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Tutorial time : 9.00 Friday Room 356
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Assignment 1

1. let $\underset{\sim}{u} = [7, 4]$ and $\underset{\sim}{v} = [3, 1]$

a. Find the cosine of the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$

We know that $|u.v| = \|u\| . \|v\| . \cos\theta$

$$\Rightarrow \cos\theta = \frac{|u.v|}{\|u\| . \|v\|}$$

$$= \frac{7.3 + 4.1}{\sqrt{7^2+4^2} . \sqrt{3^2+1^2}} = \frac{25}{\sqrt{65} . \sqrt{10}} = \frac{25}{\sqrt{650}} = \frac{25}{5\sqrt{26}} = \frac{5}{\sqrt{26}}$$

Therefore $\cos\theta = \frac{5}{\sqrt{26}}$

b. Find the projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$

$$\text{proj}_{\underset{\sim}{v}} \underset{\sim}{u} = \frac{\underset{\sim}{u} . \underset{\sim}{v}}{\|\underset{\sim}{v}\|^2} . \underset{\sim}{v} = \frac{25}{(\sqrt{10})^2} . [3, 1] = \frac{25}{10} [3, 1] = \left[\frac{75}{10}, \frac{25}{10}\right] = \left[\frac{15}{2}, \frac{5}{2}\right]$$

Therefore, the projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$ is the vector $\left[\frac{15}{2}, \frac{5}{2}\right]$.

c, All unit vectors in $\mathbb{R}^2$ which are orthogonal to $\underset{\sim}{v}$

We have $\underset{\sim}{v} = [3, 1]$

To find unit vectors that are orthogonal to $\underset{\sim}{v}$, we need to find normal vectors $\underset{\sim}{n}$ that are orthogonal to $\underset{\sim}{v}$:
$[n_1, n_2]$

$$\underset{\sim}{n} . \underset{\sim}{v} = 0$$
$$\Leftrightarrow n_1 . v_1 + n_2 v_2 = 0$$
$$\Leftrightarrow 3n_1 + n_2 = 0$$

Possible coordinates for normal vector $\underset{\sim}{n}$ are : $\underset{\sim}{n_1} = [1, -3]$ and $\underset{\sim}{n_2} = [-1, 3]$

$$\Rightarrow \text{Unit vector of } \underset{\sim}{n_1} = \frac{1}{\|\underset{\sim}{n_1}\|} . \underset{\sim}{n_1} = \frac{1}{\sqrt{1^2+(-3)^2}} . [1, -3] = \frac{1}{\sqrt{10}} [1, -3] = \left[\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right]$$

$$\Rightarrow \text{Unit vector of } \underset{\sim}{n_2} = \frac{1}{\|\underset{\sim}{n_2}\|} . \underset{\sim}{n_2} = \frac{1}{\sqrt{(-1)^2+3^2}} . [-1, 3] = \frac{1}{\sqrt{10}} . [-1, 3] = \left[\frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right]$$

Therefore, unit vectors in $\mathbb{R}^2$ that are orthogonal to $\underset{\sim}{v}$ are $\left[\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right]$ and $\left[\frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right]$

2. Given $A = (1, 1, 2)$, $B = (-3, 1, 4)$, $C = (-1, -1, 0)$

a. Find point $T = (t_1, t_2, t_3)$ such that $\overrightarrow{AT} = 3\overrightarrow{AB} - \overrightarrow{AC}$

. $\overrightarrow{AT} = [t_1 - 1, t_2 - 1, t_3 - 2]$
$\overrightarrow{AB} = [-4, 0, 2] \Rightarrow 3\overrightarrow{AB} = [-12, 0, 6]$
$\overrightarrow{AC} = [-2, -2, -2]$

$$\overrightarrow{AT} = 3\overrightarrow{AB} - \overrightarrow{AC}$$

$$\begin{bmatrix} t_1 - 1 \\ t_2 - 1 \\ t_3 - 2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 6 \end{bmatrix} - \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \Leftrightarrow \begin{cases} t_1 - 1 = -10 \\ t_2 - 1 = 2 \\ t_3 - 2 = 8 \end{cases} \Leftrightarrow \begin{cases} t_1 = -9 \\ t_2 = 3 \\ t_3 = 10 \end{cases}$$

Therefore $T = (-9, 3, 10)$

b. Given point $P = (x, x+1, 3)$. Find all values x such that the lengths of $\overrightarrow{AP}$ and $\overrightarrow{BP}$ coincide

