A common singular spectrum analysis of bovine tuberculosis incidence in Great Britain

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Data

Bovine tuberculosis (bTB), caused by the *Mycobacterium bovis* pathogen, is an important disease in cattle.

VetNet/SAM database (APHA):

- January 2003 – December 2012
  - Test/Analysis: 2003–2010
  - Forecasting: 2011–2012
- Monthly confirmed herd breakdowns (OTFW)
  - England, Wales and Scotland

Purpose

1. Can we detect regular signals in the time series?

2. Are the signals from England, Wales and Scotland similar?

3. Can any similarities improve our knowledge of bTB breakdowns in Scotland?

Singular Spectrum Analysis (SSA)

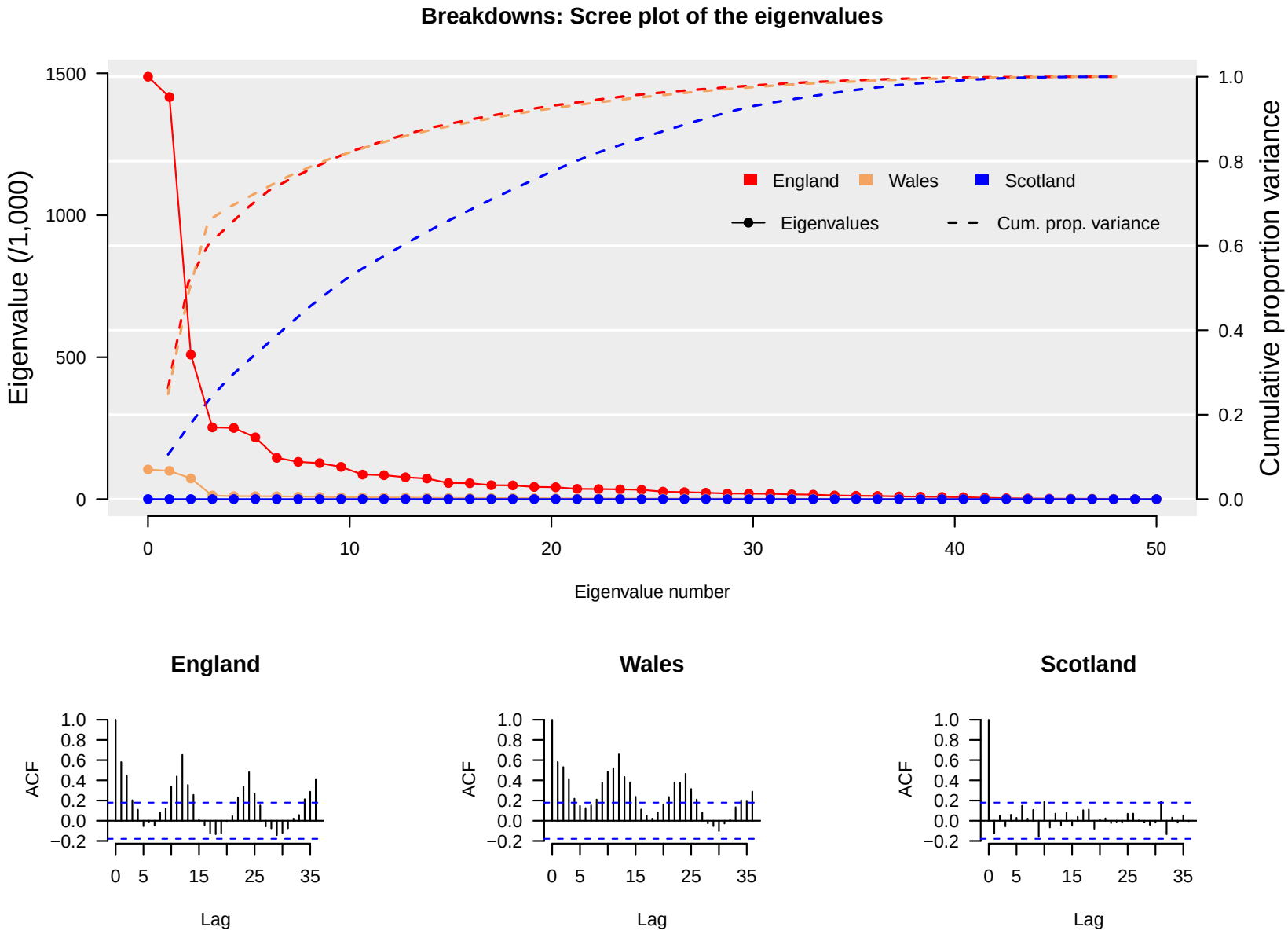
Trajectory matrix (Hankel structure)

