Assignment 1

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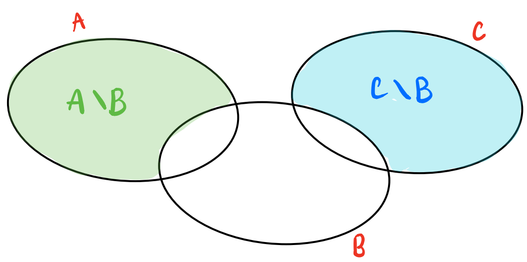
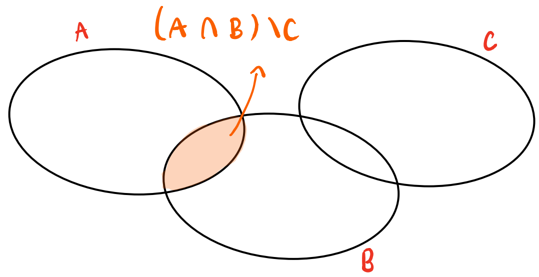
Tutorial: Carslaw 451 Monday 2-3pm

1. Let A, B, C be arbitrary sets

a. $(A \setminus B) \cup (C \setminus B) = (A \cap B) \setminus C$



$(A \setminus B) \cup (C \setminus B)$



$(A \cap B) \setminus C$

Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6, 7\}$, and $C = \{7, 8, 9\}$

$\Rightarrow$. $A \setminus B = \{1, 2\}$, $C \setminus B = \{8, 9\}$

$(A \setminus B) \cup (C \setminus B) = \{1, 2, 8, 9\}$

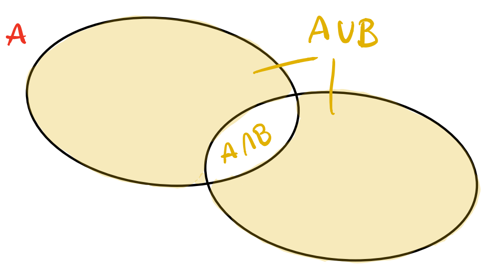
. $A \cap B = \{3\}$

$(A \cap B) \setminus C = \{3\}$

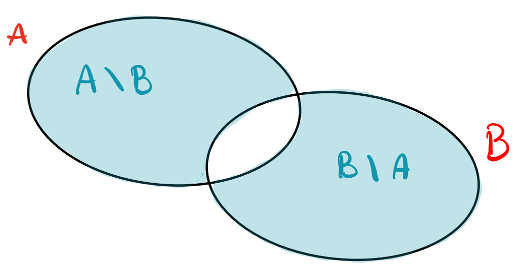
. Since $\{3\} \not\subseteq \{1, 2, 8, 9\}$ and $\{1, 2, 8, 9\} \not\subseteq \{3\}$

$\therefore (A \setminus B) \cup (C \setminus B) \neq (A \cap B) \setminus C$

b. $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$



$(A \cup B) \setminus (A \cap B)$



$(A \setminus B) \cup (B \setminus A)$

To prove this statement is true, by definition we must show that

$(A \cup B) \setminus (A \cap B) \subseteq (A \setminus B) \cup (B \setminus A)$ and

$(A \setminus B) \cup (B \setminus A) \subseteq (A \cup B) \setminus (A \cap B)$

- let $x \in (A \cup B) \setminus (A \cap B)$

  Then $x \in (A \cup B)$ and $x \notin (A \cap B)$

  Using De Morgan's law, we have:

  $$(A \cup B) \setminus (A \cap B) = [(A \cup B) \setminus A] \cup [(A \cup B) \setminus B]$$
  $$= (B \setminus A) \cup (A \setminus B)$$

  $\therefore (A \cup B) \setminus (A \cap B) \subseteq (B \setminus A) \cup (A \setminus B)$.