(meaning that finding all values x such that $\|\overrightarrow{AP}\| = \|\overrightarrow{BP}\|$)

. $\overrightarrow{AP} = [x-1, x, 1] \Rightarrow \|\overrightarrow{AP}\| = \sqrt{(x-1)^2 + x^2 + 1^2}$

$\overrightarrow{BP} = [x+3, x, -1] \Rightarrow \|\overrightarrow{BP}\| = \sqrt{(x+3)^2 + x^2 + (-1)^2}$

let $\|\overrightarrow{AP}\| = \|\overrightarrow{BP}\|$: $\sqrt{(x-1)^2 + x^2 + 1} = \sqrt{(x+3)^2 + x^2 + 1}$

$$\Leftrightarrow \begin{cases} (x-1)^2 + x^2 + 1 \geq 0 \text{ (True)} \\ (x-1)^2 + x^2 + 1 = (x+3)^2 + x^2 + 1 \end{cases}$$

$$\begin{aligned} \Leftrightarrow (x-1)^2 &= (x+3)^2 \\ \Leftrightarrow x^2 - 2x + 1 &= x^2 + 6x + 9 \\ \Leftrightarrow -8x &= 8 \\ \Leftrightarrow x &= -1 \end{aligned}$$

Therefore, the lengths of $\overrightarrow{AP}$ and $\overrightarrow{BP}$ coincide when $x = -1$.

c. Given $\underset{\sim}{u} = [2, 14, x]$
Find all values of x such that $\underset{\sim}{u}$ is a linear combination of $\overrightarrow{AB}$ and $\overrightarrow{AC}$

We know that $\overrightarrow{AB} = [-4, 0, 2]$ and $\overrightarrow{AC} = [-2, -2, -2]$

$\underset{\sim}{u}$ is a linear combination of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ when there are scalars a and b that satisfy:

$$\underset{\sim}{u} = a.\overrightarrow{AB} + b.\overrightarrow{AC}, \quad a, b \in \mathbb{R}$$

$$\begin{bmatrix} 2 \\ 14 \\ x \end{bmatrix} = a \begin{bmatrix} -4 \\ 0 \\ 2 \end{bmatrix} + b \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \text{ or } \begin{cases} 2 = -4a - 2b & (1) \\ 14 = -2b & (2) \\ x = 2a - 2b & (3) \end{cases}$$

From equation (2), we have: $14 = -2b \Leftrightarrow b = -7$
Substitute the value of b into equation (1), we have:

$$2 = -4a - 2.(-7)$$
$$2 = -4a + 14$$
$$4a = 12$$
$$a = 3$$

Substitute the value of a and b into equation (3), we have:

$$x = 2.3 - 2.(-7)$$
$$= 6 + 14 = 20$$

Rewrite the linear combination: $\begin{bmatrix} 2 \\ 14 \\ 20 \end{bmatrix} = 3\begin{bmatrix} -4 \\ 0 \\ 2 \end{bmatrix} - 7\begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$

Therefore, $x = 20$ will result in $\underset{\sim}{u}$ the linear combination of $\vec{AB}$ and $\vec{AC}$.

d. Find the area of the triangle ABC:

$$\vec{AB} \times \vec{AC} = [4, -12, 8] \Rightarrow \| \vec{AB} \times \vec{AC} \| = \sqrt{4^2 + (-12)^2 + 8^2} = \sqrt{224} = 4\sqrt{14}$$

$$\Rightarrow S_{\Delta ABC} = \frac{1}{2} \| \vec{AB} \times \vec{AC} \| = \frac{1}{2} . 4\sqrt{14} = 2\sqrt{14} \text{ (units squared).}$$

Therefore, the area of ΔABC is $2\sqrt{14}$ units squared.

e. Find a vector form and parametric equations for the line passing through A and perpendicular to ΔABC

. line perpendicular to ΔABC meaning that its direction vector $\underset{\sim}{d}$ is orthogonal to all vectors made up of ΔABC that is, $\underset{\sim}{d}$ is orthogonal to $\vec{AB}$ and $\vec{AC}$

$$\Rightarrow \underset{\sim}{d} = \vec{AB} \times \vec{AC} = [4, -12, 8]$$

. The vector form is $x = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + t\begin{bmatrix} 4 \\ -12 \\ 8 \end{bmatrix}, \quad t \in \mathbb{R}$

. Parametric form: $\left.\begin{array}{l} x = 1 + 4t \\ y = 1 - 12t \\ z = 2 + 8t \end{array}\right\} \quad t \in \mathbb{R}$