$$\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 & \dots & x_{n-L+1} \\ x_2 & x_3 & \dots & x_{n-L+2} \\ \vdots & \vdots & & \vdots \\ x_L & x_{L+1} & \dots & x_n \end{bmatrix}$$

SSA involves spectral decomposition of $\boldsymbol{X}\boldsymbol{X}^T$.

Golyandina and Zhigljavsky (2013). *Singular Spectrum Analysis for Time Series.*

SSA results


Breakdowns: Scree plot of the eigenvalues
Eigenvalue (/1,000)
Cumulative proportion variance
Eigenvalue number
England
Wales
Scotland
Eigenvalues
Cum. prop. variance
England
Wales
Scotland
ACF
Lag

Common Principal Components (CPC)

Full CPC model

$$\boldsymbol{\Sigma}_i = \boldsymbol{B}\boldsymbol{\Lambda}_i\boldsymbol{B}^T, \quad i = 1, \ldots, k$$

Partial CPC model

$$\boldsymbol{\Sigma}_i = \boldsymbol{B}_i\boldsymbol{\Lambda}_i\boldsymbol{B}_i^T, \quad i = 1, \ldots, k$$

$$\boldsymbol{B}_i = \begin{bmatrix} \boldsymbol{\beta}_1 & \ldots & \boldsymbol{\beta}_q & \boldsymbol{\beta}_{q+1}^{(i)} & \ldots & \boldsymbol{\beta}_p^{(i)} \end{bmatrix}$$

Flury (1988). *Common Principal Components and Related Multivariate Models.*

Common Singular Spectrum Analysis (CSSA)

$$\boldsymbol{X}_i \boldsymbol{X}_i^T = \boldsymbol{B} \boldsymbol{\Lambda}_i \boldsymbol{B}^T, \quad i = 1, \ldots, k$$

The k centred trajectory matrix cross products are simultaneously diagonalised by the same eigenvector matrix, $\boldsymbol{B}$.

Simultaneous diagonalisation:
Stepwise CPC algorithm

Viljoen and Nel (2010). *Common singular spectrum analysis of several time series.*

Trendafilov (2010). *Stepwise estimation of common principal components.*

CSSA results

- England and Wales:

Eigenvectors		Absolute vector correlation
England	Wales	
1	2	**0.97**
2	1	**0.92**
3	3	**0.91**
18	28	0.73
9	33	0.72
⋮	⋮	⋮

