

# Frequently Asked Questions

## Entropy Has No Direction: Mirror-State Paradox and Constraint-Reshaped Distributions

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T. Peng, *Entropy Has No Direction: A Mirror-State Paradox Against Universal Monotonic Entropy Increase and a First-Principles Proof that Constraints Reshape the Entropy Distribution*  $P_\infty(S; \lambda)$ , arXiv:2602.15369 [cond-mat.stat-mech] (2026).

<https://arxiv.org/abs/2602.15369>

The L<sup>A</sup>T<sub>E</sub>X source of the manuscript and this FAQ are available at <https://github.com/tpeng1977/entropy>. For the full argument, proofs, and references, see the manuscript `entropy.tex` / `entropy.pdf`.

### 1. What is the mirror-state paradox?

For any microstate  $A$  at time  $t_0$ , define the *mirror state*  $B = \mathcal{T}A$  (time reversal: same positions, reversed momenta). Under time-reversal invariant dynamics, the forward evolution of  $B$  is the time-reversed past of  $A$ . If a *universal* law said “entropy does not decrease” for every microstate and every time, then applied to both  $A$  and  $B$  it would imply  $S_A(t_0 + \delta t) \geq S_A(t_0)$  and  $S_A(t_0 - \delta t) \geq S_A(t_0)$  for small  $\delta t > 0$ . So  $t_0$  is a two-sided local minimum of  $S_A(t)$ . Since  $t_0$  is arbitrary, every time is a local minimum; with minimal regularity (e.g. continuity of  $S$  along trajectories), entropy must be constant on every trajectory. So a universal monotonicity claim is logically incompatible with time-reversal symmetry and would remove any entropic arrow of time.

### 2. Does this disprove the Second Law of Thermodynamics?

It shows that the Second Law *cannot* be a *universal* fundamental law in the form “entropy does not decrease” (either strict trajectory-wise or as a universal statistical principle) when microscopic dynamics are time-reversal invariant. The paper does not deny that in many practical settings entropy tends to increase; it denies that this follows as a universal, state-by-state consequence of the underlying physics. Any one-way statement about  $\Delta S$  must rely on additional structure: special initial conditions, coarse-graining, or constraints/boundaries.

### 3. What replaces “entropy has a direction”?

The consistent view: **entropy has no direction; it is described by a probability distribution**  $P(S)$ . The shape of this distribution depends on constraints and boundary conditions (encoded as  $\lambda$ ). Long-time behavior is captured by  $P_\infty(S; \lambda)$ , the entropy distribution induced by the invariant measure under those constraints.

### 4. What is $P_\infty(S; \lambda)$ ?

$P_\infty(S; \lambda)$  is the *long-time* entropy distribution: the probability distribution of (coarse-grained) entropy  $S$  in the limit of long times, under constraint parameters  $\lambda$  (geometry, boundaries, fields, etc.). It is determined by the invariant measure (e.g. microcanonical on the energy shell or canonical with a heat bath). The paper proves from first principles that changing  $\lambda$  (Hamiltonian and/or accessible phase space) changes this distribution.

#### 5. How do constraints reshape the entropy distribution?

Constraints enter via the Hamiltonian  $H(\Gamma; \lambda) = H_0(\Gamma) + V_c(\Gamma; \lambda)$  and/or the accessible set  $\mathcal{A}(\lambda)$ . They change the invariant measure (e.g. uniform on the energy shell in the microcanonical case) and hence the macrostate probabilities  $\pi_m^{(E)}(\lambda) = W_m^{(E)}(\lambda)/\Omega(E; \lambda)$ . So the induced distribution over entropy values  $P_\infty^{(E)}(S; \lambda)$  changes when the accessible macrostate volumes  $W_m^{(E)}(\lambda)$  change with  $\lambda$ .

#### 6. When does $P_\infty^{(E)}(S; \lambda)$ stay the same (up to translation)?

Only in one case: when the multiset of accessible macrostate volumes  $\{W_m^{(E)}(\lambda)\}$  is scaled by a common factor  $c > 0$  (up to permutation). Then  $P_\infty^{(E)}(S; \lambda_2)$  is just  $P_\infty^{(E)}(S - k_B \ln c; \lambda_1)$ . Otherwise the distribution changes *structurally*, not merely by shifting the entropy axis.

#### 7. What does the Qiao–Wang experiment show?

Qiao and Wang showed that in charged small nanopores (effective pore size  $d_e \approx 1$  nm, ion size  $d_i \approx 0.7$  nm, so  $d_i < d_e < 2d_i$ ) the steady-state ion distribution is intrinsically out of equilibrium: the potential difference can be nearly an order of magnitude larger than the heat-engine upper bound from the traditional Second Law. The system can produce useful work in an isothermal cycle by absorbing heat from a single thermal reservoir. This is interpreted as the asymmetric constraint (nanopore geometry) reshaping  $P_\infty(S; \lambda)$ , making spontaneous low-entropy transitions accessible without requiring compensating entropy increase elsewhere.

#### 8. Does this mean a perpetual motion machine of the second kind is possible?

The paper argues that the traditional *fundamental* impossibility of a second-kind perpetual motion machine (extracting work from a single heat reservoir in a cycle) is not a universal consequence of time-reversal invariant physics. In the corrected framework, that “impossibility” is model- and limit-dependent. Together with the Qiao–Wang result, both theoretical and experimental barriers are argued to be overcome, so that practical devices based on constraint-resaped entropy distributions are in principle feasible. Engineering and scaling remain open.

#### 9. What about heat death?

If entropy does not have a universal direction and constraints can reshape  $P_\infty(S; \lambda)$ , then a *universal* heat death of the universe is not a necessary consequence of the same first principles. The paper does not make detailed cosmological claims but notes implications for long-term sustainability and the possibility of sustained non-equilibrium phenomena under suitable constraints.

#### 10. Which definition of entropy is used?

Boltzmann (coarse-grained) entropy: a time-reversal symmetric partition of phase space into macrostates  $\{C_m\}$ , with  $S_m^{(E)} = k_B \ln(W_m^{(E)}/W_0)$  on the energy shell. The conclusions depend on this choice only in that the coarse-graining is assumed time-reversal symmetric,  $S(\mathcal{T}\Gamma) = S(\Gamma)$ .

*Prepared by Ting Peng (t.peng@ieee.org). For updates or further questions, contact the author.*