The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH2021 Vector Calculus and Differential Equations — Semester 1, 2020

Lecturer: Zhou Zhang

This assignment is due by **Sunday, 5 April 2020 at 11:50PM**.
Your paper needs to be **hand-written** and scanned for submission.
Submit your assignment using *turnitin* on Canvas.
Note: do **NOT** include your **NAME** in the title of the submission line for anonymous marking.
Please **ONLY** write the **SID** on your paper.

1. (1) In xy-plane, sketch the region $\mathcal{R}$ which is inside the circle $x^2 + y^2 = 4$ and between the two lines $y = 1$ and $y = -1$.

(2) The boundary of $\mathcal{R}$ is a curve C with a few (smooth) pieces. Write down the parametrisation for C by considering it as the motion of **unit speed** which

- starts at the point $(\sqrt{3}, -1)$, and
- goes around in the counter-clockwise direction (around the origin).

You can ignore the corners where the direction of the motion changes suddenly.

2. For the curve C in Problem 1,

(1) calculate the total arc length;

(2) calculate the line integral

$$\int_C V \cdot dr$$

for the vector field $V = (-y, x)$;

(3) calculate the line integral

$$\int_C W \cdot dr$$

for the vector field $W = (x, y)$.

(4) decide whether V and W are conservative, and explain the reason respectively.

3. For the region $\mathcal{R}$ in Problem 1,

(1) if it is the shape of a piece of material with uniform density $\frac{1}{2}$, calculate the total mass of the piece **using double integral**;

(2) the x-coordinate of the centre of mass for the piece in (1) is the following quantity,

$$x_c = \frac{\iint_{\mathcal{R}} x \cdot \frac{1}{2}\, dxdy}{\iint_{\mathcal{R}} \frac{1}{2}\, dxdy}.$$

Give an explanation of the fact that the value of x_c is 0.