- let $x \in (A \setminus B) \cup (B \setminus A)$

  Then $x \in (A \setminus B)$ or $x \in (B \setminus A)$

  if $x \in A \setminus B$ then $x \in A$ and $x \notin B$

  else if $x \in B \setminus A$ then $x \in B$ and $x \notin A$

  thus, there are only two possibilities for x, i.e. $x \in A$ or $x \notin A$

  if $x \in A$ then $x \notin B$ or

  if $x \notin A$ then $x \in B$

  which means x is either in A or B but not in A and B

  $\Rightarrow x \in (A \cup B) \setminus (A \cap B)$

  $\therefore (A \setminus B) \cup (B \setminus A) \subseteq (A \cup B) \setminus (A \cap B)$ as desired. $\square$

2.

a) Let $f : \mathbb{Z} \to \mathbb{R}$ be defined by $f(n) = \sin\left(\frac{n\pi}{4}\right)$

Find a subset $C \subseteq \mathbb{Z}$ such that $f|_C$ is injective

For $f|_C$ to be injective, every distinct element $n_1, n_2 \in C$ then $f(n_1) \neq f(n_2)$

We know that $-1 \leq \sin\left(\frac{n\pi}{4}\right) \leq 1$. Since $\sin\left(\frac{n\pi}{4}\right)$ is a periodic function, which repeats over intervals of 2π radians, we will plug n from 0 to 8 to have the value of our angle from 0 to 2π :

$n = 0 \Rightarrow \sin(0) = 0$

$n = 1 \Rightarrow \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$n = 2 \Rightarrow \sin\left(\frac{\pi}{2}\right) = 1$

$n = 3 \Rightarrow \sin\left(\frac{3\pi}{2}\right) = \frac{\sqrt{2}}{2}$

$n = 4 \Rightarrow \sin(\pi) = 0$

$n = 5 \Rightarrow \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

$n = 6 \Rightarrow \sin\left(\frac{3\pi}{2}\right) = -1$

$n = 7 \Rightarrow \sin\left(\frac{7\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

$n = 8 \Rightarrow \sin(2\pi) = 0$

Thus, $f|_C$ is injective with its domain $C = \{0, 1, 2, 5, 6\}$.

b, $f, g: A \to B$. $C, D \subset A$ such that $f|_C = g|_C$ and $f|_D = g|_C$

Consider $f, g: \mathbb{N} \to \mathbb{N}$ with $f(x) = 2^x$ and $g(x) = 2x$

Let $C = \{1, 2\}$ and $D = \{2\}$. $C, D \subset \mathbb{N}$

- We have $f|_C = \{2^x \mid x \in C\}$ and $g|_C = \{2x \mid x \in C\}$

  Since $f|_C(x) = g|_C(x)$ $\forall x \in C \Rightarrow f|_C = g|_C$

- We also have $f|_D = \{2^y \mid y \in D\}$ and $g|_D = \{2y \mid y \in D\}$

  $f|_D(y) = g|_D(y)$ $\forall y \in D \Rightarrow f|_D = g|_D$.

- Yet $f(x) \neq g(x)$ when consider in its original domain because

  $f(3) = 2^3 = 8 \neq g(3) = 2.3 = 6$.

- To guarantee $f(x) = g(x)$, their respective domains need to be subsets of $C \cup D$.

  Consider $x \in (C \cup D) \Rightarrow x \in C$ or $x \in D$

  if $x \in C \Rightarrow f(x) = g(x)$ $\forall x \in C$

  $x \in D \Rightarrow f(x) = g(x)$ $\forall x \in D$

c, Suppose $C, D \subseteq A$ such that $f: C \to B$ and $g: D \to B$ are functions.

What condition on f and g is necessary to ensure that there exists a function $h: A \to B$

such that $h|_C = f$ and $h|_D = g$?

Given the definition of restrictions, we have:

$h|_C : C \to B$ and $h|_D : D \to B$

For $h|_C = f$ requires $h|_C(x) = f(x)$, $\forall x \in C$

$h|_D = g$ requires $h|_D(z) = g(z)$, $\forall z \in D$

- Assume that f and g are not equal, in order for $h: A \to B$ to exist, the domain of f and g cannot overlap as that might result in a value of $x \in A$ maps to 2 different outputs in B which makes $h: A \to B$ not a function.