The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1023: Multivariable Calculus and Modelling — Semester 2, 2019

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1023/
Lecturers: Eduardo Altmann and Leo Tzou

This **individual** assignment is due by **11:59pm Thursday 29 August 2019**, via Canvas. Late assignments will receive a penalty of 5% per day until the closing date. A single PDF copy of your answers must be uploaded in the Learning Management System (Canvas) at https://canvas.sydney.edu.au/courses/17310. Please submit only one PDF document (scan or convert other formats). It should include your SID, your tutorial time, day, room and Tutor's name. Please note: Canvas does NOT send an email digital receipt. We strongly recommend downloading your submission to check it. What you see is exactly how the marker will see your assignment. Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 2.5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	**Grade**	**Criterion**
5	A	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
4	B	Very good work, making excellent progress, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
3	C	Good work, making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
2	D	A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.
1	E	Some attempt, with limited progress made.
0	F	No credit awarded.

Let $x(t) \in [0,1]$ be the fraction of maximum capacity of a live-music venue at time t (in hours) after the door opens. The rate at which people go into the venue is modeled by

$$\frac{dx}{dt} = h(x)(1-x), \tag{1}$$

where $h(x)$ is a function of x only.

1. Consider the case in which people with a ticket but outside the venue go into it at a constant rate $h = 1/2$ and thus

$$\frac{dx}{dt} = \frac{1}{2}(1-x).$$

(a) Find the general solution $x(t)$.

(b) The initial crowd waiting at the door for the venue to open is $k \in [0,1]$ of the maximum capacity (i.e. $x(0) = k$). How full is the venue at t?

2. Suppose people also decides whether to go into the venue depending on if the place looks popular. This corresponds to $h(x) = \frac{3}{2}x$ and thus

$$\frac{dx}{dt} = \frac{3}{2}x(1-x).$$

(a) Find the general solution $x(t)$.

(b) What should be the initial crowd $x(0)$ if the band wants to start playing at $t = 2$ hours with 80% capacity?

3. Consider the two models, A and B, both starting at 10% full capacity. Model A is governed by the process of question (1) and model B is governed by the process described in question (2).

Start this question by writing down the respective particular solutions $x_A(t)$ and $x_B(t)$.

(a) Which of the two models will first reach 50% of full capacity?

(b) Which of the two models will first reach 99% of full capacity?

(c) Plot the curves $x_A(t)$ and $x_B(t)$. Both curves should be consistent with:

(i) your answers to the two previous items;

(ii) the rate of change at $t = 0$ (i.e., $\frac{dx}{dt}$ at $t = 0$);

(iii) the values of x in the limit $t \to \infty$.