical framework presented above. Their experiment demonstrates that asymmetric constraints can reshape the entropy distribution in a way that enables a particle system to spontaneously transition toward lower entropy states, without requiring entropy increase elsewhere in the system.

6.1 The Qiao–Wang experiment

Qiao and Wang investigated nanoporous carbon electrodes in dilute aqueous CsPiv solutions where the effective nanopore size ($d_e \approx 1\,\text{nm}$) only slightly exceeds the ion size ($d_i \approx 0.7\,\text{nm}$), satisfying $d_i < d_e < 2d_i$. The steady-state ion distribution is intrinsically non-equilibrium: the measured $|\delta V|$ is nearly an order of magnitude above the upper limit set by the heat-engine statement of the second law, and the system produces useful work in an isothermal cycle by absorbing heat from a single thermal reservoir.

6.2 Interpretation: constraint-reshaped $P_\infty(S;\lambda)$

In our framework the nanopore walls act as the constraint parameter λ that reshapes $\mathcal{A}(\lambda)$. The quasi-one-dimensional confinement fundamentally alters $W_m^{(E)}(\lambda)$: ion trajectories lose full chaoticity, collisions become sparse, and the system cannot relax to global equilibrium—a structural change in $P_\infty^{(E)}(S;\lambda)$ per the sharp criterion of Sec. 5. The observed non-Boltzmannian surface-ion density $\sigma^\pm$ is a direct manifestation: the constraint prevents the system from reaching S_{eq} and instead pins it at $S_{\text{ne}} < S_{\text{eq}}$, without requiring compensating entropy increase elsewhere. It is the entropy landscape itself that has been reshaped, not the entropy balance between subsystems.

7 Conclusion

The mirror-state paradox shows that the Second Law cannot be a universal fundamental law in the form "entropy is monotone" (strictly or as a universal statistical principle) under time-reversal invariant microscopic physics. The consistent replacement is that entropy is described by a probability distribution $P(S)$, without an intrinsic direction, and that this distribution depends on constraints and boundary conditions. We proved from first principles that constraints reshape the long-time entropy distribution $P_\infty(S;\lambda)$ by altering accessible macrostate volumes and/or statistical weights, with a sharp microcanonical criterion that isolates the only degenerate case of mere translation.

A concise way to summarise the conceptual stance of this work is: *constraints set the landscape* (they fix the accessible phase space and the Hamiltonian, and thus which macrostates are possible); *dynamics set the rules* (time-reversal invariant microscopic evolution governs how particles move, independent of entropy); *stochasticity—or in quantum mechanics, intrinsic indeterminacy—determines which of the accessible states is realised*; and *entropy is only a reading*, a coarse-grained statistic of the current macrostate, not a driver of the dynamics. In that picture, the future is not predetermined by a single entropic fate: under different constraints, different futures—mixing, separation, or self-organization—become possible, and each regime remains open to design. Every day is new.

This pattern—constraints reshaping the entropy landscape and enabling spontaneous formation of structure—is familiar from star and planet formation, crystal and snowflake growth, mineral veins, and arguably the emergence of life: all instances of constraint-induced local entropy decrease, consistent with the present framework.

In this corrected framework, the traditional "impossibility" intuition behind a perpetual motion machine of the second kind is no longer a fundamental axiom but a limit-dependent, model-dependent statement; in that precise sense, one may say that the road toward a perpetual motion machine of the second kind is now faintly visible.

The experimental validation by Qiao and Wang [6] demonstrates that this road is not merely visible but has been traversed. Their work shows that asymmetric constraints can reshape the entropy distribution to enable spontaneous low-entropy transitions, allowing a system to produce useful work in a cycle by absorbing heat from a single thermal reservoir. With this experimental confirmation, both the theoretical and experimental barriers to a perpetual motion machine of the second kind have been overcome.

The path forward is clear: through iterative enhancement, engineering optimisation, and scaling of constraint-based entropy reshaping, practical devices can be developed. The implications extend far beyond energy generation. Because constraints can perpetually reshape entropy distributions and enable spontaneous order, the macroscopic world need not tend toward a featureless equilibrium: universal heat death is not a foregone conclusion but an artefact of the unconstrained limit. With this principle at hand, humanity can sustain civilisation even through extended ice ages, ensuring the perpetual flourishing and development of human culture. The macroscopic world will not end in silence; it will continuously manifest various wonderful phenomena, sustained by the ever-present possibility of constraint-induced order generation.

The era of thermodynamic fatalism is over; the age of thermodynamic design has begun.

References

[1] H. Goldstein, C. Poole, and J. Safko, *Classical Mechanics* (Addison–Wesley, 3rd ed., 2002).

[2] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics* (Wiley, 2nd ed., 1985).

[3] R. K. Pathria and P. D. Beale, *Statistical Mechanics* (Elsevier, 3rd ed., 2011).

[4] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II: Nonequilibrium Statistical Mechanics* (Springer, 2nd ed., 1991).

[5] J. L. Lebowitz, "Boltzmann's entropy and time's arrow," *Physics Today* **46**(9), 32–38 (1993).

[6] Y. Qiao and M. Wang, "Breaking the boundaries of the second law of thermodynamics: Intrinsic nonequilibrium distribution of large ions in charged small nanopores," arXiv:2407.04599 [cond-mat.soft] (2025).

[7] T. Peng, "Geometry challenges entropy: Regime-dependent rectification in nanofluidic cascades," arXiv:2602.13931 [physics.comp-ph] (2026).