- Scotland and England/Wales: No common eigenvectors

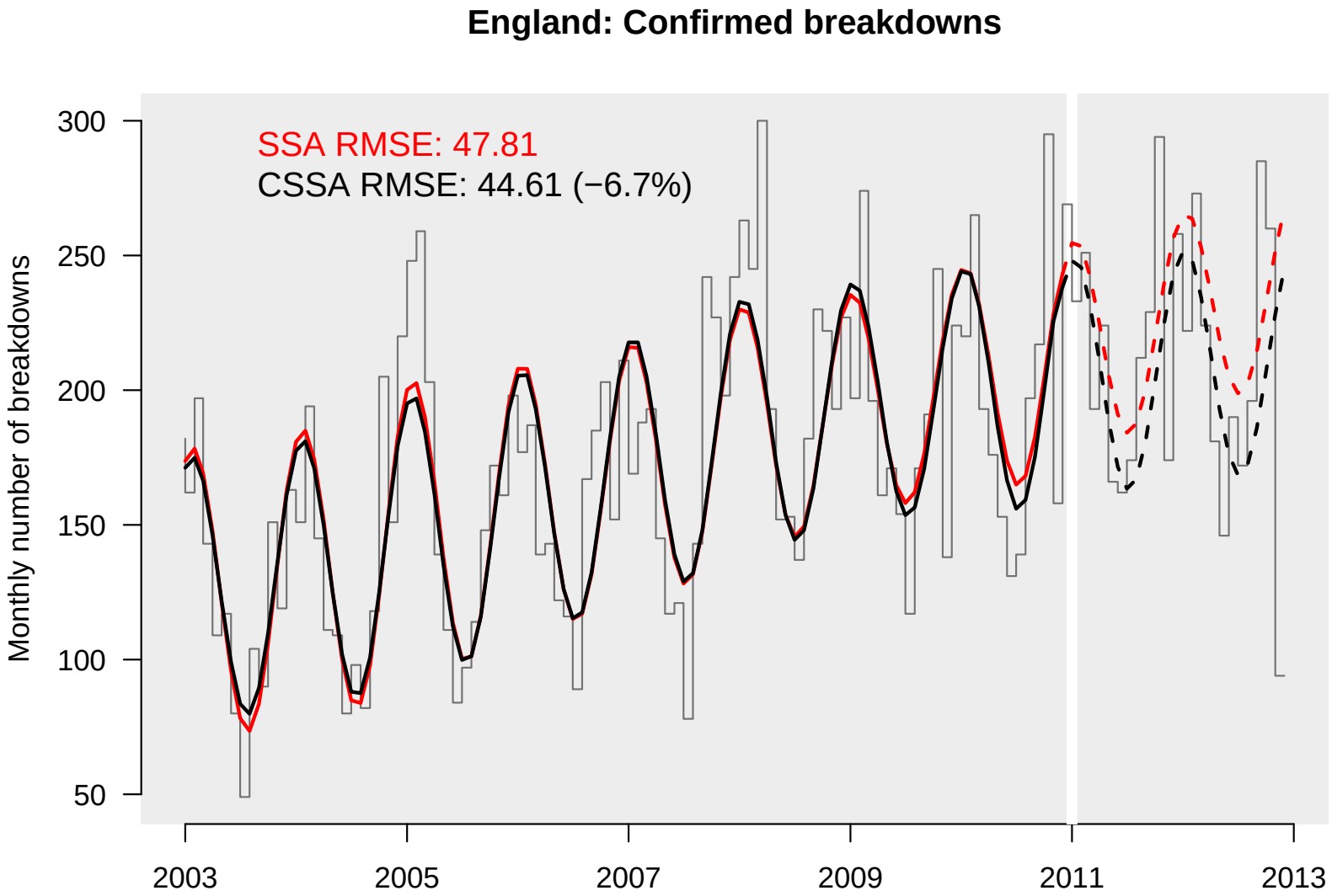
Pepler et al. (2016). A comparison of some methods for the selection of a common eigenvector model for the covariance matrices of two groups.

