The University of Sydney
School of Mathematics and Statistics

Assignment 2

MATH1004: Discrete Mathematics — Semester 2, 2019

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1004/
Lecturer: Emily Cliff

This **individual** assignment is due by **11:59pm Thursday 24 October, 2019**, via Canvas. Late assignments will receive a penalty of 5% per day until the closing date. A single PDF copy of your answers must be uploaded in the Learning Management System (Canvas) at https://canvas.sydney.edu.au/courses/17306. Please submit only one PDF document (scan or convert other formats). It should include your SID, your tutorial time, day, room and Tutor's name. Please note: Canvas does NOT send an email digital receipt. We strongly recommend downloading your submission to check it. What you see is exactly how the marker will see your assignment. Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 2.5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	**Grade**	**Criterion**
5	A	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
4	B	Very good work, making excellent progress, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
3	C	Good work, making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
2	D	A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.
1	E	Some attempt, with limited progress made.
0	F	No credit awarded.

1. Let A, B, C be sets. Let $a(x)$ be the statement "$x \in A$"; let $b(x)$ be the statement "$x \in B$"; and let $c(x)$ be the statement "$x \in C$". This allows us to use propositional logic to express statements about sets built out of A, B, and C. For example, the statement "$x \in (B \setminus C) \cup (C \setminus A)$" is given by the compound proposition $(b(x) \wedge \sim c(x)) \vee (c(x) \wedge \sim a(x))$.

(a) Let P and Q be sets, let $p(x)$ be the statement "$x \in P$", and let $q(x)$ be the statement "$x \in Q$". Suppose that $(\forall x)(p(x) \Rightarrow q(x))$ is true. Explain why this tells you that $P \subseteq Q$. Then explain why if $p(x)$ and $q(x)$ are logically equivalent (i.e. if $(\forall x)(p(x) \Leftrightarrow q(x))$ is true), then $P = Q$.

(b) For each of the following five sets S, give a compound proposition to express the statement "$x \in S$".

(i) $S_1 = (A \setminus B) \setminus C$

(ii) $S_2 = A \setminus (B \setminus C)$

(iii) $S_3 = (A \setminus B) \cup (A \cap C)$

(iv) $S_4 = (A \setminus B) \cap (A \setminus C)$

(v) $S_5 = A \setminus (B \cup C)$

(c) Determine which of the above sets must be equal to each other by using the rules of propositional logic to prove that the statements "$x \in S_i$" which you wrote down in the previous part of the question are logically equivalent.

(Note/hint: you may use other methods in your scratchwork to help you figure out which sets are equal, but in order to receive credit for this question, you *must* complete the proof using propositional logic.)

2. (a) Use induction to prove that if n is any positive integer, and $X, A_1, A_2, \ldots A_n$ are sets, then

$$X \setminus (A_1 \cup A_2 \cup \ldots \cup A_n) = (X \setminus A_1) \cap (X \setminus A_2) \cap \ldots \cap (X \setminus A_n)$$

(Note/hint: you may use results from the set-theory section of the course. You **do not** need to use propositional logic as in question 1! As ever, Venn diagrams may be good for intuition, but do not give real proofs.)

(b) Use induction to prove that 13 divides $4^{2n+1} + 3^{n+2}$ for all $n \in \mathbb{N}$.

3. (a) Suppose that $a = d \cdot k + b$, where a, b, d, k are all integers. Prove that b is divisible by d if and only if a is divisible by d.

(b) Let $x = abc$ be a three-digit number with digits a, b, c (so $a, b, c \in \{0, 1, 2, \ldots 9\}$). Prove that x is divisible by 3 if and only if $a + b + c$ is divisible by 3.

(Hint: if you're stuck on the three-digit case, try proving the claim for two-digit numbers $y = bc$ to warm up.)