The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1004: Discrete Mathematics — Semester 2, 2019

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1004/
Lecturer: Emily Cliff

This **individual** assignment is due by **11:59pm Thursday 29 August 2019**, via Canvas. Late assignments will receive a penalty of 5% per day until the closing date. A single PDF copy of your answers must be uploaded in the Learning Management System (Canvas) at https://canvas.sydney.edu.au/courses/17306. Please submit only one PDF document (scan or convert other formats). It should include your SID, your tutorial time, day, room and Tutor's name. Please note: Canvas does NOT send an email digital receipt. We strongly recommend downloading your submission to check it. What you see is exactly how the marker will see your assignment. Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 2.5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	**Grade**	**Criterion**
5	A	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
4	B	Very good work, making excellent progress, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
3	C	Good work, making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
2	D	A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.
1	E	Some attempt, with limited progress made.
0	F	No credit awarded.

1. Let A, B and C be arbitrary sets. One of the following statements is true, and one is false. Draw Venn diagrams for each side of each statement. Give a careful proof of the statement which is true, and provide a specific counterexample to the statement which is false. (This means that for the statement which is false, you should specify the sets A, B and C, and explain why, for the A, B and C you have specified, the statement is false.)

(a) $(A \setminus B) \cup (C \setminus B) = (A \cap B) \setminus C$.

(b) $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

2. Recall from Lecture 3-1 the definition of the *restriction* of a function $f : A \to B$ to a subset $C \subseteq A$, denoted $f|_C$.

(a) Let $f : \mathbb{Z} \to \mathbb{R}$ be defined by $f(n) = \sin \frac{n\pi}{4}$. Find a subset $C \subseteq \mathbb{Z}$ with as many elements as possible, such that $f|_C$ is injective.

(b) Let $f, g : A \to B$ be two functions. Suppose that C and D are subsets of A which are not equal to A, such that $f|_C = g|_C$ and $f|_D = g|_D$. Give an example of such sets A, B, C, D and functions f and g such that f and g are not equal. Find a condition on C and D that guarantees that $f = g$, and prove that your condition works. (Try not to make your condition more restrictive than necessary!)

(c) Now suppose that C, D are subsets of A, and that $f : C \to B$ and $g : D \to B$ are functions. What condition on f and g is necessary to ensure that there exists a function $h : A \to B$ such that $h|_C = f$ and $h|_D = g$?