Forecasting

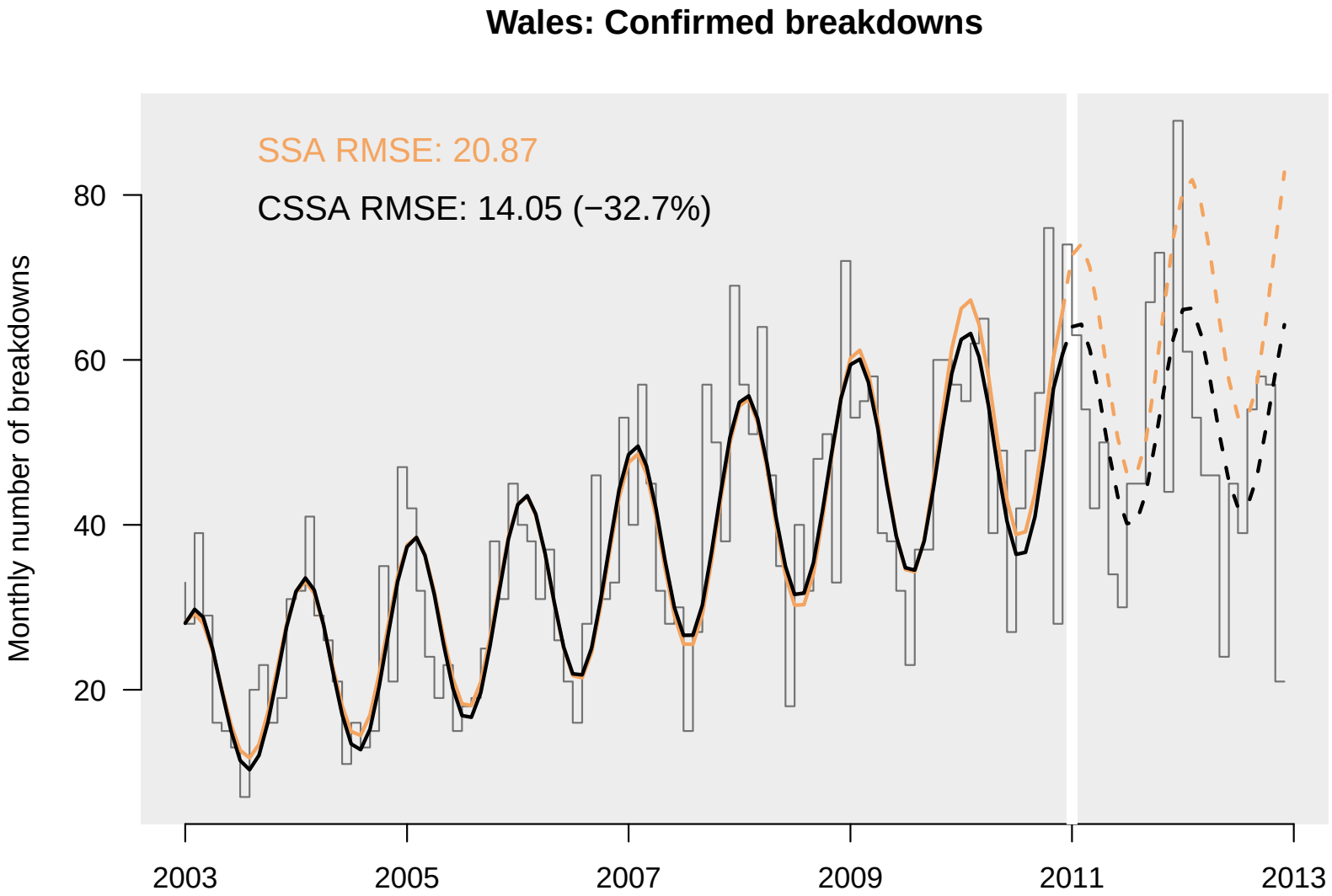
Recurrent forecasting for the period 2011–2012, using either

- first three eigenvectors (SSA)
- estimated common signal in England and Wales (CSSA)

