THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

MATH2021 Vector Calculus and Differential Equations — Semester 1, 2020

Lecturer: Zhou Zhang

- Due on **Sunday, 24 May 2020 at 11:50PM, Sydney Time**, which is the **deadline**.
- Format: **hand-written** on **physical paper** and scanned for submission. It's fine to take pictures for pages of solution paper separately and compile them into one file for submission.
- We **DON'T** accept writing on iPad or tablet, or submission in any other form.
- Submit your assignment through *turnitin* on Canvas. **Double check** the status of paper after submission.
- **DON'T** include **NAME** in the title of the submission line for anonymous marking.
- **MUST** include **SID** on paper.

1. We have the unit ball $B = \{(x, y, z) \mid x^2+y^2+z^2 \leq 1\}$ in $\mathbb{R}^3$ with the unit sphere $S = \{(x, y, z) \mid x^2+y^2+z^2 = 1\}$ as its boundary. S is the union of hemispheres $S_1 = \{(x, y, z) \mid x^2+y^2+z^2 = 1, z \geq 0\}$ and $S_2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1, z \leq 0\}$. Consider the vector field $V = (x, y, z^2)$.

1) S is a level surface. Explain that $n = (x, y, z)$ is its unit outward normal vector field.

2) Write down the surface integral $\iint_{S_1} V \cdot n \, dS$ as a double integral by considering S_1 as a graph over xy-plane. You are NOT asked to calculate this double integral.

3) Apply Gauss' Theorem to calculate $\iint_S V \cdot n \, dS$.

Hint: cylindrical coordinates and/or symmetry might be helpful.

2. In the xy-plane $\mathbb{R}^2$, we have the unit disk centred at origin, $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$, and its boundary circle C in the counter-clockwise direction. In the xyz-space $\mathbb{R}^3$ where the xy-plane is the plane $\{z = 0\}$, we have the unit upper hemisphere $S_1 = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, z \geq 0\}$ with the same boundary C.

Consider the vector field $F(x, y, z) = (y, -x, xy \sin z)$ over $\mathbb{R}^3$.

1) Apply Green's Formula/Theorem to calculate $\oint_C F \cdot dr$.

2) Calculate $(\nabla \times F) \cdot n$ over S_1, where n is the unit normal of S_1, pointing out of the unit ball centred at origin. The answer should be a function with only variables x and y.

3) Apply Stokes' Theorem to calculate $\iint_{S_1} (\nabla \times F) \cdot n \, dS$.

3. Find the particular solution for the following ODEs with unknown function $y(x)$.

1) $y' + y = 0$, $y(0) = 1$.

2) $y' + y = x$, $y'(0) = 0$.

3) $y'' + 2y' + 2y = 0$, $y(0) = y'(\pi) = 0$.