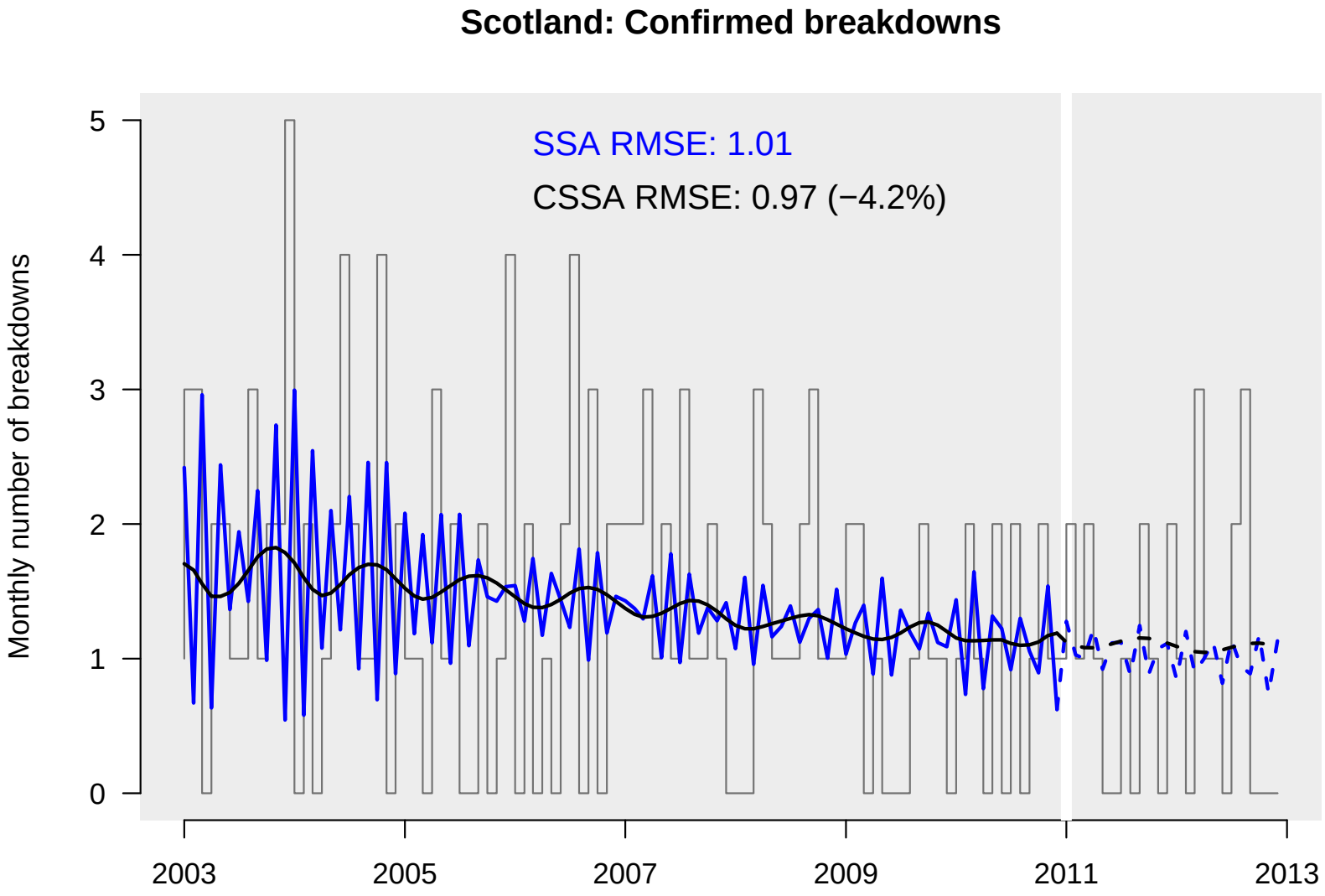
Forecasts: SSA vs. CSSA



Forecasts: SSA vs. CSSA



Forecasts: SSA vs. CSSA



Conclusions

- First three singular spectra in the trajectory matrices of England and Wales constitute common signal
- Estimated signal appears to be seasonal (annual)
- Absence of any strong signal in Scottish data
- Forecasting improved using estimated common signal, in all three regions

References

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Thank you!

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